Cooperation and equity in resource sharing: Fair intergenerational sharing of a natural resource

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Sustainable development and natural resource extraction 1/3

Definition

Sustainable development = "development that meets the needs of the present without compromising the ability of future generations to meet their own needs"

- Impossible by definition with a scarce natural resource: not enough resource to meet all generation's needs!
- Possible compromise: welfare equivalent of meeting needs but require intergenerational transfers/compensations
- Sustainable allocation of resource and compensations?

Sustainable development and natural resource extraction 2/3

- Lower bounds on welfare
  Present generations should enjoy as much welfare as if they would consume their needs given the resource constraints
- Imposes lower bounds on present generation welfare and intergenerational transfers
- Transfers should not be too high to compromise the ability of future generations to meet their own needs
- Upper bounds on welfare
  Present generation welfare should not be too high to preserve future generation needs

Sustainable development and natural resource extraction 3/3

- Fictitious economy in which agents can bargain on on resource sharing and transfers
- Lower bounds on welfare
  Any group of generations should obtain at least what it should achieve by itself
- Upper bounds on welfare
  No generation should take advantage its first-mover bargaining power to exploit future generations by getting more than their highest welfare from resource extraction

Paper overview

- Defining "sustainable development" in an intergenerational natural resource extraction economy
- Two fairness criteria: core lower bounds and aspiration welfare upper bounds
- Characterize the unique fair extraction path and intergenerational transfers (or welfare distribution)
- Discuss its existence

Literature

- Comparison of infinite utility streams
  (Basu and Mitra, 2003, Bossert et al. 2007, Asheim,2007, ...)
- Natural resource extraction with exogenous social welfare function
  (Dasgupta and Heal, 1974, Solow, 1974, Dixit et al. 1980, and subsequent works)
- Here resource extraction path and transfers (utility streams) with resource-based fairness principles
- Related to the river sharing problem
  (Ambec and Sprumont, 2002, Ambec and Ehlers 2008)

The model

- Natural resource shared by successive overlapping generations at $t \in \mathbb{N}^+$
- Resource dynamic $k_{t+1} = \rho(k_t - x_t)$ with $\rho \geq 1$
- Production function $f_t(x_t)$ strictly concave and increasing up to a maximum $\hat{x}_t$ with $f_t'(0) = +\infty$
- Each generation lives two periods and discount γ the second period
- m_t: transfer paid by generation t when young and received by generation $t-1$ when old
- Generation t's welfare: $f_t(x_t) - m_t + \gamma m_{t+1}$

A scarce resource

- If each generation t extracts up to its needs $\hat{x}_t$ then resource exhausted
- Autarky extraction: Each generation $t < \tilde{t}$ extracts $\hat{x}_t$, then $\tilde{t}$ extract the remaining stock and generations $t > \tilde{t}$ extract nothing
- Pareto improvement if first generations reduce extraction in exchange of transfers from future generations

Feasibility constraints

An allocation $\{x_t, m_t\}_{t=0,\dots,+\infty}$ must satisfy:

- $0 \leq x_t \leq k_t$
- $0 \leq m_t \leq f_t(x_t)$
- Core lower bounds
- Solidarity upper bounds

A fictitious cooperative game

- Each generation or group of generations should get at least what it would get on its own
- Fictitious cooperative game in which all generations can meet and agree on an allocation
- Any coalition S should get at least $v(S)$
- Consecutive and non-consecutive coalitions
- Generations outside the coalition S are in autarky

Value function for consecutive coalitions

$$v(S) = \max_{x_S} \sum_{t \in S} \gamma^t f_t(x_t),$$

$$s.t. \left| \begin{array}{l} k_{t+1} = \rho\left(k_t - x_t\right), \\ k_t \geq x_t \geq 0, k_t \geq 0, \\ k_{minS} = k_{minS}^{ncS} \end{array} \right.$$

where

$$k_{minS}^{ncS} \equiv \min\{\rho^{minS} k_0 - \sum_{t=0}^{minS-1} \rho^{minS-t} \hat{x}_t, 0\}.$$

Value function for any coalition S

$$v(S) = \max_{x_S} \sum_{t \in S} \gamma^t f_t(x_t),$$

$$s.t. \left| \begin{array}{l} k_{t+1} = \rho\left(k_t - x_t\right), \\ k_t \geq x_t \geq 0, k_t \geq 0, \\ k_{min T_l} = k^{ncS}_{min T_l} \text{ for } l = 1, ..., L \end{array} \right.$$

where $\{T_l\}_{l=1,...L}$ is the coarsest partition of S into connected components

Core lower bounds

Definition

An allocation $\{x_t, m_t\}$ satisfies the core lower bounds if and only if for all coalitions $S \subset \mathbb{N}^+$

$$\sum_{t \in S} \gamma^t \left(f_t(x_t) - m_t + \gamma m_{t+1} \right) \geq v(S).$$

Solidarity and aspiration welfare

- Solidarity principle introduced by Moulin (1990) for games with externalities
- Since resource is scarce, a generation suffers from a negative externality due to other generations' extraction
- *Aspiration welfare*=Welfare achieved by a generation or a coalition of generations if nobody else extract
- Resource scarcity implies that it is impossible to assign to everybody its aspiration welfare
- By solidarity, no coalition of generation should enjoy **more** then its *aspiration welfare*

Aspiration welfare for consecutive coalitions

$$w(S) = \max_{x_S} \sum_{t \in S} \gamma^t f_t(x_t),$$

$$s.t. \left| \begin{array}{l} k_{t+1} = \rho \left(k_t - x_t \right), \\ k_t \geq x_t \geq 0, k_t \geq 0, \\ k_{minS} = \rho^{minS} k_0. \end{array} \right.$$

Aspiration welfare for any coalition S

$$w(S) = \max_{x_S} \sum_{t \in S} \gamma^t f_t(x_t),$$

$$s.t. \left| \begin{array}{l} k_{t+1} = \rho\left(k_t - x_t\right), \\ k_t \geq x_t \geq 0, k_t \geq 0, \\ k_{minS} = \rho^{minS} k_0, k_{minT_l} = \rho^{(minT_{l+1} - maxT_l)} k_{maxT_l} \text{ for } l = 2, ..., L \end{array} \right.$$

where $\{T_l\}_{l=1,...L}$ is the coarsest partition of S into connected components

Solidarity upper bounds

Definition

An allocation $\{x_t, m_t\}$ *satisfies the solidarity upper bounds if and only if for all coalitions* $S \subset \mathbb{N}^+$

$$\sum_{t \in S} \gamma^t \left(f_t(x_t) - m_t + \gamma m_{t+1}\right) \leq w(S).$$

A unique allocation

- $\{x_t^*\}$ solution to the program defined by $v(\mathbb{N}^+)$
- $\{m_t^*\}$ that assigns to each generation its marginal contribution to the preceding generation

$$\gamma^t(f_t(x_t^*) - m_t^* + \gamma m_{t+1}^*) = v(Pt) - v(P^0 t)$$

where Pt ($P^0 t$) set of (strict) predecessors of t

Proposition

If $m_t^* \leq f_t(x_t^*)$ for every $t \in \mathbb{N}^+$, $\{x_t^*, m_t^*\}$ is the unique allocation that satisfies the core lower bounds and the aspiration welfare upper bounds.

Main points of the proof

1. The core lower bounds for Pj with $j \longrightarrow \infty$ leads to

$$\sum_{t=0}^{\infty} \gamma^t f_t(x_t) + \lim_{j \longrightarrow \infty} \gamma^{j+1} m_{j+1} \geq v(\mathbb{N}^+)$$

therefore $\sum_{t=0}^{\infty} \gamma^t f_t(x_t) \geq v(\mathbb{N}^+)$ which implies $\{x_t\} = \{x_t^*\}$

2. $\{x_t^*, m_t^*\}$ is unique because $v(Pj) = w(Pj)$ for any j and the fairness bounds forces
$w(Pj) \geq \sum_{t=0}^{j} \gamma^t (f_t(x_t^*) - m_t + \theta m_{t+1}) \geq v(Pj)$
3. $\{x_t^*, m_t^*\}$ satisfies the core lower bounds
4. $\{x_t^*, m_t^*\}$ satisfies the solidarity upper bounds

Efficient extraction path $\{x_t^*\}$

$\{x_t^*\}$ solution to

$$\max_{\{x_t\}} \sum_{t=0}^{\infty} \gamma^t f(x_t)$$

$$s.t. \left| \begin{array}{l} k_{t+1} = \rho(k_t - x_t), \\ x_t \geq 0, k_t \geq 0, \\ k_0 > 0 \text{ given.} \end{array} \right.$$

First-order condition

$$f_t'(x_{t-1}) = \gamma \rho f_t'(x_t)$$

Efficient extraction path $\{x_t^*\}$

Proposition

If $f_t(x) = f(x)$ for every t, the fair path of extraction $\{x_t^*\}$ and the stock of resource are:

i) *monotonically increasing if $\gamma\rho > 1$ with an asymptotical constant extraction path $x_\infty^* = \hat{x}$ and $k_\infty = \frac{\rho}{\rho - 1}\hat{x}$,*

ii) *monotonically decreasing with a stock asymptotically exhausted if $\gamma\rho < 1$,*

iii) *constant for every t if $\gamma\rho = 1$ with a constant extraction path $x_t^* = \left(1 - \frac{1}{\rho}\right) k_0$ for every t.*

Same results for production functions with technical progress $f_t(x_t) = A_t f(x_t) = A_0 \eta^t f(.)$ or $f_t(x_t) = f(A_t x_t)$

Fair transfers $\{m_t^*\}$

$$m_{t+1}^* = \frac{\sum_{i=0}^{t} \gamma^i f_i\left(x_i^{P_t}\right) - \sum_{i=0}^{t} \gamma^i f_i\left(x_i^*\right)}{\gamma^{t+1}} \geq 0$$

where $x_i^{P_t}$ is the solution of $\max_{x_i} \sum_{i=0}^{t} \gamma^i f_i\left(x_i\right)$ subject to the resource and non negativity constraints.

Feasibility condition $m_{t+1}^* \leq f_{t+1}(x_{t+1}^*)$ might be difficult to meet because m_t^* increases with t during the first generations up to $\tilde{t}$.

Example

- $\gamma\rho = 1$
- $f_t(x_t) = \sqrt{x_t}$
- $x_t^* = (1-\gamma) k_0$
- $x_i^{P_t} = (1-\gamma) k_0 / \left(1-\gamma^{t+1}\right)$

$$m_{t+1}^* = \frac{\sqrt{\frac{k_0}{(1-\gamma)}}\left(\sqrt{1-\gamma^{t+1}} - \left(1-\gamma^{t+1}\right)\right)}{\gamma^{t+1}}$$

- $m_{t+1}^* \leq f_{t+1}\left(x_{t+1}^*\right) \iff \sqrt{1-\gamma^{t+1}} \leq \left(1-\gamma^{t+2}\right)$
- Feasible for $\gamma = 0.3$ or $\gamma = 0.5$ but not $\gamma = 0.7$

Utility

Proposition

For all $t \geq 2$, $u_t^* \leq u_1^*$ *if* $f_t\left(x_t^{P_t}\right) \leq f_1\left(\hat{x}_1\right)$.

Implies that technical progress is a necessary condition for the fair allocation to keep the utilities at least constant (if $f_t = f$ then decreasing utility)

Conclusion

- Fair allocation in an overlapping generation economy of natural resource
- Two fairness principles: minimal and maximal welfare
- Unique fair allocation extracts the resource efficiently and assigns to any generation its marginal contribution to its predecessors
- Might not be feasible due to the technology and resource constraints
- Increasing transfers and decreasing utility if no technical progress