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Abstract

We characterize intertemporal utility functions over heterogeneous goods that feature (i) a constant elasticity of substitution between goods at each point in time and (ii) a constant intertemporal elasticity of substitution for at least one of the goods. We find that a standard (stationary) intertemporal utility function is consistent with these two properties if and only if it either is of the intertemporal constant elasticity of substitution (ICES) form, that is, if all elasticities of substitution are identical, or if the instantaneous utility function is Cobb-Douglas. We also characterize the families of standard intertemporal utility functions that feature either (i) or (ii), but not the respective other property. The ICES utility function offers a simple and consistent solution for applications that use constant good-specific intertemporal substitutability. This is, for example, relevant for dual discounting of market and non-market goods.

JEL-Codes: D150, D610, Q510.

Keywords: substitutability, CES, CIES, intertemporal utility, non-market goods.

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1 Introduction

The intertemporal evaluation and efficient management of heterogeneous goods is a key challenge for economics. This is particularly relevant when assessing long-lived public goods, such as infrastructure or atmospheric carbon. Such evaluation and management often crucially depends on the substitution possibilities between private and public goods in individual utility or social welfare. For example, the economic evaluation of climate policy strongly depends on the degree of substitutability between private consumption and atmospheric carbon as well as on the intertemporal degree of substitutability (Sterner and Persson 2008, Gollier 2010, Traeger 2011, Drupp and Hänsel 2019). Parsimony in modeling the effect of substitution possibilities on the economic evaluation and efficient intertemporal allocation suggests assuming a constant elasticity of substitution (CES) between private and public goods, and a constant intertemporal elasticity of substitution for the private good (CIES).

We characterize intertemporal utility functions over two heterogeneous goods, say a private and a public good, when preferences have these two properties:[1] (i) The elasticity of substitution between the two goods is constant (but not necessarily identical) at each point in time (CES). (ii) The intertemporal elasticity of substitution for consumption of one of the goods, say the private good, is constant over time (CIES). We show that a standard intertemporal utility function (Koopmans 1960) satisfies these two properties simultaneously if and only if either all elasticities are equal, that is at each point in time the CES is equal to the CIES for the private good, or if the instantaneous utility function is Cobb-Douglas. As a consequence, the intertemporal elasticity of substitution for the public good must also be constant. Thus, except for the case where the instantaneous utility function is Cobb-Douglas, the only intertemporal utility function satisfying (i) and (ii), up to monotone transformations, is a utility function with identical elasticity of substitution for any pair of goods at any two points in time. We call this the *intertemporal constant elasticity of substitution* (ICES) utility function. We also characterize the families of standard intertemporal utility functions that have either of the two properties (i) or (ii), but not the respective other one.

Our result adds to the literature on constant elasticities of substitution. For a production technology with more than two goods, Blackorby and Russell (1989) show that constant (both Allen/Uzawa and Morishima) elasticities of substitution between any pair of goods are constant only if all elasticities of substitution are identical. Also in (static) modeling of preferences over heterogeneous goods with constant elasticities, the typical assumption is that the elasticity of substitution is the same for any pair of goods (Dixit and Stiglitz 1977).

[1] Terming the two goods under consideration as "private" and "public" is for expositional clarity and points to an important field of application. It is a purely terminological choice which does not restrict the generality of our treatment. The generalization of our analysis to more than two goods is straightforward.

2 Characterizing intertemporal preferences with constant elasticities of substitution

We study a decision problem where the objects of choice are intertemporal allocations of two goods in discrete time, c_t and d_t with $t = 0, 1, \ldots$. We write $\mathbf{c}_t = (c_t, c_{t+1}, \ldots)$ and $\mathbf{d}_t = (d_t, , d_{t+1}, \ldots)$. Throughout we assume that the decision maker's preferences over allocations can be represented by an ordinal, continuous intertemporal utility function $V(\mathbf{c}_t, \mathbf{d}_t)$, and we further assume that they can be represented by

$$V(\mathbf{c}_0, \mathbf{d}_0) = \sum_{t=0}^{\infty} \beta^t \, u(c_t, d_t) \ , \tag{1}$$

where $\beta \in (0, 1)$ is the utility discount factor and $u(c_t, d_t)$ is the instantaneous utility function that is unique up to positive affine transformation. For concreteness, we refer to the good c_t as 'private goods consumption', and to the good d_t as 'public good'.

We are interested in substitution between the consumption of the goods at the same or different points in time. One elasticity of particular interest is the intertemporal substitution for private goods consumption. It is given by (Hicks 1932[1963]):

$$\vartheta_{c_t, c_{t'}}(\mathbf{c}_0, \mathbf{d}_0) := \frac{\text{MRS}_{c_t, c_{t'}}}{c_t / c_{t'}} \frac{d\,(c_t / c_{t'})}{d\text{MRS}_{c_t, c_{t'}}} \ , \quad \text{where} \quad \text{MRS}_{c_t, c_{t'}} = \frac{\frac{\partial V(\mathbf{c}_0, \mathbf{d}_0)}{\partial c_{t'}}}{\frac{\partial V(\mathbf{c}_0, \mathbf{d}_0)}{\partial c_t}} \tag{2}$$

is the marginal rate of substitution between private goods consumption at t and t'. A typical assumption is the following.

Assumption 1. *The elasticity of intertemporal substitution for private goods consumption is constant,* $\vartheta_{c_t, c_{t'}}(\mathbf{c}_0, \mathbf{d}_0) = \vartheta$ *for all* t, t', $\mathbf{c}_0$, *and* $\mathbf{d}_0$.

Intertemporal preferences over private goods consumption that satisfy Assumption 1 are termed constant intertemporal elasticity of substitution (CIES) preferences.

Another elasticity of substitution is between the two goods at the same point in time,

$$\sigma^t_{c_t, d_t}(\mathbf{c}_0, \mathbf{d}_0) := \frac{\text{MRS}_{c_t, d_t}}{c_t / d_t} \frac{d\,(c_t / d_t)}{d\text{MRS}_{c_t, d_t}} \ , \quad \text{where} \quad \text{MRS}_{c_t, d_t} = \frac{\frac{\partial V(\mathbf{c}_0, \mathbf{d}_0)}{\partial d_t}}{\frac{\partial V(\mathbf{c}_0, \mathbf{d}_0)}{\partial c_t}} \tag{3}$$

is the marginal rate of substitution between private goods consumption and the public good in a given period t. In the static context, a common assumption is that of a constant elasticity of substitution (CES) between the two goods. In the dynamic context, this can be formulated as follows.

Assumption 2. *For each period* $t = 0, 1, ...$, *the elasticity of substitution between contemporary private and public goods is constant,* $\sigma^t_{c_t, d_t}(\mathbf{c}_0, \mathbf{d}_0) = \sigma_t$ *for all* $\mathbf{c}_0$, *and* $\mathbf{d}_0$.

Preferences satisfying Assumption 2 are termed constant elasticity of substitution (CES) preferences.

The main result of this paper is that Assumptions 1 and 2 are compatible, but only if all elasticities of substitution are equal, $\vartheta = \sigma_t$ for all $t \geq 0$, or if the instantaneous utility function is Cobb-Douglas. We formulate this as:

Proposition 1. *A utility function* (1) *satisfies Assumptions 1 and 2 if and only if it is either of the form*

$$V(\mathbf{c}_0, \mathbf{d}_0) = \sum_{t=0}^{\infty} \beta^t \, \frac{\theta}{\theta - 1} \, \left(\alpha \, c_t^{\frac{\theta-1}{\theta}} + (1-\alpha) \, d_t^{\frac{\theta-1}{\theta}} - 1\right) \ , \tag{4}$$

with $\theta > 0$ and $\alpha \in (0,1)$, or a monotone transformation of (4)*, or of the form*

$$V(\mathbf{c}_0, \mathbf{d}_0) = \sum_{t=0}^{\infty} \beta^t \, \frac{\theta}{\theta - 1} \, \left(c_t^{\alpha} \, d_t^{1-\alpha}\right)^{\frac{\theta-1}{\theta}} \ , \tag{5}$$

with $\theta > 0$, or a monotone transformation of (5)*.*

Proof. See Appendix A.4. □

This means that, although Assumptions 1 and 2 per se allow for different elasticities of substitution, the two assumptions are consistent with an intertemporal utility function (1) if and only if all elasticities are identical, or if the instantaneous utility function is Cobb-Douglas. We call the function (4) an *intertemporal constant elasticity of substitution (ICES)* utility function.

In the ICES utility function (4), the parameter $\theta > 0$ is both the constant intertemporal elasticity of substitution for private goods consumption and the constant elasticity of substitution between contemporary private and public goods, such that $\theta = \vartheta = \sigma_t$ for all $t = 0, 1, \ldots$. Proposition 1 also implies that the intertemporal elasticity of substitution for the public good is constant as well. In the case of the form (4) it is equal to θ, i.e. identical to both the intertemporal elasticity of substitution of private consumption and the elasticity of substitution between the two goods. In the other case it is constant as well, but depends on α as well.

If one conducts a monotone transformation of (4) by adding -1 to the term in brackets, the limit for $\theta \to 1$ exists, and is the Cobb-Douglas function:

$$V(\mathbf{c}_0, \mathbf{d}_0) = \sum_{t=0}^{\infty} \beta^t \, \left(\alpha \, \ln(c_t) + (1-\alpha) \, \ln(d_t)\right) \ . \tag{6}$$

For $\theta < 1$, the utility function (4) captures the case where both goods are intertemporal complements and complements to each other, with the case of perfect complements, or maximin over goods and time, for the limit $\theta \to 0$. For $\theta > 1$, the utility function (4) captures the case where both goods are intertemporal substitutes and substitutes for each other, with the case of perfect substitutes for the limit $\theta \to \infty$.

To explore what drives the result in Proposition 1, we briefly study what would be the form of the utility function if either Assumptions 1 or 2 are relaxed. We first consider the case where Assumption 1 is relaxed.

Corollary 1. *A utility function* (1) *satisfies Assumption 2 if and only if it is of the form*

$$V(\mathbf{c}_0, \mathbf{d}_0) = \sum_{t=0}^{\infty} \beta^t \, v\left(\frac{\sigma}{\sigma - 1}\left(\alpha\, c_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\, d_t^{\frac{\sigma-1}{\sigma}} - 1\right)\right), \tag{7}$$

with $\alpha \in (0,1)$, $\sigma > 0$ and some monotonically increasing function $v(\cdot)$, or a monotone transformation of (7)*.*

Proof. See Appendix A.2. □

The utility function (7) exhibits a constant elasticity of substitution of σ between private goods consumption and public good in period t. As, due to stationarity, the instantaneous utility function $u(c_t, d_t)$ in (1) is the same for all t, the elasticity of substitution between the two goods, σ, has to be constant over time. However, Assumption 1 is violated, unless $v(\cdot)$ is linear. In particular, for a nested CES-CIES function, where $v(x) = \frac{1}{1-\eta}\left(\frac{\sigma-1}{\sigma}\, x\right)^{\frac{\sigma-1}{\sigma}(1-\eta)}$ with $\eta > 0$, the intertemporal elasticities of substitution for the private consumption good and the public good will depend on the quantities of the respective other goods consumed and are thus, in general, not constant. As Proposition 1 shows, the only exception is the case where $\sigma = 1$, i.e. where the instantaneous utility function is Cobb-Douglas.

If we relax Assumption 2, we obtain the following characterization.

Corollary 2. *A utility function* (1) *satisfies Assumption 1 if and only if it is of the form*

$$V(\mathbf{c}_0, \mathbf{d}_0) = \sum_{t=0}^{\infty} \beta^t \left(\frac{\vartheta}{\vartheta - 1}\, a(d_t)\, c_t^{\frac{\vartheta-1}{\vartheta}} + w(d_t)\right), \tag{8}$$

with scalar functions $a(d_t)$ and $w(d_t)$, some $\vartheta > 0$, or a monotone transformation of (8)*.*

Proof. See Appendix A.3. □

The utility function (8) exhibits a constant intertemporal elasticity of substitution of ϑ for private goods consumption. This is standard in the literature when only market consumption goods are concerned (e.g. Emmerling et al. 2017). But unless $a(d_t) = \alpha$ and $w(d_t) = \frac{\vartheta}{\vartheta-1}(1-\alpha) d_t^{\frac{\vartheta-1}{\vartheta}}$, or positive affine transformations of these, Assumption 2 is violated: the elasticity of substitution between c_t and d_t is in general not constant, nor is the intertemporal elasticity of substitution for the public good constant.

3 Conclusion

We have characterized the *intertemporal constant elasticity of substitution* (ICES) utility function over heterogeneous goods as the only specification (up to monotone transformations) of a stationary intertemporal utility function that is consistent with the two properties of (i) a constant elasticity of substitution between goods at each point in time and (ii) a constant intertemporal elasticity of substitution for at least one of the goods. The only exception to this case is if the instantaneous utility function is Cobb-Douglas. We have also characterized the families of stationary intertemporal utility functions that have either one of the two properties, but not the respective other one, such as the special case of the nested CES-CIES function.

ICES utility is relevant for several applications. One particular example is the economic policy evaluation when heterogeneous goods are considered, as is the case in the discounting guidelines of The Netherlands (Groom and Hepburn 2017). The most often used intertemporal utility function for evaluating changes of the relative price of public vis-a-vis private goods over time in such settings is the special CES-CIES, with CES across goods and CIES with respect to instantaneous utility of the aggregate consumption bundle (Sterner and Persson 2008, Gollier 2010, Traeger 2011). This CES-CIES case implies that good-specific intertemporal substitution is non-constant. Since the aggregate CIES is usually based on estimates of the CIES for the private goods consumption only, this specification may lead to intertemporal mis-calibration and thus a distorted savings profile regarding private consumption. The current literature does not take this into account, or uses ad-hoc measures to re-calibrate effects on savings decisions (Drupp and Hänsel 2019). In contrast, ICES utility functions offer a clean solution for applications where good-specific intertemporal substitutability is supposed to be constant.

The intertemporal valuation of environmental public goods is only one example where ICES utility functions may be useful. More generally, ICES provides a parsimonious specification for utility functions that allow analyzing effects of intra- and intertemporal substitutability and complementarity between different goods.

A Proofs

A.1 Derivation of Lemma 1

Lemma 1. *At period t the elasticity of substitution between private goods consumption, c_t, and the public good, d_t, is constant if and only if utility function* (1) *is represented by*

$$V(\mathbf{c}_0, \mathbf{d}_0) = \frac{\sigma_t}{\sigma_t - 1} \left(a_t \, c_t^{\frac{\sigma_t - 1}{\sigma_t}} + b_t \, d_t^{\frac{\sigma_t - 1}{\sigma_t}} + A_t \right), \tag{A.9}$$

with some $a_t > 0$ and $A_t \in \mathbb{R}$ that are independent of c_t and d_t (but may depend on any $c_{t'}$ with $t' \neq t$, or $d_{t''}$ with $t'' \neq t$), and constant elasticity of substitution $\sigma_t > 0$, or any monotone transformation of (A.9).[2]

Proof. We abbreviate $x_t := c_t/d_t$ and consider the marginal rate of substitution as a function of x_t. With the assumption of a constant elasticity of substitution in period t, σ_t, the definition (3) becomes a differential equation for $\mathrm{MRS}_{c_t,d_t}(x_t)$ that can be solved by separation of variables as follows. Using $\sigma_t = \sigma^t_{c_t,d_t}(\mathbf{c}_t, \mathbf{d}_t)$ in (3), we can write

$$\sigma_t \, \frac{d\mathrm{MRS}_{c_t,d_t}}{\mathrm{MRS}_{c_t,d_t}} = \frac{dx_t}{x_t} \tag{A.10}$$

Integration yields

$$\sigma_t \, \ln(\mathrm{MRS}_{c_t,d_t}) = \ln(x_t) + \ln(a_t/b_t), \tag{A.11}$$

with some constant of integration $a_t/b_t > 0$. We now solve for MRS_{c_t,d_t} to obtain

$$\mathrm{MRS}_{c_t,d_t} = (a_t \, x_t/b_t)^{\frac{1}{\sigma_t}} . \tag{A.12}$$

Finally, we insert the definitions of x_t and MRS_{c_t,d_t} to obtain the partial differential equation

$$(b_t \, d_t)^{\frac{1}{\sigma_t}} \, \frac{\partial V(\mathbf{c}_0, \mathbf{d}_0)}{\partial d_t} = (a_t \, c_t)^{\frac{1}{\sigma_t}} \, \frac{\partial V(\mathbf{c}_0, \mathbf{d}_0)}{\partial c_t}, \tag{A.13}$$

which is solved by (A.9). □

[2] Note that another familiar representation of a CES utility function is obtained by applying the monotone transformation $\varphi(x) = \left(\frac{\sigma_t - 1}{\sigma_t} \, x \right)^{\frac{\sigma_t}{\sigma_t - 1}}$ to (A.9).

A.2 Proof of Corollary 1

Under Assumption 2, i.e. if $\sigma_{c_t,d_t}(\mathbf{c}_0, \mathbf{d}_0) = \sigma_t$ for all $t \geq 0$, we can apply Lemma 1 for all $t \geq 0$. Thus, the utility function (1) can be written as

$$V(\mathbf{c}_0, \mathbf{d}_0) = \sum_{t=0}^{\infty} v_t \left(\frac{\sigma_t}{\sigma_t - 1} \left(a_t\, c_t^{\frac{\sigma_t - 1}{\sigma_t}} + b_t\, d_t^{\frac{\sigma_t - 1}{\sigma_t}} - 1 \right) \right), \tag{A.14}$$

where the $v_t(\cdot)$ are any increasing functions $v_t : \mathbb{R} \to \mathbb{R}$. Stationarity, i.e. that $u(\cdot, \cdot)$ in (1) is independent of t, implies that there must be some $v(\cdot)$ such that $v_t(\cdot) = \beta^t\, v(\cdot)$ and that a_t, b_t, and σ_t are independent of time. By normalization, we can write $a_t = \alpha$ and $b_t = 1 - \alpha$, and use $\sigma = \sigma_t$ for all t. Hence, under Assumption (2), the utility function (1) must be of the form (7).

A.3 Proof of Corollary 2

Applying Lemma 1 to the case of constant elasticity of intertemporal substitution ϑ between c_t and $c_{t'}$ for all t and t', we conclude that the intertemporal utility (1) must be of the form

$$V(\mathbf{c}_0, \mathbf{d}_0) = \sum_{t=0}^{\infty} \frac{\vartheta}{\vartheta - 1}\, a_t\, c_t^{\frac{\vartheta - 1}{\vartheta}} + A, \tag{A.15}$$

where $a_t > 0$ and $A \in \mathbb{R}$ are independent of any c_t, $t = 0, 1, \ldots$, but may depend on all d_t, $t = 0, 1, \ldots$. Stationarity implies $a_t = \beta^t\, \alpha\, a(d_t)$ and $A = \sum_{t=0}^{\infty} \beta^t\, w(d_t)$. Thus, Assumption 1 implies that (1) must be of the form (8).

A.4 Proof of Proposition 1

Under both Assumptions 1 and 2, (1) must be represented by both (7) and (8). As $u(\cdot, \cdot)$ in (1) is unique up to positive affine transformation, this means that it must be

$$\frac{\vartheta}{\vartheta - 1}\, a(d_t)\, c_t^{\frac{\vartheta - 1}{\vartheta}} + w(d_t) = A\, v \left(\frac{\sigma}{\sigma - 1} \left(\alpha\, c_t^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha)\, d_t^{\frac{\sigma - 1}{\sigma}} - 1 \right) \right) + B, \tag{A.16}$$

with some constants $A > 0$ and $B \in \mathbb{R}$ to allow for the affine transformations.

Because this holds as identity, also the derivative of the left-hand-side must equal the derivative of the right-hand-side. Differentiating both sides of (A.16) with respect to c_t, and multiplying by c_t gives

$$a(d_t)\, c_t^{\frac{\vartheta - 1}{\vartheta}} = A\, \alpha\, c_t^{\frac{\sigma - 1}{\sigma}}\, v' \left(\frac{\sigma}{\sigma - 1} \left(\alpha\, c_t^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha)\, d_t^{\frac{\sigma - 1}{\sigma}} - 1 \right) \right). \tag{A.17}$$

Using (A.17) in (A.16), we obtain

$$w(d_t) = A\, v\left(\frac{\sigma}{\sigma-1}\left(\alpha\, c_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\, d_t^{\frac{\sigma-1}{\sigma}} - 1\right)\right) + B$$
$$- A\,\frac{\vartheta}{\vartheta-1}\,\alpha\, c_t^{\frac{\sigma-1}{\sigma}}\, v'\left(\frac{\sigma}{\sigma-1}\left(\alpha\, c_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\, d_t^{\frac{\sigma-1}{\sigma}} - 1\right)\right). \quad \text{(A.18)}$$

The right-hand side is independent of c_t if and only if either $v(\cdot)$ is linear and $\sigma = \vartheta$, or if $\sigma \to 1$ — i.e., $\frac{\sigma}{\sigma-1}\left(\alpha\, c_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\, d_t^{\frac{\sigma-1}{\sigma}} - 1\right)$ becomes the Cobb-Douglas function $c_t^{\alpha}\, d_t^{1-\alpha}$ — and $v(x) = x^{\frac{\vartheta-1}{\alpha\,\vartheta}}$, as then

$$w(d_t) = A\, v\left(c_t^{\alpha}\, d_t^{1-\alpha}\right) + B - A\,\frac{\vartheta}{\vartheta-1}\,\alpha\, c_t^{\alpha}\, d_t^{1-\alpha}\, v'\left(c_t^{\alpha}\, d_t^{1-\alpha}\right) = B. \quad \text{(A.19)}$$

For the second equality, $w(d_t) = B$, we have used the assumption $v(x) = x^{\frac{\vartheta-1}{\alpha\,\vartheta}}$. With this assumption and $\sigma \to 1$, it follows that $a(d_t) = d_t^{\frac{1-\alpha}{\alpha}\,\frac{\vartheta-1}{\vartheta}}$. Using $\sigma = 1$, $v(x) = x^{\frac{\vartheta-1}{\alpha\,\vartheta}}$, and $\theta := \frac{\alpha\,\vartheta}{1-\vartheta\,(1-\alpha)}$ in (A.14), and applying a positive affine transformation (multiplying by α) we obtain (5).

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