Lecture 2: Multidimensionality, PVM and BPM

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June 8, 2010

Today's program

- Multidimensionality
- Probabilistic voting model
- A first differentiated candidates model: The binary policy model

Multidimensional issues

Example

- *Legislature with 3 members has to divide \$100 among members (districts) for "pork" projects (local public goods)*
- *A proposal is a* 3×1 *vector whose entries sum to no more than 100 (i.e., a feasible distribution of the money to members)*
- *Each legislator prefers more money for his own district, does not care about allocation of the remaining money (i.e.* $x \succ_i y \iff x_i > y_i$*)*

Many issues/ spatial setting

Example

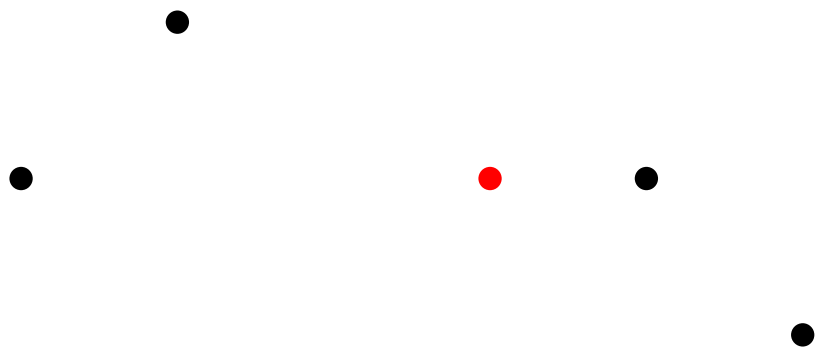
Dimension 1: Fiscally conservative/big spender
Dimension 2: Socially conservative/liberal

Is the "most central voter" (the median in each direction) again decisive?
$\Rightarrow$ No. Generically, no Condorcet winner exists if the issue space is multidimensional, even if all voters have single peaked preferences.

McKelvey–Plott–Theorem

Theorem (McKelvey, Plott)

For a Condorcet winner x^* in a multidimensional setting to exist when voter indifference curves are circles, the voters' bliss points must be radially symmetric around x^*.



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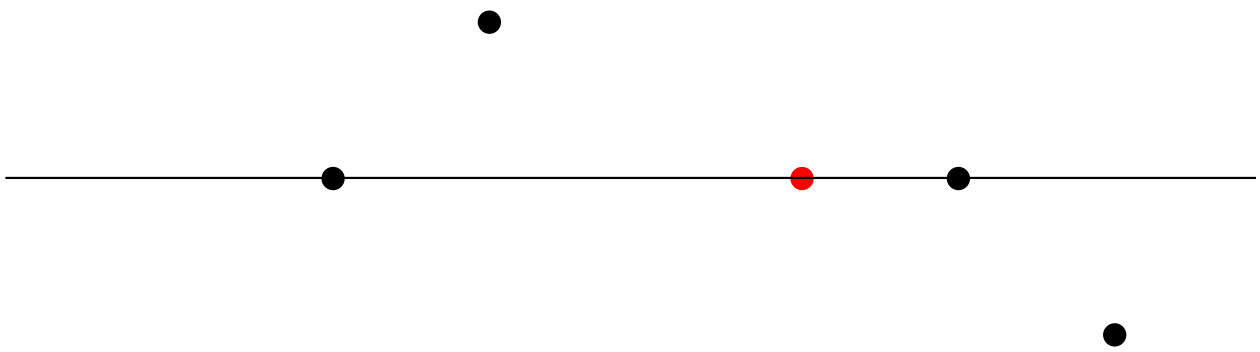

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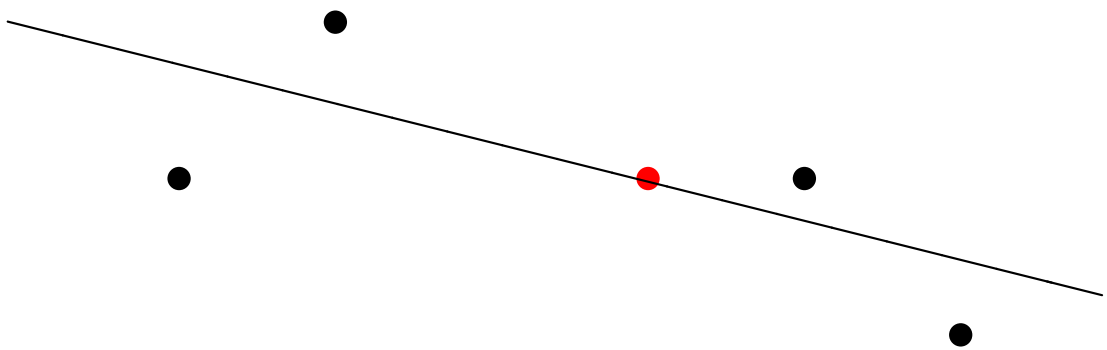

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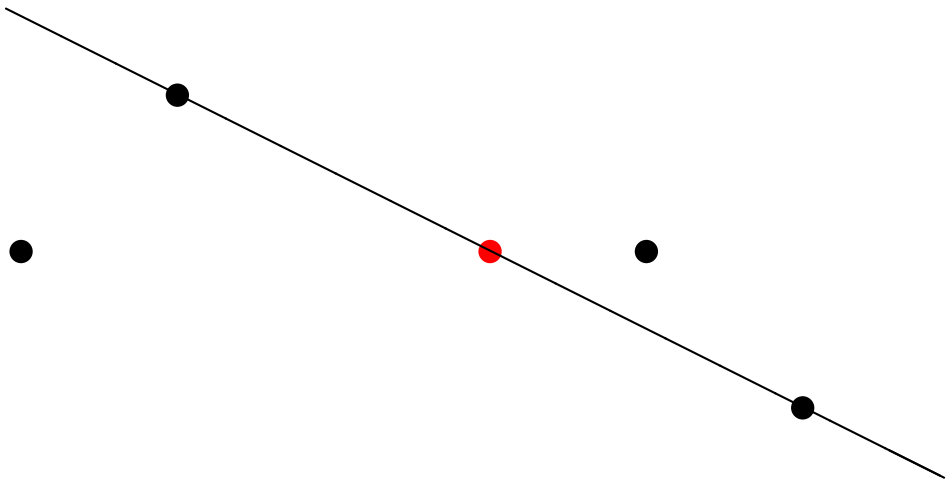
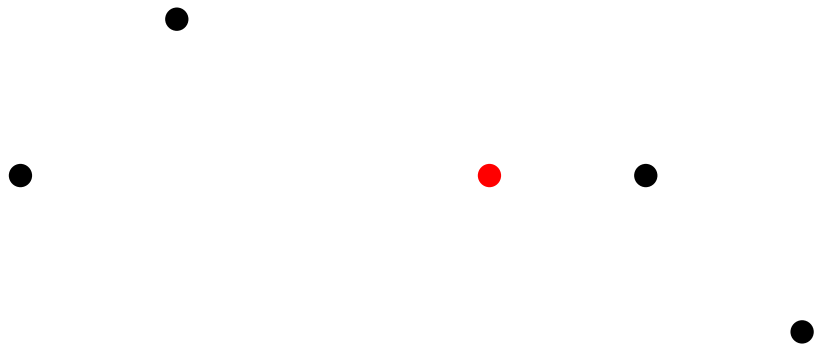


Without radial symmetry



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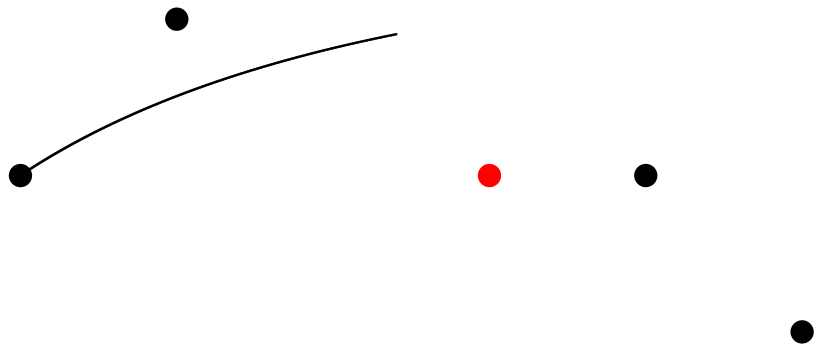

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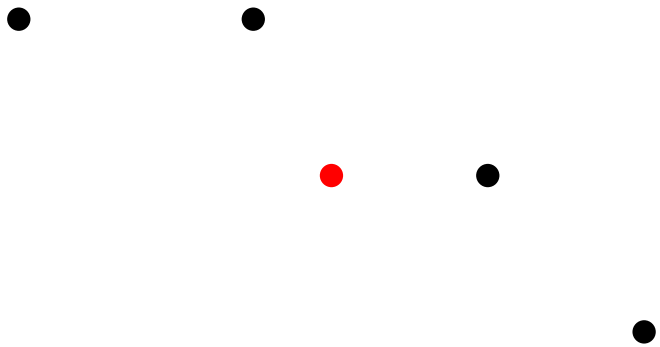

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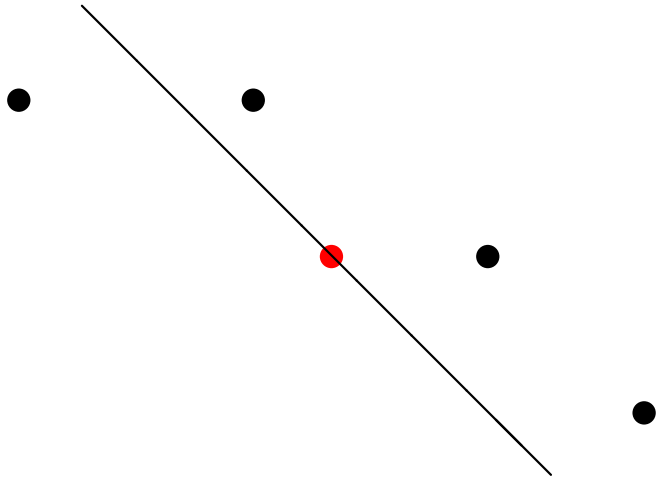

Probabilistic voting

Important tool to "smooth a model"

Deterministic voting: Vote for the candidate with the nearer platform (with certainty)

Probabilistic voting: For each voter i there is a cdf $F^i(\cdot)$ that gives the probability of voting for candidate A.

Depends on $W_A^i - W_B^i$.

$W_A^i = W^i(q_A)$: utility of i when A's platform is implemented.

Possible micro-foundation: There are other components of policy that cannot be changed, and voters may have a preference for one or the other candidate.

Probabilistic voting

A: Maximize expected votes:
$\sum_{i=1}^{I} F^i(W^i(q_A) - W^i(q_B))$
B *minimizes* this expression
First order conditions:
$\sum_{i=1}^{I} f^i(W^i(q_A) - W^i(q_B))\frac{\partial W^i}{\partial q_A} = 0$
Symmetric equilibrium ($q_A = q_B \Rightarrow W^i(q_A) - W^i(q_B) = 0$)
Equivalent to maximization of a weighted sum of utilities, where $f^i(0)$ is the weight of voter i.
Voters that can likely be swayed by a favorable platform receive more weight than (likely) partisans.

Krasa and Polborn (JET 2010)

Weighted-issue preferences

$$u(a^j, \theta) = -\sum_{i=1}^{\infty} \lambda_i \left| a_i^j - \theta_i \right|$$

- Voters care about specific issues
- Weight of each issue may depend on preference type
- Concept behind internet-based platform comparisons (e.g., www.wahl-o-mat.de, smartvote.ch, stemwijzer.nl)

Majority-efficiency

Definition

$a \in A^j$ is majority-efficient if, for all other policies a' that candidate j could choose, a majority prefers a to a'.

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- Positive interpretation: Generalization of "moving to the middle" in spatial model
- Candidate-related concept: Condorcet winner among those policies that a candidate can choose
- Majority-efficiency measures *institutional efficiency* of democracy: Does the majority get what they want ?

Existence of majority-efficient positions

Proposition (3)

Assume initial configuration of preferences has a majority-efficient policy. Shifting weight to preference types that agree with the majority-efficient policy on more issues preserves majority-efficiency.

Example

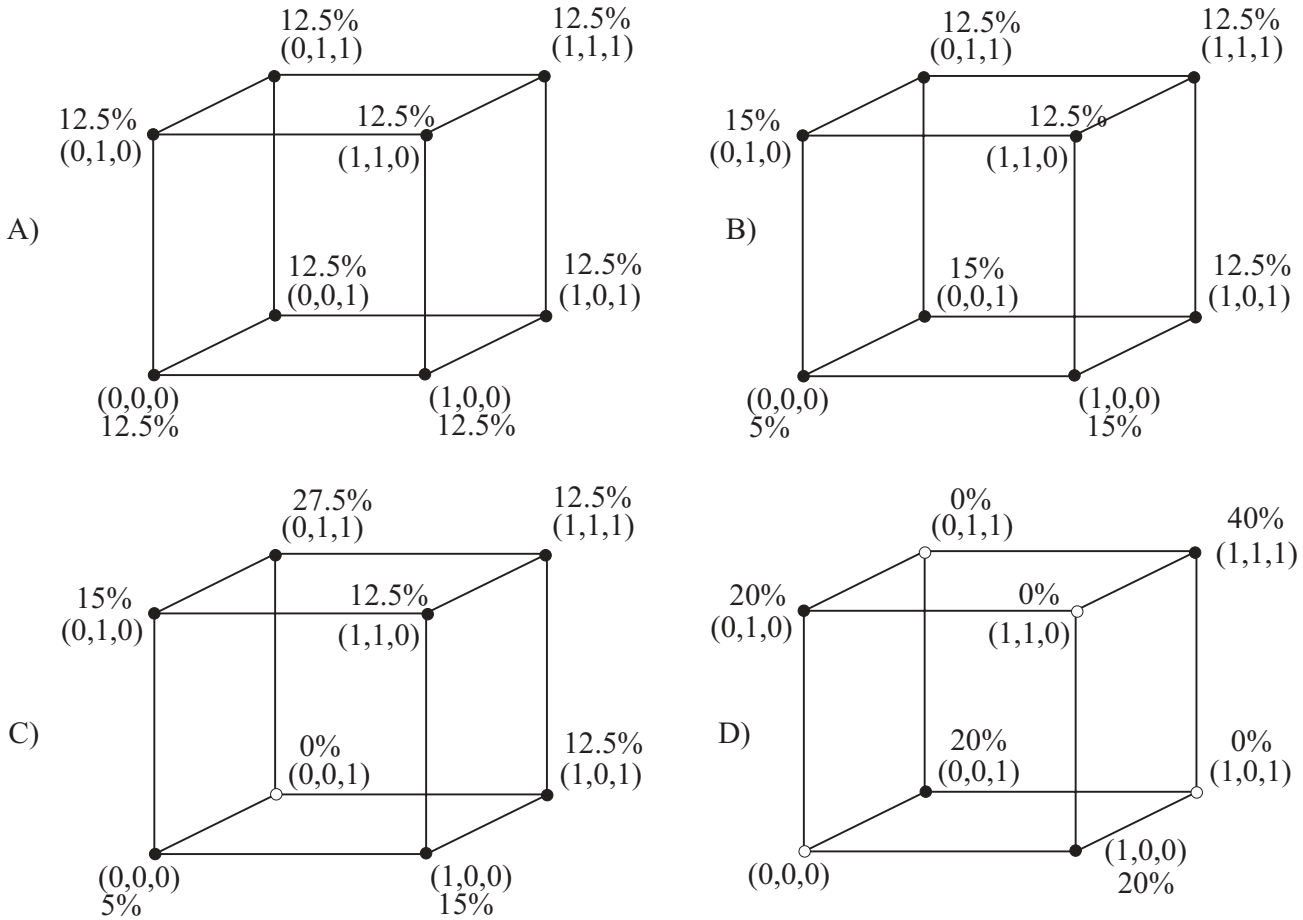
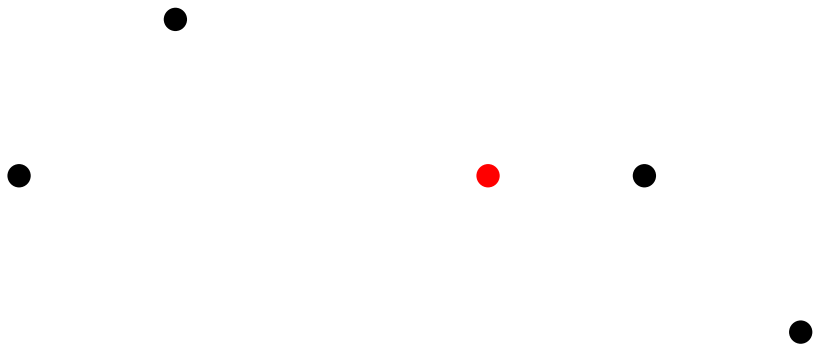


Figure: Existence and Non-Existence of Majority-efficient Policies

Isn't this obvious?

No: Contrast with Plott (1967), in which CW exists if and only if voter bliss points are radially symmetric around a "median in all directions"



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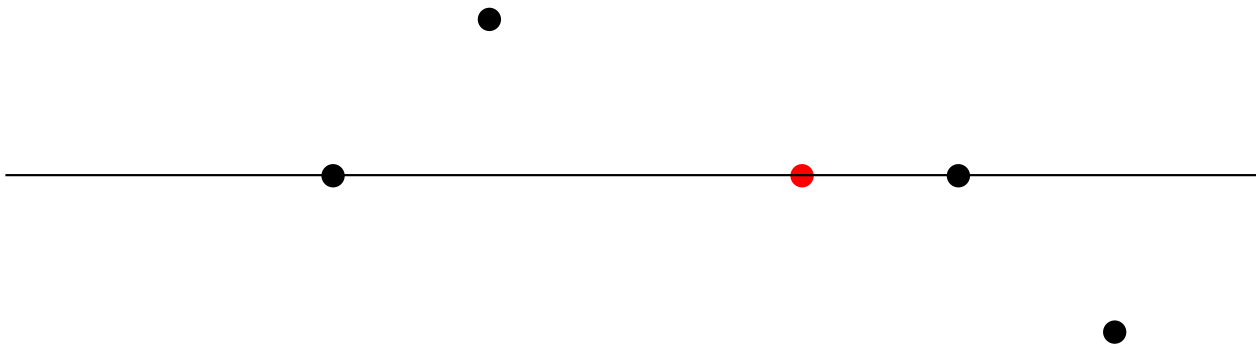

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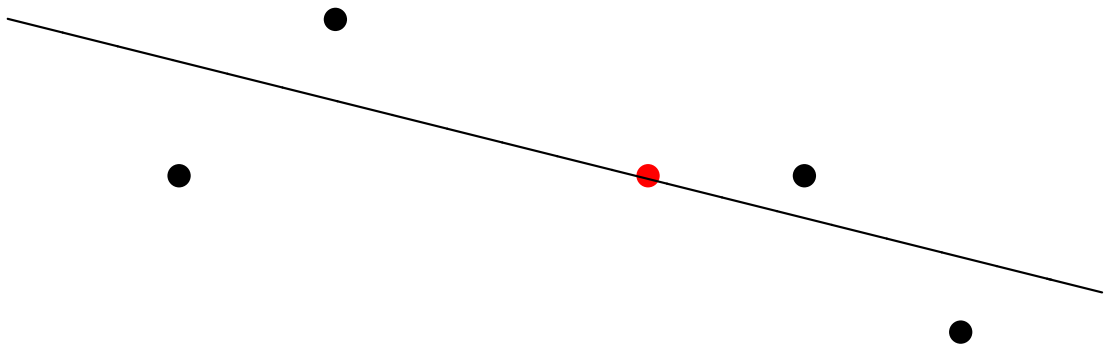

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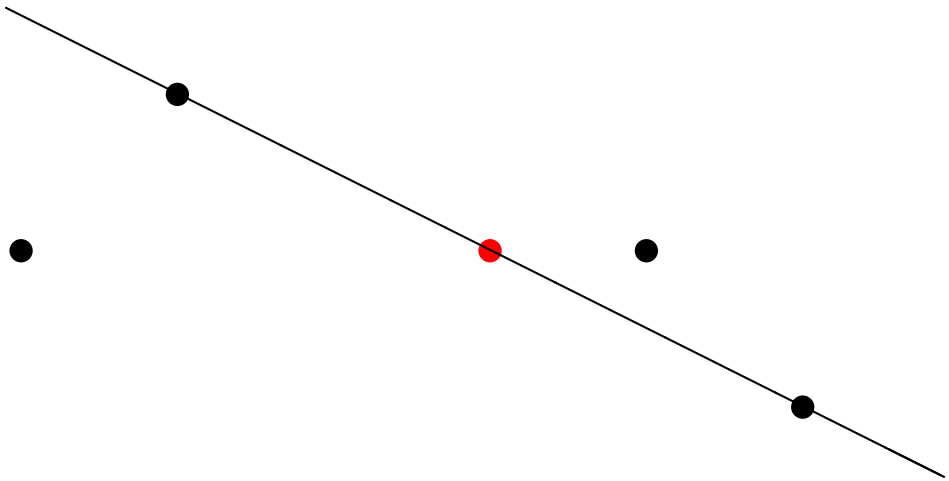

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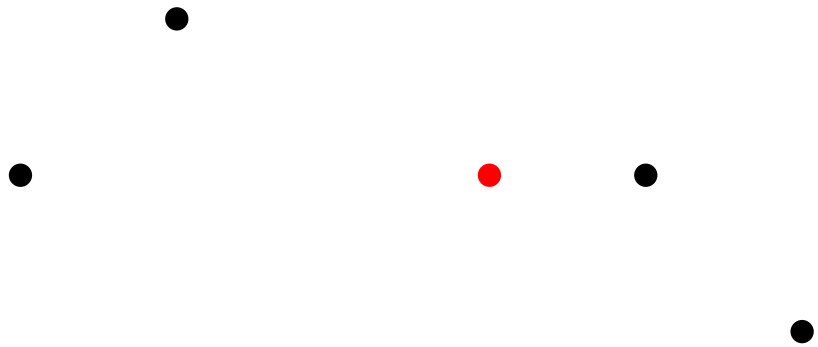
Now: Shift some voter closer to the median. This usually destroys radial symmetry.



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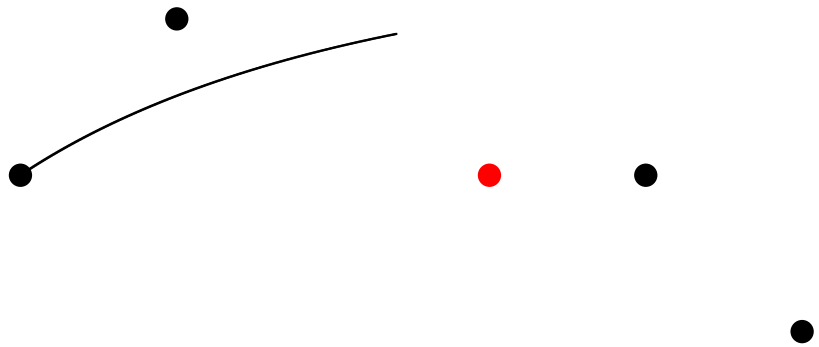

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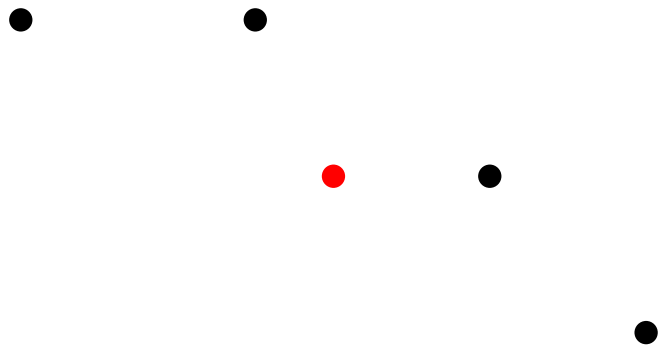

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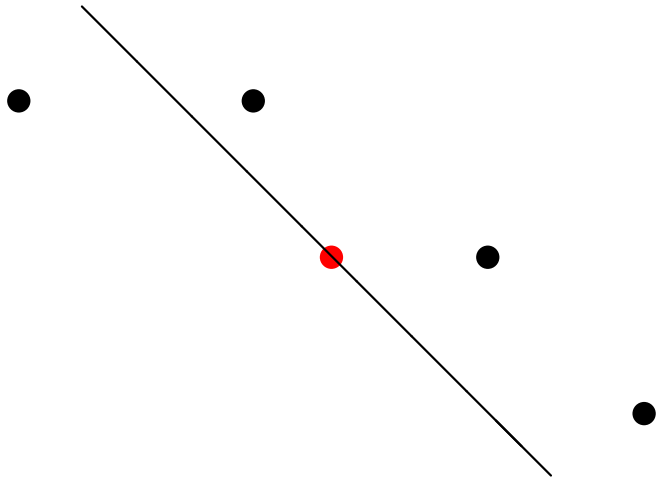

Other existence results

- If there are one or two flexible issues, a majority-efficient policy always exists
- If the distribution of ideal points is independent across issues, a majority-efficient policy always exists

Do candidates choose majority-efficient positions? – Identical candidates

Proposition (4)

Suppose that $A^0 = A^1$:
Then (a^0, a^1) is a pure strategy equilibrium if and only if both a^0 and a^1 are majority-efficient.

Same fixed positions + pure strategy equilibrium $\Rightarrow$ Majority-efficiency

Do candidates choose majority-efficient positions? – Nonidentical candidates

- Fixed issue differences between candidates generate *core supporters* and *swing voters*.

Definition

Suppose one flexible issue. Voter type $\tau = (\lambda, \theta)$ is a swing voter for Candidate j if

$$-\sum_{k=1}^{I-1} \lambda_k |\theta_k - a_k^{-j}| < -\sum_{k=1}^{I-1} \lambda_k |\theta_k - a_k^{j}| < -\sum_{k=1}^{I-1} \lambda_k |\theta_k - a_k^{-j}| + \lambda_I .$$

- Politicians care whether a position they take wins or loses more voters.
- Don't care about infra-marginal individuals.

Majority-efficiency when one policy is flexible

Proposition (5)

Suppose: Candidates are flexible only in issue I;
ξ_j: Proportion of Candidate j's swing voters who prefer $a_I = 0$.

- $(0,0)$ *is a vote-maximizing equilibrium if and only if*

$$\frac{\mathrm{SV}_1}{\mathrm{SV}_0} \in \left[\frac{1-\xi_0}{\xi_1}, \frac{\xi_0}{1-\xi_1}\right]. \tag{1}$$

- *Analogous condition for* $(1,1)$
- $(0,1)$ *and* $(1,0)$ *are almost never pure strategy equilibria*

Reason 1 for majority-inefficiency

Nonrepresentativeness of swing voters.

Example

Issue 1: Candidate 0 fixed at 0, Candidate 1 fixed at 1.
Issue 2: Both candidates flexible.
Suppose that all voters who prefer 0 on issue 2, and none of those who prefer 1, are swing voters. ($\xi_0 = \xi_1 = 1$)
$\Rightarrow$
Candidates choose $a_2 = 0$ (even if 0 is the minority position)

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Example

Suppose $\xi_0 = \xi_1 = 0.6$ (and equal to population at-large)
$\Rightarrow$ 0 is majority-efficient
$(0,0)$ is a vote-maximizing equilibrium if and only if

$$\frac{\mathrm{SV}_1}{\mathrm{SV}_0} \in \left[\frac{2}{3}, \frac{3}{2}\right].$$

Reason 2 for majority-inefficiency

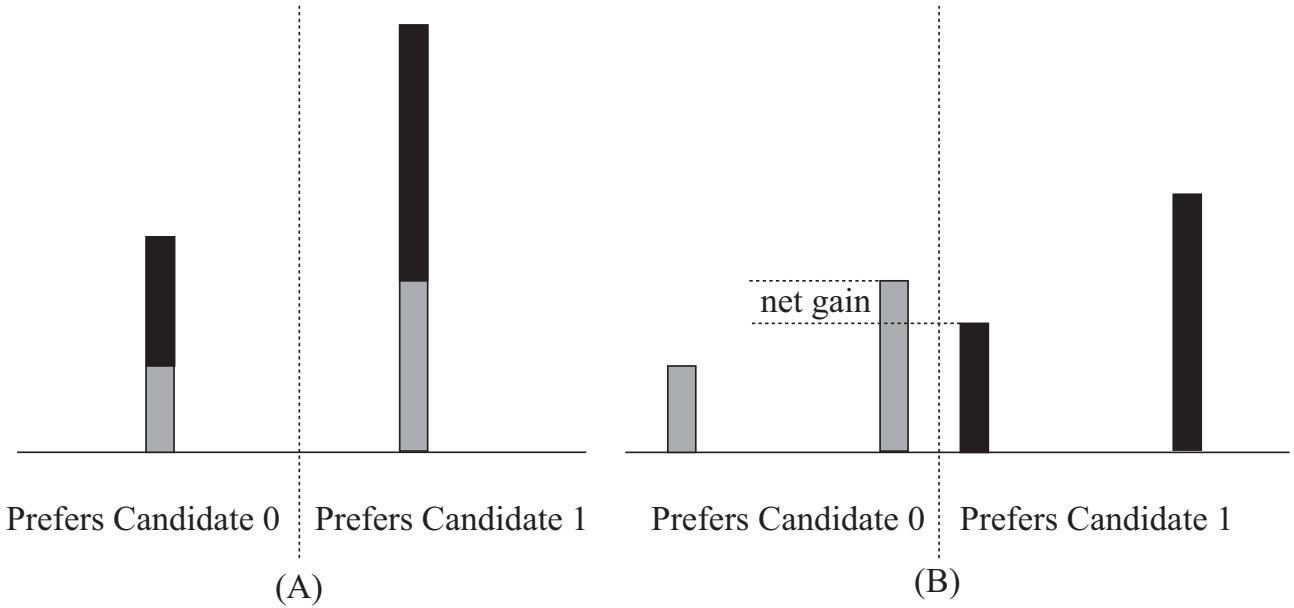

Figure: Net Preference Distribution for Candidate 1

Conditions for majority-(in)efficiency

Proposition (6)

Suppose ideal points are independently distributed across issues.
Furthermore, both candidates are fixed on the same set of issues F.

1. *Suppose C0 is "stronger" on fixed issues in terms of both size of majority support and importance of issues (matching condition).*
   Then Candidate 0 wins by choosing a majority-efficient policy.
2. *Whenever this condition is not satisfied, there are robust cases such that, if Candidate 1 selects a majority-efficient policy, then Candidate 0 wins if and only if he selects a majority-inefficient policy.*

Plurality vs. runoff election systems

- Questions:
  - What is the effect of "spoiler candidates"?
  - Which electoral system is better? (More likely to produce majority-efficient outcomes)

Plurality vs. runoff – results

- Without fixed positions, the result under runoff rule is at least as good for a majority than the result under plurality rule.
- With only fixed positions, the result under runoff rule is at least as good for a majority than the result under plurality rule.
- With both fixed and flexible positions, the result under plurality rule may be better than the result under runoff rule.

Plurality vs. runoff – Intuition in the binary policy model

- Setting: Two "main" candidates, one "spoiler" (unacceptable for majority, but liked by a minority)
- Under runoff rule, the main candidates ignore the spoiler who will disappear after the first round
  $\Rightarrow$ choose positions according to set of swing voters in 2-way race
- Plurality rule: Spoiler removes some voters from the set of swing voters between the main candidates.
  Smaller set of swing voters may (or may not) be more representative of electorate at-large
  $\Rightarrow$ May lead the main candidates to adopt more moderate positions
- Spoiler may be good for efficiency!

Plurality vs. runoff – Significance

- There is a large literature about "optimal" voting systems (e.g., approval voting, Maskin on extended simple majority rule)
- All these papers keep the candidates fixed as they change the voting system
- But: Voting system can be expected to influence the available choices as well
- Even a system that does "very well" in terms of selecting the CW *from a given set of candidates* may not necessarily be that good *overall*