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NON-PARAMETRIC EFFICIENCY, PROGRESS AND REGRESS MEASURES FOR PANEL DATA: METHODOLOGICAL ASPECTS

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Abstract

This purely methodological paper deals with the role of time in non-parametric efficiency analysis. Using both FDH and DEA technologies, it first shows how each observation in a panel can be characterized in efficiency terms vis-a-vis three different kinds of frontiers: (i) "contemporaneous", (ii) "sequential" and (iii) "intertemporal". These are then compared with window analysis.

Next, frontier shifts "outward" and "inward", interpreted as progress or regress are considered for the two kinds of technologies, and computational methods are described in detail for evaluating such shifts in either case. These are also contrasted with what is measured by the "Malmquist" productivity index.

Finally, an alternative way of identifying progress and regress, independent of the frontier notion and referring instead to some "benchmark" notion, is extended here to panel data.

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1. Introduction: Production Sets and the Efficiency Concept

The concept of efficiency in production has received a precise meaning in economics when Koopmans and Debreu introduced in 1951 the *production set* notion in the theory of production. A production set, in their terminology[2], is a collection Y of pairs of vectors (x, u) – called hereafter production plans – where $x = (x_1, \dots, x_i, \dots, x_n)$ is a vector of quantities of I inputs and $u = (u_1, \dots, u_i, \dots, u_n)$ a vector of quantities of J outputs, that have the property of being *feasible* ones. By "feasible" it is meant that the quantities are such that the output bundle u can physically be produced by making use of the input bundle x. In formal terms,

$$Y = \{ (x, u) \mid x \in R^{I}_{+}, u \in R^{J}_{+} \ ; (x, u) \text{ is feasible} \} .$$

The usefulness of the set notion for our purposes comes from the fact that it brings along two further notions, namely *(i)* that of the *boundary* (or frontier) of the set, and *(ii)* that of the *interior* of the set. This indeed permits one to distinguish between production plans that belong to the interior of the production set, which are called inefficient, and those that do belong to (a particular part of) the frontier, and are called efficient.

Inefficiency so defined yields itself to measurement once the property of belonging to the interior is translated in terms of a distance – that is, a real number – between the production plan under consideration and the boundary of the set. In this setting, efficiency of the plan amounts to zero distance, and inefficiency to a strictly positive distance.

2. Constructing Alternative Reference Production Sets From Observed Data: The Postulates Behind FDH, DEA and Other Methodologies

In most empirical analyses of efficiency, the analyst must construct a representation of the production set from the data he has available on input and output quantities, because the set itself, being an abstract concept, is not

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2 In BANKER, CHARNES and COOPER 1984, and PETERSEN 1990 the term used is that of "production possibilities set". FÄRE, GROSSKOPF and LOVELL 1985 and GROSSKOPF 1986 prefer to call a production set a "technology". In this paper, we take all these expressions as synonyms.

observable. He then uses the set's frontier to measure the efficiency of the activities reported by the data. GROSSKOPF 1986 aptly calls "reference" set[3] the production set so constructed from empirical data. In so doing, any production plan that is included in the production set *and is not observed* necessarily results from some postulate, explicit or implicit, as to its feasibility. Hence, depending upon which postulates are retained, a different reference set is constructed.

Formally, let

$$Y_o = \{(x^k, u^k) \mid x^k \in R^I_+, u^k \in R^J_+, k = 1, 2, \ldots, n\} \cup \{(0^I, 0^J)\} \quad (1)$$

denote a set of n actually observed production plans, to which the origin of the input-output space is added by convention[4] (0^I and 0^J are the I- and J-dimensional null vectors); for brevity we call Y_o the observations set or the data set. Let also $Y(Y_o)$ denote a reference production set constructed from Y_o. If the following two postulates[5] are used:

***Postulate (i)**: Every observed production plan belongs to the constructed production set*, and

***Postulate (ii)**: Every non observed production plan, that is weakly dominated in inputs and/or in outputs by some observed production plan also belongs to the constructed production set,*

then a "free disposal hull (FDH) reference production set", Y_{FDH}, constructed from Y_o can be written as follows

$$Y_{FDH}(Y_o) = \left\{ \begin{bmatrix} u \\ x \end{bmatrix} \in R^{I+J}_+ \,\middle|\, \begin{bmatrix} u \\ x \end{bmatrix} = \begin{bmatrix} u^h \\ x^h \end{bmatrix} + \sum_{i=1}^{I} \mu_i \begin{bmatrix} 0^J \\ e^I_i \end{bmatrix} - \sum_{j=1}^{J} v_j \begin{bmatrix} e^J_j \\ 0^I \end{bmatrix}, \right.$$
$$\left. (x^h, u^h) \in Y_o, \quad \mu_i \geq 0, v_j \geq 0, i = 1, \ldots I, j = 1, \ldots, J \right\} \quad (2)$$

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3 in her words, "reference technology".

4 The reason for including the origin in Y_0 will appear below, in particular when we shall discuss the DEA case.

5 We expand here on a formulation originally used in DEPRINS, SIMAR and TULKENS 1984, and further developed in TULKENS 1993.

where e_i^I denotes an I-dimensional zero vector with the ith component equal to 1 and similarly e_j^J denotes a J-dimensional zero vector with the jth component equal to 1.

The first postulate makes the ensuing methodology of efficiency measurement[6] a deterministic one. The methodology would indeed be stochastic if this belonging property were formulated in probabilistic terms. Postulate *(ii)* is the one of free disposal of inputs and of outputs, from which the name of the constructed reference set is derived[7]. We express it here in terms of "dominance", a term that for our present purposes may be defined as follows:

Definition: *In (2), observation (x^h, u^h) weakly dominates (x, u) in inputs if $\exists\, i\!: \mu_i > 0$, and in outputs if $\exists\, j\!: v_j > 0$.*

The values taken by the variables μ_i (denoting disposal of the inputs) and v_j (disposal of the outputs) thus express the free disposal assumption. Notice that an equivalent way to write (2), useful for what follows, is:

$$Y_{FDH}(Y_o) = \left\{ \begin{bmatrix} u \\ x \end{bmatrix} \in R_+^{I+J} \;\middle|\; \begin{bmatrix} u \\ x \end{bmatrix} = \sum_{h=1}^{n} \gamma^h \begin{bmatrix} u^h \\ x^h \end{bmatrix} + \sum_{i=1}^{I} \mu_i \begin{bmatrix} 0^J \\ e_i^I \end{bmatrix} - \sum_{j=1}^{J} v_j \begin{bmatrix} e_j^J \\ 0^I \end{bmatrix}, \right.$$
$$(x^h, u^h) \in Y_o,\; \gamma^h \in \{0, 1\}, h = 1,\dots,n,\; \sum_{h=1}^{n} \gamma^h = 1,$$
$$\left. \mu_i \geq 0, i = 1,\dots I,\; v_j \geq 0, j = 1,\dots,J \right\}. \tag{3}$$

In the DEA literature, efficiency is measured by referring to production sets similarly constructed from the data, and using a third postulate in addition to the two mentioned above. This third postulate differs, however, depending upon which DEA model is used: the one of BANKER,

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[6] The actual computation of efficiency is dealt with in section 4.

[7] This terminology was introduced in efficiency analysis by THIRY and TULKENS 1992 (p. 46). In private communication with one of the authors, Professor Cooper expresses a preference for "efficiency domination", which had already used in an unpublished note (BOWLIN, BRENNAN, CHARNES, COOPER and SUEYOSHI 1984). Notice that free disposal is defined, here as in DEBREU 1959 (p. 42) as a property of the production set, rather than of market conditions — which we find irrelevant — as in the discussion by BANKER, CHARNES and COOPER 1984 (p. 1082).

CHARNES and COOPER 1984, the one of DEPRINS and SIMAR 1983 or the original one of FARRELL 1957 and CHARNES, COOPER and RHODES 1978. Let us consider each of these in turn, in that chronologically reverse order.

Postulate (iii-1): *Every non observed production plan that is a convex combination of production plans induced by postulate (i), excluding the origin, and by postulate (ii), also belongs to the constructed production set.*

Using the above notation, postulates *(i)*, *(ii)* and *(iii-1)* induce a reference production set that reads:

$$Y_{DEA-ID}(Y_o) = \left\{ \begin{bmatrix} u \\ x \end{bmatrix} \in R_+^{l+J} \;\middle|\; \begin{bmatrix} u \\ x \end{bmatrix} = \sum_{h=1}^{n} \gamma^h \begin{bmatrix} u^h \\ x^h \end{bmatrix} + \sum_{i=1}^{I} \mu_i \begin{bmatrix} 0^J \\ e_i^I \end{bmatrix} - \sum_{j=1}^{J} \nu_j \begin{bmatrix} e_j^J \\ 0^I \end{bmatrix}, \right.$$
$$(x^h, u^h) \in Y_o \setminus \{(0^I, 0^J)\}, \quad \gamma^h \geq 0, h = 1, \ldots, n, \quad \sum_{h=1}^{n} \gamma^h = 1,$$
$$\left. \mu_i \geq 0, i = 1, \ldots I, \; \nu_j \geq 0, j = 1, \ldots, J \right\}, \tag{4}$$

to be called the "*convex* free disposal hull" of the data. Because the origin is excluded from the data set, the constructed set does not contain the origin either. The piecewise linear frontier of this set is often interpreted as exhibiting "variable" returns to scale (VRS). As BANKER, CHARNES and COOPER 1984 clearly show, this is only true however in the special sense that returns are increasing (I) only in the lower range of the inputs up to some point, and are decreasing (D) beyond. As there are other forms conceivable of variable returns[8], we think it more accurate to use "ID" rather than "VRS"[9] in the subscripts used to identify the set constructed here.

Postulate (iii-2): *Every non observed production plan that is a convex combination of production plans induced by postulates (i) and (ii) also belongs to the constructed production set.*

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[8] Typically, the FDH sets just considered may exhibit *any* form of returns to scale: returns are are thus in some sense "more" variable than in the ID case. This is analogous to the situation with parametric frontiers, where the flexibility of translog functions allows for more variability in the returns than ID type of production functions.

[9] e.g. by FØRSUND 1992, or LOVELL 1993 (pp. 29-30).

Wih this postulate *(iii-2)*, postulates *(i)* and *(ii)* induce a reference production set that reads:

$$Y_{DEA-CD}(Y_o) = \left\{ \begin{bmatrix} u \\ x \end{bmatrix} \in R_+^{I+J} \;\middle|\; \begin{bmatrix} u \\ x \end{bmatrix} = \sum_{h=1}^{n} \gamma^h \begin{bmatrix} u^h \\ x^h \end{bmatrix} + \sum_{i=1}^{I} \mu_i \begin{bmatrix} 0^J \\ e_i^I \end{bmatrix} - \sum_{j=1}^{J} \nu_j \begin{bmatrix} e_j^J \\ 0^I \end{bmatrix}, \right.$$
$$(x^h, u^h) \in Y_o,\quad \gamma^h \geq 0,\ h=1,\ldots,n,\ \sum_{h=1}^{n} \gamma^h = 1,$$
$$\left. \mu_i \geq 0,\ i=1,\ldots I,\ \nu_j \geq 0,\ j=1,\ldots,J \right\}. \tag{5}$$

Here too, the constructed set is the convex free disposal hull of the data, but the fact that it contains the origin implies that its (piecewise linear) frontier can only exhibit constant returns (in the lower range of the inputs, up to some point) and then decreasing returns to scale (which is what DEPRINS and SIMAR 1983 (p. 136) aimed at introducing). Hence our "CD" specification in the identifying subscripts.

Postulate (iii-3): *Every non observed production plan that is a linear combination of production plans induced by postulates (i) and (ii) also belongs to the constructed production set.*

In our notation, the reference production set $Y_{DEA\text{-}C}$ constructed from Y_o with postulates *(i), (ii) and (iii-3)* reads as follows:

$$Y_{DEA-C}(Y_o) = \left\{ \begin{bmatrix} u \\ x \end{bmatrix} \in R_+^{I+J} \;\middle|\; \begin{bmatrix} u \\ x \end{bmatrix} = \sum_{h=1}^{n} \gamma^h \begin{bmatrix} u^h \\ x^h \end{bmatrix} + \sum_{i=1}^{I} \mu_i \begin{bmatrix} 0^J \\ e_i^I \end{bmatrix} - \sum_{j=1}^{J} \nu_j \begin{bmatrix} e_j^J \\ 0^I \end{bmatrix}, \right.$$
$$(x^h, u^h) \in Y_o,\quad \gamma^h \geq 0, h=1,\ldots,n,$$
$$\left. \mu_i \geq 0, i=1,\ldots I,\ \nu_j \geq 0, j=1,\ldots,J \right\}. \tag{6}$$

Because of the proportionality allowed by the weights of the linear combinations, this set is a cone, to be called the "free disposal *cone*" constructed from the data. As well noted by both FARRELL 1957 and CHARNES, COOPER and RHODES 1978, the linearity of its frontier implies that it exhibits constant returns to scale, hence our subscript "C".

It can easily be seen[10] that for a given data set Y_o,

$$Y_{FDH} \subset Y_{DEA-ID} \subset Y_{DEA-CD} \subset Y_{DEA-C}. \tag{7}$$

This means that the postulates *(i)* through *(iii-3)* have been stated in order of increasing strength, this term being understood as referring to the importance of the parts of each reference set that are not made of observed points. Formally, the relative strength of the postulates is expressed by the alternative restrictions put on the weights γ_h whereby the observations are combined, on the sum of these weights, and on the convention to include or not the origin in the observations set.

The convexity assumption, expressed in or implied by the postulates *(iii)* that induce some form of DEA, deserves a special further comment. The above exposition of the FDH methodology clearly shows that it can be dispensed with, without loosing the economically essential element of efficiency analysis, namely a well defined reference production set. Hence, the question of whether this assumption should be retained in this area becomes an essentially empirical one.

The non necessary character of convexity in efficiency analysis has found a further confirmation in the recent contribution by PETERSEN 1990, who introduces two other kinds of non convex piecewise linear production sets, different from FDH. It can be shown that these reference production sets also derive from stronger postulates than *(i)* and *(ii)*, among which some form of proportionality. They lie somehow in between the FDH and the DEA-C reference sets.

3. Constructing Reference Production Sets From Alternative Time-Related Observations Subsets

3.1 *Panels*

The n observed production plans referred to in the above definition of the set Y_o are most naturally thought of as a cross-section, *i.e.* plans

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[10] A diagrammatic presentation of these inclusions is given in TULKENS 1993, pp. 185-186.

achieved by different firms[11], each one indexed by k and all observed at the same point in time. Alternatively, the n production plans may be a time series of successive such plans realized by a single firm, the index k then standing for a time index. When these two situations are combined, Y_0 is a panel. Two indexes are then better associated with each observation: k to denote the firm, and t to denote the point in time when the observation is made. An observation now reads (x^{kt}, u^{kt}) and the data set is described as

$$Y_o^{KT} = \{(x^{kt}, u^{kt}) \mid x^{kt} \in R_+^l,\ u^{kt} \in R_+^l,\ k = 1,2,\ldots,n;\ t = 1,2,\ldots,m\} \cup \{(0^l, 0^l)\}, \quad (8)$$

where the superscripts K and T refer to the sets of firms and of observation times, respectively, and with the same convention as in (1) as to the origin.

The presence of the time dimension in the data introduces a new variety of ways of evaluating the efficiency of observed productive behavior. Indeed, besides the necessity of choosing one among the several alternative reference production sets that we have seen in the previous section, there may arise reasons to construct it not from the full data set Y_o^{KT}, but instead from only alternative subsets of it, that we shall call *"reference observations subsets"*. In TULKENS 1986 (section 6) the following possibilities were described[12] in this respect:

(a) A first one is to construct a reference production set at each point in time t, from the observations made *at that time only*. In this case, a different reference observations subset, denoted as

$$Y_o^{Kt} = \{(x^{kt}, u^{kt}) \mid k = 1,2,\ldots,n\} \cup \{(0^l, 0^l)\} \quad (9)$$

is used at each point in time $t = 1,2,\ldots,m$. Over the whole observation period, a sequence of m reference production sets is constructed, one for each time t. We call *"contemporaneous"* each of these production sets and denote them as

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[11] or more generally "decision-making units", as nicely formulated by CHARNES, COOPER and RHODES 1978.

[12] With an empirical application to monthly data, extending over 12 months, from the Belgian Post Office, with p = FDH.

$$Y_p(Y_o^{Kt}), \quad t = 1,2,\ldots,m,$$

where the subscript p stands for the alternative sets of postulates that imply FDH, DEA-ID, -CD or -C reference production sets.

Contrary to those that will be defined next, successive production sets so constructed, even if they are of the same type p, are essentially unrelated to one another; that is, they may or may not overlap in any possible way.

(b) A second possibility is to construct a reference production set at each point in time t also, using the observations made *from the point in time s=1 up until s=t*. The reference observations subsets at each $t = 1,2,\ldots,m$ being here denoted as

$$Y_o^{K(1,t)} = \{(x^{ks},u^{ks}) \mid k = 1,2,\ldots,n; s = 1,2,\ldots,t\} \cup \{(0^l,0^j)\} \qquad (10)$$

m successive reference production sets are thereby constructed, different of course from those under *(a)* above, that we call "*sequential* " and denote as

$$Y_p(Y_o^{K(1,t)}), \quad t = 1,\ldots,m.$$

Here too, the subscript p stands for FDH, DEA-ID, -CD or -C, according to which postulates of section 2 are retained for their construction. However, whatever the shape thus obtained for these sets, since

$$Y_o^{K(1,t)} = \bigcup_{s=1}^{t} Y_o^{Ks},$$

successive sequential reference production sets have the property of being nested into one another, that is:

$$Y_p(Y_o^{K(1,t+1)}) \supseteq Y_p(Y_o^{K(1,t)}), \text{ for every } t = 1,2,\ldots,m-1. \qquad (11)$$

(c) A third possibility is to construct a single production set from the observations made throughout the whole observation period. The reference observations subset is here simply Y_o^{KT}, as defined at the

beginning of this subsection. We call the resulting reference production set *"intertemporal"* and denote it as

$$Y_p(Y_o^{KT}).$$

With p = FDH, DEA-ID, -CD or -C, its shape depends once more upon the postulates of section 2 that are retained. Notice that

$$Y_p(Y_o^{KT}) \equiv Y_p(Y_o^{K(1,m)}) \quad \forall p,$$

that is, intertemporal production sets are also the sequential production sets associated with the observation(s) made at the last point in time in the data set.

3.2 Time Series

Further reference observations subsets can be specified by treating each unit k separately, thereby singling out k-specific time series. This yields:

(a) "k-specific sequential production sets", denoted as $Y_p(Y_o^{k(1,t)})$, $t = 1, ..., m$, and derived at each t from the observations set consisting of the time series of the activities of the sole unit k from $s = 1$ up to $s = t$; and

(b) "k-specific intertemporal production sets", denoted as $Y_p(Y_o^{kT})$. There is in this case a single time series of m observations of firm k. Notice that the indices k and T are here reversed with respect to K and t in the definition of Y_o^{Kt} given at the beginning of this section.

Efficiency analysis of time series observations sets has been dealt with by FÄRE, GRABOWSKI and GROSSKOPF 1985 in terms of sequential production sets with p = DEA-ID, and with an application to yearly data (one output, four inputs) extending over twenty years on Philippine agriculture. Both the sequential and the intertemporal approaches were used in THIRY and TULKENS 1992, and pursued in TULKENS & VANDEN EECKAUT 1991 in the same spirit as that of the present paper, with applications to monthly data of an urban transit firm in Belgium extending over several years. We shall therefore limit ourselves in what follows to the panel situations covered by cases *(a)-(c)* above.

3.3 Window Analysis

Finally, a connection should be made between the above and what is called "window analysis", as initiated by CHARNES, CLARK, COOPER and GOLANY 1985. Window analysis is in fact a special case of *(b)*. Indeed, a (time) window of size $s \in \{1,2,\ldots,r; r < m\}$ at time t is defined as a subset of adjacent points in time $T^{st} = \{\tau \mid \tau = t, t+1, \ldots, t+s; t \le m-s\}$ whose observations are used to construct an intertemporal reference production set, valid for the time period [t, $t+s$] only. Successive such windows, defined for $t=1, 2,\ldots, m-s$, yield a sequence of reference production sets, that we might call "locally intertemporal".

These sets are not nested however, as it is the case with sequential analysis in *(b)* above. In contrast with this last methodology, whose spirit is to assume that what was feasible in the past remains feasible for ever, the treatment of time in window analysis is more in the nature of an averaging over the periods of time covered by the window. For the size of the window, as well as for the fact that part of the past is ignored, it seems to be hard to find more than an *ad hoc* justification.

To summarize this section, for measuring the efficiency of observations (x^{kt}, u^{kt}) in a panel, there is a double variety of ways to proceed: first, according to the postulated production set used as reference — FDH, DEA-ID, -CD or -C, and second according to the selected observations subset used to construct the production set — contemporaneous, sequential, intertemporal, or "window". This makes for 16 possibilities! While the best choice to be made between them surely varies with the empirical application to be dealt with, their main interest lies perhaps in the fact that they also pave the way for the progress and regress analyses that are to follow in sections 5 and 6.

4. Mathematical Programming Formulations of Efficiency Measurement From Contemporaneous, Sequential or Intertemporal Frontiers.

We now move to the techniques of numerical measurement of efficiency when the data are in panel form. The main purpose is to show that measurement can be made in all cases by means of mathematical programming techniques that are by now well established for computing efficiency from statistical data on inputs and outputs. This provides a unifying framework for the various approaches defined in the preceding two sections.

We use a presentation similar to the one in TULKENS 1993, which is in turn close to those of PETERSEN 1990, GROSSKOPF 1986 and many others. To the usual distinction between input and output orientations we find it useful to add the graph orientation proposed in FÄRE, GROSSKOPF and LOVELL 1985 (chap. 5)[13], and we call efficiency *"degrees"* the resulting measures. These will be contrasted below (see section 6) with what we shall call there *"indexes"*.

4.1 Alternative Reference Observations Subsets

To allow for a flexible use of the various reference observations subsets we have defined, we wish to define a further symbol, $\overline{Y}_o$, to denote for any set Y_0 the set of indexes whereby its observed elements are identified (thus, ignoring the origin). In the case of double indexes as with panel data sets, $\overline{Y}_o$ refers to index pairs; for instance, for $Y_o = Y_o^{KT}$ as defined in (7), $\overline{Y}_o = \left\{(kt) \middle| (x^{kt}, u^{kt}) \in Y_o^{KT}\right\}$.

For any observation (x^{kt}, u^{kt}), its input, output and graph efficiency degrees, relative to an FDH reference production set and to some reference observations subset Y_0, are determined from the values θ^{kt*}, λ^{kt*} and δ^{kt*}, respectively, of the objective functions at the optimal solutions of the following mixed integer programs:

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[13] The input and output measures are radial ones whereas the graph orientation measure is a hyperbolic one .

Problem 1.1 (input orientation)

$$\min_{\{\theta^{kt},\,\gamma^{hs},(hs)\in\overline{Y}_o\}} \theta^{kt}\ \text{, subject to}$$

$$\sum_{(hs)\in\overline{Y}_o} \gamma^{hs} u_j^{hs} \geq u_j^{kt} \qquad j=1,\ldots,J$$

$$\theta^{kt} x_i^{kt} - \sum_{(hs)\in\overline{Y}_o} \gamma^{hs} x_i^{hs} \geq 0, \qquad i=1,\ldots,I$$

$$\sum_{(hs)\in\overline{Y}_o} \gamma^{hs} = 1 \ \text{ and } \ \gamma^{hs} \in \{0,1\}, \quad (hs)\in\overline{Y}_o.$$

Problem 1.2 (output orientation)

$$\max_{\{\lambda^{kt},\,\gamma^{hs},(hs)\in\overline{Y}_o\}} \lambda^{kt}\ \text{, subject to}$$

$$\lambda^{kt} u_j^{kt} - \sum_{(hs)\in\overline{Y}_o} \gamma^{hs} u_j^{hs} \leq 0 \qquad j=1,\ldots,J$$

$$\sum_{(hs)\in\overline{Y}_o} \gamma^{hs} x_i^{hs} \leq x_i^{kt}, \quad i=1,\ldots,I$$

$$\sum_{(hs)\in\overline{Y}_o} \gamma^{hs} = 1 \ \text{ and } \ \gamma^{hs} \in \{0,1\}, \quad (hs)\in\overline{Y}_o.$$

Problem 1.3 (graph orientation)

$$\min_{\{\delta^{kt},\,\gamma^{hs},(hs)\in\overline{Y}_o\}} \delta^{kt}\ \text{, subject to}$$

$$\frac{u_i^{kt}}{\delta^{kt}} - \sum_{(hs)\in\overline{Y}_o} \gamma^{hs} u_j^{hs} \leq 0 \qquad j=1,\ldots,J$$

$$\delta^{kt} x_i^{kt} - \sum_{(hs)\in\overline{Y}} \gamma^{hs} x_i^{hs} \geq 0, \qquad i=1,\ldots,I$$

$$\sum_{(hs)\in\overline{Y}_o} \gamma^{hs} = 1 \ \text{ and } \ \gamma^{hs} \in \{0,1\}, \quad (hs)\in\overline{Y}_o.$$

where

(a) $\overline{Y}_o = \{(hs) | (x^{hs}, u^{hs}) \in Y_o^{Kt}\}$

or

(b) $\overline{Y}_o = \{(hs) | (x^{hs}, u^{hs}) \in Y_o^{K(1,t)}\}$

or

(c) $\overline{Y}_o = \{(hs) | (x^{hs}, u^{hs}) \in Y_o^{KT}\}$.

In these problems we have $\theta^{kt*} \leq 1$, $\lambda^{kt*} \geq 1$ and $\delta^{kt*} \leq 1$, respectively because in problem 1.1, $\theta^{kt} = \gamma^{kt} = 1$, $\gamma^{hs} = 0\ \forall (hs) \neq (kt)$ is always a feasible solution; similarly in problem 1.2, $\lambda^{kt} = \gamma^{kt} = 1,\ \gamma^{hs} = 0\ \forall (hs) \neq (kt)$ is also a feasible solution; and in problem 1.3, $\delta^{kt} = \gamma^{kt} = 1,\ \gamma^{hs} = 0\ \forall (hs) \neq (kt)$ is a feasible solution. Therefore,

— if *(a)* applies, the numbers $E_x^C(kt;FDH) = \theta^{kt*}$, $E_u^C(kt;FDH) = [\lambda^{kt*}]^{-1}$ and $E_{(x,u)}^C(kt;FDH) = \delta^{kt*}$, all three ≤ 1, measure the *contemporaneous* FDH-efficiency degrees, respectively in input, in output and in the graph measure, of observation (x^{kt}, u^{kt}) ;

— if *(b)* applies, the three numbers $E_x^S(kt;FDH) = \theta^{kt*}$, $E_u^S(kt;FDH) = [\lambda^{kt*}]^{-1}$ and $E_{(x,u)}^S(kt;FDH) = \delta^{kt*}$, all ≤ 1, measure the *sequential* FDH-efficiency degrees, respectively in input, in output and in the graph measure, of this observation; and

— if *(c)* applies, the numbers $E_x^I(kt;FDH) = \theta^{kt*}$, $E_u^I(kt;FDH) = [\lambda^{kt*}]^{-1}$ and $E_{(x,u)}^I(kt;FDH) = \delta^{kt*}$, again all ≤ 1, measure the *intertemporal* FDH-efficiency degrees, in input, in output and in the graph measure, of the same observation.

To cover all the observations in the panel, $n \times m$ problems, *i.e.* one for each pair *(kt)*, are to be solved in cases *(a)* and *(b)*, in each one of the input, the output and the graph orientations; in case *(c)*, only n problems are involved in each orientation, *i.e.* one for each unit *k*.

4.2 *Alternative Reference Production Sets*

Problems 1.1 to 1.3 are all formulated for gauging the efficiency of an observation relative to an FDH reference production set as defined in (3). However, if in either one of these problems, the last line is replaced by :

$$\sum_{(hs)\in \overline{Y}_o} \gamma^{hs} = 1, \quad \text{and} \quad \gamma^{hs} \geq 0,\ (hs) \in \overline{Y}_o \qquad (12)$$

the efficiency degrees in input, output and graph are those relative to DEA-ID production sets as described by (4). We denote them as $E_x^r(kt;DEA-ID)$, $E_u^r(kt;DEA-ID)$ and $E_{(x,u)}^r(kt;DEA-ID)$, respectively, with the superscript $r = C, S,$ or I according to whether (a), (b) or (c) applies.

If instead of line (12) the following is used:

$$\sum_{(hs)\in\overline{Y}_o} \gamma^{hs} \le 1, \quad \text{and} \quad \gamma^{hs} \ge 0, \ (hs) \in \overline{Y}_0 \tag{13}$$

the efficiency degrees obtained are those relative to the DEA-CD reference production sets defined in (5), that we denote as $E^r_x(kt;DEA-CD)$, $E^r_u(kt;DEA-CD)$ and $E^r_{(x,u)}(kt;DEA-CD)$. Notice that the fact that the origin was included in (5) and excluded from (4) is reflected here in the weak inequality *vs.* the equality with respect to 1 that is imposed in (13) and (12), respectively, on the sum of the weights γ^{hs} whereby the observation (*kt*) is expressed as a convex combination of other observations.

Finally, replacing (13) by

$$\gamma^{hs} \ge 0, \ (hs) \in \overline{Y}_0 \tag{14}$$

implies assuming that the reference production set is of the DEA-C type as defined in (6), *i.e.* the smallest Farrell cone containing the data. The efficiency degrees are then naturally denoted by $E^r_x(kt;DEA-C)$, $E^r_u(kt;DEA-C)$ and $E^r_{(x,u)}(kt;DEA-C)$.

Of course, the choice of the alternative observations sets referred to with (*a*), (*b*) and (*c*) is also offered when anyone of these three alternative forms of DEA is used.

For computing the solutions of these mathematical programs there exist today appropriate algorithms and computer codes. In this respect, it should be mentioned that in spite of the integrality constraint that makes problems 1.1-1.3 mixed integer ones, they are in fact much simpler to solve than the linear programs obtained when any of the constraints (12), (13) or (14) are introduced. Indeed, an algorithm of complete enumeration, described in TULKENS 1993 (pp. 189) and consisting essentially in pairwise dominance comparisons of all input-output vectors in the data set, does the job[14].

We refer to this simple algorithm because it points to an important difference in the logic that lies behind FDH *vs.* DEA methodologies. The rôle of the integrality constraint is indeed essentially to identify a *dominance*

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[14] A computer code for large panel data sets, allowing for comparisons of FDH results with those obtained with each one of the DEA variants, and operating on Macintosh is currently being prepared by the second author.

relation between observed production plans. On the one hand, an observation is declared "efficient" — and considered as lying on the boundary of the reference production set — if it is *undominated*: undomination, belonging to the frontier and efficiency are thus synonyms in FDH methodology. On the other hand, an observation is declared "inefficient", *i.e.* it lies in the interior of the set, if it is *dominated* by one or several other ones; and in this case the mixed integer program identifies a "most dominating" observation that serves as a reference to compute the efficiency degree, either in input or in output.

By contrast, the linear programs used in DEA seek to compute a distance (in input, in output or graph) with respect to the frontier of a convex *envelope* of the data. While dominance plays also some rôle in identifying this envelope, the additional requirement of convexity induces the possibility that undominated observations be declared inefficient because they do not lie on the convex envelope of the data.

5. Measuring Progress And Regress From Frontiers Shifts

5.1 *Progress And Regress With Nonparametric Frontiers*

Over the time period covered by a panel, the question naturally arises whether the boundary of the reference production set can shift. The analyst's answer to this question is contained in the reference observations subset he decides to select from within the panel data he has available: *(i)* at one extreme, if he uses the full data set to construct a single intertemporal production set, he makes the assumption of no shift at all; *(ii)* at the other extreme, if he uses at each t the contemporaneous observations only, to construct contemporaneous production sets, he is answering the question in the affirmative, by allowing the reference production sets at each point in time to be completely different from one another, without there being a priori any relation between them; and *(iii)* with the construction of sequential production sets, which are also different from one another over time, there is postulated some form of dependence between these sets, formalized by the property that they are nested. The dependence consists in assuming that "what was possible in the past remains always possible in the future". This amounts to allowing only for *outward* shifts of the frontier over time, that is, enlargements of the constructed production possibilities set.

In economics, beginning with SOLOW 1957, the notion of "progress" — more specifically called "technical progress" — has for a long time been associated with outward shifts of production frontiers. Symmetrically, inwards shifts refer to "regress". While this literature deals essentially with parametric production functions, there is no reason why these notions of progress and regress could not be extended to shifts of non parametric representations of the boundary of production sets. It is indeed not only a natural complement to non parametric efficiency measures, but also an inescapable one when the data involve time. Our purpose in this section is therefore to provide suitable concepts and methods for measuring frontier shifts of nonparametric frontiers.

Starting with the remark that observations inducing shifts of a frontier are by definition lying at some distance away from where this frontier was before these observations were made, it follows that measuring this distance is precisely what a measure of progress or regress should do. For this task, the techniques of efficiency measurement are of direct use, as we shall show presently. Frontier shifts can indeed be gauged by programming models that are somehow mirror images of those used to gauge efficiency; and we shall show how to compute "progress-regress degrees", just parallel to the "efficiency degrees" considered so far. The methods vary, however, according to the postulated reference production set, *i.e.* FDH or DEA.

5.2 *Defining And Measuring Progress And Regress With FDH Frontiers*

a) Definitions

As was suggested above, a logic of dominance — more than one of envelopment — governs the construction of FDH production sets as well as the formulation of the ensuing efficiency measurement. To be consistent with that, the description of shifts of the frontier of these sets should be done in the same spirit. We therefore propose the following concepts.

Definition 1: *an observation (x^{kt}, u^{kt}) is said to induce "progress" if it is (i) undominated at time t, and (ii) dominating one or several observations, made at some time $s < t$ and found undominated at s.*

Since undomination is the same as belonging to the frontier, the distance between the observation made at time t and one of those (appropriately chosen if there are several of them) undominated made at the earlier time s provides a measure of the outward frontier shift locally achieved by the former. Time s as used here will henceforth be called "reference time". In many recent applications, s is taken as t-1, but it need not be so in all cases.

Definition 2: *an observation (x^{kt}, u^{kt}) is said to induce "regress" if it is (i) undominated at time t, and (ii) dominated by one or several observations, made at times $s < t$ tand found undominated at s*

As above, the distance between the observation at time t and one of those made earlier (appropriately chosen if there are several of them) provides a way to measure the (local) inwards frontier shift induced by the former.

These definitions are incomplete however, to the extent that they do not specify the observations reference sets with respect to which the above domination-undomination properties are considered. In correcting for that, when this specification is introduced the following distinction needs to be made. On the one hand, if domination-undomination at times t and $s < t$ are specified with respect to Y_o^{Kt} and Y_o^{Ks}, respectively, progress and regress are defined in terms of a relation between the two *contemporaneous* frontiers that these observations subsets induce; call them "contemporaneous progress and regress".

On the other hand, if domination-undomination are specified with respect to $Y_o^{K(1,t)}$ and $Y_o^{K(1,s)}$, progress and regress should be defined in terms of a relation between the two *sequential* frontiers that these observations subsets induce, and be called "sequential progress and regress". Sequential regress is however a logical impossibility: indeed, going back to the definition, if an observation is *sequentially* undominated at time t, it cannot be dominated by any observation made at $s < t$. Thus with sequential reference sets, only progress can ever occur. What would be interpreted as regress with contemporaneous reference sets is in fact treated as inefficiency in a sequential context[15].

Bearing this distinction in mind, we now turn to how to compute degrees of progress or regress, with FDH reference sets.

b) Measurement With Contemporaneous FDH Frontiers.

For any observation (x^{kt}, u^{kt}) to be characterized *vis-à-vis* the contemporaneous frontier at reference time s, proceed according to the following five steps (stated explicitly only for input measures; to alleviate this text, the parallel procedures for output and graph measures are given in the Appendix):

Step 1.- Compute first the FDH efficiency in input of (x^{kt}, u^{kt}) according to problem 1.1 above, with $Y_0 = Y_o^{Kt}$ as reference observations subset.

Step 2.- If (x^{kt}, u^{kt}) is found inefficient, it is not a frontier observation and thus cannot be used for frontier shift measurement; the computation is therefore terminated as far as observation (x^{kt}, u^{kt}) is concerned[16]. If (x^{kt}, u^{kt}) is found efficient, go to step 3.

Step 3.- Consider $Y_{o*}^{Ks}(x;FDH)$, *i.e.* the subset of observations in Y_o^{Ks} that were found FDH input efficient at reference time s, and compute the optimal solution of the following

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[15] In the context of nonparametric efficiency analysis of time series, the above "progress" notion was formulated first in THIRY and TULKENS 1992 (section 4, pp. 56-57), but only for sequential observations subsets. Indeed in that context, contemporaneous efficiency is trivially achieved by all observations (since there is only one at each t), making the progress as well as the regress measures from contemporaneous production sets not meaningful.

[16] In spite of being inefficient, observation (x^{kt}, u^{kt}) might well lie outside of the reference production set constructed from the observations subset $Y_{o*}^{Ks}(x;FDH)$ to be defined in the following step 3: the observation itself is then in progress *vis-à-vis* the frontier at time s, and this progress can also be measured by the procedure that follows; this measure is however not one of a frontier shift.

Problem 2.1

$$\max_{\{\theta^{kt},\, \gamma^{hs},\, (hs)\in\overline{Y}_o\}} \theta^{kt}, \quad \text{subject to}$$

$$\sum_{(hs)\in\overline{Y}_o} \gamma^{hs} u_j^{hs} \le u_j^{kt},\ j = 1,\dots,J$$

$$\theta^{kt} x_i^{kt} - \sum_{(hs)\in\overline{Y}_o} \gamma^{hs} x_i^{hs} \le 0, \quad i = 1,\dots,I$$

$$\sum_{(hs)\in\overline{Y}_o} \gamma^{hs} = 1 \quad \text{and} \quad \gamma^{hs} \in \{0,1\},\ (hs)\in\overline{Y}_o$$

where

$$\overline{Y}_o = \left\{(hs)\middle| s < t;\ (x^{hs}, u^{hs}) \in Y_{o*}^{Ks}(x;FDH)\right\} \cup \{(kt)\}. \quad (15)$$

For this problem again, the values $\theta^{kt} = \gamma^{kt} = 1,\ \gamma^{hs} = 0\ \forall (hs) \neq (kt)$ of the variables are always a feasible solution. Therefore, one has $\theta^{kt*} \geq 1$ for the value of the objective function at the optimum. Call this value $P_x^C(kt,s;FDH)$, a short notation for denoting the "progress degree (P) of observation (kt)relative to an FDH frontier at time s, measured in input (x) from contemporaneous data (C)".

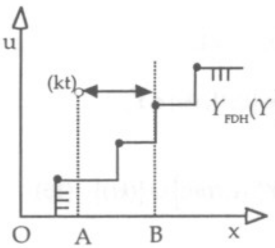
If $P_x^C(kt,s;FDH) > 1$, no further computation needs to be done as far as observation (x^{kt}, u^{kt}) is concerned. *Progress is indeed measured by this number in terms of the proportion of how much more of all*[17] *inputs unit k at time t would have used if it had used the same input bundle as the unit it dominates most (in input) in* Y_o^{Ks}. In figure 1.a, such a value of $P_x^C(kt,s;FDH)$ is illustrated by the ratio 0B/0A. If $P_x^C(kt,s;FDH) = 1$, go to step 4.

Step 4.- With $P_x^C(kt,s;FDH) = 1$, there is no progress, but there may be regress. To find that out, compute the optimal solution of Problem 1.1 above, *with* $\overline{Y}_o$ *defined as in* (15). Here, one has $\theta^{kt*} \le 1$ (since as stated there, $\theta^{kt} = \gamma^{kt} = 1,\ \gamma^{hs} = 0\ \forall (hs) \neq (kt)$ is always feasible) and call $R_x^C(kt,s;FDH)$ that value of the objective function at the optimum.

$R_x^C(kt,s;FDH) < 1$ *measures regress, in terms of the fraction of the actual input usage made by unit k at time t that was used at time s by the unit that dominates k most (in input).* On figure 1.b, $R_x^C(kt,s;FDH)$ is illustrated by the

[17] This is indeed a radial measure.

ratio 0B/0A. With this value the computation is also terminated. If $R_x^C(kt,s;FDH) = 1$, go to step 5.



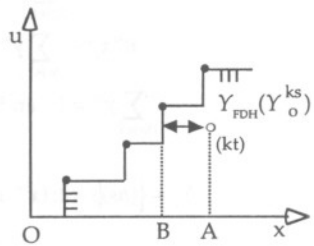
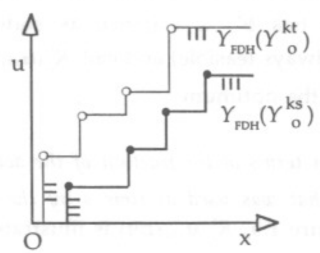


Figure 1.a *Figure* 1.b

Progress and regress with contemporaneous FDH frontiers

Step 5.- Finally, if $P_x^C(kt,s;FDH) = R_x^C(kt,s;FDH) = 1$, neither progress nor regress prevail: the frontier observation (x^{kt}, u^{kt}) is neither dominating nor dominated by any of the frontier observations at *s* and there is therefore no frontier shift. This completes the progress *vs* regress characterization of observation (x^{kt}, u^{kt}).

Applying the above procedure to all observations in Y_o^{Kt} yields a complete description of the frontier shift achieved at time *t*. Figures 2.a and b show two typical cases: in the former, one of the two reference sets contains the other; in the latter, the two sets only partially overlap, exhibiting regress for low values of the output and progress for large ones.



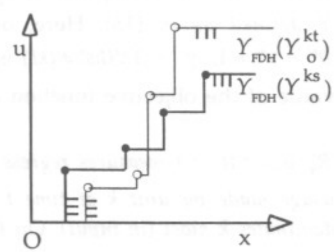


Figure 2.a *Figure* 2.b

Alternative shifts of contemporaneous FDH frontiers

In comparison with efficiency measurement, we notice three differences in the evaluation of progress *vs.* regress:

(i) Progress *vs.* regress may require up to three computations for each observation, namely those described at steps 1, 3 and 4.

(ii) The elements of the reference observations subsets used in the programming models of steps 3 and 4 are characteristically the observations that make up the frontier *at time t-1* — plus (x^{kt}, u^{kt}) itself which ensures that these problems always have a solution. In problems 1.1-1.3, the reference observations subsets used were containing only observations made at time *t*.

(iii) In step 4 the problem whose solution measures regress actually has a structure identical to that of problem 1.1, designed to measure efficiency; only the reference observations subset is different. But this is only natural, since regress amounts to inefficiency *vis-à-vis* the the frontier at *t-1*. By contrast in step 3 the problem 2.1 that measures progress is quite different from problem 1.1, but it is somehow a mirror image of it with the operator *max* substituted for *min*, and all inequalities reversed. Again, this is natural since the dominance relation that is sought for is in the reverse order compared with the one involved when efficiency is measured.

c) Measurement With Sequential FDH Frontiers.

Similar computations can be done for measuring progress with sequential frontiers as defined in section 3. In this case, however, the reference time *s* can only be *t-1*. Fot the rest, the steps to be applied to each observation (x^{kt}, u^{kt}) are only slightly modified as follows:

Step 1: Use $Y_0 = Y_o^{K(1,t)}$ as reference observations subset.
Step 2: Unchanged.
Step 3: Consider $Y_{o*}^{K(1,t-1)}(x;FDH)$, as the subset of observations found FDH sequentially input efficient at time *t-1*, and compute the solution of *problem 2.1* with $\overline{Y}_o$ defined as $\overline{Y}_o = \{(hs) | (x^{hs}, u^{hs}) \in Y_{o*}^{K(1,t-1)}(x;FDH)\} \cup \{(kt)\}$. Call $P_x^S(kt, t-1; FDH)$ (where the superscript *S* stands for "sequential") the value of the objective function at the solution. If $P_x^S(kt, t-1; FDH) > 1$, it

measures progress, in the same way that $P_x^C(kt,t-1;FDH)$ would do, and the computation is terminated. If $P_x^S(kt,t-1;FDH) = 1$, go to step 4.

Step 4: With $P_x^S(kt,t-1;FDH) = 1$ there is no progress. Since there can be no regress with sequential reference production sets of this type, the observation (x^{kt}, u^{kt}) is only sequentially efficient. Its characterization is thereby completed.

Repeating the procedure for all elements of Y_o^{Kt} provides a complete characterization of the sequential frontier shift between *t-1* and *t*. A graphical illustration is given on Figure 3. It shows that progress has a "local" character, in the sense that the whole frontier does not necessarily shift. This is typical of the nonparametric approach in its sequential form[18], quite in contrast with the description of progess with the parametric methods alluded to in section 5.1.

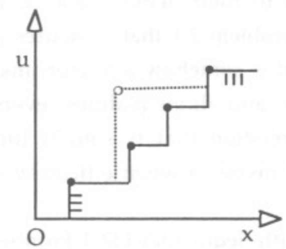


Figure 3

Local FDH progress

5.3 *Measuring Progress And Regress With DEA Frontiers*

With DEA, the logic of envelopment by means of the boundary of a convex set prevails. This implies that progress and regress are to be described by shifts of that boundary *per se*, more than by referring only to the observations themselves, as is the case with dominance on which FDH rests. As a formal definition of frontier shift is hardly necessary in this case, we directly proceed to the computational procedures.

18 Also encountered by THIRY and TULKENS 1992 in their efficiency analysis of time series.

a) Measurement with Contemporaneous DEA Frontiers

The procedure is basically similar to the one proposed above for FDH frontiers. However, specific problems arise at some points, that entail important differences. In this exposition we account for the fact that DEA can take on different forms according to the assumed returns to scale by using as before the index p = {DEA-ID, DEA-CD, *or* DEA-C}. For an observation (x^{kt}, u^{kt}), to be characterized in input terms:

Step 1.- Compute first its DEA efficiency according to *problem 1.1*, with the last line replaced by (12), (13) or (14) according to the value chosen for p (returns to scale option), and with $Y_o = Y_o^{Kt}$ as reference observations subset.

Step 2.- If (x^{kt}, u^{kt}) is found inefficient, *i.e.* not a frontier observation at time t, the computation is terminated as far as this observation is concerned. If it is found efficient, go to step 3.

Step 3.- Consider $Y_{o\bullet}^{Ks}(x;p)$, *i.e.* the subset of observations in Y_o^{Ks} that were found efficient in input at reference time s according to the same returns to scale option p, and compute the optimal solution of problem 1.1 with

$$\overline{Y}_o = \left\{(hs) \middle| s < t, (x^{hs}, u^{hs}) \in Y_{o\bullet}^{Ks}(x;p)\right\}. \tag{16}$$

and the last line replaced by (12), (13) or (14) according to the value chosen for the returns to scale option p.

If a solution exists[19], one of three cases can occur:

— If one has $\theta^{kt^*} > 1$ for the value of the objective function at the optimum, call it $P_x^C(kt,s;p)$. It measures the progress achieved by observation (x^{kt}, u^{kt}) over the DEA-p frontier at time s (ratio 0B/0A on figure 4.a).

[19] It was spotted by FÄRE, GROSSKOPF, LINDGREN and ROOS 1989 that there may not exist a solution to problem 1.1 with (16), when the constraints (12) or (13) are used. We return to this point in step 4, in comment *(ii)* and in Section 5.4 below.

— If one has $\theta^{kt^*} < 1$, call it $R_x^C(kt,s;p)$; it measures regress *vis-à-vis* the DEA-p frontier at time s (ratio 0B/0A on Figure 4.b).

— Finally, $\theta^{kt^*} = 1$ denotes absence of frontier shift, *i.e.* the observation (x^{kt}, u^{kt}) lies on the frontier at s.

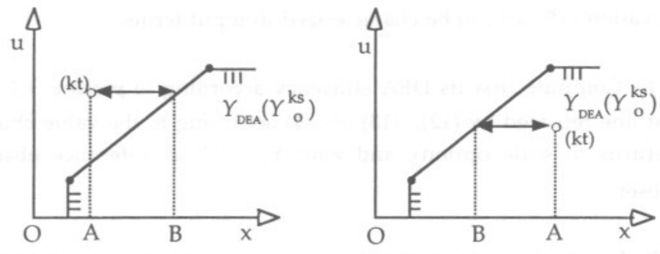


Figure 4.a *Figure* 4.b

Progress and regress with DEA contemporaneous frontiers

When one of these three cases occurs, the computation is terminated as far as observation (x^{kt}, u^{kt}) is concerned. On the other hand if no solution exists, go to step 4.

Step 4.- If problem 1.1 with (16) has no solution, redefine $\overline{Y}_o$ as

$$\overline{Y}_o = \left\{(hs)\middle| s < t, (x^{hs}, u^{hs}) \in Y_{o^*}^{Ks}(x;p)\right\} \cup \left\{(kt)\right\} \qquad (17)$$

that is, the same observations subset except that observation (x^{kt}, u^{kt}) is now included, and compute now the optimal solution of problem 1.1 with this expression of $\overline{Y}_o$, and with the last line replaced by (12) or (13) in accordance with the constraint that was used in previous problem . Here, the value θ^{kt^*} of the objective function at the optimum is ≥ 1, because $\gamma^{hs} = 1 \; for \; (hs) = (kt)$, yielding $\theta^{kt} = 1$ is always a feasible solution for this form of problem 2.1.

Therefore, as in step 3, if $\theta^{kt^*} > 1$ for the value of the objective function at the optimum, call it $P_x^C(kt,s;p)$, as it measures progress (ratio 0B/0A on figure 5.a). If $\theta^{kt^*} = 1$, no progress is revealed by the input measure (see figure 5.b) and observation (x^{kt}, u^{kt}) is only efficient. This terminates the progress *vs.* regress characterization of a single observation with DEA-type reference production sets.

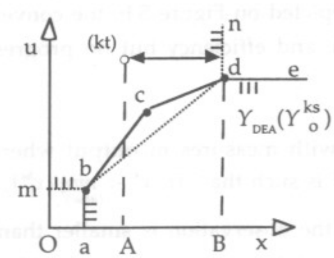


Figure 5.a

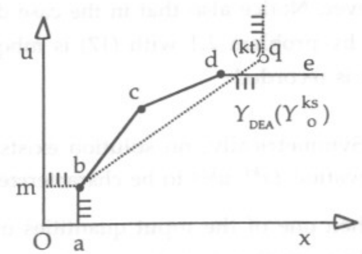


Figure 5.b

Progress and efficiency measurement in case of no solution to problem 1.1 with (16)

As it was the case with FDH, applying the above procedure to all observations in Y_o^{Kt} yields a complete description of the DEA frontier shift achieved at time *t*.

Parallel to our comments above on contemporaneous progress *vs.* regress with FDH reference sets, we note here the following:

(i) Progress *vs.* regress evaluation basically requires only two computations with DEA because the second one (step 3) directly sorts out between progress and regress. A third computation is needed only in the case of no solution. This is however not an exceptional case as will be seen in the next paragraph.

(ii) The way we propose to get around the non existence of a solution at step 4 deserves special attention. When does this phenomenon occur? Problem 1.1 with (12) or (13) has no solution when the observation (x^{kt}, u^{kt}) to be characterized is such that $\exists\, j : u_j^{kt} > \max_{(hs)\in Y_o} \{u_j^{hs}\}$, that is, when one of the output quantities of the observation is larger than the largest similar output reached by some observation in the reference observations set. This is clearly seen in Figure 5.a: to find a distance in input to the right of observation (x^{kt}, u^{kt}) by means of problem 1.1 is indeed impossible since the frontier the problem specifies is abcde. Problem 2.1 with (17), by contrast, uses as reference the convex set whose frontier is labelled mbdn, of which

the observation (x^{kt}, u^{kt}) happens to be an eolement. This is why it always has a solution. This convex set hardly has an economic interpretation, however. Notice also that in the case depicted on Figure 5.b, the convex set used by problem 2.1 with (17) is mbqn, and efficiency but no progress in input is recorded.

Symmetrically, no solution exists with measures in output when the observation (x^{kt}, u^{kt}) to be characterized is such that $\exists\, i : x_i^{kt} < \min_{(hs)\in \bar{Y}_o} \{x_i^{hs}\}$, that is, when one of the input quantities of the observation is smaller than the smallest similar input quantity reached by some observation in the reference observations set. This is illustrated in Figure 6.a, together with the solution 0C/0D (revealing progress) provided by the solution of the equivalent of problem 2.1 in the output orientation (that is, problem 2.2 in the appendix). Figure 6.b, finally, exhibits a case that combines the two preceding ones, where a solution to problem 1.1 exists neither in output nor in input, while progress can be measured by problem 2.1 with (17) and its equivalent in the output orientation.

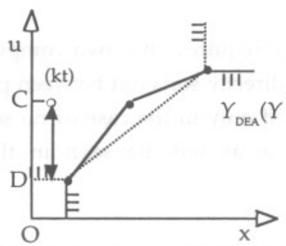


Figure 6.a

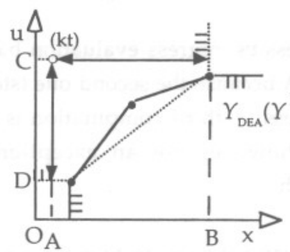


Figure 6.b

Measurement in two further "no solution" cases

b) Measurement With Sequential DEA Frontiers

Using sequential frontiers, the procedure just expounded for contemporaneous ones is to be modified as follows (recall that only $s = t\text{-}1$ is valid here as reference time):

Step 1: Use $Y_0 = Y_o^{K(1,t)}$ as reference observations subset.

Step 2: Unchanged.

Step 3: Consider $Y_{o*}^{K(1,t-1)}(x;p)$, the subset of observations found DEA-*p* sequentially efficient in input at time *t-1*, and compute the solution of problem 1.1 with $\overline{Y}_o$ defined as

$$\overline{Y}_o = \{(hs) | s < t, (x^{hs}, u^{hs}) \in Y_{o*}^{K(1,t-1)}(x;p)\} \tag{18}$$

instead of (16). If there is a solution, call $P_x^S(kt;p)$ the value of the objective function at the solution. If $P_x^S(kt;p) > 1$, it measures progress, in the same way that $P_x^C(kt,s;p)$ with $s = t-1$ would do, and the computation is terminated. If $P_x^S(kt;p) = 1$, go to step 4. If there is no solution, go to step 5.

Step 4: With $P_x^S(kt;p) = 1$ there is no progress, the observation (x^{kt}, u^{kt}) is only sequentially efficient, and the computation is terminated.

Step 5: If there is no solution at step 3, redefine $\overline{Y}_o$ in (18) as

$$\overline{Y}_o = \{(hs) | s < t, (x^{hs}, u^{hs}) \in Y_{o*}^{K(1,t-1)}(x;p)\} \cup \{(kt)\} \tag{19}$$

and compute the solution of problem 1.1 with $\overline{Y}_o$ so defined and the last line replaced by (12) or (13). The properties of the solution and its interpretation are as in step 5 of the procedure with contemporaneous frontiers. The characterization of an observation (x^{kt}, u^{kt}) in terms of sequential progress with DEA reference frontiers is thereby completed.

Repeating the procedure for all elements of Y_o^{Kt} provides a complete characterization of the sequential frontier shift between *t-1* and *t*. A graphical illustration is given on Figure 7. As we observed with FDH sequential analysis, it shows the"local" character of the frontier shift. This particular form of progress was already obtained, in a different context, by ATKINSON and STIGLITZ 1969 (p. 573) who present an identical diagram.

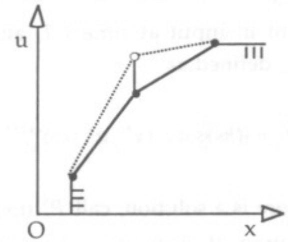


Figure 7

Local DEA progress

5.4 *On Frontier Shift Measures Used In Malmquist Indexes*

We want to conclude this section by comparing our approach to frontier shifts with the "Malmquist index" recently developed by FÄRE, GROSSKOPF, LINDGREN and ROOS 1989 and 1992 (hereafter referred to as FGLR), also designed to characterize changes over time in productive activities described by panel data. A basic difference lies in the fact that the Malmquist index, denoted here as $M(k; t\text{-}1,t; p)$ measures the *productivity gain* (if $M(k; t\text{-}1,t; p) > 1$) or *loss* (if $M(k; t\text{-}1,t; p) < 1$) achieved by a given unit k from time $t\text{-}1$ to time t.[20] As is well known, productivity is a notion different from efficiency. However, recognizing the fact that the unit whose productivity change is to be assessed may be efficient or inefficient (relative to some reference frontier of type p, to be specified shortly) at either one of the two observation times, the authors show that the productivity index can be decomposed into two factors that we presently recall.

The first factor does indeed measure the "efficiency gain or loss" of the unit from $t\text{-}1$ to t, that is, in our notation, the extent to which (x^{kt}, u^{kt}) is closer to or farther away from the frontier of $Y_p(Y_o^{Kt})$ than (x^{kt-1}, u^{kt-1}) was *vis-à-vis* the frontier of $Y_p(Y_o^{Kt-1})$. Using in each case the computation prescribed by problem 1.1 above[21], this is measured by the ratio

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20 BERG, FØRSUND and JANSEN 1991 use alternative points in time s in lieu of $t\text{-}1$. The issues raised by the choice of alternative reference times are of the nature of those arinsing in choosing a base for any index number.

21 This problem yields a Malmquist index in input; parallel measures in output or graph are easily defined.

$E_x^C(kt;p)/E_x^C(kt-1;p)$: a value larger than 1 measures an efficiency gain, a value smaller than 1 an efficiency loss;

The other factor measures "frontier shift" in the sense that has been treated here, that is, the extent to which the frontier used to measure the efficiency of (x^{kt}, u^{kt}) has shifted away from the one that has served to measure the efficiency of (x^{kt-1}, u^{kt-1}). Now this can be measured in two ways : either from (x^{kt}, u^{kt}) by a ratio $S_x^C(kt,t-1;p)/E_x^C(kt;p)$, or from (x^{kt-1}, u^{kt-1}) by a ratio $E_x^C(kt-1;p)/S_x^C(kt-1,t;p)$, where the numerators and denominators are derived as follows from our previous developments:

$S_x^C(kt,t-1;p) = P_x^C(kt,t-1;p)$ if a progress-regress evaluation of observation (x^{kt}, u^{kt}) with respect to the cotemporaneous p-frontier at time $t-1$ results in $P_x^C(kt,t-1;p) > 1$;

$= R_x^C(kt,t-1;p)$ if the same evaluation results in $R_x^C(kt,t-1;p) < 1$;

$= 1$ if the same evaluation yields $P_x^C(.) = R_x^C(.) = 1$;

similarly for $S_x^C(kt-1,t;p)$, where $t-1$ and t replace t and $t-1$, respectively, in the previous definition;

and $E_x^C(kt;p)$ and $E_x^C(kt-1;p)$ are the efficiency degrees already referred to, introduced here to correct for the possible inefficiency of the observations (x^{kt}, u^{kt}) and (x^{kt-1}, u^{kt-1}).

With this notation, the FGLR decomposition of the Malmquist index reads[22]:

$$M(k;t-1,t;p) = \frac{E_x^C(kt;p)}{E_x^C(kt-1;p)} \times \left[\frac{S_x^C(kt,t-1;p)}{E_x^C(kt;p)} \times \frac{E_x^C(kt-1;p)}{S_x^C(kt-1,t;p)} \right]^{\frac{1}{2}} \qquad (20)$$

Frontier shift as measured by the expression in the square brackets thus appears as being only a component of productivity as evaluated by the Malmquist index. The way this measure is done raises two problems, however. One is the issue of "where" to measure the frontier shift: from the

[22] For ease of comparison with the rest of this paper, the formula is presented here in terms of efficiency and progress-regress degrees instead of the distance functions actually used by the authors in their original presentation.

current observation (x^{kt}, u^{kt}) to the frontier of the past *i.e.* of $Y_p(Y_o^{Kt-1})$ *i.e.* the first factor in the square brackets, or from the past observation (x^{kt-1}, u^{kt-1}) to the frontier of the current reference set $Y_p(Y_o^{Kt})$ *i.e.* the second factor? The authors' compromising choice is to take a geometric mean of the two. How is that related with what we have done in the preceding sections? The other problem is that the authors confine themselves to DEA-C reference production sets when they need to use both input and output measures, because with DEA-CD and DEA-ID production sets output and/or input measures may not exist in certain cases (as recorded with problem 1.1 with (16) above, which is the one they use), and they offer no solution for that.

On these two issues, we have the following points to make:

(i) Our perspective in the preceding analysis was to measure frontier shift only, and this appeared not to need, *per se*, any kind of averaging. Of course, averaging in the productivity index is only due to the necessity to stick with the same unit k at times t-1 and t. Our approach just reminds one that imposing this constraint is not necessary for measuring technical progress in terms of a frontier shift, and shows how to do it.

(ii) Because problem 1.1 with (16) may have no solution in some important cases, both FGLR 1989 and 1992 restrict themselves to p = DEA-C reference production sets. Our present claims in that respect are (1°) that by using FDH instead of DEA reference production sets, the programming problems always have a solution, as was seen in subsection 5.2; and (2°) that with DEA models of all types, the no solution cases can be solved by following what we propose in step 4 of section 5.3. Admittedly, our solution is in the spirit of substituting a domination-undomination criterion for the one of convex envelopment, which may be seen as an inacceptable change of philosophy within a given evaluation methodology. However we feel that there lies a strong justification for our proposal in the fact that the no solution cases *always* arise in parts of the input-output space where it is not the convexity assumption that creates the problem but well the free disposal one: remember indeed that both DEA-CD and ID construct convex *free disposal* hulls, and this is the source of the "horizontal" and "vertical" segments in

the boundary of the corresponding production sets, as for instance in figures 6a and 6b.

6. Measuring Progress And Regress From A "Benchmark Production Correspondence"

It must be recognized that either one of the two ways of measuring progress-regress by frontier shifts, *i.e.* from contemporaneous or from sequential frontiers, has important drawbacks. When contemporaneous frontiers are used, it is hard to avoid arbitrariness in the choice of the reference time *s*; and if *s* is chosen to be always equal to $t\text{-}1$[23], it is characteristically of short memory: only the last frontier counts, and whatever progress (or regress) was achieved earlier is not accounted for in the numerical evaluation at time *t* With sequential progress, on the other hand, there is no room for regress, neither conceptually, nor in terms of measurement. Regress, in this setting, is simply assimilated with inefficiency because the production frontier is postulated to only move outwards and not inwards.

To overcome these drawbacks we call here[24] upon a new concept, namely the "benchmark correspondence". The characteristic of this approach is to abandon the frontier concept, and to replace it by a special form of the input-output relation, derived from the dominance notion, that we call the "benchmark production correspondence", and by reference to which both progress and regress will be defined. As a corollary, however, the distinction between efficiency and inefficiency, understood as belonging or not to a frontier, fades away since there is no frontier anymore! Hence our substitution of the word benchmark for those of frontier or hull.

The essence of the approach is to associate with the data not a whole production set, but instead only a subset of it — technically called the graph of some correspondence — this subset being *not* postulated to be a frontier. The subset is induced from the data in an evolving way, that is, it is

23 If *s* is chosen < $t\text{-}1$, all the intermediary information is ignored.

24 What follows is actually the panel data version of a methodology expounded in greater detail for time series in our companion paper (TULKENS and VANDEN EECKAUT 1991) that is entirely devoted to this topic.

gradually modified over time, on the basis of the information revealed by the observations made at each time $t=1,2, ..., m$, in a typically sequential spirit.

At every t, a subset of the observations in the panel is thus singled out, that we call the "benchmark observations subset at time t". From these observations, the benchmark correspondence and its graph are defined. Finally, all observations made at t are characterized in terms of progress or regress *vis-à-vis* this benchmark, by means of an index computed by a procedure very close to the one used before in sequential efficiency and progress analysis. We shall now describe these three steps more precisely. Since this is a progress-regress measure, we first specify what is to be done at time $t=1$, where there are no earlier observations to compare the observations (x^{k1}, u^{k1}) with; then we move to any time $t \geq 2$.

Stage $t=1$

Step 1.1: Consider the set Y_o^{K1} of the data observed at time 1, and identify by means of problem 1 the subset Y_{o*}^{K1} of these observations that are contemporaneously FDH efficient. Denote henceforth this last subset Y_{oB}^{K1} and call it the "benchmark observations set at time 1".

Step 1.2: With every observation $(x^{k1}, u^{k1}) \in Y_{oB}^{K1}$, associate the set

$$D_i(x^{k1},u^{k1}) = \left\{ \begin{bmatrix} u \\ x \end{bmatrix} \in R_+^{I+J} \middle| \begin{bmatrix} u \\ x \end{bmatrix} = \begin{bmatrix} u^{k1} \\ x^{k1} \end{bmatrix} + \sum_{i=1}^{I} \mu_i \begin{bmatrix} 0^J \\ e_i^I \end{bmatrix} + \sum_{j=1}^{J} v_j \begin{bmatrix} e_j^J \\ 0^I \end{bmatrix}, \mu_i \geq 0, v_j \geq 0, \right\}$$
$$\cup \left\{ \begin{bmatrix} u \\ x \end{bmatrix} \in R_+^{I+J} \middle| \begin{bmatrix} u \\ x \end{bmatrix} = \begin{bmatrix} u^{k1} \\ x^{k1} \end{bmatrix} - \sum_{i=1}^{I} \mu_i \begin{bmatrix} 0^J \\ e_i^I \end{bmatrix} - \sum_{j=1}^{J} v_j \begin{bmatrix} e_j^J \\ 0^I \end{bmatrix}, \mu_i \geq 0, v_j \geq 0, \right\} \quad (21)$$

that is, the set of points in the whole input-output space (thus, non observed as well as observed points) that neither dominate nor are dominated *strongly*[25] by (x^{kt}, u^{kt}); in other words, points that are in "dominance indifference" with respect to (x^{kt}, u^{kt}) — whence the subscript "i" appended to D. With Y_{oB}^{K1}, itself, associate then the set

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[25] Referring to weak domination in Section 2, an observation strongly dominates another one in inputs if $\mu_i > 0 \; \forall \, i$ and in output if $v_j > 0 \; \forall \, j$.

$$D_i\left(Y_{oB}^{K1}\right) = \bigcap_{k\left|\left(x^{k1},u^{k1}\right)\in Y_{oB}^{K1}\right.} D_i\left(x^{k1},u^{k1}\right).$$

This set may be seen as the graph (illustrated by the dashed area on Figure 8) of the mapping

$$\beta_1 : R_+^l \rightarrow R_+^l : x \rightarrow U(x) = \{u|(x,u) \in D_i(Y_{oB}^{K1})\}$$

which we call the "benchmark production corresponcence *at time* 1".

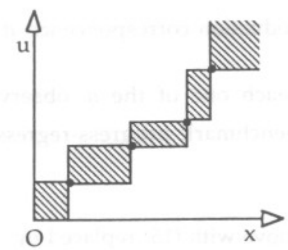

Figure 8
Graph of the BPR correspondence

Step 1.3: Finally, each one of the n observations in Y_o^{K1} conventionally receives a numerical progres-regress evaluation — called "BPR index" — equal to its FDH efficiency degree as computed at step1.1.

General stage $t \geq 2$.

Step t.1: At any time $t \geq 2$, consider the data Y_o^{Kt} and identify its subset of contemporaneously FDH efficient observations Y_{o*}^{Kt}. From defining

$$Y_{oB}^{Kt} = Y_{o*}^{Kt} \cap D_i\left(Y_{oB}^{K(1,t-1)}\right)$$

we obtain

$$Y_{oB}^{K(1,t)} = \bigcup_{s=1}^{t} Y_{oB}^{Ks},$$

which we call the "benchmark observations set for the period (1,t)"

Step t.2: With every observation $(x^{kt}, u^{kt}) \in Y_{oB}^{K(1,t)}$, associate a set $D_i(x^{kt}, u^{kt})$ defined as in (21), and with the set $Y_{oB}^{K(1,t)}$ itself associate

$$D_i(Y_{oB}^{K(1,t)}) = \bigcap_{(ks) \mid s \le t; (x^{ks}, u^{ks}) \in Y_{oB}^{K(1,t)}} D_i(x^{kt}, u^{kt})$$

This set is also the graph of the mapping

$$\beta_t : R_+^l \to R_+^l : x \to U(x) = \{u \mid (x,u) \in D_i(Y_{oB}^{K(1,t)})\}$$

i.e. the "benchmark production corresponcence *at time t*".

Step t.3: Finally, for each one of the *n* observations (x^{kt}, u^{kt}) in Y_o^{Kt} a numerical index of "benchmark progress-regress" — the BPR index — is computed as follows:

— Solve problem 2.1 above with (15) replaced by

$$\overline{Y}_{oB}^{K(1,t)} = \{ (h,s) \mid (x^{hs}, u^{hs}) \in Y_{oB}^{K(1,t)} \} \cup \{(kt)\} \qquad (22)$$

i.e. the set of index pairs that denote the elements of the benchmark observations set at time t [26] and the observation (x^{kt}, u^{kt}) itself. One has $\theta^{kt^*} \ge 1$ for the value of the objective function at the optimum. If $\theta^{kt^*} > 1$, call it $P_x(kt; BPR)$ and the characterization of observation (x^{kt}, u^{kt}) is terminated, as this measures the progress (in input) achieved by this observation relative to the benchmark correspondence at time t.

— If $\theta^{kt^*} = 1$ in problem 2.1 then move to problem 1.1 and solve it with $\overline{Y}_o$ also defined as in (22). Here, $\theta^{kt^*} \le 1$. If $\theta^{kt^*} < 1$, call it $R_x(kt; BPR)$ and the characterization of observation (x^{kt}, u^{kt}) is also terminated, as this measures the regress (in input) achieved by the observation relative to the same benchmark.

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[26] A superscript x or u should be added to these sets to allow for possible differences in their definition in input or in output terms, respectively. In the text, we stick to our convention of dealing only with input measures.

— If $\theta^{kt^*} = 1$ in both problems 1.1 and 2.1 $\theta^{kt^*} = 1$, there is neither regress nor progress achieved by the observation (x^{kt}, u^{kt}) with respect to the benchmark correspondence at time t, and observation (x^{kt}, u^{kt}) belongs to the benchmark observation subset $Y_{oB}^{K(1,t)}$. A conventional BPR index equal to 1 is attached to it.

With the corresponding computation in output terms given in the appendix this completes our description of progress and regress measurement by means of a "BPR index".

7. Conclusion

We have presented a wide variety of ways to measure nonparametrically efficiency, progress and regress from panel data. Yet our developments show that this variety reduces to two alternatives basic formal structures : the one of problems 1.1 - 1.3 and the one of problems 2.1 - 2.3. While the former is designed to evaluate efficiency, the latter is designed to evaluate progress (revealing under way that evaluating regress is formally identical to evaluating inefficiency, and thus formally belongs to the category of problems 1.1 - 1.3).

Nevertheless, this wealth of approches may leave the impression that in empirical studies, the choice of the most appropriate method must be quite an intricate one. We would argue, however, that several methods are better than only a few: our experience is indeed that by analyzing a same data set in various ways, one gets a much better understanding of what these data contain, and one finally reaches conclusions that more strongly founded.

Appendix

1. Progress and Regress Measurement With Contemporaneous FDH Frontiers

a) Ouput measures

Step 1.- Compute first the FDH efficiency in output of (x^{kt}, u^{kt}) according to problem 1.2 above, with $Y_0 = Y_o^{Kt}$ as reference observations subset.

Step 2.- If (x^{kt}, u^{kt}) is found inefficient, it is not a frontier observation and the computation is therefore terminated. If (x^{kt}, u^{kt}) is found FDH efficient, go to step 3.

Step 3.- Consider $Y_{o^*}^{Kt-1}(u;p)$, *i.e.* the subset of observations in Y_o^{Kt-1} that were found FDH output efficient at time *t-1*, and compute the optimal solution of the following

Problem 2.2

$$\min_{\{\theta^{kt},\, \gamma^{hs},\, (hs)\in \overline{Y}_o\}} \lambda^{kt}, \quad \text{subject to}$$

$$\lambda^{kt} u_j^{kt} - \sum_{(hs)\in \overline{Y}_o} \gamma^{hs} u_j^{hs} \geq 0,\ j = 1,\ldots,J$$

$$\sum_{(hs)\in \overline{Y}_o} \gamma^{hs} x_i^{hs} \geq x_i^{kt}, \quad i = 1,\ldots,I$$

$$\sum_{(hs)\in \overline{Y}_o} \gamma^{hs} = 1 \quad \text{and} \quad \gamma^{hs} \in \{0,1\},\ (hs)\in \overline{Y}_o$$

where

$$\overline{Y}_o = \left\{(hs) \middle| (x^{hs}, u^{hs}) \in Y_{o^*}^{Kt-1}(u;FDH)\right\} \cup \{(kt)\}. \tag{A.1}$$

Write as $P_u^C(kt)$ the inverse value of the objective function at the optimum, that is: $P_u^C(kt) = [\lambda^{kt*}]^{-1}$. Notice that $P_u^C(kt) \geq 1$, because $\lambda^{kt} = \gamma^{kt} = 1,\ \gamma^{hs} = 0\ \forall (hs) \neq (kt)$ is always a feasible solution for problem 2.2.

If the number $P_u^C(kt)$ is strictly larger than 1, no further computation needs to be done as far as observation (x^{kt}, u^{kt}) is concerned. $P_u^C(kt)$ indeed measures progress in terms of the proportion of how much more of all outputs[27] unit k at time t produces than if it had made no progress.

Step 4.- If $P_u^C(kt)$ is found equal to 1, there is no progress, but there may be regress. To find that out, compute the optimal solution of Problem 1.2, with $\overline{Y}_o$ defined as in (A.1)above, and write as $R_u^C(kt)$ the value of the objective function at the optimum, that is: $R_u^C(kt) = \lambda^{kt*}$. Here, one has $R_x^C(kt) \leq 1$ since $\lambda^{kt} = \gamma^{kt} = 1,\ \gamma^{hs} = 0\ \forall (hs) \neq (kt)$ is also always a feasible solution for problem 2.2.

A value strictly less than 1 for $R_u^C(kt)$ measures regress. This value also terminates the computation.

Step 5.- Finally, if both $P_u^C(kt)$ and $R_u^C(kt)$ are found to be equal to 1, neither progress nor regress prevail: the observation (x^{kt}, u^{kt}) is also a frontier observation at $t - 1$ and there is thus no frontier shift. This completes the progress *vs* regress in output characterization of observation (x^{kt}, u^{kt}).

b) Graph measures.

Step 1.- Compute first the FDH graph efficiency of (x^{kt}, u^{kt}) according to problem 1.3 above, with $Y_0 = Y_o^{Kt}$ as reference observations subset.

Step 2.- If (x^{kt}, u^{kt}) is found inefficient, it is not a frontier observation and the computation is therefore terminated. If (x^{kt}, u^{kt}) is found FDH efficient, go to step 3.

Step 3.- Consider $Y_{o*}^{Kt-1}(x,u;p)$, *i.e.* the subset of observations in Y_o^{Kt-1} that were found FDH graph efficient at time t-1, and compute the optimal solution of the following

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27 This is indeed a radial measure.

Problem 2.3

$$\max_{\{\delta^{kt},\,\gamma^{hs},(hs)\in\overline{Y}_o\}} \delta^{kt} \text{ , subject to}$$

$$\frac{u_i^{kt}}{\delta^{kt}} - \sum_{(hs)\in\overline{Y}_o} \gamma^{hs} u_j^{hs} \geq 0 \qquad j=1,\dots,J$$

$$\delta^{kt} x_i^{kt} - \sum_{(hs)\in\overline{Y}} \gamma^{hs} x_i^{hs} \leq 0, \qquad i=1,\dots,I$$

$$\sum_{(hs)\in\overline{Y}_o} \gamma^{hs} = 1 \text{ and } \gamma^{hs} \in \{0,1\}, \ (hs)\in\overline{Y}_o.$$

where

$$\overline{Y}_o = \{(hs) | (x^{hs}, u^{hs}) \in Y_{o*}^{Kt-1}(x,u;FDH)\} \cup \{(kt)\}. \qquad \text{(A.2)}$$

Write as $P^C_{x,u}(kt)$ the value of the objective function at the optimum, that is: $P^C_{x,u}(kt) = \delta^{kt*}$. Notice that $P^C_{x,u}(kt) \geq 1$, because $\lambda^{kt} = \gamma^{kt} = 1$, $\gamma^{hs} = 0 \ \forall (hs) \neq (kt)$ is always a feasible solution for problem 2.3.

If the number $P^C_{x,u}(kt)$ is strictly larger than 1, no further computation needs to be done as far as observation (x^{kt}, u^{kt}) is concerned. $P^C_{x,u}(kt)$ indeed measures progress in terms of the proportion of how much more of all outputs unit k at time t produces and, simultaneously the proportion of how much less of all inputs unit k at time t uses than if it had made no progress.

Step 4.- If $P^C_{x,u}(kt)$ is found equal to *1*, there is no progress, but there may be regress. To find that out, compute the optimal solution of Problem 2.3, with $\overline{Y}_o$ defined as in (A.2)above, and write as $R^C_{x,u}(kt)$ the value of the objective function at the optimum, that is: $R^C_{x,u}(kt) = \delta^{kt*}$. Here, one has $R^C_{x,u}(kt) \leq 1$ since $\lambda^{kt} = \gamma^{kt} = 1$, $\gamma^{hs} = 0 \ \forall (hs) \neq (kt)$ is also always a feasible solution for problem 2.3.

A value strictly less than *1* for $R^C_{x,u}(kt)$ measures regress. This value also terminates the computation.

Step 5.- Finally, if both $P^C_{x,u}(kt)$ and $R^C_{x,u}(kt)$ are found to be equal to *1*, neither progress nor regress prevail: the observation (x^{kt}, u^{kt}) is also a frontier observation at $t-1$ and there is thus no frontier shift. This completes the progress *vs* regress in graph characterization of observation (x^{kt}, u^{kt}).

2. Progress and Regress Measurement with Contemporaneous DEA Frontiers

Output and graph measures are obtained from our description in section 5.3 of the computational procedure in input just by making the following substitutions :

Description of the procedure in the input orientation	Description of the procedure in the output orientation	Description of the procedure in the graph orientation[28]
For...	**Substitute...**	**Substitute...**
Problem 1.1	Problem 1.2	Problem 1.3
Problem 2.1	Problem 2.2	Problem 2.3
$Y_{o\bullet}^{Ks}(x;p)$	$Y_{o\bullet}^{Ks}(u;p)$	$Y_{o\bullet}^{Ks}(x,u;p)$
$\theta^{kt\bullet}$	$[\lambda^{kt\bullet}]^{-1}$	$\delta^{kt\bullet}$
$P_x^C(kt,s;p)$	$P_u^C(kt,s;p)$	$P_{x,u}^C(kt,s;p)$
$R_x^C(kt,s;p)$	$R_u^C(kt,s;p)$	$R_{x,u}^C(kt,s;p)$

3. Progress and Regress Measurement from A "Benchmark Production Correspondence"

Output and graph measures are likewise obtained from our description in section 6 of the computational procedure in input with the following substitutions :

Description of the procedure in the input orientation	Description of the procedure in the output orientation	Description of the procedure in the graph orientation
For...	**Substitute...**	**Substitute...**
Problem 1.1	Problem 1.2	Problem 1.3
Problem 2.1	Problem 2.2	Problem 2.3
$\theta^{kt\bullet}$	$[\lambda^{kt\bullet}]^{-1}$	$\delta^{kt\bullet}$
$P_x(kt;BPR)$	$P_u(kt;BPR)$	$P_{x,u}(kt;BPR)$
$R_x(kt;BPR)$	$R_u(kt;BPR)$	$R_{x,u}(kt;BPR)$

[28]For the graph orientation, problem 2.3 is never used because problem 1.3 always has a solution.

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