Lecture 1: Spatial model and median voter theorem

Mattias Polborn (Illinois)

June 1, 2010

Introduction and motivation

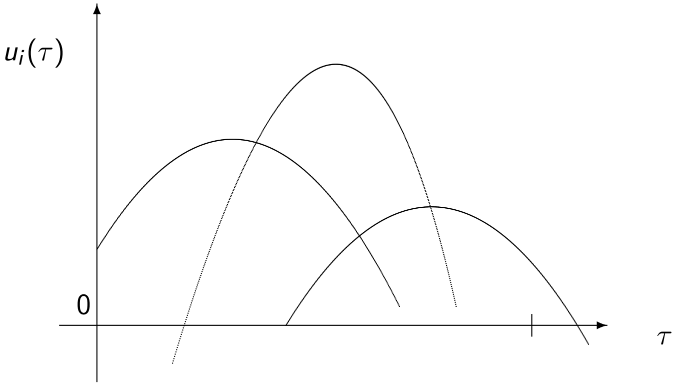
- A long history of analyzing desirable and undesirable properties of voting systems (Condorcet, Borda)
- Around 1950: Arrow's theorem. There is no social preference aggregation mechanism that simultaneously satisfies
  - Completeness
  - Transitivity
  - Pareto efficiency
  - Independence of irrelevant alternatives
  - No dictator

Introduction and motivation

- Completeness axiom requires that mechanism "works" *for any* constellation of preferences that voters might have
- Change of completeness axiom may admit aggregation procedures that satisfy T, P, IIA, ND.
- Of course: The class of admissible preferences should still be large and empirically important!
- Duncan Black (1948): Single-peaked preferences on one policy variable

The spatial model – single-peaked preferences

Example: A community chooses a proportional tax rate τ and uses the revenue to buy a public good. Preferences over tax rates and implied spending are plausibly *single peaked*.



The spatial model – Single-peaked preferences

Example: A community chooses a proportional tax rate τ and uses the revenue to buy a public good. Preferences over tax rates and implied spending are plausibly *single peaked*. Generally, an individual's preferences can be made "single-peaked" (re-label the options). The "single-peakedness assumption" means that the ordering is the same for all voters.

The spatial model – Median voter theorem

Definition

A **Condorcet winner** *is a proposal that would beat every other proposal in a pairwise election.*

The following theorem is due to Black (1948)

Median Voter Theorem

Suppose that every voter's preferences are single peaked with respect to the same ordering of options, and order voters with respect to their bliss points.
Then the bliss point of the median voter is the Condorcet winner.

Voting on public good provision

Example

N voters, everyone has the same preferences
$U(x, G) = x + \ln(G)$
x: private consumption
G: public good
y_i: income of individual i proportional taxation: $G = \sum \tau y_j$
(state budget constraint)

Indirect utility of individual i:
$V_i(\tau) = (1 - \tau) y_i + \ln(\tau \sum y_j)$
Optimal tax rate for i:
$-y_i + \frac{1}{\tau} = 0 \Rightarrow \tau_i^* = \frac{1}{y_i}$
Richer voters prefer a smaller tax rate. The median rich individual determines the equilibrium tax rate, $\tau = 1/y_m$.

Candidate competition

Two candidates, plurality rule
One-dimensional policy space

Politicians make promises as to which policy they will implement if elected

In equilibrium, both politicians will try to get as close to the median voter as possible, because the median voter is decisive for which candidate wins.

Note that the reason for this is not that there are necessarily many people with moderate policy preferences!

Sequential position games (Osborne)

- Variations in timing: Candidates choose sequentially from $\{Out\} \cup [0, 1]$.
- Candidates run for office whenever they have a positive probability of being elected, but have a higher utility from not running than losing for sure.
-
    - 2 candidates: Both choose median
    - $3, \ldots, N$ candidates: Osborne conjecture: 1 and N choose the median, the rest of the candidates does not enter.

Sequential position games (Osborne)

Evidently, for 2 candidates, the unique SPE is that both choose the median (say, 1/2)
Consider 3 candidates

- If C1 chooses 1/2, C2 cannot enter at a point where he has a positive winning probability (1/2? C3$\rightarrow$ $1/2+\varepsilon$! ; $1/2-\varepsilon$? C3$\rightarrow$ $1/2+\varepsilon/2$!)
  In the subgame following $x_1 = 1/2$, the equilibrium path is $x_2 =$ Out and $x_3 = 1/2$.
- Can C1 choose anything better than $x_1 = 1/2$?
  If C2 keeps out after $x_1 \neq 1/2$, then $x_3 = 1/2$ and C1 loses for sure
  If C2 can respond with any x_2 that keeps C3 out, C1 loses for sure $x_1 = 1/2 - \varepsilon$, where ε is small? $x_2 = 1/2 + \varepsilon - \delta$! ($\delta$ very small), $x_3 =$ Out

Sequential position games (Osborne)

Interesting variations on sequential timing:

- Finite population
  - 3 candidates: If C1 enters at M, C2 can enter at M-1 and C3 at M+1 (tie between C2 and C3, $\Rightarrow$ equilibrium in the subgame after 1's entry.
    $\Rightarrow$ C1 cannot enter at M
  - If M-1 and M+1 are symmetric around 1: equilibrium with entry deterrence
  - non-symmetric: C1 has to choose Out
  - Does this generalize for more than 3 potential candidates?
- Runoff rule
  - 3 candidates: 2 candidates can deter entry of the third candidate by locating symmetrically around the median voter
  - 4+ candidates: Same result, as the third candidate always loses and therefore does not enter (crucial: continuity)

Downsian divergence with policy motivation (Calvert, APSR 1985)

- Candidates do not know the exact location of the median. Median is distributed according to density $f(x)$
- Candidates care (only) about the policy implemented after the election

Utility of the right candidate (policies x, ideal points θ)

$$-F\left(\frac{x_L+x_R}{2}\right)(x_L-\theta_R)^2-\left(1-F\left(\frac{x_L+x_R}{2}\right)\right)(x_R-\theta_R)^2$$

First-order condition (where $\bar{x}=\frac{x_L+x_R}{2}$)

$$\frac{1}{2}f(\bar{x})\left[(x_R-\theta_R)^2-(x_L-\theta_R)^2\right]-2\left(1-F(\bar{x})\right)(x_R-\theta_R)=0$$

Divergence with policy-motivated candidates and "valence" (Groseclose, AJPS 2001)

Reasons for valence difference ε:

- competence
- personality
- Track record/name recognition
- non-pledgable issues

Favored candidate: L

If utility functions are concave, then there exists a cutoff c such that everyone to the left of the cutoff votes for the left candidate and vice versa. (not true for non-concave utility functions)

Favorite position of the median voter is unknown, but distributed according to $f(\cdot)$.

Candidates are policy and office motivated. Weight $\lambda \in [0, 1]$ on office, $1 - \lambda$ on policy.

Divergence with policy-motivated candidates and "valence"

Case 1: $\lambda = 1$, $\varepsilon = 0$:
Both candidates choose the median
Case 2: $\lambda = 1$, $\varepsilon > 0$:
No pure strategy equilibrium: Candidate L would like to take the same position as R, R would always like to differentiate himself from L.
Mixed strategy equilibrium: Aragones and Palfrey
Case 3: $\lambda < 1$ (e.g. $\lambda = 0$):
What happens without valence advantage? (Unknown median position; politicians are policy motivated)

Aragones and Palfrey (JET, 2002)

- Incumbent's valence is larger than challenger's (known)
- Median voter position is unknown (small uncertainty)
- Candidates are office-motivated ($\lambda = 1$)

Results:

- Incumbent has an incentive to move close to challenger, challenger has to move away from incumbent in order to win
- Only mixed strategy equilibrium exists
- Difficult to characterize, but discrete version converges to positions near to the expected MV when uncertainty about MV's position becomes small.

Aragones and Palfrey (JET, 2002)

Other possibility to model uncertainty: Valence realization is unknown (possibly not symmetrically distributed around 0, but one candidate has a stochastic advantage)
Incentive to move to the median
(If MV position is known, both candidates will adopt it and only the better candidate receives votes)

Divergence with policy-motivated candidates and "valence" (Groseclose, AJPS 2001)

$$\max -(x-\theta_L)^2 F\left(\frac{x+y}{2}\right) - (y-\theta_L)^2\left(1 - F\left(\frac{x+y}{2}\right)\right)$$

First order condition

$$-2(x-\theta_L)F(\frac{x+y}{2}) + [(y-\theta_L)^2 - (x-\theta_L)^2]f(\frac{x+y}{2}) = 0$$

When ε grows from 0 to a small positive value, L moves towards the center and R moves away from the center. ($\not\Leftrightarrow$ marginality hypothesis)

Divergence with policy-motivated candidates and "valence" (Groseclose, AJPS 2001)

Reason: As $\varepsilon \uparrow$, with x and y unchanged, c increases.
$\rightarrow$ The marginal benefit of moving closer to the middle has increased for the left candidate and decreased for the right candidate
$\rightarrow$ left candidate moves closer to the center, right candidate moves farther away.
($\not\Leftrightarrow$ marginality hypothesis)

Divergence with policy-motivated candidates and "valence" (Groseclose, AJPS 2001)

- Result true if voter utility functions are sufficiently concave and uncertainty about the median position is sufficiently large (Groseclose, Prop4).
- If uncertainty about MV is too small, candidates are already very close together → a policy motivated candidate will use his advantage to move his platform closer to his preferred point.
- If the left candidate has a larger advantage, he may also move out again.
- Empirical test (?): Do incumbents take more moderate policy positions than challengers (assuming incumbency is related to valence)

Entry deterrence as a reason for divergence

Palfrey (1983):
Entry deterrence by two "incumbent" parties facing one potential entrant
Incumbents choose platform simultaneously, then potential entrant chooses platform
Parties maximize vote share (not: probability of winning)
Not an equilibrium that both incumbent parties locate at the median
? $\rightarrow$ entrant locates slightly to the left (or right) and gets almost 50%, while the established parties get 25% each.
$\rightarrow$ one incumbent can do better by going slightly to the left $\rightarrow$ entrant locates to the right of the median
Note: Depends a lot on vote share maximization!
Uniform distribution on $[0, 1]$: Incumbent parties locate at $(1/4, 3/4)$

Entry deterrence as a reason for divergence

Callander (2005)
Electing a legislature: One platform for all candidates of one party
Different MVs in different districts
Incumbent parties maximize the share of districts they win
One potential entrant in each district, chooses platform after the national parties
Enters only if positive chance of winning the district
Equilibrium platform distance is twice the distance of the median voters in the most extreme districts (if districts are not too different)