Cooperation and equity in resource sharing: Sharing a common resource fairly

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General topic

- Common-pool natural resources: water, forest, fisheries, fossil fuel, clean air,...
- How to share the resource? the benefit from resource extraction?
- Cooperative approach
- Equity issues

Methodology

- Define principles (axioms) for sharing welfare from resource management applied to particular problem
- Satiation, spatial and temporal issues
- Characterize sharing rules / Welfare definition / Sharing agreements
- Mechanisms to implement those welfare distribution: market, negotiation rules,...

Frameworks

- Lecture 1: Sharing a common resource fairly
  Based on the SCW paper
- Lecture 2: Cooperation and equity in the river sharing problem
  Based on paper in GEB and book chapter with Lars Ehlers
- Lecture 3: Intergenerational sharing of a natural resource
  Based on MMS paper with Hippolyte d'Albis

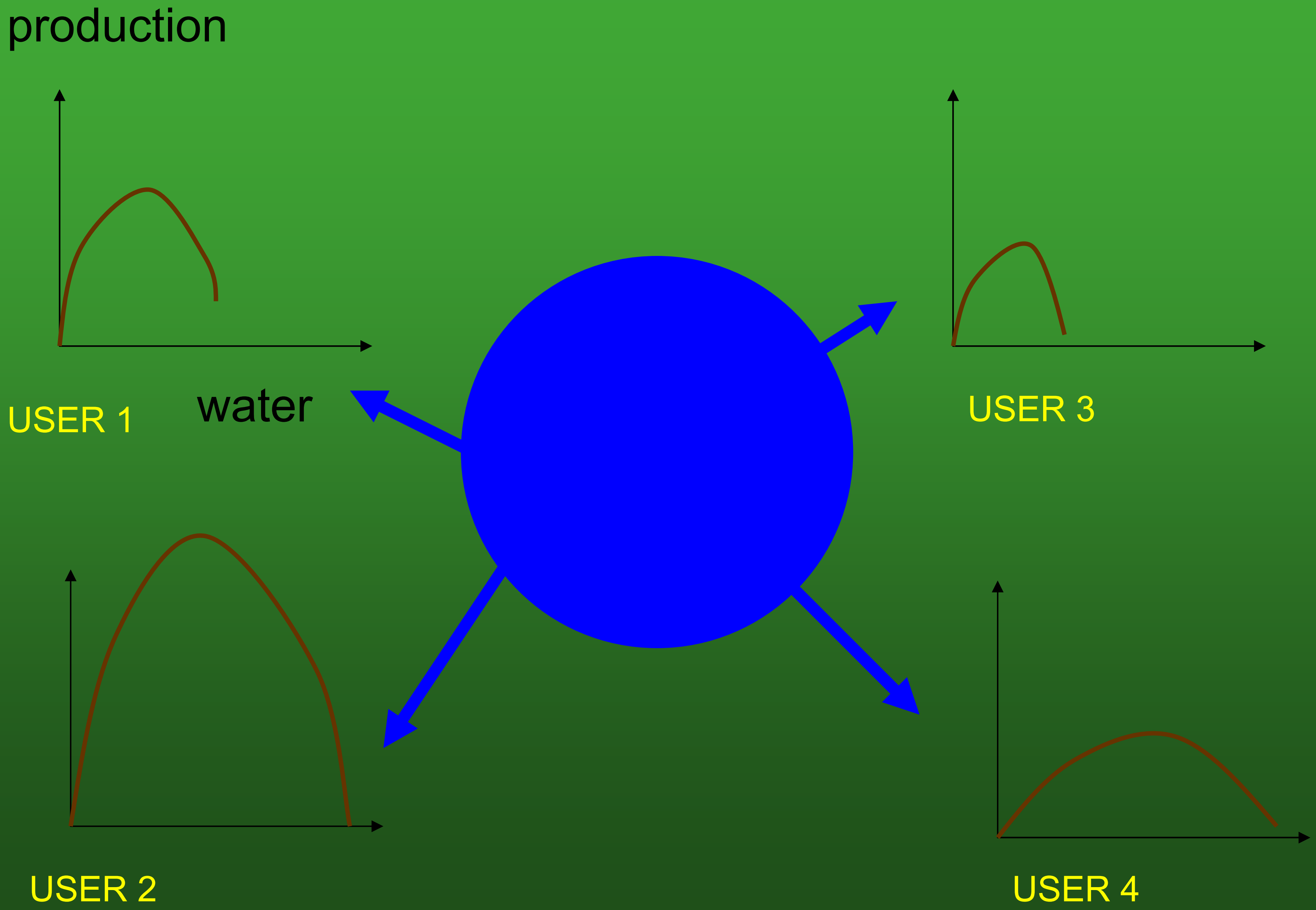
Related literature

- On the axiomatic approach to fair division
  Survey from William Thomson on "Fair allocation rules"
- Literature on common-pool resource sharing in practice leaded by Elinor Ostrom
- International agreements for river water sharing in practice (e.g. Ariel Dinar)
- On the axiomatic approach to fair division
  Survey from Geir Asheim on "intergenerational equity"
- Cooperation and equity for the design of international environmental agreements

Lecture 1: Sharing a resource with concave benefit

- A common resource X shared by a set of agents
- Equal access / Equal rights
- Heterogeneous increasing and concave benefit of resource extraction with satiation
- Scarce resource
- Fair division of the total welfare of resource extraction

SET-UP



Literature

- Sharing with single-peak preferences but without side payment
  Sprumont (1991), Ching (1992),...
- Fairness and efficiency in general equilibrium but with non-satiated preferences
  Foley (1967), Schmeidler and Vind (1972), Varian (1974), ...
  Or more general preferences
  Zhou (1991), Thomson and Zhou (1993), Barbera and Jackson (1995)

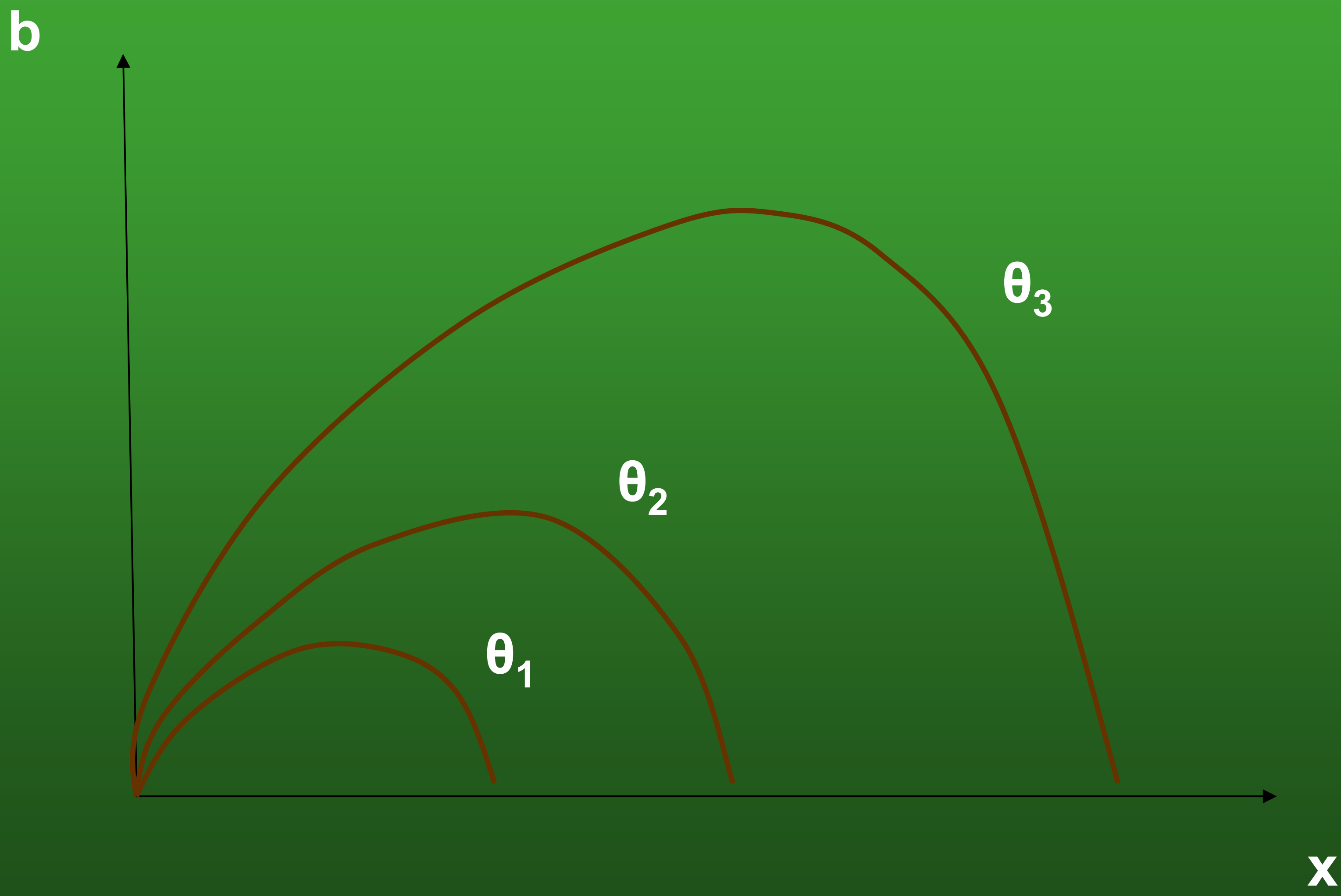
The model 1/2

- X to be shared
- Continuum of agents $\theta \in \Theta = [\underline{\theta}, \bar{\theta}]$ of mass 1 with distribution f and cumulative F
- Agent θ welfare's with resource consumption x and transfer t:

$$b(x, \theta) + t$$

- b increasing up to $\hat{x}_\theta$
- $\frac{\partial^2 b}{\partial \theta \partial x}(x, \theta) > 0$
- $b(0, \theta) = 0$ and $\frac{\partial b}{\partial x}(0, \theta) \geq k$

BENEFIT FUNCTIONS FOR $\theta_1 < \theta_2 < \theta_3$



The model 2/2

- The resource is scarce:

$$\int_{\Theta} \hat{x}_\theta dF(\theta) > X.$$

- An allocation $\{x_\theta, t_\theta\}_{\theta \in \Theta}$ is feasible if

$$\int_{\Theta} x_\theta dF(\theta) \leq X,$$

and budget-balanced if

$$\int_{\Theta} t_\theta dF(\theta) \leq 0.$$

No-envy or Incentive-Compatibility

$\{x_\theta, t_\theta\}$ satisfies no-envy iff

$$b(x_\theta, \theta) + t_\theta \geq b(\min\{x_{\theta'}, \hat{x}_\theta\}, \theta) + t_{\theta'} \text{ for every } \theta' \in \Theta$$

for every $\theta \in \Theta$.

Similar to incentive-compatible or strategy-proofness.

Equal-Sharing individual rationality

$\{x_\theta, t_\theta\}$ is Equal-Sharing Individual Rational (ESIR) iff

$$b(x_\theta, \theta) + t_\theta \geq b(\min\{X, \hat{x}_\theta\}, \theta)$$

for every $\theta \in \Theta$.

Efficient allocation

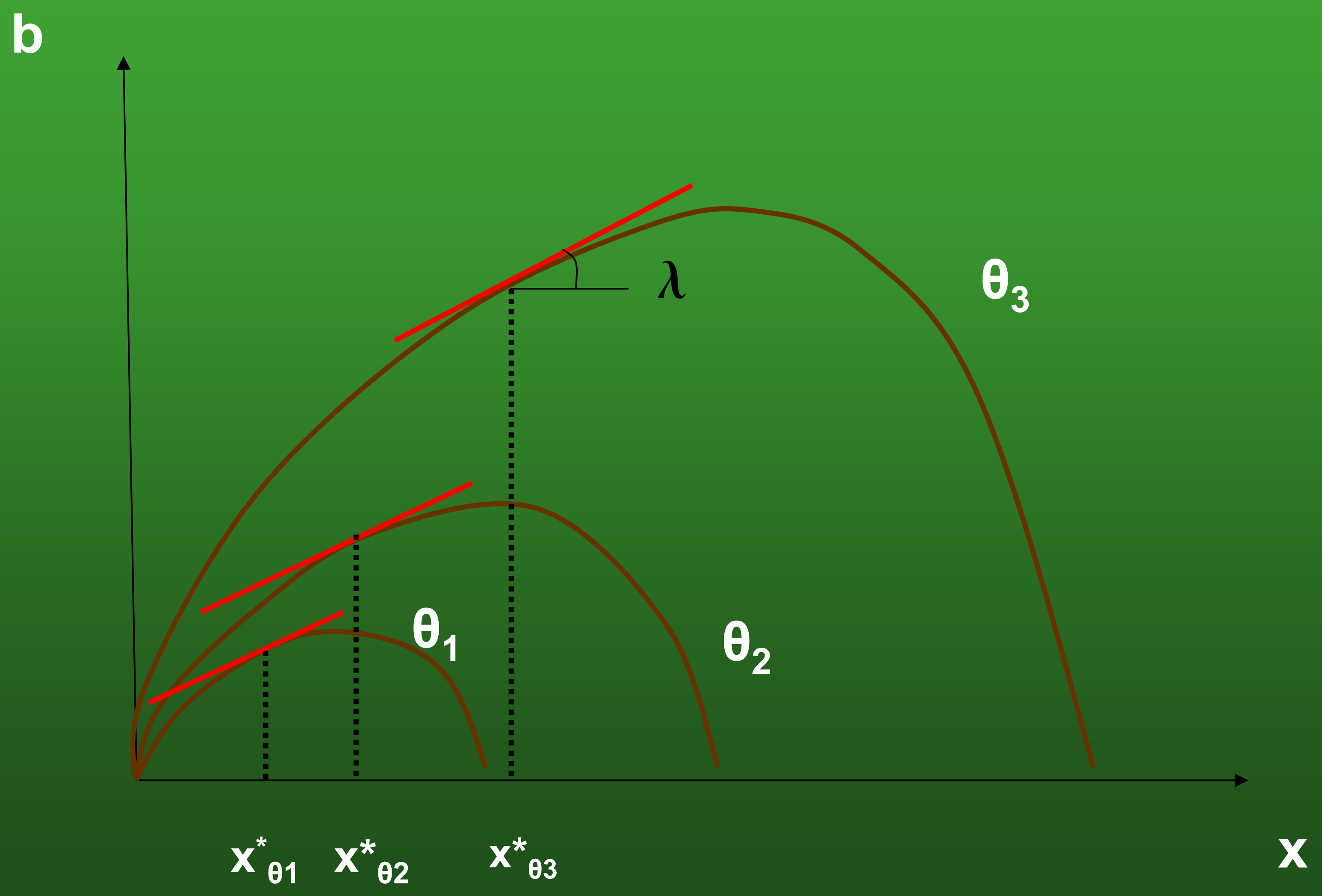
$\{x_\theta^*\}$ solution to

$$\max_{\{x_\theta\}} \int_\Theta b(x_\theta, \theta) dF(\theta) \text{ subject to } \int_\Theta x_\theta dF(\theta) \leq X.$$

Foc:

$$\frac{\partial b}{\partial x}(x_\theta^*, \theta) = \lambda \quad \text{for every} \quad \theta \in \Theta \quad \text{with} \quad \lambda > 0.$$

EFFICIENT RESOURCE ALLOCATION



Walrasian allocation from equal endowment

The *Walrasian allocation from equal endowment* $\{x^*_\theta, t^*_\theta\}$ is the market allocation if X divided equally among agents

It leads to $t^*_\theta = \lambda(X - x^*_\theta)$ and assigns

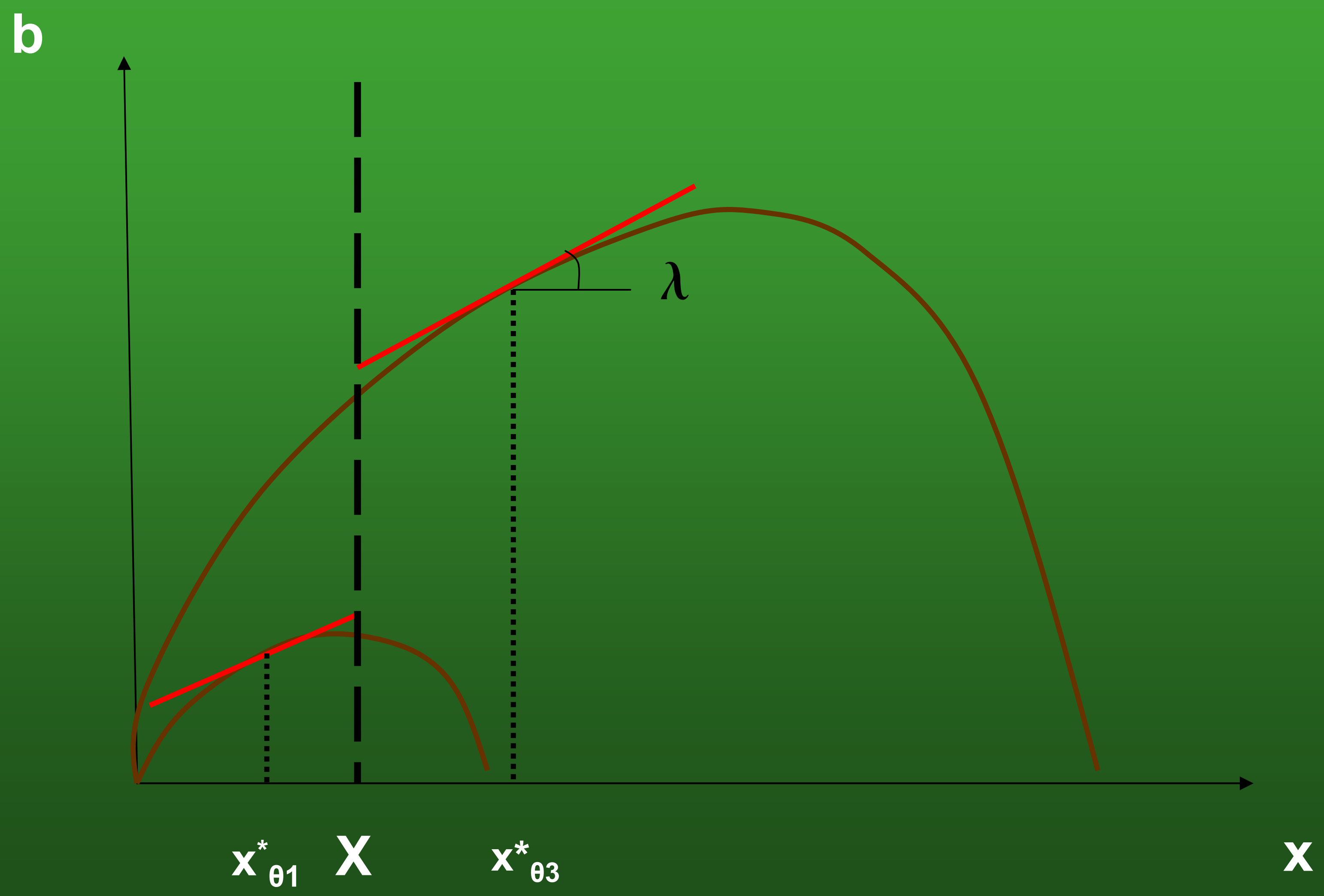
$b(x^*_\theta, \theta) + \lambda(X - x^*_\theta)$ to θ

Theorem

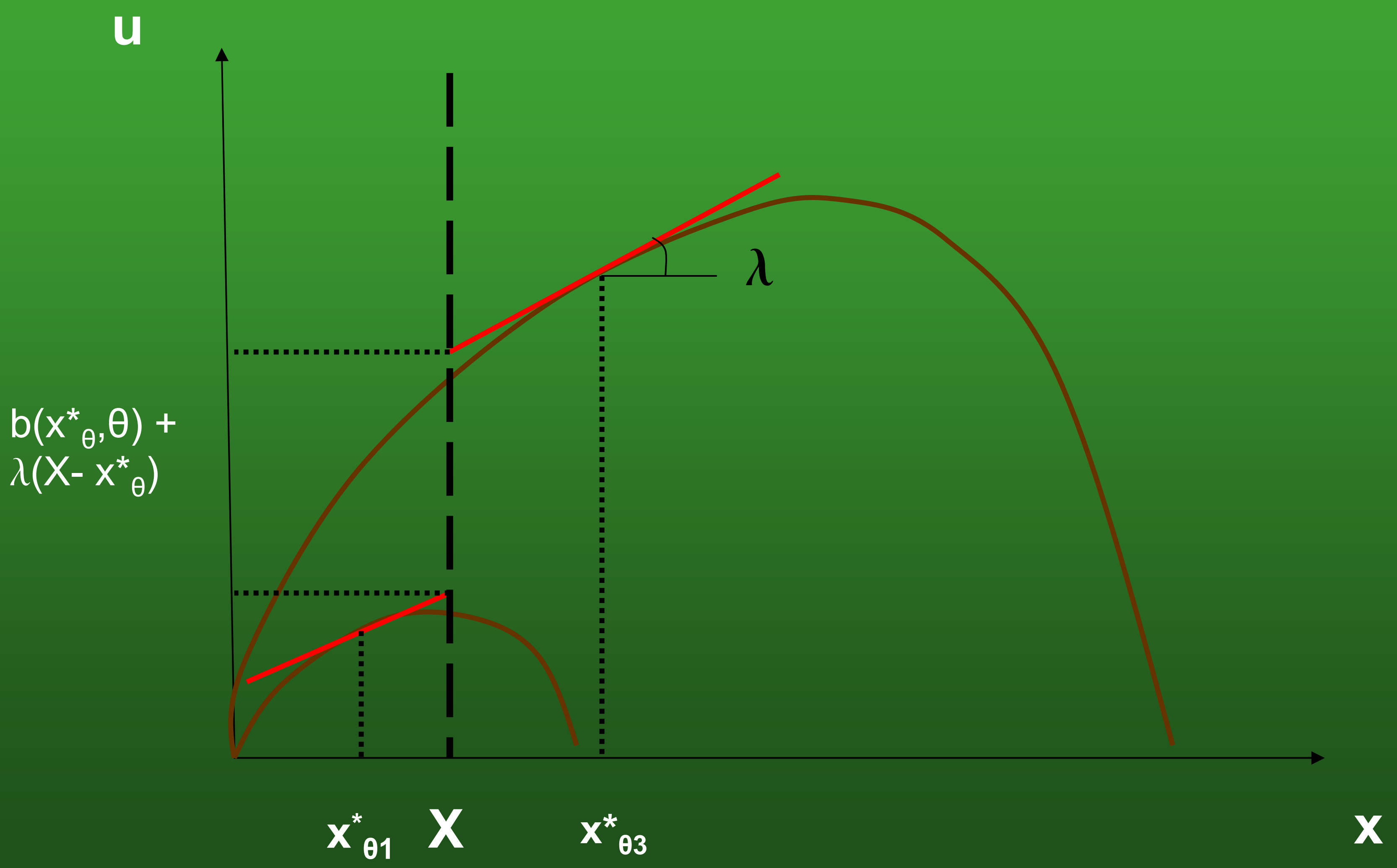
The Walrasian allocation from equal endowments $\{x^*_\theta, t^*_\theta\}$ *is the only allocation that is efficient, satisfies no-envy and equal-sharing individual rationality*

Decentralized by assigning equal property rights on X in a competitive market or by selling the resource at price λ and redistributing equally the money collected

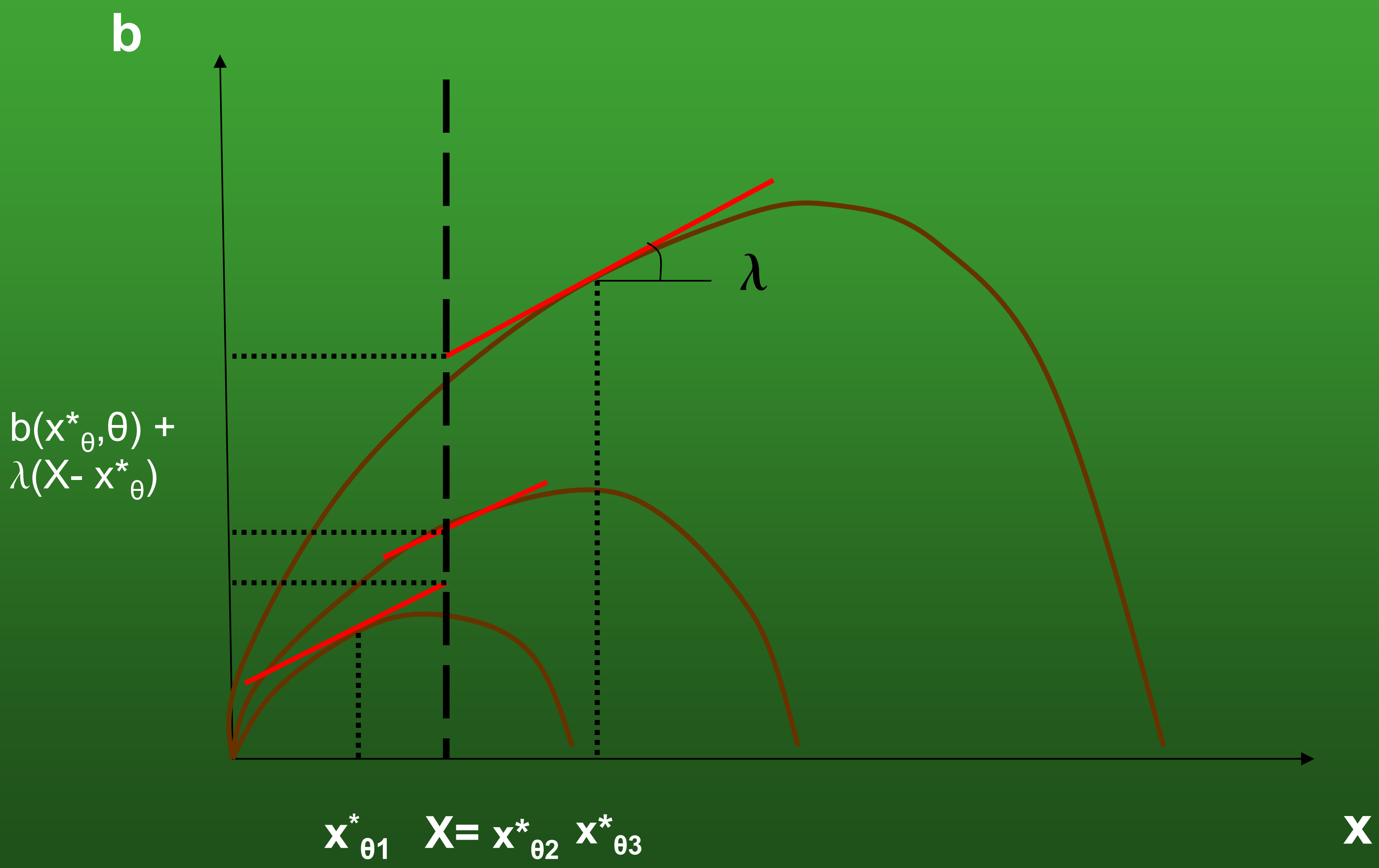
PROOF RESULT 1



PROOF RESULT 1



PROOF RESULT 1



Peak upper bound 1/2

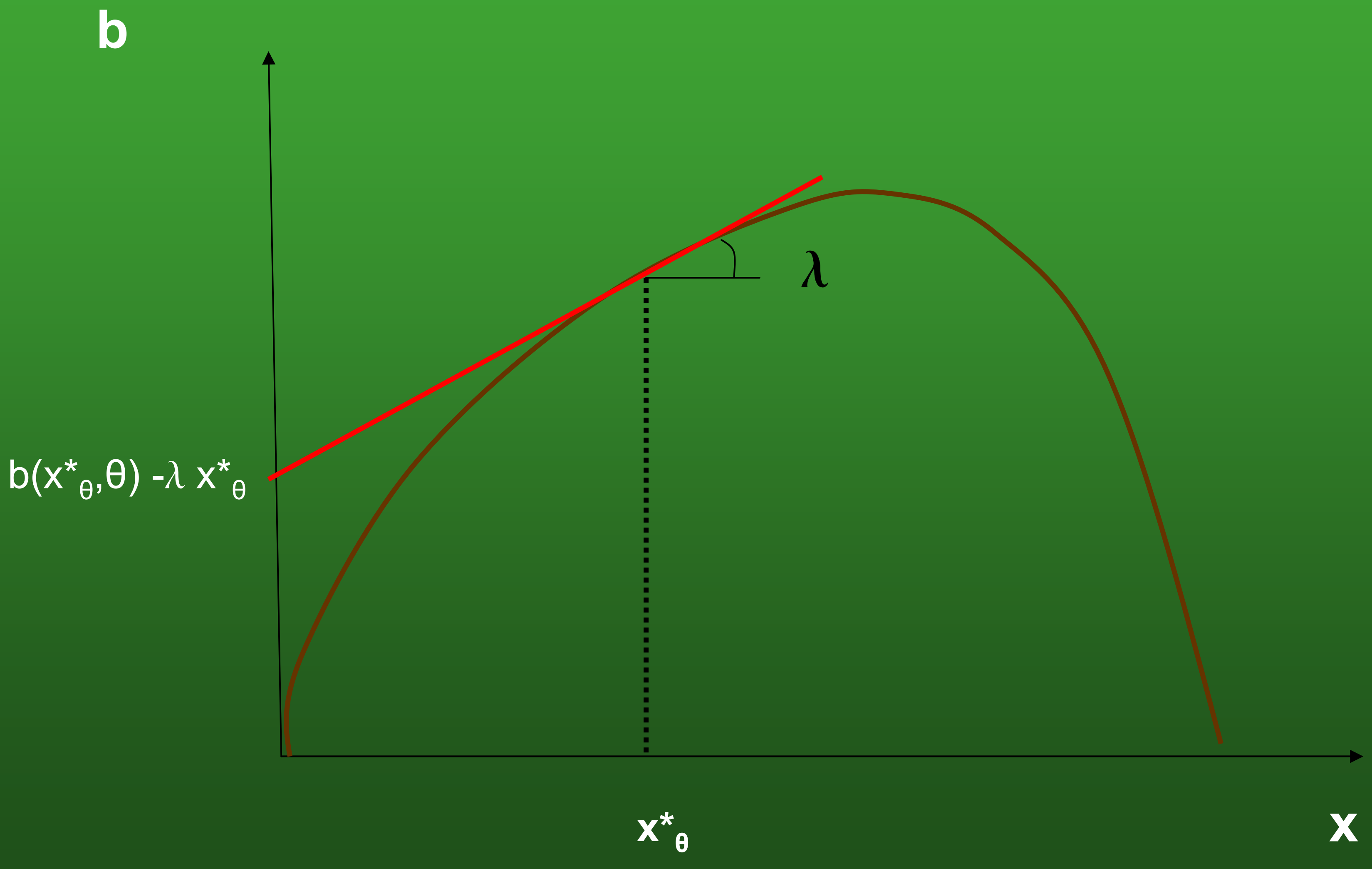
Since no all agents can enjoy its peak benefit $b(\hat{x}_\theta, \theta)$ due to resource scarcity, by solidarity, no agent should get strictly more than that

$\{x_\theta, t_\theta\}$ satisfies the peak upper bound (PUB) if $b(x_\theta, \theta) + t_\theta \leq b(\hat{x}_\theta, \theta)$ for every $\theta \in \Theta$

Peak upper bound 2/2

- The Walrasian allocation with equal endowment $\{x_\theta^*, t_\theta^*\}$ fails to satisfy the PUB
- The allocation $\{x_\theta^*, -\lambda x_\theta^*\}$ satisfies PUB, efficiency, No-envy and individual rationality $b(x_\theta, \theta) + t_\theta \geq 0$ for every $\theta \in \Theta$
- Decentralized by pricing the resource λ at not redistributing the money collected

INDIVIDUAL RATIONALITY



Consistency 1/2

An allocation is consistent if it assigns the same bundles to the "reduced" economy obtained when some agents leave with their assign bundle

Consistency 2/2

$X_\Omega = \int_\Omega x^*_\theta dF(\theta)$ and $T_\Omega = \int_\Omega t^*_\theta dF$ to be shared among agents in $\Omega \in \Theta$

The Walrasian allocation with equal endowment in the reduced economy $\{x^\Omega_\theta, t^\Omega_\theta\}_{\theta\in\Omega}$ is such that $x^\Omega = x^*_\theta$ and $t^\Omega_\theta = \lambda(X - x^*_\theta) + T_\Omega = t^*_\theta$ for every $\theta \in \Omega$.

Strict no envy

- An allocation satisfies strict no envy if no agent prefers the average holding of any group of agents
- The allocation $\{x_\theta^*, -\lambda x_\theta^*\}$ satisfies strict no envy.

NEXT

NEXT: What if unequal access to the resource like in a river or during time?