The Theory of Currency Speculation

by Hans-Werner Sinn

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Section B
The Theory of Currency Speculation

In this section the theory of economic decision making under uncertainty developed above is applied to the choice problem of the currency speculator. In contrast to portfolio theory, the analysis is only an exercise in positive theory. Since speculators are important people it is worth-while trying to understand their behavior, but the aim of speculation theory is not to show them how to increase their profits.

The analysis draws partly on the approaches of GRUBEL (1966), FELDSTEIN(1968), LELAND (1971), and HOCHGESAND (1974). However, in so far as this study integrates the problem with a multiperiod approach and explores the particular implications of Weber's law and the BLOOS rule, it goes beyond them.

1. *The Basic Problems of the Spot and Forward Speculators*

The decision problem of the currency speculators is analyzed in a highly idealized model of currency markets with perfectly flexible exchange rates. It is assumed that all transactions are carried out at fixed dates between which nothing happens. At each date there is a spot and a forward market. In the latter the conditions for an exchange in currency on the subsequent transactions day are settled[1]. The analysis is confined to the two-country case. The domestic country is the United States, the foreign country Germany; accordingly the currencies are $ and DM. The exchange rate is the dollar price of one Deutschmark.

At the decision date, speculators know the current spot rate w_0^K and the current forward rate w_0^T, but the rates that will obtain after one period, W_1^K and W_1^T, as well as all other future rates, are unknown. Decisions therefore have to be made on the basis of equivalent objective probabilities.

1.1. *Forward Speculation*

The forward speculator buys or sells forward currency, planning to carry out a compensating transaction in the spot market when the delivery date comes and he has to meet his obligations. Consider first the case where he buys, i.e., where he has a long position in currency futures. If he buys h DM 'today' in the forward market then 'to-

[1] The possibility of multiple forward markets, each concerned with a different time in the future, is excluded. A model with multiple forward markets was developed by SOHMEN (1966 and 1973).

morrow' he has to spend \$ $h\ w_o^T$ to meet his obligations, but after exchanging the h DM received through the forward contract he has a dollar revenue of $h\ W_1^K$. His expected profit in dollars after one period therefore is

$$(1) \qquad X = h(W_1^K - w_0^T).$$

Next consider the case of a short position. If the speculator 'today' sells k DM in the forward market then he 'tomorrow' receives \$ $k\ w_0^T$, but to meet his obligations he has to spend \$ $k\ W_1^K$ for a purchase of Deutschmarks in the spot market. Hence his profit is \$ $k(w_0^K - W_1^K)$. If we set $k = -h$ so that a forward sale of Deutschmarks is interpreted as a negative purchase, then his profit is again given by equation (1).

Forward speculators, in principle, do not need capital. It could, therefore, be conjectured that a speculator can make his commitment h as large as he wishes. But this is not so. In practice an institutional rule has developed that limits his commitments, and we shall see that there are good reasons for this rule. The banking companies carrying out the forward contracts for their customers usually require a safety margin of between 10 and 20 percent of the dollar value of the forward commitment, since they are liable to the foreign trading partners. The level of the safety margin in general seems to be independent of whether speculators sell short or buy long. As interest is paid by the banks on the value of the safety margin this rule does not involve costs to the speculator but merely links the maximum speculative commitment with his personal wealth[2].

Given this information, the opportunity set of end-of-period wealth distributions (V) attainable by the speculator can easily be described. For simplicity we assume that, apart from the speculative profit, the speculator does not have further random income flows. As usual, non-random flows are admissible, however. It is assumed that their present value is a part of wealth and can be used to provide the safety capital required by the banks. Let β denote the proportion of safety capital in the dollar volume of the commitment, a the speculator's wealth after subtracting period consumption, and $q-1$ the safe market rate of interest. Then the opportunity set sought obviously is given by

$$(2) \qquad V = aq + h(W_1^K - w_0^T), \ |h| \leq \frac{aq}{\beta w_0^T}.$$

[2] The information was given on July 27, 1977, by the Deutsche Bank, Mannheim. GRUBEL (1965, p. 252) reports a 10% safety margin, but his information refers to the time where the exchange rate could only fluctuate within the narrow official band.

1.2. *Spot Market Speculation*

Next, consider the case of speculation in the spot market. In contrast to forward speculation, a speculation in the spot market is associated with an international movement of capital. Spot-market speculators import or export capital without protecting themselves in the forward market. Suppose a spot-market speculator plans, for one period, to buy foreign fixed-interest bonds for h^* dollars and to invest the remainder of his wealth, $a-h^*$, in fixed-interest domestic bonds. Then at the end of the speculation period his wealth is

$$(3) \qquad V=(a-h^*)q^I+h^*\frac{W_1^K}{w_0^K}q^A$$
$$=aq^I+\frac{h^*q^A}{w_0^K}\left[W_1^K-\frac{q^I w_0^K}{q^A}\right]$$

where q^I-1 is the domestic and q^A-1 the foreign rate of interest. If, however, the speculator plans to borrow k^* dollars from foreigners in order to invest them in the domestic country then his end-of-period wealth is $aq^I+k^*q^I-q^A W_1^K k^*/w_0^K$ or, if with $k^*=-h$ we interpret a capital import as a negative capital export, it is again given by equation (3).

1.3. *Interest Arbitrage as the Link between Spot and Forward Speculation*

A comparison of the end-of-period wealth distributions (2) and (3) of the forward-market and the spot-market speculators may, at first sight, give the impression that both kinds of speculation are significantly different from each other. This impression is, however, wrong. This can be seen very clearly when the role of interest arbitrage is considered in addition to speculation[3]. Like spot-market speculators, interest arbitragers are capital importers or exporters. The difference is simply that arbitragers protect themselves in the forward market while speculators do not.

Suppose, from the viewpoint of the arbitragers, domestic and foreign fixed-interest bonds are perfect substitutes. Then we must have[4]

[3] Interest arbitragers and speculators are not necessarily different people, the latter can participate in arbitrage with that part of their wealth which is safely invested.

[4] With equation (4) we assume an infinitely elastic 'arbitrage schedule' of the kind assumed in the classical theory of interest parity. GRUBEL (1966, pp. 18–21) questioned this assumption, pointing out that even arbitrage is subject to political risk. Cf. also SCHRODER (1969, pp. 30–32).

$$q \equiv q^I = q^A \frac{w_0^T}{w_0^K} \tag{4}$$

for, if $q^I > q^A w_0^T/w_0^K$, there would be an enormous inflow of risk free capital which would reduce w_0^K and/or raise w_0^T, and if $q^I < q^A w_0^T/w_0^K$, a corresponding capital outflow would induce the opposite adjustments.

By using the arbitrage equation (4), equation (3) now can be written as

$$V = aq + \frac{h^* q^A}{w_0^K}(W_1^K - w_0^T). \tag{5}$$

This equation already resembles equation (2). The only difference is that in (2) the speculative commitment h was measured in foreign currency (DM) while in (5) the commitment is denoted by h^* which is the dollar value of the capital to be exported. This difference, however, does not matter. If we measure the commitment of the spot-market speculator by the Deutschmark value of the redemption by setting $h \equiv q^A h^*/w_0^K$, then (5) takes on exactly the same form as (2). This surprising result originates from TSIANG (1959) who showed that spot-market speculation is, in economic terms, the same as a combination of forward speculation and pure interest arbitrage.

Thus it seems that from now on we only have to consider the forward speculator. However, the constraint $|h| \leq aq/(\beta w_0^T)$ has yet to be examined. If the capital exporting spot-market speculator uses his total wealth to buy foreign bonds or if the capital importing spot-market speculator takes on a debt up to the value of his wealth, then we have $|h^*| = a$, i.e., $|h| = aq/w_0^T$. It is not very likely that a speculator could succeed in making an even higher commitment by taking on additional debt for, in this case, some creditors would have to lend without security. Thus, for values of β that realistically are significantly below unity, the opportunity set of the forward speculator who makes use of banks specializing in speculation is larger than that of the spot-market speculator. Since, however, the latter can always become a forward speculator, expression (2) can be used quite meaningfully to describe the opportunity set of any type of currency speculator.

1.4. *Integrating the Speculation Problem into the Basic Multiperiod Approach*

Provided with this attractive result, an attempt is now made to integrate the decision problem of the speculator into the basic model of stochastic multiperiod planning. First we check whether the opportunity set described by (2) satisfies the requirement of stochastic constant returns to scale and write it in the form

$$(6) \qquad V = aQ$$

$$\text{where } Q \equiv q + \gamma\left(\frac{W_1^K - w_0^T}{w_0^T}\right), \quad \left|\gamma \equiv \frac{h w_o^T}{a}\right| \leq \frac{q}{\beta}.$$

This expression isolates an opportunity set of standard risk projects. Stochastic constant returns to scale prevail if this opportunity set is independent of the level of wealth, a. Since the admissible range for γ is obviously independent of wealth, the condition is satisfied only if, in addition, the level of the speculative commitment has no influence on the current forward rate, w_0^T, and the probability distribution of the future spot rate, W_0^K. We ensure this by the assumption of a competitive market structure.

A second assumption in the basic model is that the distributions Q at different points in time are stochastically independent of each other. If it is assumed that the speculators understand the operation of the market sufficiently well to take account of the arbitrage equation (4), then, in (6), w_0^T can be replaced by $w_0^K q^I/q^A$. As in the case of portfolio analysis, our assumption therefore implies that the speculators expect stochastically independent growth rates, that is, a random walk in the exchange rate[5]. The assumption implies that, after an increase in the current exchange rate, the speculators do not expect either that there will be a relatively smaller rise in the exchange rate than they conjectured before this increase or that the observed change is simply a sign of even greater relative changes in the future. In short, a unitary expectation elasticity is assumed[6].

Another condition required for the multiperiod planning model was that the opportunity set should contain at least one element that avoids with certainty the loss of all wealth. This condition is clearly satisfied, since each h in the range $0 \leq h < aq/w_0^T$ gives the desired protection.

With this, the integration of a wide class of speculation problems into our basic model is almost complete. We have only to assume additionally that, at each transactions date, the decision maker, after completing

[5] If the present approach is interpreted as referring to speculation in commodities futures then an assumption concerning the kind of price movement is unnecessary since there is no connection between forward and spot prices similar to (4).

[6] We thus decide for an intermediate solution between two extreme assumptions that have been favored in the literature. Cf. FRIEDMAN (1953, p. 175), ALIBER (1970, esp. pp. 304–306), and Nurkse, R., International currency experience. Princeton 1944. The last is cited according to ALIBER (1970, p. 304) and SOHMEN (1973, p. 73) since it was not available in the West German library system.

the previous contracts, thinks not only about his new commitment but also about the level of withdrawals for current consumption and that he attempts to maximize the multiperiod preference functional derived from the laws of Weber and Fechner. Then his implicit short-run aim is

$$\max_h E[U(aq + h(W_1^K - w_0^T))], \quad |h| \leq \frac{aq}{\beta w_0^T}. \tag{7}$$

where $U(.)$ is one of the time-dependent Weber functions. The implications of this aim will be discussed in the following sections.

2. *Optimal Speculation in the Ideal Case*

2.1. *The Two-Sided* (μ, σ) *Diagram*

To solve the maximization problem (7), the (μ, σ) diagram is considered again. Thus, from (2) the needed distribution parameters

$$E(V) = aq + h[E(W_1^K) - w_0^T] \tag{8}$$

and[7]

$$\sigma(V) = h \operatorname{sgn} h \, \sigma(W_1^K). \tag{9}$$

are calculated.

As is known, for an exact representation of the choice problem in a (μ, σ) diagram it is necessary for all distributions in the opportunity set to belong to the same linear class. To check this condition, calculate the standardized random variable $Z = [V - E(V)]/\sigma(V)$:

$$Z = \operatorname{sgn} h \frac{W_1^K - E(W_1^K)}{\sigma(W_1^K)}. \tag{10}$$

Equation (10) shows that, in general, there are *two* linear distribution classes rather than one. If a long position is taken ($\operatorname{sgn} h = +1$), then the distribution class to which the future spot rate (W_1^K) belongs applies, but if a short position is taken ($\operatorname{sgn} h = -1$), the distribution class defined by the 'mirror image' of the future spot rate applies. Only if W_1^K is symmetrically distributed, will these two distribution classes coincide.

[7] For the definition of the 'sign' function cf. footnote 36 in chapter II D.

But there is no reason to expect symmetry. Rather, the exchange rate distribution seems to be right skewed. This is already suggested by the fact that, at the level of zero, the exchange rate has a lower bound while there is no obvious upper bound. If a symmetry assumption is suitable at all, then it should refer to the logarithm of the exchange rate so that W_1^K and $1/W_1^K$ are equally distributed. Hence, the usual (μ, σ) diagram cannot be used to find a solution. But what about considering two diagrams?

This is done in Figure 6, where two indifference-curve systems of the kind depicted in Figure 7 in chapter III A are put together in an appropriate way. In this figure, it is assumed that the wealth distributions are bounded to the left, which, according to (10), implies that W_1^K is bounded from above and from below[8]. For the time being, the ranges of abnormal indifference curves where the BLOOS rule comes into operation are left out. Accordingly, it is temporarily assumed that the opportunity locus does not intersect with these ranges. In section B 3 other possibilities are considered in detail.

The right-hand section of the indifference-curve system refers to a long position $(h>0)$, and the left, that is the mirror image of the normal representation, refers to a short position $(h<0)$. Because of the asymptotic efficiency of the variance[9], the indifference curves are nearly symmetrical with respect to the ordinate when the coefficients of variation are small. But the higher the standard deviation for any given mean, the greater the effect the difference between the two distribution classes has on the indifference-curve shapes. When the distribution of W_1^K exhibits the described asymmetry, the end-of-period wealth distribution is right skewed in the case of a long position and left skewed in the case of a short position. In connection with the preference for right skewed distributions, suggested by Weber's law[10], this implies that the indifference curves are more curved in the left section of the diagram than in the right[11].

Since the points where the indifference curves enter the ordinate indicate the corresponding certainty equivalents, the indifference curves of

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[8] With W_1^K being unbounded from above, in the case of strong risk aversion $(\varepsilon \geq 1)$ there would be lexicographic pseudo indifference curves in the left section of the diagram. Under weak risk aversion $(0<\varepsilon<1)$, however, even in the case of an unbounded distribution of W_1^K, in the neighborhood of the ordinate there is always a range where the indifference curves have the normal shapes provided that, for $w_1^K \rightarrow \infty$, the density converges at least as fast as that of a normal distribution. Cf. the analysis towards the end of section III B 1.2.

[9] Cf. chapter II D 2.2.1.

[10] Cf. the corresponding remarks in the last third of section III A 2.3.2.

[11] That long and short speculation cannot be treated symmetrically was recognized by KENEN (1966, pp. 151 and 166).

both sections of the diagram that enter the ordinate at the same point can be considered as single indifference curves extending over both sections. Whenever the points representing end-of-period wealth distributions are situated on the same connected indifference curve, they are evaluated as being equal, no matter whether they are in the right or the left sections of the diagram.

Strictly speaking two arrows should be shown in the diagram, which are labelled $\sigma(V)$, start at the origin of the abscissa, and go in opposite directions. However, to obtain a scale that goes in one direction over the whole abscissa, the left-hand part is indicated as $-\sigma(V)$ and the right-hand part as $+\sigma(V)$, that is, in general as $\operatorname{sgn} h\,\sigma(V)$. This way, the indifference curves define a preference structure over the distribution parameters $E(V)$ and $\operatorname{sgn} h\,\sigma(V)$ that is identical with the one implied by the expected-utility criterion. Thus, for arbitrary distribution classes of W_1^K, the goal function (7) can be replaced by

$$\text{(11)} \qquad \max_h U[E(V), \operatorname{sgn} h\,\sigma(V)], \quad |h| \leq \frac{aq}{\beta w_0^T}.$$

By using the $(\mu, \operatorname{sgn} h\,\sigma)$ diagram the optimal speculative commitment

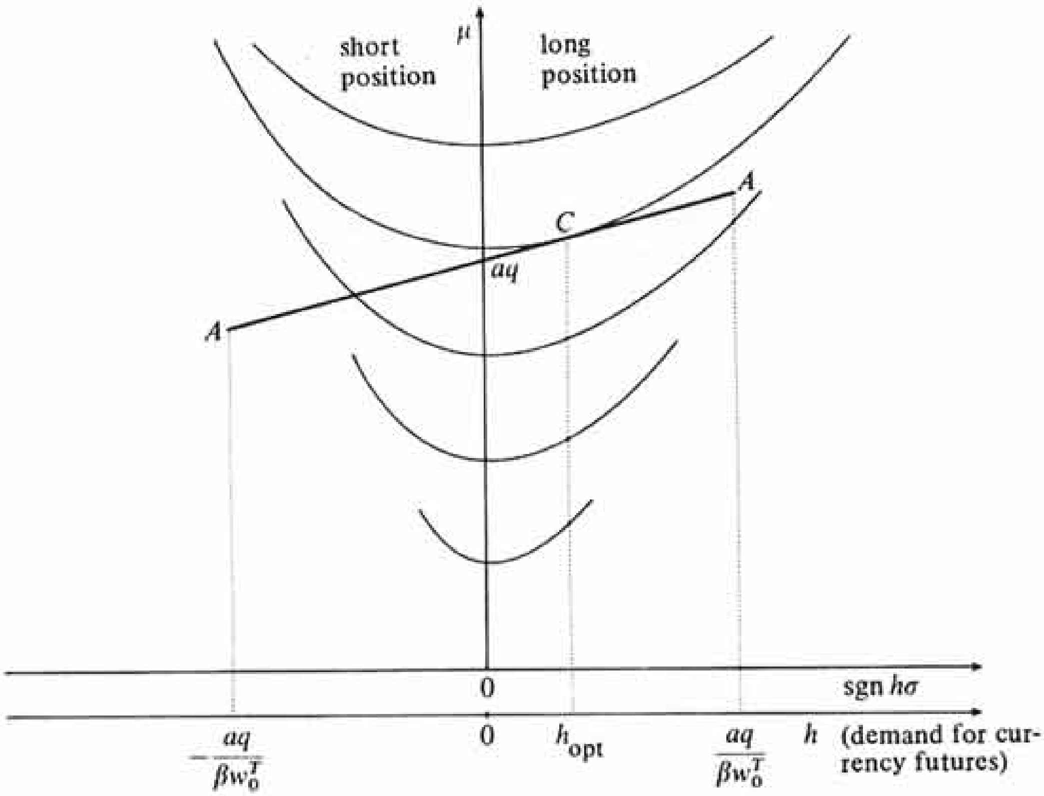

Figure 6

can easily be determined if the opportunity set is known. This set will now be considered. Calculating

$$h = \frac{\operatorname{sgn} h\, \sigma(V)}{\sigma(W_1^K)} \tag{12}$$

from (9) and inserting this expression into (8) we find that the opportunity locus is given by a straight line,

$$\mu = E(V) = aq + \operatorname{sgn} h\, \sigma(V) \frac{E(W_1^K) - w_0^T}{\sigma(W_1^K)}, \tag{13}$$

where account has to be taken of the constraint

$$\sigma(V) \leq \frac{aq}{\beta} \frac{\sigma(W_1^K)}{w_0^T} \tag{14}$$

that corresponds to the constraint $|h| \leq aq/(\beta w_0^T)$.

As an example for the case $E(W_1^K) > w_0^T$, the opportunity locus is represented by the line *AA* of Figure 6. Since, by assumption, this line does not enter the range of abnormal indifference curves, there are two possibilities.

Either, as in the diagram, the optimal point is determined by a tangency solution or it coincides with the right-hand end of the 'opportunity line'. In the latter case the speculator buys as many Deutschmarks in the forward market as the bank allows. In general, the number of Deutschmarks bought is shown by the ray parallel to the abscissa which was constructed by using the proportionality between $\operatorname{sgn} h\, \sigma(V)$ and h as given by (12).

2.2. *The Reaction of the Demand for Forward Currency to Changes in Expectations*

The influence speculators have on the spot and forward exchange rates is crucially determined by what they expect the future spot rate to be. For an evaluation and possible regulation of speculative trade, it is therefore useful on the one hand to have a theory concerning the formation of expectations and, on the other, to have a theory explaining how speculators react to a change in expectations. The former is beyond the scope of this book. The latter, however, is implicit in the approach developed above.

2.2.1. Changes in the Expected Spot Rate

It has already been shown in Figure 6 that a long position is advantageous to the speculator if $E(W_1^K) > w_0^T$. The kind of commitment that is chosen in the cases $E(W_1^K) < w_0^T$ and $E(W_1^K) = w_0^T$ can easily be determined.

According to (13) and (14) a reduction of $E(W_1^K)$ turns on the line AA in a clockwise direction, while at the same time its length changes, since the ends of the line move vertically. If $E(W_1^K) = w_0^T$, the line is horizontal. Because the connected indifference curves have a slope of zero at the ordinate, in this case the point of tangency coincides with the ordinate and hence $h_{opt} = 0$. If the expected spot rate is below the forward rate then the line AA slopes downwards to the right and the point of tangency C is in the left section of the diagram: a short position is advantageous ($h_{opt} < 0$). The result is summarized in the following expression

$$(15) \qquad h \left\{ \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} \right\} 0 \Leftrightarrow E(W_1^K) \left\{ \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} \right\} w_0^T .$$

Although (15) shows that the demand for forward Deutschmarks *globally* is a rising function of its expected spot rate, we do not know whether this function is monotonic. FELDSTEIN (1968, pp. 186 f.) pointed out that there may be counteracting income and substitution effects of a change in $E(W_1^K)$ so that there is a possibility that the demand for forward Deutschmarks is not everywhere a rising function of the expected Deutschmark spot rate[12].

For a general evaluation of speculation, the question is of great importance regardless of whether the speculators' abilities in forecasting the proper spot rate are estimated optimistically or pessimistically. The pessimist would stress that the expectation of a speculator as described by $E(W_1^K)$ is usually wrong and is subject to large fluctuations so that, from his point of view, it would be desirable if $\partial h / \partial E(W_1^K) = 0$, for then the transmission mechanism between expectations and forward rates is interrupted. The optimist, on the other hand, believes that speculators link changes in the forward rate with changes in the actual future spot rate, which requires $\partial h / \partial E(W_1^K) > 0$ if speculators are well-informed.

The reason for the indeterminateness mentioned by Feldstein is the generality of the preference hypothesis he used, which required nothing more than risk aversion. Fortunately, Weber's law provides us with additional information that gets rid of the indeterminateness: the pessimist's hopes are dashed and the optimist's hopes are confirmed.

To see why, consider Figure 7. There the original opportunity line AA with the point of tangency C moves counterclockwise towards BB since

[12] Cf. also LELAND (1971, pp. 260 f.) and HOCHGESAND (1974, pp. 116 f. and 128 f.).

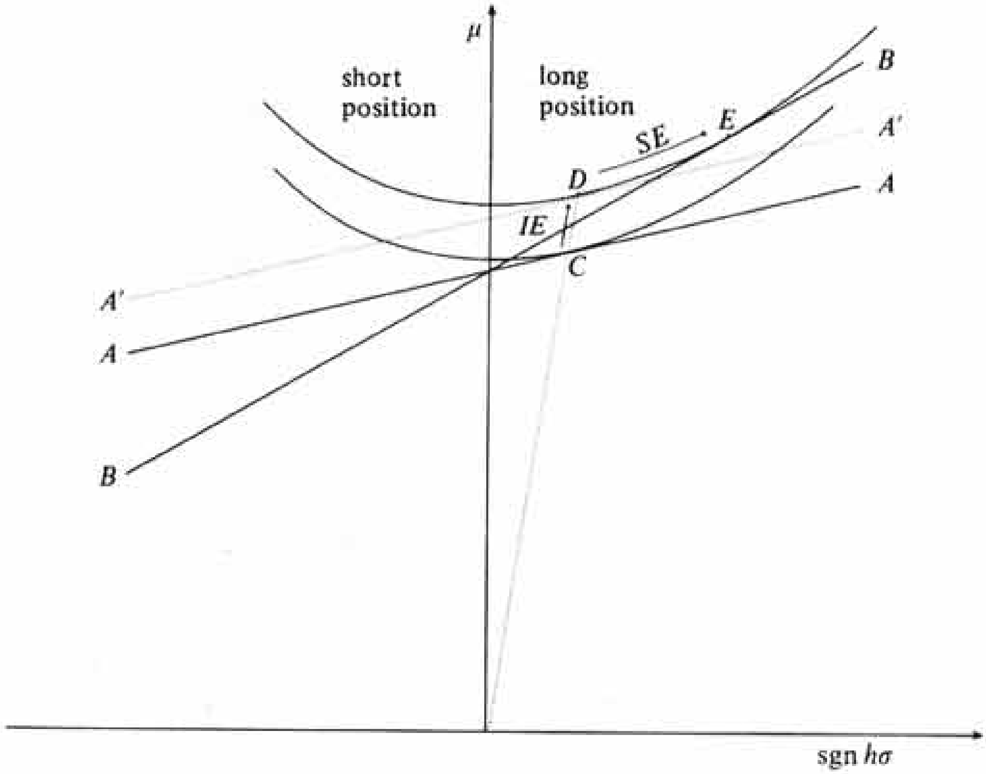

Figure 7

an increase in the expected spot rate is assumed. The new point of tangency is E. The movement from C to E can be divided into an income and a substitution effect. The income effect (IE) is represented by a parallel shift of the line AA to the position AA' and by the corresponding shift in the point of tangency from C to D. Because of the homotheticity of the indifference-curve system as implied by Weber's law, point D is to the right of point C: the income effect is positive. The substitution effect is represented by a movement of the opportunity line from position $A'A'$ to BB along a given indifference curve. The point of tangency accordingly moves from D to E. Because of the convexity of the indifference curves, E clearly is to the right of D. Hence the income and substitution effects reinforce each other and so we have $\partial h/\partial E(W_1^K)>0$.

In an analogous way, this reasoning can be used for the case of a short position. If the supply of forward currency is interpreted as a negative demand, the unambiguous result emerges that, concerning tangency solutions of the kind C and E, the demand for forward currency is a strictly monotonically increasing function of the expected spot rate. By inspection of (12) and (14), it is easy to see that this result cannot be maintained in the case of a corner solution. Speculators who have committed as much as their banks allow do not react to marginal changes in the expected spot rate. On the other hand these speculators are only a small proportion of all speculators; they cannot have an influence on the qualitative aspects of the market reactions to changes in expectations.

2.2.2. Changes in the Variance of the Future Spot Rate

Speculators base their behavior on a conjectured distribution of the future spot rate. The standard deviation of this distribution can be interpreted as a measure of their degree of confidence in the estimation of the mean of this distribution. Thus, from an allocative point of view, it is to be hoped that the influence that speculators have on the exchange rates is smaller the higher the standard deviation, for the less confident speculators are about their own forecasts, the higher the probability that they will create, rather than reduce, fluctuations in the time path of the exchange rate.

By reference to equations (12) and (13), it can easily be shown that the model speculator does not disappoint this hope.

With regard to the change in the initial tangency solution, there are two effects that, thanks to Weber's law, reinforce each other[13]. One is that, according to (13), the opportunity line in the $(\mu, \operatorname{sgn} h\sigma)$ diagram gets flatter when $\sigma(W_1^K)$ rises. It takes place regardless of whether the point of tangency was initially in the left or right section of the diagram. As is known from the previous section, as a reaction to this movement in the opportunity line, the point of tangency moves unambigously towards the ordinate. Thus $\sigma(V)$ is getting smaller. This effect, that, by itself, reduces the optimal commitment h, is reinforced by a second effect: according to (12), after an increase in $\sigma(W_1^K)$, each value of $\sigma(V)$ is associated with an absolutely lower value of h than before.

If, initially, the solution point is situated at an end of the opportunity line, then, for marginal changes in $\sigma(W_1^K)$, the speculative commitment is not affected. Independently of $\sigma(W_1^K)$ we then have $h = aq/\beta w_o^T$ or $h = -aq/\beta w_0^T$.

Since a corner solution, as a rule, does not occur for all speculators, we again find a clear-cut conclusion for the aggregate. If all speculators expect higher variances of the future spot rate the commitments in the aggregate are reduced regardless of whether these are long or short.

2.3. *The Wealth Effect and the Stability Problem*

From the application of our basic model to the portfolio problem (A 3.3.), we know that the optimal portfolio structure is independent of

[13] Cf., however, FELDSTEIN (1968, p. 187).

the decision maker's wealth. In the case of speculation a similar result can be found.

In Figure 6, an increase in the level of wealth (a) available after a subtraction of period consumption implies a parallel shift and lengthening of the opportunity line so that its ends (A) move upwards along rays through the origin. This can be checked directly by inspecting (13) and (14). Because of the homotheticity of the indifference-curve system, this also means that the point of tangency C moves upward along a ray through the origin. Hence the *demand* for Deutschmark futures rises in strict proportion to wealth. If initially, in contrast to Figure 6, $h_{opt} < 0$ had been the case, then analogous reasoning would have shown that the optimal *supply* of Deutschmark futures ($-h_{opt}$) rises in proportion to wealth.

This wealth effect has some relevance for the stabilizing effects of speculation[14]. If the speculator was already committed in the previous period, then at the beginning of the period his wealth before consumption depends on the current spot rate. Because of the constancy of the marginal propensity to consume out of wealth, this means that the level of the funds to be reinvested also depends on this rate. With a long position it rises, with a short position it falls. Hence the wealth effect implies that the current demand for currency futures depends on the current spot rate.

Suppose, before the decision point in time 0, the speculator expects $E(W_1^K) > w_0^T$. In this case, he plans to take a long position and, because of the assumption that the expectation elasticity is one, he sticks to this plan regardless of what the variates of the current spot and forward rates, w_0^K and w_0^T, happen to be. If the speculator's previous commitment was long, then, at point in time 0, his demand for currency futures is a rising function of the spot rate and, because of the arbitrage condition (4), also of the forward rate. Obviously, in this case, the wealth effect is destabilizing. If, however, the speculator was previously in a short position, then the reverse is the case. At point in time 0, his demand for Deutschmark futures is a falling function of the forward rate. The wealth effect is stabilizing.

Analogous reasoning can be applied to the case where, at point in time 0, the speculator decides to sell short. Thus we reach the general conclusion that the wealth effect has a stabilizing influence on the exchange market if speculators switch between long and short positions, and has a destabilizing influence if they stay with a given type of speculation.

[14] As far as is known, the wealth effect has been disregarded in the extensive literature on the problem of whether or not speculation is stabilizing. Reviews of the literature are given by HOCHGESAND (1974) and STEINMANN (1970).

3. *On the Possibility of an Excessively Short Position*

In this section, an aspect of the speculator's decision problem is studied that may bring about a particular preference for short positions. It is the BLOOS rule[15] that is responsible for this preference for it allows the speculator to shift part of the speculation risk on to the shoulders of others.

As we know, the BLOOS rule comes into operation only if the gross wealth distribution extends partly over the negative half of the wealth axis. Then, in the usual (μ, σ) diagram, the distribution is represented by a point below the border line[16] $\mu = \underline{k}\sigma$, where $-\underline{k}$ is the highest lower bound to the standardized end-of-period wealth distribution. We therefore need to think about where this border line is located in the two-sided (μ, σ) diagram and what shape the indifference curves have beyond it.

Since, in the present case, there are *two* standardized end-of-period wealth distributions according to whether the speculator holds a long or a short position, two lower bounds, $\underline{k}_L$ and $\underline{k}_S$, have to be distinguished. By using (10), these bounds can be derived from the distribution of the future spot rate W_1^K. Assume, to take a plausible[17] example, logarithmically symmetrical bounds:

$$\frac{E(W_1^K)}{1+\lambda} \leq W_1^K \leq (1+\lambda)E(W_1^K), \quad 0 < \lambda \leq \infty. \tag{16}$$

Then

$$\underline{k}_L = \frac{E(W_1^K)}{\sigma(W_1^K)} \frac{\lambda}{1+\lambda} \tag{17}$$

and

$$\underline{k}_S = \frac{E(W_1^K)}{\sigma(W_1^K)} \lambda, \tag{18}$$

so that we find the following border lines in the $(\mu, \operatorname{sgn} h\,\sigma)$ diagram:

$$E(V) = \underline{k}_L \operatorname{sgn} h\,\sigma(V), \tag{19}$$

$$E(V) = -\underline{k}_S \operatorname{sgn} h\,\sigma(V). \tag{20}$$

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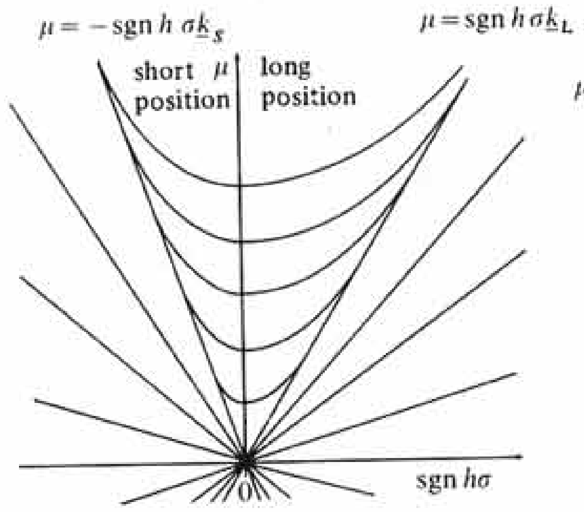
[15] Cf. chapter III B.

[16] Cf. expression (III A 44).

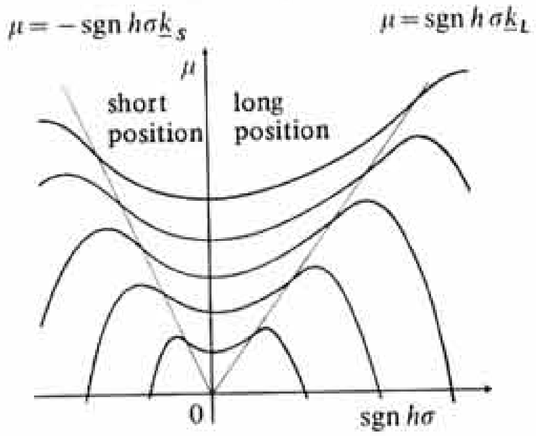
[17] Cf. the remarks in section 2.1.

They are both depicted in Figures 8 and 9. Beyond these border lines two types of indifference curves are possible[18]. In the case of strong risk aversion ($\varepsilon \geq 1$; Figure 8), there are pseudo indifference curves in the form of straight lines through the origin. In the case of weak risk aversion ($\varepsilon < 1$; Figure 9) there are genuine indifference curves that become concave at some stage, change the signs of their slopes, and eventually intersect the abscissa.



$\varepsilon \geq 1$

Figure 8



$0 < \varepsilon < 1$

Figure 9

The question now is, under which conditions does the opportunity set available to the decision maker contain end-of-period wealth distributions that map beyond the border lines (19) and (20). According to (13) and (14), the safety margin parameter β required by the banks is of crucial importance for this question. If use is made of this parameter, the question can be posed more precisely by asking which values of β define critical levels below which the opportunity line given by (13) and (14) goes beyond the right (19) or left (20) border lines. To calculate these levels, call them β_L^* and β_S^*, first combine (14) with (19) or (20), so that

$$\frac{E(V)}{\underline{k}} = \frac{aq}{\beta^*} \frac{\sigma(W_1^K)}{w_0^T}, \quad \underline{k} = \underline{k}_L, \underline{k}_S; \; \beta^* = \beta_L^*, \beta_S^*.$$

[18] Cf. Figures 10 and 12 in chapter III B. In Figure 8 above it is assumed that the funnel edges are tangent to the indifference curves. As we know from the discussion of Figure 12 in chapter III B this is a particular property which does not hold for all types of distribution if $1 < \varepsilon < 2$. The property is irrelevant for the present discussion.

Now replace $E(V)$ according to (8) and substitute

$$h = \operatorname{sgn} h \frac{aq}{\beta^* w_0^T}, \qquad \beta^* = \beta_L^*, \beta_S^*.$$

This is possible because the constraint $|h| < aq/(\beta w_0^T)$ from (7) is equivalent to (14). Finally, specify $\underline{k}$ by using the values given alternatively by (17) and (18). Then the result is

$$\text{(21)} \qquad \beta_L^* = \frac{E(W_1^K)}{w_0^T} \frac{\lambda}{1+\lambda}$$

and

$$\text{(22)} \qquad \beta_S^* = \frac{E(W_1^K)}{w_0^T}(1+\lambda) - 1.$$

In order to find out under which conditions the required safety margin satisfies its purpose, that is, when it is above the critical values given by (21) and (22), we assume for the time being that all agents involved estimate the same probability distribution for the future spot rate (idealized uncertainty) and consider two *extreme* cases.

1. Suppose $E(W_1^K) > w_0^T$. Then $\beta_L^* \leq 0$ if λ is small enough to render $E(W_1^K)/(1+\lambda) \geq w_0^T$. In this case, no safety margin need to be required by the banks, since the smallest possible spot rate exceeds the forward rate. It is true, since $\beta_S^* > 0$, a safety margin would be necessary to exclude negative gross wealth in the case of a short position. However, even if the banks did not require a safety margin, no one would engage in this type of speculation, for, at best, wealth would be maintained at the same level while the most likely outcome would be that it is reduced. This result holds irrespective of the fact that the speculator can shift some of the risk on to his bank's shoulders. An analogous argument can be given in the case $E(W_1^K) < w_0^T$ and $E(W_1^K)(1+\lambda) \leq w_0^T$ so that $\beta_S^* \leq 0$. Here, too, we find that for λ sufficiently small no safety margin is required.

2. Another extreme case is $\lambda \to \infty$. For a long position a safety margin of 100% ($\beta_L^* = 1$) will then be needed. If, in the worst of all cases, the value of the foreign currency falls to zero, the wealth of even the most courageous speculator would be just enough to buy, as contracted, the devalued foreign currency and to throw it into the waste paper basket. The situation is very different in the case of a short position. Here, in

the limiting case, an infinitely high safety margin is needed; this means that the bank should not allow short speculation at all.

By their very nature, both of the extreme situations illustrated do not reflect normal expectations about changes in the exchange rate. Nevertheless, there are more realistic examples, which do confirm the observation that, in the case of a short position, it is hardly possible to avoid transferring some of the speculator's risk to other's shoulders. Suppose that a doubling or a halving of the spot rate are considered to be the most extreme possibilities ($\lambda = 1$) and assume, for simplicity, $E(W_1^K) \approx w_0^T$. Then for a long position a minimum safety margin of about 50% is needed while about 100% is needed for a short position. Comparing this with the, in practice, more realistic margin of 20%, we find from $|h| \leq aq(\beta w_0^T)$ that, in the case of a long position, the speculative commitment can be 2 1/2 times and, in the case of a short position, 5 times as large as it would have to be if a risk transfer were to be excluded. Formally this means that the opportunity line in the $(\mu, \operatorname{sgn} h\sigma)$ diagram exceeds the right-hand border line by 2 1/2 and the left-hand border line by 5 times the corresponding distance to the ordinate!

In the light of this dramatic change in the previous assumption that the opportunity line does not go beyond the range of normal indifference curves, the previous results definitely need to be rechecked. Little happens, when there is strong risk aversion ($\varepsilon \geq 1$; cf. Figure 9). Since all pseudo indifference curves outside the funnel are subordinate to those inside, a solution is only possible within the funnel, in the extreme case at the edges (cf. footnote 18). Equation (15) will then continue to be true. Speculators in this case are so afraid of losing their wealth that, being able to avoid some of their obligation in the case of a total disaster, has no appeal for them. Unfortunately, it was this very hypothesis of strong risk aversion that was shown to be rather unrealistic[19]. Under weak risk aversion ($0 < \varepsilon < 1$), optimal solutions outside the funnel are clearly possible.

Figure 10 shows a particularly curious situation. There, $E(W_1^K) > w_0^T$, so that a long position with the point of tangency C could be expected to be optimal. But, in fact, the opportunity line reaches the highest indifference curve at its left end, at point P. Not a moderate long position but a short position of the highest possible extent is optimal. This is the case mentioned in the introduction.

The reason for this result is the assumption of logarithmically symmetrical bounds to the probability distribution of the spot rate. It implies

[19] Cf. section III B 2, towards the middle, and section IV B 2.3.2.

that, in the case of a short position, the gross wealth distribution is strongly left skewed. As we know, this property is a disadvantage if the gross and the net distributions coincide, an aspect that is represented by the stronger curvature of the indifference curves in the left section of the figure[20]. However, if the gross distribution can take on negative values, then, because of the BLOOS rule, this disadvantage can be compensated or even overcompensated.

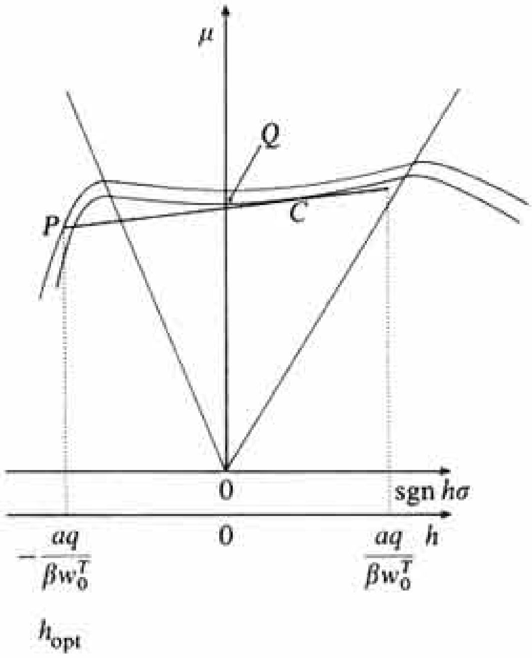


Figure 10

That there is the possibility of an overcompensation can be shown by a thought experiment. Consider, as a first step, the case of a risk neutral decision maker ($\varepsilon = 0$). For him, the indifference curves within the funnel are horizontal and outside they bend downwards, since the kink in the utility curve, brought about by the BLOOS rule, clearly implies risk loving behavior[21]. Assume that the opportunity line the decision maker faces is also horizontal, $E(W_1^K) = w_0^T$, and that, at its right-hand side, it just ends at the edge of the funnel ($\beta = \beta_L^*$). Then, since (21) and (22) give $\beta_S^* = \lambda > \lambda/(1+\lambda) = \beta_L^* = \beta$, the opportunity line has to go beyond the left-hand edge of the funnel. The situation is illustrated in Figure 11. Obviously, the optimal point is at the left end of the opportunity line, i.e., at point P. It is clearly better than, for example, point Q which is on the ordinate.

[20] Cf. section 2.1 above.

[21] To see that there is a negative slope consult equation (III B 5).

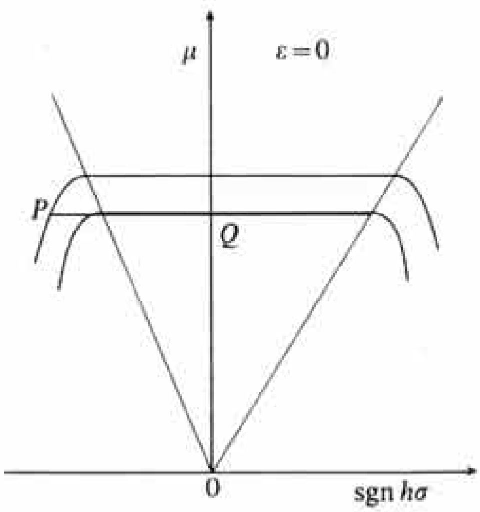


Figure 11

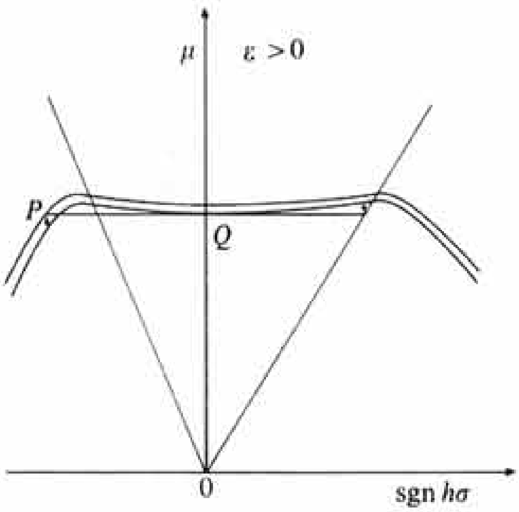


Figure 12

Now consider the second step. Rather than setting $\varepsilon = 0$, we assume ε is slightly above 0. Then, for any given ratio $\mu/(\text{sgn}\, h\sigma)$, in the right-hand section of the diagram, the indifference-curve slope is higher and, in the left-hand section, it is lower than before, while at the ordinate it remains zero. With a strong increase in ε, it could happen that P is situated on the same, or even on a lower, indifference curve than Q. But, for a sufficiently small increase in ε, the indifference curve passing through Q will, as in Figure 11, still be below P. Figure (12) represents the new situation.

In the third and last step, change the position of the opportunity line by reducing the initial wealth a and increasing the expected spot rate $(E(W_1^K) > w_0^T)$. Via the movements of the ends of the opportunity line, as indicated by the arrows in Figure 12, we are then indeed able to reproduce the situation depicted in Figure 10.

This raises considerable doubt concerning the functioning of the market in currency futures. If speculators choose a short position simply because they hope they will not have to meet their liabilities in certain cases, they cannot be expected to properly link the forward rate with the future spot rate. But, in fact, the problem is even more complex.

The suspicion of a malfunctioning of the market seems to vanish if we take into account the behavior of banking firms. Banks will have a strong interest in avoiding being burdened with the speculator's insolvency risk, so they require a safety margin just high enough to prevent this from happening. Actually, the safety parameter β could be considered as a measure of the most extreme changes in the spot rate that the banks think possible. Unfortunately, however, there are two reasons why this argument cannot really eliminate the doubts about the performance of the market raised by the above analysis.

The first is that the banks themselves, particularly when they are facing insolvency problems, may have an incentive to take advantage of an excessively short speculation. The recent bankruptcy of the West German *Herstatt Bank* that had bad luck in an excessively short speculation seems to be a good example.

The second is that banks and speculators might have different ideas about the possible spot rates. The safety margins banks require at best indicate the changes in the spot rate they consider possible, but these margins do not provide information on what the speculators think. If both types of agents calculate with different probability distributions, it may well be that speculators believe the situation is as depicted in Figure 10 while banks believe that they are secure.

There does not seem to be a straight-forward way of evaluating this possibility from a welfare point of view, even if we confine ourselves to the Pareto criterion. From an extremely subjectivist position, an excessively short speculation is not a disadvantage, either from the viewpoint of the speculator or from the viewpoint of the bank; otherwise a contract would not have been made. The Pareto criterion may, however, also be used in a more objective and less tautological form. Suppose, after an exchange of information between bank and speculator, both parties agree on the probability distribution of the spot rate. Then, the decision in favor of a speculative commitment that, for the speculator and the bank together, brings about an expected net loss must indicate an objective deterioration for at least one of them. If the exchange of information really took place the excessively short speculation would no longer occur. But it seems hard to imagine such an exchange of information taking place, for the speculator who plans an excessively short commitment has no incentive to reveal his information.

4. *Summary*

With the theory of currency spot and forward speculation another area has been investigated to which the previously developed approach can be meaningfully applied. For this application it seemed useful to derive a two-sided (μ, σ) diagram by means of which the optimal speculative commitment, whether long or short, can easily be found without assuming a particular distribution class for the spot rate estimated by speculators. A number of results were achieved that, although formulated with respect to forward speculation, are equally relevant for the behavior of spot market speculators, since these may be interpreted as forward speculators engaged in interest arbitrage.

When sufficiently high safety margins are required by the banks, a long position is advantageous to the speculator if the expected spot rate exceeds the observed forward rate, and a short position is preferable in the reverse case. Because of the preference structure implied by Weber's law, the demand for forward currency reacts normally to a change in the expected spot rate, that is, it rises when the expected spot rate increases. An increase in the assumed variance around the expected spot rate leads to a reduction in the speculative commitment, irrespective of whether the speculator takes a long position or a short position.

If the safety margin required by the bank is not high enough to make negative variates of the speculator's gross wealth distribution impossible, or if the bank itself engages in speculation, then these results may not hold. The speculator may well prefer to risk an excessively short commitment although his expectation of the future spot rate is above the forward rate and although his preferences are characterized by a concave von Neumann–Morgenstern utility function. This is another implication of the BLOOS rule.

The speculator's demand for forward currency *ceteris paribus* is proportional to his wealth. This wealth dependence is important for the question of whether speculation has a stabilizing or a destabilizing effect on the spot and forward rates, since, in the case of repetitive speculation, current wealth is determined by the current level of the spot rate. The wealth effect stabilizes if the speculation changes between long and short positions. It destabilizes when a particular type of commitment is maintained.

Section C
Theory of Insurance Demand

One of the most important and most obvious accomplishments of risk theory is to explain why the insurance business is rewarding for both the insurance purchaser and the insurance company. For this reason, the situation of the insurance purchaser has frequently been used in this book to illuminate the discussion[1]. Now an attempt is made to give a more systematic and comprehensive analysis of insurance demand. Section C 1 considers the determinants of insurance demand for given risks and section C 2 extends the analysis to the case of endogenous risks where the household can decide not only, as usual, the optimal rate of consumption, but also can choose between alternative loss prevention policies and insurance contracts.

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[1] Cf. chapter II C 1.2, II C 1.3, and III B 1.1.