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Feldstein Meets George: Land Rent Taxation and Socially Optimal Allocation in Economies with Environmental Externality

Abstract

We consider an overlapping generations (OLG) economy with land as a fixed factor of production and an environmental externality on production in which tax revenue from land rent and/or from other schemes such as labor income, capital income, and production taxation can be used for environmental protection through investment in emission mitigation. We show that, for any given target of stationary stock of pollution, the land rent taxation scheme leads to a higher steady state capital accumulation than the other schemes, and hence the steady state consumption of agents when young under this scheme is also higher than under the others. In addition, under an ambitious mitigation target when the efficiency of the mitigation technology is relatively high compared to the dirtiness of production, the land rent taxation also provides a higher steady state consumption when old, resulting in higher social welfare, than the others. In the second part of the paper, we propose a period-by-period balanced budget policy, which includes land rent and capital income taxes with intergenerational transfers, to decentralize the socially optimal allocation during the transitional phase to the social planner's steady state.

JEL-codes: H230, I310, Q500.

Keywords: overlapping generations economy, land rent, taxation, socially optimal allocation.

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1 Introduction

The crucial purpose of pollution mitigation is to reduce the long-term damage of climate change and hence to improve the social welfare towards sustainable development. The Paris Climate Agreement on limiting global warming in this century and beyond to well below 2 degrees centigrade above pre-industrial level indeed requires strict long-term and persistent global emission mitigation efforts. These are essentially financed through taxation schemes, and hence probably lessen capital accumulation and economic growth. These efforts, however, may contradict the poverty reduction goal unless an appropriate climate policy is designed. This requires an efficient tax policy to support the calls for climate change mitigation from both scientists and policy makers. We engage in this discussion by presenting an OLG economy with *land* as a fixed factor of production and an environmental externality on productivity in which pollution is a by-product of production. Within this framework, we will evaluate the long-term impacts of climate change mitigation policy on capital accumulation and social welfare under alternative tax policies, and consider the possibility of obtaining the socially optimal allocation.

Our paper integrates two theoretical strands in the environmental economics and public finance literature. The first is a sizable literature concerning environmental quality and long-term growth, as well as the decentralization of socially optimal allocation. Papers by Howarth and Norgaard (1992) and John and Pecchennino (1994) are two of the pioneers in this field which is recently revisited and advanced by Dao and Dávila (2014) and Dao and Edenhofer (2018). The second is the literature of incidence of rent taxation on fixed factor of production (Feldstein 1977, Calvo et al. 1979, Chamley and Wright 1987, and recently Edenhofer et al. 2015). While the first strand focuses on correcting environmental externalities, it ignores land as a factor of production, and tax on its rent can be a source for public spending. The second strand does not take environmental issues into account. If we consider the fact that pollution is a by-product of production, then enhancing capital accumulation will degrade the environment which will have negative feedback on production in the future. Therefore, it is reasonable to think that the environmental protection policy needs to be combined with land rent taxation in order to guarantee an improvement in social welfare. This paper attempts to fill this gap in the literature. We will point out that, for a given target of stationary stock of pollution under the climate change mitigation agreement, land rent taxation can be an efficient instrument for mitigation. We argue that it will lead to higher stationary capital accumulation and consumption, hence greater stationary welfare, than other taxation schemes. In addition, the land rent tax can be combined with appropriate capital income tax and intergenerational transfers in order to decentralize the socially optimal allocation.

The role of tax on land was indeed addressed extensively by classical economists. In particular, the American political economist Henry George (1839 - 1897) is famous for the idea that land (or resource) rent should be taxed for public use or share. George (1879) proposed a single tax on land in lieu of distortion taxes on labor and productive investment. According to George's (1879) perspective of justice, the economic value derived from natural resources, including land, should belong equally to all citizens of a society.[1] In addition, in contrast with taxation on other factors of production, tax on land does not change the size of land and may enhance capital accumulation through the portfolio effect of owning land and holding capital. Indeed, in an OLG model without an environmental externality or the effect of infrastructure, Feldstein (1977) points out that capital accumulation can be enhanced, and thus welfare improved, through the taxation of the rent from land. That is because the taxing of land rent will shift the savings of a household's portfolio towards capital accumulation.[2] The theoretical results in this paper consolidate George's ideas in the context

[1] For more discussion about this single tax, see George (1890).

[2] This tax incidence in Feldstein (1977) is then analyzed in refined models such as Calvo et al. (1979) which introduces the bequest motive, and Chamley and Wright (1987) which analyzes both the static and dynamic effects of land rent tax. Petrucci (2006) and Koethenbuerger and Poutvaara (2009) also consider the effects of land rent taxation on capital accumulation and improving welfare but they study this in the framework of small and open economies.

of investing in climate change mitigation.

The paper also relates to the literature of land tax and investment in public goods or infrastructure (Arnott and Stiglitz 1979, Rangel 2005, Mattauch et al. 2013, and Behrens et al. 2015). Arnott and Stiglitz (1979) and Behrens et al. (2015) focus on static models investigating the relationship between aggregate urban land rent and local public good provision in which the authors identify general conditions under which the Henry George theorem holds. Rangel (2005) sets up a two-period two-generation political economy to study institutional implementation over alternative taxation and debt regimes, including a *land-tax-only scheme*, to obtain Pareto improvement through investing in intergenerational public goods. In a continuous time Ramsey model, Mattauch et al. (2013) show that when land rent is sufficiently high, the social optimum can be obtained by using the land rent tax revenue to invest in public infrastructure. Such investment is less rigorous in the *laissez-faire* economy. Our paper differs from this body of literature by incorporating an environmental externality, in an OLG model à la Diamond (1965), to study the incidence of land rent taxation on environmental protection and welfare. We will compare this to other schemes such as capital income, labor income and production taxation. In addition, we propose a combination of land rent tax, capital income tax, and intergenerational transfers in order to decentralize the socially optimal allocation during the transitional phase as a competitive outcome.

There is a sizable literature relating climate change to fossil energy extraction. A gradual rise in carbon tax hasten the extraction of fossil energy as its owners face a sharp decline in demand for this energy in the long-term. This increases carbon emissions in the short-term, creating a so-called *Green Paradox* which is pioneered by Sinn (2008). Grafton et al. (2012) and Van der Ploeg and Withagen (2012) then characterize the conditions and timing of the carbon tax which cause such a Green Paradox. Our paper abstracts from fossil energy extraction and hence we do not address the Green Paradox. We do not, however, necessarily contradict this literature since we focus on different aspects. Particularly, we consider that in the long run, it can be efficient for the revenue from non-distortion land rent tax to be invested in emission mitigation. The consideration of these topics together should be addressed in a different setup.

The rest of the paper is organized as follows. The basic model is presented in section 2. The competitive equilibrium and steady state are defined in section 3. Section 4 examines the effects of taxation schemes on steady state stock of pollution and capital accumulation. We compare these effects on capital accumulation and welfare of taxation schemes in section 5. Section 6 characterizes the socially optimal allocation from the viewpoint of the social planner. A strategy decentralizing the socially optimal allocation is proposed in section 7. Finally, section 8 concludes the paper.

2 The model

We consider a privately owned OLG economy with *land* as a fixed factor of production and an environmental externality on production. The final output is produced from capital, labor, and land. Agents or households live for two periods —say young and old. They work when young to earn income for consumption during their youth, savings in terms of capital and land ownership. They buy land from the previous generation and sell it to the subsequent generation. Their capital and land are rented to producing firms. The capital savings with its returns, land rent, and sale of land are consumed in the second period of life. We do not consider the bequest motives of agents. We adopt some standard assumptions, which are widely used in the literature, namely the perfect competition in the final good sector and the constant returns to scale property of final good production function over its factor of productions. We, however, make discussions in Appendix A6 about relaxing these assumptions and about the robustness of the paper's results.

2.1 Dynamics of pollution stock and production

We assume the dynamics of pollution stock as follows

$$E_t = (1-\delta)E_{t-1} + \xi Y_t - \gamma M_t$$

where $E_t \in \mathbb{R}$ is the index of pollution stock (the so-called carbon concentration) in period t; ξY_t is the pollution arising from the production as an inevitable by-product; γM_t is pollution abatement coming from the mitigation effort M_t. The $\xi > 0$ is the dirtiness coefficient of production and $\gamma > 0$ is the effectiveness coefficient of mitigation technology. The $\delta \in (0,1]$ is the decay rate of pollution stock which measures the convergent speed of pollution stock to the natural state $E = 0$.

The final good firms operate under perfect competition and follows production function

$$Y_t = z(E_{t-1})F(K_t, L_t, X) \tag{1}$$

where $z(E_{t-1})$ is total factor productivity in t which depends on the stock of pollution in the previous period. Note that the past stock of pollution affects the current productivity, reflecting the long-term effect of pollution. We assume that $z(E) = \bar{z} > 0$ for $E \leq 0$,[3] $z'(E) < 0$ for $E > 0$, $z'(0^+) = 0$, and $\lim_{E \to +\infty} z(E) = 0$. The F exhibits constant returns to scale over capital K, labor L, and land X; and $F_i(K,L,X) > 0$, $F_{ii}(K,L,X) < 0$, $F_{ij}(K,L,X) > 0$ where $i,j \in \{K,L,X\}$, $i \neq j$. The returns to factors of production are determined by their marginal productivity, i.e.

$$r_t = z(E_{t-1})F_K(K_t, L_t, X) \tag{2}$$

$$w_t = z(E_{t-1})F_L(K_t, L_t, X) \tag{3}$$

$$x_t = z(E_{t-1})F_X(K_t, L_t, X) \tag{4}$$

where r_t, w_t, and x_t are respectively returns to capital, labor, and land. For simplicity without loss of generality, we follow Feldstein (1977) by assuming that capital does not depreciate.

2.2 Agents

We consider an economy without population growth consisting of L_t identical agents. Without lost of generality, we normalize $L_t = 1$ for all $t \in \mathbb{N}$. Each agent t lives for two periods.[4] In the first period of life, the agent t is endowed with one unit of labor which he supplies inelastically to the market to earn labor income. He allocates that income to consumption when young c_t^y, and savings in terms of physical capital k_{t+1}, which will be rented to producing firms, and savings in terms of holding land X, which will be rented to producing firms and sold to the succeeding generation in the next period. These savings will be fully consumed in the second period of life. The agent t has preferences over consumptions when young and old $(c_t^y, c_{t+1}^o) \in \mathbb{R}_+^2$, represented by $U(c_t^y, c_{t+1}^o) = u(c_t^y) + v(c_{t+1}^o)$ with $u', v' > 0$, $u'', v'' < 0$, and u and v satisfy Inada conditions.[5]

The life-time utility maximization problem of the agent is

[3]For simplification, we assume $z(E)$ is a positive constant for $E \leq 0$. One may argue that $z'(E) > 0$ for $E < 0$. Indeed, capturing this assumption does not crucially change the analytical results of this paper. Moreover, the case $E < 0$ is not a focus of our paper.

[4]We assign the time index t to an agent, say agent t, implying that the agent becomes adult (or young) in period t and old in period $t+1$. A similar idea is applied for a generation as a whole.

[5]We can extend the model by introducing the disutility of pollution in the preference. The utility function can be presented as $\tilde{U}(c_t^y, c_{t+1}^o, E_{t+1}) = u(c_t^y) + v(c_{t+1}^o) + \pi(E_{t+1})$ where for all $E > 0$, $\pi' < 0$ and $\pi'' > 0$. We assume the affect of each agent t is negligible to the stock of pollution, hence it has no incentive to internalize the environmental externality and always treats E_{t+1} as given in solving its optimization problem. However, the qualitative results do not change when we model preference in this way. So, in order to illustrate the ideas, we keep the model simple and thus abstract disutility of pollution from the utility function.

$$\max_{c_t^y, k_{t+1}, c_{t+1}^o, X} u(c_t^y) + v(c_{t+1}^o) \tag{5}$$

$$\text{subject to :} \qquad c_t^y + k_{t+1} + p_t X \leq (1-\tau^L) w_t \tag{6}$$

$$c_{t+1}^o \leq \left[1 + (1-\tau^K) r_{t+1}\right] k_{t+1} + \left[(1-\tau^X) x_{t+1} + p_{t+1}\right] X \tag{7}$$

given w_t, perfect foreseen r_{t+1}, and the prices of land p_t and p_{t+1} in t and $t+1$, respectively; $\tau^X, \tau^K, \tau^L \in [0,1]$ are the tax rates on land rent, capital income, labor income respectively. Equations (6) and (7) describe the budget constraints of the agent in young and old period, respectively.[6]

Under standard assumptions on u and v above, guaranteeing the concavity of the objective function and the interiority of the optimal solution, the choice of the agent is characterized by

$$\begin{pmatrix} u'(c_t^y) \\ 0 \\ v'(c_{t+1}^o) \\ 0 \end{pmatrix} = \lambda_t \begin{pmatrix} 1 \\ 1 \\ 0 \\ p_t \end{pmatrix} + \mu_t \begin{pmatrix} 0 \\ -1-(1-\tau^K) r_{t+1} \\ 1 \\ -(1-\tau^X) x_{t+1} - p_{t+1} \end{pmatrix} \tag{8}$$

for some $\lambda_t, \mu_t > 0$, and the budget constraints (6) and (7) are binding; i.e.

$$\frac{u'(c_t^y)}{v'(c_{t+1}^o)} = 1 + (1-\tau^K) r_{t+1} \tag{9}$$

$$c_t^y + k_{t+1} + p_t X = (1-\tau^L) w_t \tag{10}$$

$$c_{t+1}^o = \left[1 + (1-\tau^K) r_{t+1}\right] k_{t+1} + (1-\tau^X) x_{t+1} X + p_{t+1} X \tag{11}$$

$$\left[1 + (1-\tau^K) r_{t+1}\right] p_t = (1-\tau^X) x_{t+1} + p_{t+1} \tag{12}$$

where the last equation is the no-arbitrage condition between savings in terms of land and capital; equation (9) is Euler equation reflecting the optimal allocation between consumption and savings.

3 Equilibrium and steady state

Without loss of any generality, we normalize $X = 1$. For lightening the notation, we denote the function $F(K_t, L; X)$ and its derivatives as F^t, F_i^t, and F_{ij}^t where $i, j \in \{K_t, L, X\}$. Since we normalize the size of population by 1, then $K_t = k_t$ for all t. We assume that all tax revenue will be used for emission mitigation. Hence, the mitigation in period t is

$$M_t = \tau^K z(E_{t-1}) F_K^t k_t + \tau^L z(E_{t-1}) F_L^t + \tau^X z(E_{t-1}) F_X^t = z(E_{t-1}) F^t \sum_i \sigma_t^i \tau^i$$

where σ_t^i and τ^i are respectively the share in final output and the tax rate on the income of production factor $i \in \{K, L, X\}$. We define the equilibrium and steady state as follows:

Equilibrium: Under the given tax rates τ^X, τ^K, τ^L and sequence of land prices $\{p_t\}_{t=0}^{+\infty}$ satisfying the non-arbitrage condition (12), the competitive equilibrium is characterized by: (i) the agent's utility maximization (5) under the budget constraints (6) and (7); (ii) the returns of the production factors as in (2), (3), and (4); (iii) the dynamics of the pollution stock. The competitive equilibrium

[6] Note that, from this section until section 5, we assume that all tax revenue is used for emission mitigation. Hence, there are no lump-sum transfers in the budget constraints. In section 7, such lump-sum transfers will appear in implementing the optimal allocation.

allocation $\{c_t^y, k_{t+1}, c_{t+1}^o, E_t\}_t$, which fully characterizes the competitive equilibrium of the economy, is the solution to the following system of equations, given E_{t-1}, k_t, p_t:[7]

$$\frac{u'(c_t^y)}{v'(c_{t+1}^o)} = 1 + (1-\tau^K)z(E_t)F_K^{t+1} \tag{13}$$

$$c_t^y + k_{t+1} + p_t = (1-\tau^L)z(E_{t-1})F_L^t \tag{14}$$

$$c_{t+1}^o = \left[1 + (1-\tau^K)z(E_t)F_K^{t+1}\right](k_{t+1} + p_t) \tag{15}$$

$$E_t = (1-\delta)E_{t-1} + (\xi - \gamma\sum_i \sigma^i\tau^i)z(E_{t-1})F^t \tag{16}$$

By doing some simple substitutions and transformations, the competitive equilibrium of the economy can be fully characterized by the following system of the Euler equation, dynamics of pollution stock, and no arbitrage condition, given E_{-1}, k_0, p_0:

$$\frac{u'\left((1-\tau^L)z(E_{t-1})F_L^t - k_{t+1} - p_t\right)}{v'\left(\left[1 + (1-\tau^K)z(E_t)F_K^{t+1}\right](k_{t+1} + p_t)\right)} = 1 + (1-\tau^K)z(E_t)F_K^{t+1} \tag{17}$$

$$E_t = (1-\delta)E_{t-1} + (\xi - \gamma\sum_i \sigma^i\tau^i)z(E_{t-1})F^t \tag{18}$$

$$p_{t+1} = \left[1 + (1-\tau^K)z(E_t)F_K^{t+1}\right]p_t - (1-\tau^X)z(E_t)F_X^{t+1} \tag{19}$$

Competitive steady state: Under the tax rates $\tau^X, \tau^K, \tau^L \in [0,1]$, the competitive steady state of the economy is characterized by Euler equation (20) and equation (21) which equalizes the decay of pollution and flow of pollution, while the stationary land price is $p = \frac{(1-\tau^X)F_X}{(1-\tau^K)F_K}$,

$$\frac{u'\left((1-\tau^L)z(E)F_L - k - \frac{(1-\tau^X)F_X}{(1-\tau^K)F_K}\right)}{v'\left([1 + (1-\tau^K)z(E)F_K][k + \frac{(1-\tau^X)F_X}{(1-\tau^K)F_K}]\right)} = 1 + (1-\tau^K)z(E)F_K \tag{20}$$

$$\delta E = (\xi - \gamma\sum_i \sigma^i\tau^i)z(E)F. \tag{21}$$

4 Effects of taxation schemes

We now study the *long-term* effects of each taxation scheme on stock of pollution and capital accumulation. In this section, for the purpose of exposition, we specify the functional forms of utility and production. We consider a benchmark model with the logarithm preference $U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$ where $\beta \in (0,1)$ is the time preference parameter of the agent, and the Cobb-Douglas production function $F^t = K_t^{\alpha_K}L^{\alpha_L}X^{\alpha_X} \equiv k_t^{\alpha_K}$, $0 < \alpha_K, \alpha_L, \alpha_X < 1$, $\alpha_K + \alpha_L + \alpha_X = 1$. For these functional forms, the system of equations characterizing the steady state now boils down to

$$\left[1 + \frac{(1-\tau^X)\alpha_X}{(1-\tau^K)\alpha_K}\right]k - \frac{\beta\alpha_L(1-\tau^L)}{1+\beta}z(E)k^{\alpha_K} = 0 \tag{22}$$

[7] Equation (15) is derived from equations (11) and (12). Note that, we normalize the size of land $X = 1$, thus the notation X does not appear in equation (15).

$$\delta E - (\xi - \gamma \sum_i \alpha^i \tau^i) z(E) k^{\alpha_K} = 0, \quad i \in \{K, L, X\} \tag{23}$$

Note that for the Cobb-Douglas production function, α_i is the share in final output of production factor $i \in \{K, L, X\}$. The existence and uniqueness of the steady state, as well as the dynamic system of the economy and its convergence to the steady state are presented in Appendix A0.

4.1 *Land rent taxation*

Under the land rent taxation scheme only, the system (22) - (23) becomes

$$\frac{\alpha_K + (1 - \tau^X)\alpha_X}{\alpha_K} k_X - \frac{\beta \alpha_L}{1 + \beta} z(E_X) k_X^{\alpha_K} = 0 \tag{24}$$

$$\delta E_X - (\xi - \gamma \tau^X \alpha_X) z(E_X) k_X^{\alpha_K} = 0 \tag{25}$$

where E_X and k_X are respectively steady state stock of pollution and capital per capita under the land rent taxation scheme.

Proposition 1 below states the existence and uniqueness of the steady state under the land rent taxation scheme and its effects on the steady state stock of pollution and capital accumulation.

Proposition 1. *In the OLG economies with $F = k^{\alpha_K}$ and $U(c^y, c^o) = \ln c^y + \beta \ln c^o$ set up above: If $\alpha_K < \frac{1}{2}$ and $\frac{\xi}{\gamma} < 1 - \alpha_K + \alpha_X$ then $\frac{\partial E_X}{\partial \tau^X} < 0$ and $\frac{\partial k_X}{\partial \tau^X} > 0$;*

Proof. See Appendix A1. □

Proposition 1 implies that if the share of capital to final output is not too high, particularly $\alpha_K < 1/2$, and the mitigation is sufficiently effective, particularly $\xi/\gamma < 1 - \alpha_K + \alpha_X$, then the higher the land rent tax $\tau^X \in [0, 1]$ the lower the steady state stock of pollution and the higher the steady state capital per capita. This is because, when the mitigation technology is relatively effective compared to the dirtiness of production, the effect of mitigation will be strong in reducing the pollution stock. The improved environmental quality then increases the output and hence enhances capital accumulation, which in turn, degrades the environment. On the other hand, as the share of capital to final output is not too high then the effect of capital accumulation on carbon concentration will be dominated by the mitigation which is supported by land rent tax revenue.

4.2 *Capital income taxation*

Under the capital income taxation scheme only, the system (22) - (23) becomes

$$\frac{\alpha_X + (1 - \tau^K)\alpha_K}{(1 - \tau^K)\alpha_K} k_K - \frac{\beta \alpha_L}{1 + \beta} z(E_K) k_K^{\alpha_K} = 0 \tag{26}$$

$$\delta E_K - (\xi - \gamma \tau^K \alpha_K) z(E_K) k_K^{\alpha_K} = 0 \tag{27}$$

where E_K and k_K are respectively the steady state stock of pollution and capital per capita under the capital income taxation scheme.

Proposition 2. *In the OLG economies with $F = k^{\alpha_K}$ and $U(c^y, c^o) = \ln c^y + \beta \ln c^o$ set up above: $\frac{\partial E_K}{\partial \tau^K} < 0$ while $\frac{\partial k_K}{\partial \tau^K}$ is ambiguous.*

Proof. See Appendix A1. □

Proposition 2 addresses the effects of the capital income tax on the steady state stock of pollution and capital accumulation. The higher tax rate leads to a lower steady state stock of pollution. This result is rather intuitive. Capital income taxation provides two reinforcing effects on environmental protection. First, the tax revenue from capital income is used for mitigation, hence it directly decreases the stock of pollution. Second, when capital income is taxed, agents will shift the savings of their portfolio towards land value ownership, thereby reducing capital accumulation, which in turn leads to lower pollution. It is interesting to note that although under the capital income taxation scheme agents tend to shift the savings of their portfolio towards land value ownership, the steady state capital per capita under this scheme is not necessarily lower than that under a *laissez-faire* economy without any intervention. Indeed, an increase in τ^K has two opposite effects on steady state capital. Firstly, emission mitigation supported by tax revenue enhances the total factor productivity of the economy, and hence improves the labor income. This has the effect of accelerating capital accumulation. Secondly, taxing capital income causes agents to partially shift their savings to land value ownership, hence lessening the capital accumulation. Therefore, the net effect depends on the responsiveness of the damage function $z(E)$ to the change in stock of pollution and on the level of the tax rate τ^K.

4.3 Labor income taxation

Under the labor income taxation scheme only, the system (22) - (23) becomes

$$\frac{\alpha_K + \alpha_X}{\alpha_K} k_L - \frac{\beta(1-\tau^L)\alpha_L}{1+\beta} z(E_L) k_L^{\alpha_K} = 0 \tag{28}$$

$$\delta E_L - (\xi - \gamma \tau^L \alpha_L) z(E_L) k_L^{\alpha_K} = 0 \tag{29}$$

where E_L and k_L are respectively the steady state stock of pollution and capital per capita under the labor income taxation scheme.

Proposition 3. *In the OLG economies with $F = k^{\alpha_K}$ and $U(c^y, c^o) = \ln c^y + \beta \ln c^o$ set up above: $\frac{\partial E_L}{\partial \tau^L} < 0$ while $\frac{\partial k_L}{\partial \tau^L}$ is ambiguous.*

Proof. The proofs are quite similar to the proofs of proposition 2. □

The effect of a labor income taxation scheme on steady state stock of pollution is similar to that of capital income taxation. Labor income taxation also has two reinforcing effects on emission mitigation. First, the tax revenue is used for emission mitigation hence it directly decreases the stock of pollution. Second, the labor income tax decreases capital accumulation by decreasing the disposable income of agents. In this way, it causes the final output to decline and hence reduces emissions. However, the general equilibrium effect of this scheme on capital accumulation is ambiguous because the lower stock of pollution increases the total factor productivity which enhances capital accumulation by increasing the labor income. So the effect of labor income taxation on steady state capital accumulation depends on the tax rate and the responsiveness of the damage function $z(E)$ to the change in pollution stock. Note that unlike the effects of capital income taxation, the change in portfolio of savings between capital and land is unclear because the labor income taxation has symmetric effects on savings in land and in capital.

4.4 Production taxation

Now we suppose that the government collects tax revenue on production, say a production tax, in order to invest in mitigation. Under this taxation scheme $\tau^p \in [0, 1)$ alone, the steady state of the economy is characterized by

$$\frac{\alpha_K + \alpha_X}{\alpha_K} k_P - \frac{\beta \alpha_L (1 - \tau^P)}{1 + \beta} z(E_P) k_P^{\alpha_K} = 0 \tag{30}$$

$$\delta E_P - (\xi - \gamma \tau^P) z(E_P) k_P^{\alpha_K} = 0 \tag{31}$$

where E_P and k_P are respectively the steady state stock of pollution and capital per capita under the production taxation scheme.

Proposition 4. *In the OLG economies with $F = k^{\alpha_K}$ and $U(c^y, c^o) = \ln c^y + \beta \ln c^o$ set up above: $\frac{\partial E_P}{\partial \tau^P} < 0$ while $\frac{\partial k_P}{\partial \tau^P}$ is ambiguous.*

Proof. The proofs are quite similar to the proofs of proposition 2. □

In line with the other schemes above, the steady state stock of pollution also decreases in the production taxation scheme. That is because this scheme has at least two reinforcing effects on emission mitigation. First, the tax revenue is used for mitigation, hence the pollution stock is directly reduced. Second, the production tax decreases the labor income received by the agents, hence reducing the capital accumulation. Therefore, it indirectly decreases the pollution stock. As with the effect of capital income taxation on capital accumulation, we need to be cautious in evaluating the effect of production taxation scheme on capital accumulation. Although the production tax directly reduces the net labor income of agents, hence reducing the savings, the steady state capital per capita is not necessarily lower than that under a *laissez-faire* economy without any intervention. On the one hand, as stated in proposition 4, this scheme leads to a lower stationary stock of pollution, hence improves the total factor productivity. In this way, this policy increases the labor income, enhancing the capital accumulation. On the other hand, the policy directly decreases the net labor incomes by imposing a tax rate $\tau^P > 0$ on the revenue of the producing firms. Which effect becomes dominant depends on the responsiveness of the damage function $z(E)$ to the change in E and on the level of the tax rate τ^P.

5 Comparison of taxation schemes

The idea of this section is inspired by the Paris agreement which requires that the long-term global temperature be limited to well below 2 degree centigrade (and even ambitiously limited below 1.5 degree centigrade) above pre-industrial level. Such a policy goal can be translated into a stationary carbon concentration target set by governments. In order to obtain this target, a government implements emission mitigation through its tax policies as described above. However, these tax policies, in achieving a government's target, may lead to different long-term capital accumulation, thus different consumption and welfare. In this section, we study the long-term effects of each taxation scheme on capital accumulation, given a target of stationary stock of pollution. We then compare these effects on consumption and welfare between taxation schemes. We suppose that the government has a target E for the long-term stationary pollution stock. This target can be determined through one of the tax and mitigation schemes detailed above.

Proposition 5. *In the OLG economies with $F = k^{\alpha_K}$ and $U(c^y, c^o) = \ln c^y + \beta \ln c^o$ set up above, given a target on steady state stock of pollution $E \in [0, \tilde{E})$, where $\tilde{E}$ is the steady state stock of pollution in case of no mitigation, then:*
(i) $k_X > \max\{k_K, k_P\}$;
(ii) $k_P > k_L$;
(iii) if $1 - \tau^K > (=)(<)\ \alpha_X(\alpha_L - \alpha_K)/\alpha_K^2$ *then* $k_K > (=)(<)\, k_L$;
(iv) if $\alpha_X \geq \alpha_K^2/(1 - \alpha_K)$ *then* $k_P > k_K$.

Proof. See Appendix A2. □

The results (i) and (ii) in proposition 5 imply that the land rent taxation will lead to the highest capital accumulation. Importantly, they indicate that the land rent taxation scheme may be more efficient than the other schemes in combating climate change and obtaining better social welfare in the long term. That is because, unlike other schemes, it does not decrease the agents' disposable income when young. Moreover, it causes agents to shift their savings towards capital accumulation, whereas capital income taxation has the opposite effect. The production and labor income taxation schemes directly decrease the disposable income and hence lessen the capital accumulation compared to the land rent taxation scheme.

Statement (iii) compares the capital accumulation under capital income taxation and labor income taxation schemes. Which scheme leads to higher capital accumulation depends on the extent of climate policy ambition. When the steady state stock of pollution is set at an unambitious level, requiring the tax rate on capital income or labor income to be not too high, in particular $\tau^K < 1 - \alpha_X(\alpha_L - \alpha_K)/\alpha_K^2$, then the former leads to higher capital accumulation than the latter and vice versa. This result can be interpreted as follows. When the climate policy is not sufficiently ambitious, both tax rates on capital income and labor income for emission mitigation corresponding to these two taxation schemes will be low. The effect of the low capital income tax in adjusting the savings of the agent's portfolio is not strong enough to cause a greater decrease in capital accumulation than that under the labor income taxation scheme. The latter generates a decrease in capital accumulation through a direct decrease in the disposable incomes of agents. Therefore, in this case, $k_K > k_L$. The situation is reversed when the climate policy is sufficiently ambitious.

Statement (iv) compares the production taxation and capital income taxation schemes under a given condition. When the share of land to output is large enough compared to that of capital, in particular when $\alpha_X \geq \alpha_K^2/(1 - \alpha_K)$ holds,[8] the production taxation scheme always leads to a higher steady state capital accumulation than the capital income taxation scheme, given a steady state pollution stock. This is because when the share of capital to final output is relatively low compared to that of land, then under the capital income taxation scheme the tax rate needs to be high to obtain the given target of pollution stock. Therefore, the price of land under the capital taxation scheme will be relatively high compared to the income of agents. As a consequence, the capital accumulation is lower than that achieved in the case of production taxation.

The heterogenous effects of taxation schemes on capital accumulation may be the cause of the difference in the steady state welfare between schemes. Capital accumulation is highest under the land rent taxation, which results in the highest final output and flow of pollution. This scheme, however, requires a higher investment in pollution mitigation than the others. So, it may be difficult to evaluate the welfare effects between these schemes. The following proposition provides some comparisons about consumption which may be useful for welfare analyses.

Proposition 6. *In the OLG economies with $F = k^{\alpha_K}$ and $U(c^y, c^o) = \ln c^y + \beta \ln c^o$ set up above, given a target on steady state stock of pollution $E \in [0, \tilde{E})$, we have:*
(i) $c_X^y > \max\{c_P^y, c_K^y, c_L^y\}$;

[8] The condition $\alpha_X \geq \alpha_K^2/(1 - \alpha_K)$ satisfies for $\alpha_K = 1/3$ and $\alpha_X \geq 1/6$ which may hold in the reality.

(ii) if $\xi/\gamma \le \alpha_X + \alpha_K$*, then* $c_X^o > c_K^o$*;*

(iii) if $\tau^P \le 1 - \left[\frac{\alpha_K+\alpha_X}{\alpha_K+\alpha_X(1-\tau^K)^{-1}}\right]^{\alpha_K}$*, then* $c_P^y \ge c_K^y$ *and* $c_P^o > c_K^o$*.*

(iv) if $\tau^P \ge 1 - \left[\frac{\alpha_X(1-\tau^X)+\alpha_K}{\alpha_X+\alpha_K}\right]^{1-2\alpha_K}$*, then* $c_X^o > c_P^o$*;*

(v) if $\tau^L \ge 1 - \left[\frac{\alpha_X(1-\tau^X)+\alpha_K}{\alpha_X+\alpha_K}\right]^{\frac{1-2\alpha_K}{\alpha_K}}$*, then* $c_X^o > c_L^o$*.*

Proof. See Appendix A3.

□

Proposition 6 provides interesting results on comparisons of consumption between taxation schemes. These results straightforwardly concern comparisons of welfare. Under given conditions, the land rent taxation scheme provides a higher steady state welfare than the other schemes. In statement (i), the consumption when young is highest under the land rent taxation scheme simply because this scheme leads to the highest capital accumulation, hence the greatest disposable labor income, while the savings rate under logarithm utility is constant.

In statement (ii), when the mitigation technology is effective enough, in particular $\xi/\gamma \le \alpha_X+\alpha_K$,[9] then the consumption when old under land rent taxation is higher than that under capital income taxation. That is because, for a given target of stationary pollution stock, the better the mitigation technology lowers the tax rates imposed on land rent and capital income corresponding to these two schemes. Hence, the relative capital gap, k_X/k_K, is closer to 1,[10] but always greater than 1. The lower relative gap k_X/k_K has two opposite effects on the relative gap of consumption c_X^o/c_K^o. On the one hand, the lower relative gap k_X/k_K leads to a lower relative gap of labor income, and hence a lower relative gap of savings between these two taxation schemes. On the other hand, the lower k_X/k_K decreases the difference in returns on capital, $[1+z(E)\alpha_K k_X^{\alpha_K-1}]/[1+(1-\tau^K)z(E)\alpha_X k_K^{\alpha_K-1}]$, between these schemes. When the mitigation technology is effective enough, the difference in returns on capital, under these taxation schemes, is strictly dominated by the corresponding difference in savings. As a consequence, $c_X^o > c_K^o$.

Statement (iii) provides a sufficient condition relating to production and capital income tax rates in comparing the consumption under these schemes. If the former is relatively low compared to the latter, in particular $\tau^P \le 1 - \left[\frac{\alpha_K+\alpha_X}{\alpha_K+\alpha_X(1-\tau^K)^{-1}}\right]^{\alpha_K}$ holds,[11] then the former leads to higher stationary consumption for both the young and the old. As the proof for this statement shows, when the condition above holds, the disposable income of an agent when young and the net return to savings under the production taxation scheme are higher than those under the capital income taxation scheme. This also results in higher consumption, when young and old, under the production taxation scheme. It implies that, with an ambitious target of stationary pollution stock, the steady state welfare under the production taxation will be higher than that under capital income taxation.

Statements (iv) and (v) state that if production and labor income tax rates are relatively high compared to the land rent tax rate, then the steady state consumption when old under land rent taxation scheme is higher than those under production and labor income taxation schemes. That is because the relatively high production tax rate not only reduces disposable income, and hence savings, but also sufficiently lowers the net return on capital. The relatively high labor income tax

[9] Note that this condition is just a sufficient condition under which $c_X^o > c_K^o$ holds.

[10] Indeed, in the proof of proposition 5 we know that $\frac{k_X}{k_K} = \left[\frac{\alpha_K+\alpha_X(1-\tau^K)^{-1}}{\alpha_K+\alpha_X(1-\tau^X)}\right]^{\frac{1}{1-\alpha_K}}$ which is increasing in τ^K and τ^X.

[11] This condition may hold for an ambitious target for stationary pollution stock. For instance, suppose that the target for stationary stock of pollution is zero, $E = 0$. For this extreme target, that condition boils down

$$\frac{\xi}{\gamma} \le 1 - \left[\frac{\alpha_K + \alpha_X}{\alpha_K + \alpha_X\gamma\alpha_K/(\gamma\alpha_K - \xi)}\right]^{\alpha_K}$$

which is satisfied when $\gamma\alpha_K$ is very close to ξ.

rate leads to low disposable income, hence low savings. So the steady state consumption when old under these schemes will be lower than that under the land rent taxation scheme.

The next proposition characterizes a condition under which the land rent taxation provides highest steady state welfare among four schemes.

Proposition 7. *In the OLG economies with* $F = k^{\alpha_K}$ *and* $U(c^y, c^o) = \ln c^y + \beta \ln c^o$ *set up above, under plausible values of parameters that* $\alpha_L < 2\alpha_K < 3 - \sqrt{5}$,[12] *if*

$$\frac{\xi}{\gamma} \leq (\alpha_K + \alpha_X)\left[1 - \left(\frac{1-2\alpha_K}{1-\alpha_L}\right)^{\frac{1}{2\alpha_K}}\right] \tag{32}$$

then there exists ambitious targets of steady state stock of pollution E *under which* $c_X^o > \max\{c_K^o, c_P^o, c_L^o\}$.

Proof. See Appendix A3.

□

Proposition 7 implies that if the efficiency of mitigation technology γ is relatively high compared to the dirtiness of production ξ, particularly the condition (32) holds, then under a sufficiently ambitious target of steady state stock of pollution the land rent taxation scheme will lead to the highest steady stead consumption when old, i.e. $c_X^o > \max\{c_K^o, c_P^o, c_L^o\}$. As apparent in the proof of this proposition, the condition (32) makes conditions in (iv) and (v) of proposition 6 hold under a sufficiently ambitious target of steady state stock of pollution.[13] Hence, the mechanisms leads to the highest steady state consumption when old under the land rent taxation scheme are analyzed within the statements (iii), (iv), and (v) of proposition 6. By combining this result with the result in statement (i) of proposition 6, we observe that the land rent taxation scheme may provide a strictly higher steady state welfare than other schemes.

6 Social planner's allocation

In this section, we consider the optimal allocation from the viewpoint of a benevolent social planner, who seeks to maximize the social welfare of all generations. The social planner's problem is

$$\max_{\{c_t^y, c_t^o, k_{t+1}, M_t, E_t\}_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} \frac{u(c_t^y) + (1+R)v(c_t^o)}{(1+R)^t} \tag{33}$$

subject to, $\forall t = 0, 1, 2, ...$

$$c_t^y + c_t^o + k_{t+1} + M_t \leq z(E_{t-1})F^t + k_t \tag{34}$$

$$E_t = (1-\delta)E_{t-1} + \xi z(E_{t-1})F^t - \gamma M_t \tag{35}$$

for given initial conditions $k_0 > 0$, E_{-1}; and $R > 0$ is the subjective discount rate of the planner. The objective function (33) is the social welfare function, equations (34) and (35) represent the feasibility constraint of society and the dynamics of pollution stock determined in t, respectively.

The social planner's allocation $\left\{c_t^{y,s}, c_t^{o,s}, k_{t+1}^s, M_t^s, E_t^s\right\}_{t=0}^{+\infty}$ and sequence of Lagrangian multipliers $\{\mu_t, \eta_t\}_{t=0}^{+\infty}$, where μ_t and η_t are respectively Lagrangian multipliers (or shadow prices) associated with constraints (34) and (35), are characterized by

[12] The first inequality guarantees ξ/γ in the condition (32) to be non negative. The second one guarantees the condition (32) to belong to set of sufficient conditions ensuring $c_X^o > c_L^o$ at a sufficiently ambitious target of stationary pollution stock. A proof for this argument is available upon request.

[13] Note that the condition (32) trivially makes the conditon in (iii) of proposition 6 hold, therefore $c_X^o > c_K^o$ for all target of steady state stock of pollution $E \in [0, \tilde{E})$.

$$\frac{u'(c_t^{y,s})}{v'(c_{t+1}^{o,s})} = 1 + \frac{\gamma - \xi}{\gamma} z(E_t^s) F_K^{t+1,s} \tag{36}$$

$$c_t^{y,s} + c_t^{o,s} + k_{t+1}^s + M_t^s - z(E_{t-1}^s) F^{t,s} - k_t^s = 0 \tag{37}$$

$$E_t^s - (1-\delta) E_{t-1}^s - \xi z(E_{t-1}^s) F^{t,s} + \gamma M_t^s = 0 \tag{38}$$

$$z'(E_t) F^{t+1,s} + \frac{(1+R)\eta_t}{\mu_{t+1} - \eta_{t+1}(1+\xi-\delta)} = 0 \tag{39}$$

$$u'(c_t^{y,s}) = \mu_t = \eta_t \gamma = (1+R) v'(c_t^{o,s}) \tag{40}$$

$\forall t \in \mathbb{N}$, where $E_{-1}^s = E_{-1}$ and $k_0^s = k_0$ are given. This allocation is fully derived in Appendix A4.

The social planner's choice $(c^{y,s}, c^{o,s}, k^s, M^s, E^s)$ at the steady state is characterized by

$$\frac{u'(c^{y,s})}{v'(c^{o,s})} = 1 + R \tag{41}$$

$$z(E^s) F_K^s = \frac{\gamma R}{\gamma - \xi} \tag{42}$$

$$z'(E^s) F^s = \frac{R + \delta}{\xi - \gamma} \tag{43}$$

$$c^{y,s} + c^{o,s} + M^s = z(E^s) F^s \tag{44}$$

$$\delta E^s = \xi z(E^s) F^s - \gamma M^s \tag{45}$$

while the steady state competitive allocation $(c^{y,c}, c^{o,c}, k^c, M^c, E^c)$ is characterized by[14]

$$\frac{u'(c^{y,c})}{v'(c^{o,c})} = 1 + z(E^c) F_K^c \tag{46}$$

$$c^{y,c} + k^c + \frac{F_X^c}{F_K^c} = z(E^c) F_L^c \tag{47}$$

$$c^{o,c} = \left[1 + z(E^c) F_K^c\right] (k^c + \frac{F_X^c}{F_K^c}) \tag{48}$$

$$M^c = 0 \tag{49}$$

$$\delta E^c - \xi z(E^c) F^c = 0 \tag{50}$$

The social planner's steady state $(c^{y,s}, c^{o,s}, k^s, M^s, E^s)$ differs from the competitive steady state $(c^{y,c}, c^{o,c}, k^c, M^c, E^c)$. Indeed, this difference not only comes from the imperfect altruism in the competitive OLG model compared to the social planner's Ramsey model, but also from the ability of the social planner to internalize the negative effect of pollution on production. In this case the competitive steady state allocation is a sub-optimal allocation.

[14]For convenience in denoting variables and functions, we use upper scripts "s" and "c" to denote variables and functions under the social planner's allocation and pure competitive allocation, respectively.

7 Implementation of the social planner's optimal allocation

In this section, we introduce taxes on land rent and capital income as well as investment in mitigating pollution to correct the inefficiency of the competitive economy and obtain the social planner's allocation. The mitigation is always set at the optimal choice of the social planner. The land rent tax and capital income tax are designed such that the agent chooses the correct saving to achieve the social planner's capital accumulation. These taxes and mitigation are balanced by the intergenerational transfers.

Let T_t^y and T_{t+1}^o be respectively a lump-sum tax (if negative) levied on agent t's income when young and a lump-sum subsidy (if positive) to the same agent when old in period $t+1$, along with land rent tax rate τ_{t+1}^X and capital income tax rate τ_{t+1}^K. The problem of the agent t is then

$$\max_{c_t^y, k_{t+1}, c_{t+1}^o} u(c_t^y) + v(c_{t+1}^o)$$

subject to

$$c_t^y + k_{t+1} + p_t \leq z(E_{t-1})F_L^t + T_t^y$$

$$c_{t+1}^o \leq [1 + (1 - \tau_{t+1}^K)z(E_t)F_K^{t+1}]k_{t+1} + (1 - \tau_{t+1}^X)z(E_t)F_X^{t+1} + p_{t+1} + T_{t+1}^o$$

where, under the emission mitigation investment, M_t, chosen by the social planner,

$$E_t = (1-\delta)E_{t-1} + \xi z(E_{t-1})F^t - \gamma M_t$$

The balanced budget constraint in each period t implies

$$T_t^y + T_t^o + \tau_t^K z(E_{t-1})F_K^t k_t + \tau_t^X z(E_{t-1})F_X^t = M_t$$

where the left-hand side is net tax revenue, the right-hand side is emission mitigation investment.

Under the policy $(T_t^y, T_{t+1}^o, \tau_{t+1}^K, \tau_{t+1}^X, M_t)$, the competitive equilibrium is characterized , again, by the Euler equation, the life-time budget constraints which are binding, the no-arbitrage condition, and dynamics of pollution stock:

$$\frac{u'(c_t^y)}{v'(c_{t+1}^o)} = 1 + (1 - \tau_{t+1}^K)z(E_t)F_K^{t+1}$$

$$c_t^y + k_{t+1} + p_t = z(E_{t-1})F_L^t + T_t^y$$

$$c_{t+1}^o = [1 + (1 - \tau_{t+1}^K)z(E_t)F_K^{t+1}]k_{t+1} + (1 - \tau_{t+1}^X)z(E_t)F_X^{t+1} + p_{t+1} + T_{t+1}^o$$

$$[1 + (1 - \tau_{t+1}^K)z(E_t)F_K^{t+1}]p_t = (1 - \tau_{t+1}^X)z(E_t)F_X^{t+1} + p_{t+1}$$

$$E_t = (1-\delta)E_{t-1} + \xi z(E_{t-1})F^t - \gamma M_t$$

One difficulty in decentralizing the planner's allocation is how to determine the whole sequence prices of land. These prices are linked together via the no-arbitrage conditions. In this way, the price of land today depends on that in the future and vice versa. In the pure competitive economy, it is necessary to assume that the price sequence of land is given, and an agent in any period t has perfect foresight on the price of land in $t+1$. This price information affects the agent's decisions in allocating his savings portfolio in terms of capital and land, and hence affects the capital accumulation for production. In this section, we propose a policy under which the sequence of land prices coincides with that of the pure competitive economy.

Suppose that in some period T, the social planner starts to implement the optimal allocation $\left\{(c_t^{y,s}, c_t^{o,s}, k_{t+1}^s, M_t^s, E_t^s)\right\}_{t=T}^{+\infty}$ given $k_T^s = k_T$, $E_{T-1}^s = E_{T-1}$. At the time that the policy is first introduced, the price of land under the pure competitive economy is p_T^c. This price is predetermined between generations T and $T-1$. The price p_T^c links to p_{T-1}^c via the no-arbitrage condition under the pure competitive regime. Introducing policy $(T_T^y, T_{T+1}^o, \tau_{T+1}^k, \tau_{T+1}^x, M_T)$ to generation T will affect the land price in period $T+1$, p_{T+1}^s. We first identify the relationship between τ_{t+1}^K and τ_{t+1}^X for all $t \geq T$, under perfect foresight on the returns on land and capital, the price of land p_{t+1}^s, under the policy $(T_t^y, T_{t+1}^o, \tau_{t+1}^k, \tau_{t+1}^x, M_t)_{t\geq T}$, is also the price of land under pure competitive regime, i.e. $p_{t+1}^s = p_{t+1}^c \; \forall t \geq T$. The following lemma presents this relation.

Lemma 1. *Under the policy $(T_t^y, T_{t+1}^o, \tau_{t+1}^K, \tau_{t+1}^X, M_t)_{t\geq T}$ in decentralizing the social planner's allocation $\left\{c_t^{y,s}, c_t^{o,s}, k_{t+1}^s, M_t^s, E_t^s\right\}_{t=T}^{+\infty}$, in order of the sequence of land prices $(p_{t+1}^s)_{t=T}^{+\infty}$ to coincide with the sequence of land prices $(p_{t+1}^c)_{t=T}^{+\infty}$ under the scenario of no policy intervention, then the following relationship between τ_{t+1}^K and τ_{t+1}^X must hold for all $t \geq T$, given p_T^c:*

$$1 - \tau_{t+1}^X = \frac{z(E_t^c)F_X^{t+1,c} - [z(E_t^c)F_K^{t+1,c} - (1-\tau_{t+1}^K)z(E_t^s)F_K^{t+1,s}]p_t^c}{z(E_t^s)F_X^{t+1,s}} \tag{51}$$

Proof. See Appendix A5. □

The capital income tax and land rent tax introduced in lemma 1 have two opposite effects which completely offset each other, making $(p_{t+1}^s)_{t=T}^{+\infty}$ coincide with $(p_{t+1}^c)_{t=T}^{+\infty}$.

The following proposition states the strategies of decentralizing the social planner's allocation.

Proposition 8. *The social planner's optimal allocation from any period $T \geq 0$ onward can be obtained as a competitive outcome by following a period-by-period balanced budget tax and transfer policy and mitigation: announcing in any period $t \geq T$ the policy instruments $(T_t^y, T_{t+1}^o, \tau_{t+1}^K, \tau_{t+1}^X, M_t)_{t\geq T}$:*

$$\tau_{t+1}^K = \frac{\xi}{\gamma} \equiv \tau_*^K$$

$$\tau_{t+1}^X = 1 - \frac{z(E_t^c)F_X^{t+1,c} - [z(E_t^c)F_K^{t+1,c} - (1-\tau_*^K)z(E_t^s)F_K^{t+1,s}]p_t}{z(E_t^s)F_X^{t+1,s}}$$

$$T_t^y = c_t^{y,s} + k_{t+1}^s + p_t - z(E_{t-1}^s)F_L^{t,s}$$

$$T_{t+1}^o = c_{t+1}^{o,s} - [1 + (1-\tau_*^K)z(E_t^s)F_K^{t+1,s}](k_{t+1}^s + p_t)$$

$$M_t = M_t^s$$

given $k_T^s = k_T$ and $E_{T-1}^s = E_{T-1}$, will be implemented. Note that in period T, the lump-sum transfer to the old is $T_T^o = M_T^s - T_T^y$, which guarantees the government budget in T to be balanced.

Proof. See Appendix A5. □

Under the policy $(T_t^y, T_{t+1}^o, \tau_{t+1}^K, \tau_{t+1}^X, M_t)_{t\geq T}$ introduced in proposition 8, the environmental externality and the imperfect altruism between generations are fully corrected during the entire transitional phase. As a consequence, the social planner's optimal allocation is implemented.

Note that by construction, under the implementation, the allocation from period $T+1$ onward, $\left\{(c_t^{y,s}, c_t^{o,s}, k_{t+1}^s, M_t^s, E_t^s)\right\}_{t=T+1}^{+\infty}$, is a Pareto optimum. In this implementation, the generation $T-1$ may not be worse off when the stock of pollution E_{T-1} is low and capital accumulation k_T is sufficiently high so that the labor income of an agent T is sufficiently high and the transfer T_T^y is strictly greater than the mitigation M_T^s. In this case the transfer to the agent $T-1$, T_T^o will be non-negative. The consumption when young and savings of generation T decrease but the welfare of this generation does not necessarily decrease. That is because the positive effect of a lower stock of pollution on productivity improves the return on capital and hence increases the consumption when old, possibly offsetting the welfare loss from the decrease in consumption when young. These results depend on the state of the economy, level of capital accumulation and environmental quality, at the time of the implementation and responsiveness of damage function $z(E)$ to the stock of pollution E. If it is the case, and if the economy starts with initially low stock of pollution E_{-1} and sufficiently high capital k_0 then the timing for triggering implementation is important to avoid the conflict between the old generation at the time of triggering implementation and future generations. Delaying implementation increases stock of pollution and may reduce capital. Hence the generation $T-1$ will be worse off as they have to pay $-T_T^o > 0$ from their old-age income in order the economy to obtain pollution mitigation M_T^s.

8 Conclusion remarks

We set up an OLG economy with land as a fixed factor of production and an environmental externality to study the heterogenous long-run effects of alternative tax and mitigation policies on capital accumulation and welfare. Four taxation schemes are examined: land rent, capital income, labor income, and production taxation. The land rent taxation scheme leads to the highest steady state capital accumulation and highest steady state consumption when young for any target of stationary stock of pollution. Given a relatively high efficiency of mitigation technology compared to the dirtiness of production and a sufficiently ambitious target of stationary stock of pollution, land rent taxation also provides the highest steady state consumption when old, and hence the highest steady state welfare of all the schemes. This result implies that land rent taxation could be an efficient instrument for combating climate change. In addition, we describe the period-by-period balanced budget policy to decentralize the socially optimal allocation as a competitive outcome during the entire transitional phase to the social planner's steady state allocation.

The paper could be extended in at least two ways. Firstly, the current paper solely considers the model of homogenous agents, so it would be interesting to extend the model to incorporate the issue of inequality. In other words, agents are heterogenous in land ownership and incomes, and hence the land rent taxation may generate double dividends in combating climate change and reducing inequality. Secondly, we could consider the directed technical change by introducing multiple sectors of intermediate production which include clean and dirty sectors. The aggregate land rent tax may be used along with the tax imposed on dirty sectors in order to subsidize clean innovations. This may allow us to study the conditions and possibilities of avoiding a Green Paradox as mentioned in Sinn (2008). In addition, although the paper provides interesting results in comparing the effects of taxation schemes stated in propositions 5 and 6, it purely focuses on these effects at the steady state. The analyses of these effects on capital accumulation, consumption and welfare during the transition to the steady state would be challenging and promisingly interesting. These ideas are left for further research.

9 Appendix

A0. Existence, uniqueness of the steady state, and convergence to the steady state

With the functional forms, $F^t = k_t^{\alpha_K}$ and $U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$, the competitive equilibrium of the economy is characterized by the following system of nonlinear equations:

$$k_{t+1} - \frac{\beta}{1+\beta}(1-\tau_L)\alpha_L z(E_{t-1})k_t^{\alpha_K} + p_t = 0 \tag{52}$$

$$E_t - (1-\delta)E_{t-1} - (\xi - \gamma\sum_i \alpha^i\tau^i)z(E_{t-1})k_t^{\alpha_K} = 0 \tag{53}$$

$$p_{t+1} - \left[1 + (1-\tau^K)\alpha_K z(E_t)k_{t+1}^{\alpha_K - 1}\right]p_t + (1-\tau^X)z(E_t)\alpha_X k_{t+1}^{\alpha_K} = 0 \tag{54}$$

for given k_t, E_{t-1}, p_t. This equilibrium follows a dynamics represented by a first-order difference equation since the Jacobian matrix of the left-hand side of the system above with respect to k_{t+1}, E_t and p_{t+1} is regular.[15] That is to say,

$$(k_{t+1},\, E_t,\, p_{t+1}) = \phi\,(k_t,\, E_{t-1},\, p_t) \tag{55}$$

where ϕ is an one to one mapping from $\mathbb{R}^3_+$ onto itself. It is obvious that ϕ is invertible and both ϕ and ϕ^{-1} are continuously differentiable. Hence, $\phi : \mathbb{R}^3_+ \to \mathbb{R}^3_+$ is called a C^1 diffeomorphism (see Galor 2011). The steady state is characterized by

$$k_* = \left[\frac{\beta(1-\tau^K)\alpha_K(1-\tau_L)\alpha_L z(E_*)}{(1+\beta)\left[(1-\tau^K)\alpha_K + (1-\tau^X)\alpha_X\right]}\right]^{\frac{1}{1-\alpha_K}}$$

$$\frac{\delta E_*}{z(E_*)} = (\xi - \gamma\sum_i \alpha^i\tau^i)k_*^{\alpha_K}$$

$$p_* = \frac{(1-\tau^X)\alpha_X}{(1-\tau^K)\alpha_K}k_*$$

Let substitute the first equation into the second equation and make a simple transformation, the existence and uniqueness of the steady state is guaranteed by the existence and uniqueness of the solution E_* to following equation

$$z(E)^{\frac{1}{\alpha_K - 1}}E = \frac{\xi - \gamma\sum_i\alpha^i\tau^i}{\delta}\left[\frac{\beta(1-\tau^K)\alpha_K(1-\tau_L)\alpha_L}{(1+\beta)\left[(1-\tau^K)\alpha_K + (1-\tau^X)\alpha_X\right]}\right]^{\frac{\alpha_K}{1-\alpha_K}} \tag{56}$$

It is obvious that, for $\tau^K, \tau^L, \tau^X \in [0,1)$, the right-hand side of (56) is bounded, while, from the assumptions on $z(E)$, the left-hand side is continuous and monotonically increasing in E and

$$\lim_{E\to-\infty}\left(z(E)^{\frac{1}{\alpha_K-1}}E\right) = -\infty \qquad \text{and} \qquad \lim_{E\to+\infty}\left(z(E)^{\frac{1}{\alpha_K-1}}E\right) = +\infty$$

Therefore, there always exists a unique solution E_* to equation (56), which implies that there always exists a unique steady state.

Lets linearize the equation (55) in the vicinity of its steady state. The associated Jacobian matrix evaluated at the steady state is

$$J_* = \begin{pmatrix} \alpha_K + \frac{1-\tau^X}{1-\tau^K}\alpha_X & \frac{\beta\alpha_L(1-\tau^L)}{1+\beta}z'(E_*)k_*^{\alpha_K} & -1 \\ \frac{(\xi-\gamma\sum_i\alpha^i\tau^i)(1+\beta)\left(\alpha_K+\frac{1-\tau^X}{1-\tau^K}\alpha_X\right)}{\beta\alpha^L(1-\tau^L)} & 1-\delta+(\xi-\gamma\sum_i\alpha^i\tau^i)z'(E_*)k_*^{\alpha_K} & 0 \\ \frac{(1+\beta)\alpha_X(1-\tau^X)}{-\beta\alpha_L(1-\tau^L)}\left[1+\frac{(1-\tau^X)\alpha_X}{(1-\tau^K)\alpha_K}\right]^2 & 0 & 1+\frac{(1-\tau^K)\alpha_K+(1-\tau^X)\alpha_X}{\frac{\beta}{1+\beta}(1-\tau^L)\alpha_L} \end{pmatrix}$$

When the center eigenspace of the steady state (k_*, E_*, p_*) is empty —*i.e.* all eigenvalues of

[15] It is fairly straightforward to derive this Jacobian matrix and its determinant is 1.

the Jacobian matrix J_* are of modulus different than 1— then (k_*, E_*, p_*) is call a *hyperbolic fixed point*. Indeed, it is easy to find conditions under which the steady state is hyperbolic.[16] When it is hyperbolic, by applying the *Stable Manifold Theorem*,[17] there exists a *local stable manifold* $W^s_\ell(k_*, E_*, p_*)$. That is to say, for all initial conditions $(k_0, E_{-1}, p_0) \in W^s_\ell(k_*, E_*, p_*)$ then

$$\lim_{n\to+\infty} \phi^{\{n\}}(k_0,\, E_{-1},\, p_0) = (k_*,\, E_*,\, p_*)$$

where $\phi^{\{n\}}(k_0,\, E_{-1},\, p_0)$ is the n^{th} iteration over (k_0, E_{-1}, p_0) under the mapping ϕ.

So there is room for us to study an economy whose initial conditions $(k_0, E_{-1}, p_0) \in W^s_\ell(k_*, E_*, p_*)$ —*i.e.* the economy will converge to its steady state.[18]

A1. Proofs of propositions 1 and 2

Proposition 1: In effect, from (24) and (25) we have

$$z(E_X)^{\frac{1}{\alpha_K-1}} E_X = \frac{\xi - \gamma\tau^X\alpha_X}{\delta}\left[\frac{\beta\alpha_L\alpha_K}{(1+\beta)\left[\alpha_K + (1-\tau^X)\alpha_X\right]}\right]^{\frac{\alpha_K}{1-\alpha_K}} \equiv g(\tau^X) \tag{57}$$

Applying the implicit function theorem for (57) we have

$$\frac{\partial E_X}{\partial \tau^X} = \frac{(1-\alpha_K)z(E_X)^{\frac{2-\alpha_K}{1-\alpha_K}}}{(1-\alpha_K)z(E_X) - z'(E_X)E_X} g'(\tau^X)\,; \quad \text{where} \quad \operatorname{sign}\frac{\partial E_X}{\partial \tau^X} \equiv \operatorname{sign} g'(\tau^X) \tag{58}$$

We have $g'(\tau^X) = \left(\frac{\beta\alpha_L\alpha_K}{(1+\beta)[\alpha_K+(1-\tau^X)\alpha_X]}\right)^{\frac{\alpha_k}{1-\alpha_k}} \frac{\alpha_X}{\delta}\left[\frac{\alpha_K}{1-\alpha_K}\hat{g}(\tau^X) - \gamma\right]$ and for all $\tau^X \in [0,1)$,

$$\operatorname{sign} g'(\tau^X) \equiv \operatorname{sign}\left[\frac{\alpha_K}{1-\alpha_K}\hat{g}(\tau^X) - \gamma\right] \quad \text{where} \quad \hat{g}(\tau^X) = \frac{\xi - \gamma\tau^X\alpha_X}{\alpha_K + (1-\tau^X)\alpha_X} \tag{59}$$

We have $\hat{g}'(\tau^X) = \frac{\xi - \gamma(\alpha_K+\alpha_X)}{[\alpha_K+(1-\tau^X)\alpha_X]^2}\alpha_X > (=)(<)\, 0 \quad \Leftrightarrow \quad \frac{\xi}{\gamma} > (=)(<)\, \alpha_K + \alpha_X$.

(a) If $\frac{\xi}{\gamma} > \alpha_K + \alpha_X$ then $\hat{g}'(\tau^X) > 0$, hence $\forall \tau^X \in [0,1)$, $\hat{g}(\tau^X) < \lim_{\tau^X\to1^-}\hat{g}(\tau^X) = \frac{\xi-\gamma\alpha_X}{\alpha_K}$, therefore

$$\frac{\alpha_K}{1-\alpha_K}\hat{g}(\tau^X) - \gamma < \frac{\alpha_K}{1-\alpha_K}\frac{\xi-\gamma\alpha_X}{\alpha_K} - \gamma = \frac{\xi - \gamma(1-\alpha_K+\alpha_X)}{1-\alpha_K} < 0 \tag{60}$$

because of the assumption $\frac{\xi}{\gamma} < 1 - \alpha_K + \alpha_X$. So, from (58), (59), and (60) we have $\frac{\partial E_X}{\partial \tau^X} < 0$.

(b) If $\frac{\xi}{\gamma} = \alpha_K + \alpha_X$ then $\hat{g}'(\tau^X) = 0$, hence $\hat{g}(\tau^X) = \frac{\gamma(\alpha_K+\alpha_X) - \gamma\tau^X\alpha_X}{\alpha_K+(1-\tau^X)\alpha_X} = \gamma$, therefore

$$\frac{\alpha_K}{1-\alpha_K}\hat{g}(\tau^X) - \gamma = \gamma\left(\frac{\alpha_K}{1-\alpha_K} - 1\right) < 0 \qquad \text{since} \quad \alpha_K < \frac{1}{2} \tag{61}$$

So, from (58), (59), and (61) we have $\frac{\partial E_X}{\partial \tau^X} < 0$.

(c) If $\frac{\xi}{\gamma} < \alpha_K + \alpha_X$ then $\hat{g}'(\tau^X) < 0$, hence $\forall \tau^X \in [0,1]$, $\hat{g}(\tau^X) < \hat{g}(0) = \frac{\xi}{\alpha_K+\alpha_X} < \gamma$, therefore

[16] For instance, we choose the tax policy such that $\xi - \gamma\sum_i \alpha^i\tau^i = 0$, then one of the eigenvalues is $\lambda_1 = 1 - \delta \in (0,1)$ and two other real eigenvalues are $\lambda_{2(3)} = \frac{a+b\pm\sqrt{(a-b)^2+4c}}{2}$, where $a = \alpha_K + \frac{1-\tau^X}{1-\tau^K}\alpha_X$, $b = 1 + \frac{1+\beta}{\beta}\frac{(1-\tau^K)\alpha_K+(1-\tau^X)\alpha_X}{(1-\tau^L)\alpha_L}$, and $c = \frac{(1+\beta)\alpha_X(1-\tau^X)}{\beta\alpha_L(1-\tau^L)}\left[1 + \frac{(1-\tau^X)\alpha_X}{(1-\tau^K)\alpha_K}\right]^2$ which all depend on the parameters of the model. That is to say, there is freedom for us to choose suitable parameters to guarantee $\lambda_{2(3)} \neq \pm 1$.

[17] For the Stable Manifold Theorem, see Galor (2011).

[18] The *global stable manifold* $W^s(k_*, E_*, p_*)$ can be obtained by the union of all backward iteration under the mapping ϕ over the local stable manifold, i.e. $W^s(k_*, E_*, p_*) = \cup_{n\in\mathbb{N}}\{\phi^{-\{n\}}(W^s_\ell(k_*, E_*, p_*))\}$.

$$\frac{\alpha_K}{1-\alpha_K}\hat{g}(\tau^X) - \gamma < \gamma\left(\frac{\alpha_K}{1-\alpha_K} - 1\right) < 0 \qquad \text{since} \quad \alpha_K < \frac{1}{2} \tag{62}$$

So, from (58), (59), and (62) we have $\frac{\partial E_X}{\partial \tau^X} < 0$.

We know that $k_X = \left[\frac{\beta\alpha_L\alpha_K z(E_X)}{(1+\beta)[\alpha_K+(1-\tau^X)\alpha_X]}\right]^{\frac{1}{1-\alpha_K}}$, where as in (57), E_X can be represented as a function of τ^X only, then $\forall \tau^X \in [0,1)$, it holds

$$\frac{\partial k_X}{\partial \tau^X} = \left[\frac{\beta\alpha_L\alpha_K z(E_X)}{(1+\beta)\left[\alpha_K+(1-\tau^X)\alpha_X\right]}\right]^{\frac{1}{1-\alpha_K}} \frac{\frac{z'(E_X)}{z(E_X)}\frac{\partial E_X}{\partial \tau^X}\left[\alpha_K+(1-\tau^X)\alpha_X\right] + \alpha_X}{(1-\alpha_K)\left[\alpha_K+(1-\tau^X)\alpha_X\right]} > 0 \tag{63}$$

Proposition 2: From (26) and (27) we have

$$z(E_K)^{\frac{1}{\alpha_K-1}} E_K = \frac{\xi - \gamma\tau^K\alpha_K}{\delta}\left(\frac{\beta\alpha_L\alpha_K(1-\tau^K)}{(1+\beta)[\alpha_X+\alpha_K(1-\tau^K)]}\right)^{\frac{\alpha_K}{1-\alpha_K}} \equiv \tilde{g}(\tau^K) \tag{64}$$

Applying the implicit function theorem for (64) with respect to E_K and τ^K, we have (note that it is straightforward to derive $-\infty < \tilde{g}'(\tau^K) < 0$),

$$\frac{\partial E_K}{\partial \tau^K} = \frac{(1-\alpha_K)z(E_K)^{\frac{2-\alpha_K}{1-\alpha_K}}\tilde{g}'(\tau^K)}{(1-\alpha_K)z(E_K) - z'(E_K)E_K} < 0 \tag{65}$$

Lets examine the impact of capital income taxation on capital accumulation. From (26)

$$k_K = \left[\frac{\beta\alpha_L\alpha_K(1-\tau^K)}{(1+\beta)[\alpha_X+(1-\tau^K)\alpha_K]}z(E_K)\right]^{\frac{1}{1-\alpha_K}} \tag{66}$$

$$\frac{\partial k_K}{\partial \tau^K} = \left[\frac{\beta\alpha_L\alpha_K(1-\tau^K)z(E_K)}{(1+\beta)[\alpha_X+(1-\tau^K)\alpha_K]}\right]^{\frac{1}{1-\alpha_K}} \frac{\left[\frac{z'(E_K)}{z(E_K)}\frac{\partial E_K}{\partial \tau^K} - \frac{1}{1-\tau^K}\right]\left[\alpha_X+(1-\tau^K)\alpha_K\right] + \alpha_K}{(1-\alpha_K)\left[\alpha_X+(1-\tau^K)\alpha_K\right]} \tag{67}$$

From (67) we find that in contrast to the effect of land rent taxation on capital accumulation (which is stated in proposition 1), the effect of capital income taxation is ambiguous and the direction of this effect seems to depend on the level of the tax rate on capital income τ^k. Since $-\infty < \tilde{g}'(\tau^K) < 0$ then $\partial E_K/\partial \tau^K$ as determined in (65) is always bounded. Hence, the numerator $\left[\frac{z'(E_K)}{z(E_K)}\frac{\partial E_K}{\partial \tau^K} - \frac{1}{1-\tau^K}\right]\left[\alpha_X+(1-\tau^K)\alpha_K\right] + \alpha_K$, which determines the sign of $\partial k_K/\partial \tau^K$, on the right hand side of (67) may be positive when τ^K is sufficiently low (depending on the responsiveness of $z(E_K)$ to the change in E_K), and it approaches $-\infty$ when $\tau^K \to 1^-$.

A2. Proof of proposition 5:

(i) We compare (24) to (26) by equalizing the stock of pollution in both equations. So, we have

$$\frac{k_K}{k_X} = \left[\frac{\alpha_K+\alpha_X(1-\tau^X)}{\alpha_K+\alpha_X(1-\tau^K)^{-1}}\right]^{\frac{1}{1-\alpha_K}} \tag{68}$$

Since $\tau^X, \tau^K \in (0,1)$ then it is obvious that $k_K/k_X < 1$, i.e. $k_K < k_X$. Similarly, we derive

$$\frac{k_P}{k_X} = \left[\frac{(1-\tau^P)\left[\alpha_K + (1-\tau^X)\alpha_X\right]}{\alpha_K+\alpha_X}\right]^{\frac{1}{1-\alpha_K}} \tag{69}$$

Since $\tau^X, \tau^P \in (0,1)$ then it is obvious that $k_P/k_X < 1$, i.e. $k_P < k_X$.

(ii) We have

$$\frac{k_P}{k_L} = \left(\frac{1-\tau^P}{1-\tau^L}\right)^{\frac{1}{1-\alpha_K}} \tag{70}$$

Suppose that $k_P \leq k_L$ then, from the last equation, $\tau^P \geq \tau^L$. Since $E_L = E_P$ then it holds

$$(\xi - \gamma\tau^L\alpha_L)k_L^{\alpha_K} = (\xi - \gamma\tau^P\alpha_P)k_P^{\alpha_K} = \frac{\delta E}{z(E)}$$

But for $k_P \leq k_L$ and $\tau^P \geq \tau^L$ we obtain $(\xi - \gamma\tau^L\alpha_L)k_L^{\alpha_K} > (\xi - \gamma\tau^P\alpha_P)k_P^{\alpha_K}$, which contradicts the last equality. Therefore, $k_P > k_L$.

(iii) We have

$$\frac{k_L}{k_K} = \left[\frac{1-\tau^L}{1-\tau^K}\frac{\alpha_X + (1-\tau^K)\alpha_K}{\alpha_X+\alpha_K}\right]^{\frac{1}{1-\alpha_K}} \tag{71}$$

Hence, $k_L > (=)(<)\, k_K$ is equivalent to

$$\frac{1-\tau^L}{1-\tau^K}\frac{\alpha_X + (1-\tau^K)\alpha_K}{\alpha_X+\alpha_K} > (=)(<)\, 1 \quad \Leftrightarrow \quad \tau^L\left[1 + (1-\tau^K)\frac{\alpha_K}{\alpha_X}\right] < (=)(>)\, \tau^K$$

(a) First, we show that if $1-\tau^K = \frac{\alpha_X(\alpha_L-\alpha_K)}{\alpha_K^2}$ then $k_K = k_L$. Suppose a contradiction that $k_K \neq k_L$. Without lost of generality, we assume that $k_K < k_L$. Hence, $\tau^L > \frac{\alpha_K}{\alpha_L}\tau^K$ since $(\xi - \gamma\tau^L\alpha_L)k_L^{\alpha_K} = (\xi - \gamma\tau^K\alpha_K)k_K^{\alpha_K}$. That is to say

$$\tau^L > \frac{\alpha_K}{\alpha_L}\left[1 - \frac{\alpha_X(\alpha_L-\alpha_K)}{\alpha_K^2}\right] \tag{72}$$

So from (71), (72), and given that $1-\tau^K = \frac{\alpha_X(\alpha_L-\alpha_K)}{\alpha_K^2}$, we have

$$\left(\frac{k_L}{k_K}\right)^{1-\alpha_K} = (1-\tau^L)\frac{\alpha_X(1-\tau^K)^{-1}+\alpha_K}{\alpha_X+\alpha_K} < \left(1 - \frac{\alpha_K}{\alpha_L}\left[1-\frac{\alpha_X(\alpha_L-\alpha_K)}{\alpha_K^2}\right]\right)\frac{\frac{\alpha_K^2}{\alpha_L-\alpha_K}+\alpha_K}{\alpha_X+\alpha_K} = 1$$

i.e. $k_L < k_K$, which contradicts the initial assumption that $k_L > k_K$. An analogous logic will applied in the case in which we assume initially that $k_K > k_L$. Therefore, it holds $k_L = k_K$.

(b) Second, we show that if $1-\tau^K > \frac{\alpha_X(\alpha_L-\alpha_K)}{\alpha_K^2}$ then $k_K > k_L$. Indeed, suppose that $k_K \leq k_L$ then $\tau^L \geq \frac{\alpha_K}{\alpha_L}\tau^K$, and from (71), it would hold

$$\psi(\tau^K) = \frac{1-\frac{\alpha_K}{\alpha_L}\tau^K}{1-\tau^K}\frac{\alpha_X+(1-\tau^K)\alpha_K}{\alpha_X+\alpha_K} \geq \frac{1-\tau^L}{1-\tau^K}\frac{\alpha_X+(1-\tau^K)\alpha_K}{\alpha_X+\alpha_K} \geq 1$$

We will show a contradiction that $\psi(\tau^K) < 1$. Indeed,

$$\psi'(\tau^K) = -\frac{\alpha_K}{\alpha_L}\frac{\alpha_X(1-\tau^K)^{-1}+\alpha_K}{\alpha_X+\alpha_K} + \left(1-\frac{\alpha_K}{\alpha_L}\tau^K\right)\frac{\alpha_X}{(\alpha_X+\alpha_K)(1-\tau^K)^2}$$

and, by simple transformation, sign $\psi'(\tau^K) \equiv$ sign $\left[\frac{\alpha_X(\alpha_L-\alpha_K)}{\alpha_K^2} - (1-\tau^K)^2\right]$. So for all $\tau^K \in (0,1)$,

$$\psi'(\tau^K) > (=)(<)\, 0 \quad \Longleftrightarrow \quad 1-\tau^K < (=)(>)\, \frac{\sqrt{\alpha_X(\alpha_L-\alpha_K)}}{\alpha_K}$$

Hence the function $\psi(\tau^K)$ get unique minimum at $\tau^K = 1 - \frac{\sqrt{\alpha_X(\alpha_L-\alpha_K)}}{\alpha_K} \in (0, 1-\frac{\alpha_X(\alpha_L-\alpha_K)}{\alpha_K^2})$. In addition, we have $\psi(0) = \psi\left(1-\frac{\alpha_X(\alpha_L-\alpha_K)}{\alpha_K^2}\right) = 1$. Therefore, for all $\tau^K \in (0, 1-\frac{\alpha_X(\alpha_L-\alpha_K)}{\alpha_K^2})$, i.e. $1-\tau^K > \frac{\alpha_X(\alpha_L-\alpha_K)}{\alpha_K^2}$, we have $\psi(\tau^K) < 1$ which contradicts the result $\psi(\tau^K) \geq 1$. Hence, if $1-\tau^K > \frac{\alpha_X(\alpha_L-\alpha_K)}{\alpha_K^2}$ then $k_K > k_L$.

Finally, similar to the previous proof, we have, if $1-\tau^K < \frac{\alpha_X(\alpha_L-\alpha_K)}{\alpha_K^2}$ then $k_K < k_L$.

(iv) When both capital taxation and production taxation schemes leads to some given steady state stock of pollution, it holds

$$\frac{\delta E}{z(E)} = (\xi - \gamma\tau^P)k_P^{\alpha_K} = (\xi - \gamma\alpha_K\tau^K)k_K^{\alpha_K} \tag{73}$$

On the other hand we have from (68) and (69) that

$$\left(\frac{k_K}{k_P}\right)^{1-\alpha_K} = \frac{(1-\tau^K)(\alpha_K+\alpha_X)}{(1-\tau^P)[\alpha_K(1-\tau^K)+\alpha_X]} \tag{74}$$

Suppose that $k_K \geq k_P$ then from (73) we have

$$\tau^P \leq \alpha_K\tau^K \tag{75}$$

From (74) we have $\frac{(1-\tau^K)(\alpha_K+\alpha_X)}{(1-\tau^P)[\alpha_K(1-\tau^K)+\alpha_X]} \geq 1$. In fact, from (75) we have

$$\frac{(1-\tau^K)(\alpha_K+\alpha_X)}{(1-\tau^P)[\alpha_K(1-\tau^K)+\alpha_X]} \leq \frac{(1-\tau^K)(\alpha_K+\alpha_X)}{(1-\alpha_K\tau^K)[\alpha_K(1-\tau^K)+\alpha_X]}$$

We will complete the proof with a contradiction through proving that

$$\frac{(1-\tau^K)(\alpha_K+\alpha_X)}{(1-\alpha_K\tau^K)[\alpha_K(1-\tau^K)+\alpha_X]} < 1$$

Indeed, the last inequality is equivalent to

$$(1-\tau^K)(\alpha_K+\alpha_X) < (1-\alpha_K\tau^K)[\alpha_K(1-\tau^K)+\alpha_X] \quad \Longleftrightarrow \quad 1-\tau^K < \frac{\alpha_X(1-\alpha_K)}{\alpha_K^2}$$

which always holds because $\tau^K \in (0,1)$ and $\alpha_X \geq \frac{\alpha_K^2}{1-\alpha_K}$. Therefore, we have $k_K < k_P$.

A3. Proof of propositions 6 and 7

Proposition 6: (i) It is fairly straightforward from logarithm utility function that $c_X^y > \max\{c_P^y, c_K^y, c_L^y\}$ since, from proposition 5, $k_X > \max\{k_K, k_P, k_L\}$.

(ii) Under land rent and capital income taxation, the stationary land prices, respectively, are $p_X = \frac{(1-\tau^X)F_X^X}{F_K^X}$ and $p_K = \frac{F_X^K}{(1-\tau^K)F_K^K}$. The consumption when old under these schemes are

$$c_X^o = \left[1 + z(E)F_K^X\right]\left[k_X + \frac{(1-\tau^X)F_X^X}{F_K^X}\right] \text{ and } c_K^o = \left[1 + (1-\tau^K)z(E)F_K^K\right]\left[k_K + \frac{F_X^K}{(1-\tau^K)F_K^K}\right]$$

With the logarithm utility function, the saving rate is constant and the savings under these schemes are $\frac{\beta}{1+\beta}z(E)F_L^x$ and $\frac{\beta}{1+\beta}z(E)F_L^k$. We have

$$k_X + \frac{(1-\tau^X)F_X^X}{F_K^X} = \frac{\beta}{1+\beta}z(E)F_L^X > \frac{\beta}{1+\beta}z(E)F_L^K = k_K + \frac{F_X^K}{(1-\tau^K)F_K^K}$$

We complete the proof by showing that

$$F_K^X k_X + (1-\tau^X)F_X^X \geq (1-\tau^K)F_K^K k_K + F_X^K \tag{76}$$

$$\Leftrightarrow \quad \left(\frac{k_X}{k_K}\right)^{\alpha_K} \geq \frac{(1-\tau^K)\alpha_K + \alpha_X}{\alpha_K + (1-\tau^X)\alpha_X} = \frac{\alpha_K + \alpha_X - \tau^K\alpha_K}{\alpha_K + \alpha_X - \tau^X\alpha_X}$$

For a given target of steady state stock of pollution $E \in [0, \tilde{E})$, we have

$$\frac{\delta E}{z(E)} = (\xi - \gamma\tau^X\alpha_X)k_X^{\alpha_K} = (\xi - \gamma\tau^K\alpha_K)k_K^{\alpha_K} \quad \Longrightarrow \quad \left(\frac{k_X}{k_K}\right)^{\alpha_K} = \frac{\xi - \gamma\tau^K\alpha_K}{\xi - \gamma\tau^X\alpha_X} = \frac{\xi/\gamma - \tau^K\alpha_K}{\xi/\gamma - \tau^X\alpha_X}$$

Since $k_X > k_K$ then $\xi/\gamma - \tau^K\alpha_K > \xi/\gamma - \tau^X\alpha_X > 0$. Hence, it is straightforward that, when $\xi/\gamma \leq \alpha_X + \alpha_K$ then $\frac{\xi/\gamma - \tau^K\alpha_K}{\xi/\gamma - \tau^X\alpha_X} \geq \frac{\alpha_K + \alpha_X - \tau^K\alpha_K}{\alpha_K + \alpha_X - \tau^X\alpha_X}$, *i.e.* (76) holds. Therefore, we have $c_X^o > c_K^o$.

(iii) We have

$$c_K^y = \frac{z(E)\alpha_L k_K^{\alpha_K}}{1+\beta} \quad \text{and} \quad c_P^y = \frac{(1-\tau^P)z(E)\alpha_L k_P^{\alpha_K}}{1+\beta}, \text{ then} \quad c_P^y \geq c_K^y \quad \Leftrightarrow \quad 1 - \tau^P \geq \left(\frac{k_K}{k_P}\right)^{\alpha_K}$$

By substituting k_K/k_P determined in (74) into the last inequality we have

$$1 - \tau^P \geq \left[\frac{(1-\tau^K)(\alpha_K + \alpha_X)}{(1-\tau^P)[\alpha_K(1-\tau^K) + \alpha_X]}\right]^{\frac{\alpha_K}{1-\alpha_K}} \quad \Leftrightarrow \quad \tau^P \leq 1 - \left[\frac{\alpha_K + \alpha_X}{\alpha_K + \alpha_X(1-\tau^K)^{-1}}\right]^{\alpha_K},$$

which holds by construction.

Similarly, we have

$$c_K^o = \frac{\beta\alpha_L z(E)k_K^{\alpha_K}\left[1 + (1-\tau^K)\alpha_K z(E)k_K^{\alpha_K - 1}\right]}{1+\beta}$$

$$c_P^o = \frac{\beta(1-\tau^P)\alpha_L z(E)k_P^{\alpha_K}\left[1 + (1-\tau^P)\alpha_K z(E)k_P^{\alpha_K - 1}\right]}{1+\beta}$$

In order to prove $c_P^o > c_K^o$, we prove first that

$$(1-\tau^P)k_P^{\alpha_K - 1} > (1-\tau^K)k_K^{\alpha_K - 1} \quad \Leftrightarrow \quad \left(\frac{k_K}{k_P}\right)^{1-\alpha_K} > \frac{1-\tau^K}{1-\tau^P}$$

Substitute k_K/k_P determined in (74) into the last inequality we have $\frac{\alpha_K + \alpha_X}{\alpha_K(1-\tau^K) + \alpha_X} > 1$, which trivially holds because $\tau^K \in (0, 1]$.

We also have

$$\frac{c_P^o}{c_K^o} = \frac{(1-\tau^P)k_P^{\alpha_K}}{k_K^{\alpha_K}} \times \frac{1+(1-\tau^P)\alpha_K z(E)k_P^{\alpha_K-1}}{1+(1-\tau^K)\alpha_K z(E)k_K^{\alpha_K-1}}$$

where the first fraction $(1-\tau^P)k_P^{\alpha_K}/k_K^{\alpha_K} = c_P^y/c_K^y \geq 1$ as proved above. We complete by proving that the second fraction in the last equation is strictly greater than 1. That is straightforward because from the proof above we have $(1-\tau^P)k_P^{\alpha_K-1} > (1-\tau^K)k_K^{\alpha_K-1}$. Therefore, $c_P^o > c_K^o$.

(iv) We have

$$\frac{c_X^o}{c_P^o} = \frac{k_X^{\alpha_K}}{(1-\tau^P)k_P^{\alpha_K}} \times \frac{1+\alpha_K z(E)k_X^{\alpha_K-1}}{1+(1-\tau^P)\alpha_K z(E)k_P^{\alpha_K-1}}$$

The second fraction in the right hand side of the last equation satisfies

$$\frac{1+\alpha_K z(E)k_X^{\alpha_K-1}}{1+(1-\tau^P)\alpha_K z(E)k_P^{\alpha_K-1}} > \frac{k_X^{\alpha_K-1}}{(1-\tau^P)k_P^{\alpha_K-1}}$$

because, from (69) we have

$$\frac{\alpha_K z(E)k_X^{\alpha_K-1}}{(1-\tau^P)\alpha_K z(E)k_P^{\alpha_K-1}} = \frac{k_X^{\alpha_K-1}}{(1-\tau^P)k_P^{\alpha_K-1}} = \frac{\alpha_K+(1-\tau^X)\alpha_X}{\alpha_K+\alpha_X} < 1$$

Hence we have

$$\frac{c_X^o}{c_P^o} > \frac{k_X^{\alpha_K}}{(1-\tau^P)k_P^{\alpha_K}} \times \frac{k_X^{\alpha_K-1}}{(1-\tau^P)k_P^{\alpha_K-1}} = \left(\frac{1}{1-\tau^P}\right)^{\frac{1}{1-\alpha_K}} \left[\frac{\alpha_K+(1-\tau^X)\alpha_X}{\alpha_K+\alpha_X}\right]^{\frac{1-2\alpha_K}{1-\alpha_K}} \tag{77}$$

By the condition $\tau^P \geq 1 - \left[\frac{\alpha_K+(1-\tau^X)\alpha_X}{\alpha_K+\alpha_X}\right]^{1-2\alpha_K}$, we have $\frac{1}{1-\tau^p}\left[\frac{\alpha_K+(1-\tau^X)\alpha_X}{\alpha_K+\alpha_X}\right]^{1-2\alpha_K} \geq 1$. Therefore, $c_X^o/c_P^o > 1$, i.e. $c_X^o > c_P^o$.

(v) The proof for this statement is quite similar to the proof in statement (iv) above.

Proposition 7: We have, under condition

$$\frac{\xi}{\gamma} \leq (\alpha_K+\alpha_X)\left[1-\left(\frac{1-2\alpha_K}{1-\alpha_L}\right)^{\frac{1}{2\alpha_K}}\right] \tag{78}$$

that $c_X^o > c_K^o$ holds for any target of steady state stock of pollution $E \in [0,\tilde{E})$. This property is straightforward from the statement (ii) in proposition 6. We next prove that there exists ambitious targets on steady state stock of pollution E under which $c_X^o > \max\{c_P^o, c_L^o\}$. We now consider the first case $c_X^o > c_P^o$. From statement (iii) in proposition 6, we have $c_X^o > c_P^o$ if it holds

$$\tau^P \geq 1 - \left[\frac{\alpha_K+(1-\tau^X)\alpha_X}{\alpha_K+\alpha_X}\right]^{1-2\alpha_K}$$

We consider at the extreme target, say the government set $E=0$. At this target, the production tax rate $\tau^P = \frac{\xi}{\gamma}$, land rent tax rate $\tau^X = \frac{\xi}{\gamma\alpha_X}$, and the condition above boils down to

$$\frac{\xi}{\gamma} \geq 1 - \left[1-\frac{1}{\alpha_K+\alpha_X}\frac{\xi}{\gamma}\right]^{1-2\alpha_K} \iff 1-\frac{\xi}{\gamma} - \left[1-\frac{1}{\alpha_K+\alpha_X}\frac{\xi}{\gamma}\right]^{1-2\alpha_K} \leq 0$$

Lets consider following function $\triangle(y) = 1 - y - \left[1 - \frac{1}{\alpha_K+\alpha_X} y\right]^{1-2\alpha_K}$ for $y \in [0, \frac{\xi}{\gamma}]$. We have

$$\triangle(0) = 0 \qquad \text{and} \qquad \triangle'(y) = -1 + \frac{1-2\alpha_K}{\alpha_K+\alpha_X}\left[1 - \frac{1}{\alpha_K+\alpha_X} y\right]^{-2\alpha_K}$$

Note that $1 - \alpha_L = \alpha_K + \alpha_X$, hence,

$$\triangle'(y) < 0 \quad \Longleftrightarrow \quad y \leq (\alpha_K + \alpha_X)\left[1 - \left(\frac{1-2\alpha_K}{1-\alpha_L}\right)^{\frac{1}{2\alpha_K}}\right]$$

This implies that, under (78), $\triangle'(y) < 0$ for all $y \in (0, \frac{\xi}{\gamma}]$, and hence $\triangle(y) < 0$ for all $y \in (0, \frac{\xi}{\gamma}]$ since $\triangle(0) = 0$. That is to say, under (78) and at the extreme target $E = 0$, it holds $c_X^o > c_P^o$.

We know from propositions 1 and 4 that $E = \mathcal{E}_X(\tau^X) = \mathcal{E}_P(\tau^P)$, where $\mathcal{E}'_X(\tau^X) < 0$, $\mathcal{E}'_P(\tau^P) < 0$. In addition, we have from (77) that

$$\frac{c_X^o}{c_P^o} > \left(\frac{1}{1-\tau^P}\right)^{\frac{1}{1-\alpha_K}} \left[\frac{\alpha_K + (1-\tau^X)\alpha_X}{\alpha_K+\alpha_X}\right]^{\frac{1-2\alpha_K}{1-\alpha_K}}$$

The right hand side of the last inequality is a continuous function of the steady state stock of pollution $E \in [0, \tilde{E}]$ and can be rewritten by

$$\Upsilon(E) = \left[\frac{1}{1-\mathcal{E}_P^{-1}(E)}\right]^{\frac{1}{1-\alpha_K}} \left[\frac{\alpha_K + (1-\mathcal{E}_X^{-1}(E))\alpha_X}{\alpha_K+\alpha_X}\right]^{\frac{1-2\alpha_K}{1-\alpha_K}} \qquad \text{where} \quad \Upsilon(0) > 1.$$

Hence, there always exists an $\varepsilon > 0$ such that $\Upsilon(E) > 1$, *i.e.* $c_X^o > c_P^o$, for all $E \in [0, \varepsilon]$.

Similarly, we can prove the existence of ambitious targets on steady state stock of pollution $E \in [0, \varepsilon']$ such that $c_X^o > c_L^o$.

A4. Social planner's optimal allocation

The Lagrangian is

$$\mathcal{L} = \sum_{t=0}^{+\infty} \frac{u(c_t^y) + (1+R)v(c_t^o)}{(1+R)^t} + \sum_{t=0}^{+\infty} \frac{\mu_t\left[z(E_{t-1})F^t + k_t - c_t^y - c_t^o - k_{t+1} - M_t\right]}{(1+R)^t}$$
$$+ \sum_{t=0}^{+\infty} \frac{\eta_t\left[E_t - (1-\delta)E_{t-1} - \xi z(E_{t-1})F^t + \gamma M_t\right]}{(1+R)^t}$$

The FOCs are

$$\frac{\partial \mathcal{L}}{\partial c_t^y} = \frac{u'(c_t^y)}{(1+R)^t} - \frac{\mu_t}{(1+R)^t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_t^o} = \frac{v'(c_t^o)}{(1+R)^{t-1}} - \frac{\mu_t}{(1+R)^t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = -\frac{\mu_t}{(1+R)^t} + \frac{\mu_{t+1}}{(1+R)^{t+1}}\left[1 + z(E_t)F_K^{t+1}\right] - \frac{\eta_{t+1}}{(1+R)^{t+1}}\xi z(E_t)F_K^{t+1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial M_t} = -\frac{\mu_t}{(1+R)^t} + \frac{\eta_t \gamma}{(1+R)^t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial E_t} = \frac{\mu_{t+1} - \eta_{t+1}\xi}{(1+R)^{t+1}} z'(E_t)F^{t+1} + \frac{\eta_t}{(1+R)^t} - \frac{\eta_{t+1}(1-\delta)}{(1+R)^{t+1}} = 0$$

$$z(E_{t-1})F^t + k_t - c_t^y - c_t^o - k_{t+1} - M_t = 0$$

$$E_t - (1-\delta)E_{t-1} - \xi z(E_{t-1})F^t + \gamma M_t = 0$$

At the steady state we have

$$u'(c^y) = \mu = \eta\gamma = (1+R)v'(c^o)$$

$$z(E)F_K = \frac{\gamma R}{\gamma - \xi}$$

$$z'(E)F = \frac{R+\delta}{\xi - \gamma}$$

$$c^y + c^o + M = z(E)F$$

$$\delta E = \xi z(E)F - \gamma M$$

A5. Proofs of Lemma 1 and Proposition 8

Lemma 1: When $t = T$, without any policy intervention the no-arbitrage condition is

$$[1 + z(E_T^c)F_K^{T+1,c}]p_T^c = z(E_T^c)F_X^{T+1,c} + p_{T+1}^c \tag{79}$$

With policy intervention, the corresponding no-arbitrage condition becomes

$$[1 + (1-\tau_{T+1}^K)z(E_T^s)F_K^{T+1,s}]p_T^s = (1-\tau_{T+1}^X)z(E_T^s)F_X^{T+1,s} + p_{T+1}^s \tag{80}$$

Since $p_T^s = p_T^c$ then for $p_{T+1}^s = p_{T+1}^c$, it must hold from (79) and (80) that

$$1 - \tau_{T+1}^X = \frac{z(E_T^c)F_X^{T+1,c} - [z(E_T^c)F_K^{T+1,c} - (1-\tau_{T+1}^K)z(E_T^s)F_K^{T+1,s}]p_T^c}{z(E_T^s)F_X^{T+1,s}}$$

By the induction method, we argue that the condition (51) holds for all $t \geq T$.

Proposition 8: Indeed, in period T, by choosing $M_T = M_T^s$, we have $E_T = E_T^s$. Under the policy $(T_T^y, T_{T+1}^o, \tau_*^K, \tau_{T+1}^X, M_T^s)$, the Euler equation is

$$\frac{u'(z(E_{T-1})F_L^T + T_t^y - k_{T+1} - p_T)}{v'([1 + \frac{\gamma-\xi}{\gamma} z(E_T^s)F_K^{T+1}](k_{T+1} + p_T) + T_{T+1}^o)} = 1 + \frac{\gamma - \xi}{\gamma} z(E_T^s)F_K^{T+1} \tag{81}$$

Note that, by construction, in period T we have $F_L^{T,s} = F_L^T$ and $E_{T-1}^s = E_{T-1}$. It is obvious that if $k_{T+1} = k_{T+1}^s$ then by construction, $c_T^y = c_T^{y,s}$ and $c_{T+1}^o = c_{T+1}^{o,s}$ and hence the Euler equation (81) obviously holds because it exactly coincides with the Euler equation (36) under the allocation of the social planner. We now prove that, under the policy $(T_T^y, T_{T+1}^o, \tau_*^K, \tau_{T+1}^X, M_T^s)$, the optimal choice of agent's savings in terms of capital $k_{T+1} = k_{T+1}^s$ is unique. In effect, suppose that there existed

$\hat{k}_{T+1} \neq k^s_{T+1}$ to be also optimal choice of capital saving. Without loss of generality, suppose that $\hat{k}_{T+1} > k^s_{T+1}$, hence the corresponding consumption $\hat{c}^y_T$ and $\hat{c}^o_{T+1}$ will be

$$\hat{c}^y_T = z(E^s_{T-1})F^{T,s}_L + T^y_T - \hat{k}_{T+1} - p_T < c^{y,s}_T$$

$$\hat{c}^o_{T+1} = \left[1 + (1-\tau^K_*)z(E^s_{T-1})\hat{F}^{T+1}_K\right]\hat{k}_{T+1} + (1-\tau^X_{T+1})z(E^s_{T-1})\hat{F}^{T+1}_X + p_{T+1} > \hat{c}^{o,s}_{T+1}$$

Therefore,

$$\frac{u'(\hat{c}^y_T)}{v'(\hat{c}^o_{T+1})} > \frac{u'(c^y_T)}{v'(c^o_{T+1})} = 1 + \frac{\gamma-\xi}{\gamma}z(E^s_T)F^{T+1,s}_K > 1 + \frac{\gamma-\xi}{\gamma}z(E^s_T)\hat{F}^{T+1}_K$$

i.e. the Euler equation is no longer satisfied, contradicting $(\hat{c}^y_T, \hat{k}_{T+1}, \hat{c}^o_{T+1})$ is the optimal choice of the agent T. So $(c^{y,s}_T, k^s_{T+1}, c^{o,s}_s)$ is the unique optimal choice of a agent under the policy $(T^y_T, T^o_{T+1}, \tau^K_*, \tau^X_{T+1}, M^s_T)$.

We next prove that, from period $T+1$ onward, the policy described in proposition 8 is period-and-period budget balanced and will implement the social planner's allocation. Without loss of generality, we consider the budget constraint and allocation in period $T+1$. Under the policy $(T^y_T, T^o_{T+1}, \tau^k_*, \tau^x_{T+1}, M^s_T)$ applied for generation T, we have $k_{T+1} = k^s_{T+1}$ and $E_T = E^s_T$ are given for the generation $T+1$. Therefore, the balanced government budget constraint under the policy $(T^y_{T+1}, T^o_{T+2}, \tau^K_*, \tau^X_{T+2}, M^s_{T+1})$, applied for the generation $T+1$, requires that it holds

$$T^y_{T+1} + T^o_{t+1} + M^s_{T+1} = \tau^K_* z(E^s_T)F^{T+1,s}_K k^s_{T+1} + \tau^X_{T+1} z(E^s_T)F^{T+1,s}_X$$

Using the no arbitrage condition, the last equation is equivalent to

$$c^{y,s}_{T+1} + k^s_{T+2} + p_{T+1} - z(E^s_T)F^{T+1,s}_L + c^{o,s}_{T+1} - [1 + (1-\tau^K_*)z(E^s_T)F^{T+1,s}_K](k^s_{t+1} + p_t) + M^s_{T+1}$$

$$= \tau^K_* z(E^s_T)F^{T+1,s}_K k^s_{T+1} + z(E^s_T)F^{T+1,s}_X - [1 + (1-\tau^K_*)z(E^s_T)F^{T+1,s}_K]p_T + p_{T+1}$$

which boils down to

$$c^{y,s}_{T+1} + k^s_{T+2} + c^{o,s}_{T+1} + M^s_{T+1} = z(E^s_T)F^{T+1} + k^s_{T+1}$$

i.e., the feasibility constraint of the social planner.

Similarly to the proof about optimal choice of generation T above, we also prove that the optimal choice of generation $T+1$ under the policy $(T^y_{T+1}, T^o_{T+2}, \tau^K_*, \tau^X_{T+2}, M^s_{T+1})$ is $(c^y_{T+1}, k_{T+2}, c^o_{T+2})$. This argument can be induced for all $t \geq T+1$.

A6. Discussions about relaxing assumptions

In this section, we discuss about relaxing two basic assumptions of the model: (i) the perfect competition in final good sector and (ii) the constant returns to scale of final good sector production function over its factors of production.

(**i**) For the first assumption, we can relax it by considering the case of monopoly competition in the final good sector without changing the main quantitative results of the model. Indeed, we are working on an aggregate private-owned economy with representative households. All in all, in such an economy, the returns on factors of production and monopoly profits belong to households. That is to say, the capital accumulation and its effect on environment through production does not depend on the type of competition in the final good sector.

(**ii**) For the second assumption, we relax in the direction of increasing returns to scale. A sizable literature in the long run economic growth theory adopts this property of production which was pioneered by Romer (1986). We argue that adopting increasing returns to scale property for production function will not change the main qualitative results of our paper, particularly we focus on the long run effects of different taxation schemes on the stationary stock of pollution and on capital accumulation under a given target of stationary stock of pollution. We follow Romer (1986) in modifying our production function, guaranteeing its increasing returns to scale property, as follows

$$\bar{F}(E_{t-1}, K_t, L, X) = \bar{z}(E_{t-1}, K_t)F(K_t, L, X)$$

where K_t has a positive effect on the total factor productivity $\bar{z}(E_{t-1}, K_t)$ —*i.e.* $\bar{z}_K(E, K) > 0$ —while, as in the benchmark model, $\bar{z}_E(E, K) < 0$; $F(K_t, L, X)$ exhibits constant returns to scale over the factors of production. Under this setup with the logarithmic utility function and $F(K_t, L, X) = K_t^{\alpha_K} L^{\alpha_L} X^{\alpha_X}$ as in the benchmark model, the steady state of the economy (k_*, E_*, p_*) is characterized by the following system of equations

$$k_* - \frac{\beta}{1+\beta}(1-\tau^L)\alpha_L \bar{z}(E_*, k_*)k_*^{\alpha_K} + p_* = 0$$

$$\delta E_* - (\xi - \gamma \sum_i \alpha^i \tau^i)\bar{z}(E_*, k_*)k_*^{\alpha_K} = 0$$

$$p_* = \frac{(1-\tau^X)\alpha_X}{(1-\tau^K)\alpha_K} k_*$$

For a purpose of exposition, we assign $\bar{z}(E, k) = z(E)k^\theta$ where $z(E)$ is defined as in the benchmark model and $\theta > 0$. We will show that the main results derived from this setup are similar to those in the benchmark model. Indeed, under this setup, the steady state is determined through solving the following equation, which is modified from equation (56) by the factor θ,

$$z(E)^{\frac{1}{\alpha_K+\theta-1}} E = \frac{\xi - \gamma \sum_i \alpha^i \tau^i}{\delta} \left[\frac{\beta(1-\tau^K)\alpha_K(1-\tau_L)\alpha_L}{(1+\beta)\left[(1-\tau^K)\alpha_K + (1-\tau^X)\alpha_X\right]} \right]^{\frac{\alpha_K+\theta}{1-\alpha_K-\theta}}$$

Similar to the benchmark model, it is straightforward to show the existence and uniqueness of the steady state since the right-hand side of the last equation is bounded while the left-hand side is continuous and monotonically increasing in E, spreading from $-\infty$ to $+\infty$.

Under the condition $\theta + \alpha_K < 1/2$,[19] we can show that under the land rent taxation scheme only, it holds $\partial E_X/\partial \tau^X < 0$ and $\partial k_X/\partial \tau^X > 0$. The proofs for these statements are similar to those which are done in Appendix A1 for proposition 1. Other properties stated in propositions 2, 3, and 4 hold under this setup and the proofs for these properties follow the proof which is done for proposition 2 (see Appendix A1). This extended model also allows us to compare the long run effects of different taxation schemes on stationary stock of pollution and on capital accumulation as in the benchmark model. Interestingly, the quantitative results obtained in this extended model are consistent with those in the benchmark model. Indeed, for a given target of stationary stock of pollution, we can derive capital ratios k_K/k_X, k_P/k_X, k_P/k_L, and k_L/k_K as follows

$$\frac{k_K}{k_X} = \left(\frac{\alpha_K + \alpha_X(1-\tau^X)}{\alpha_K + \alpha_X(1-\tau^K)^{-1}} \right)^{\frac{1}{1-\alpha_K-\theta}}$$

$$\frac{k_P}{k_X} = \left(\frac{(1-\tau^P)\left[\alpha_K + (1-\tau^X)\alpha_X\right]}{\alpha_K + \alpha_X} \right)^{\frac{1}{1-\alpha_K-\theta}}$$

[19] This is just a modified condition from the condition $\alpha_K < 1/2$ in the benchmark model (see proposition 1). This condition implies that the marginal effect of capital accumulation on total factor productivity decreases rather quickly.

$$\frac{k_P}{k_L} = \left(\frac{1-\tau^P}{1-\tau^L}\right)^{\frac{1}{1-\alpha_K-\theta}}$$

$$\frac{k_L}{k_K} = \left(\frac{1-\tau^L}{1-\tau^K}\frac{\alpha_X+(1-\tau^K)\alpha_K}{\alpha_X+\alpha_K}\right)^{\frac{1}{1-\alpha_K-\theta}}$$

Interestingly, these equations are respectively similar to equations (68), (69), (70), and (71) except the exponential factors which are independent in comparing these ratios with 1 as long as $\alpha_K+\theta<1$. Therefore, the results stated in proposition 5 hold in this extended model.

These results allow us to compare consumption and hence welfare under these taxation schemes. Similar to the benchmark model, for a given target of stationary stock of pollution $E \in (0, \tilde{E})$, the level of consumption when young is highest under land rent taxation scheme because this scheme leads to the highest capital accumulation level, and hence highest disposable labor income, while the saving rate under logarithmic preference is constant. The consumption when old depends on saving rate and returns to saving. In the benchmark model, we characterize conditions under which the consumption when old under land rent taxation scheme is higher than that under other schemes. With the positive externality of capital accumulation on total factor productivity in this extended model, this result would be enhanced because the land rent taxation scheme leads to the higher capital accumulation than other schemes. Thanks to the increasing returns to scale property of final good production function, the higher capital accumulation under the land rent taxation scheme is transmitted into higher consumption when old.

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