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Liquidity provision during currency and debt crises:

A sequential game analysis*

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ABSTRACT

We model the relationship between a currency crisis and debt crisis within a four-stage sequential game framework. The government can commit already in stage 1 to bailing out part of the private sector's outstanding debt if a bad shock occurs or wait until stage 4 to give an optimal bailout. A commitment to bailing out provides a reconciliation of the multiple equilibria that result from self-fulfilling expectations. The government will also avert currency crises by committing to bailing out. In contrast to what previous literature has assumed, moral hazard effects are not only endogenized but they are also taken into account when the government minimises its loss function. Only with commitment, the optimal bailout policy is based in the trade-off between the moral hazard effects and the deterioration of the fundamentals.

1. Introduction

In light of the events that have occurred in East Asia and Latin America since 1997, many questions have been raised in not only about the causes of the twin crises (currency and financial crises) but how to deal with those crises when they occur. There is already a growing literature on both types of crisis, using different methodologies and assumptions. This study will mainly focus on the following two questions:

i) Can or cannot provision of liquidity, a) effectively ameliorate the effects that costly liquidation associated with financial crisis can have on the fundamentals, and b) avoid the currency crises that usually follow a panic in the financial sector.

ii) Are financial and/or currency crises caused by inconsistent and unsustainable macroeconomic problems, domestic foreign currency debt, and/or self-fulfilling expectations.

The role of such interventions at the macroeconomic level is still much of a debate and not final conclusions have been established yet. At the microeconomic level, the academic literature has so far focused on micro-prudential regulation such as how to deal with financial problems of individual banks. In this regard much focus has been placed on the role of deposit insurance or capital requirements. One can agree that there has been a great development in technology and financial environment such as emergence of repo markets and of real time gross payment systems facilitating monetary transfer from one institution to another, deposit insurance, higher banking competition, and strict regulation of solvency capital. The relevant literature then sustains that in these days there is no need for a LOLR or in general provision of liquidity at the microeconomic level. Still, even if this may be reasonable, it would be harder to argue that such policy is credible, and more so at the macroeconomic level. Time consistency problems are likely to arise. Is it then credible to commit to no-intervention at all when ex-post maybe be optimal to avoid systemic risk and all the costs that this involves? (Freixas (2001)).

This paper is somewhat related to Cooper and Corbae (2002) work. One of the main differences is that they consider a closed economy, which implies that they do not analyse how financial collapse could be associated with uncertainty over the exchange rate. They find first that with passive monetary authorities (no intervention), there will be multiple (sunspot) equilibria; and second, the number of equilibria is reduced with intervention of the monetary authorities (by means of money growth). They point out that without a theory of equilibrium selection, predictions about the effects of such monetary policy are impossible to make (page 183). We here also find that *when the government does not commit* ex-ante to an optimal bailout (derived from first principles)[1], the exchange rate and financial crises can be driven by self-fulfilling expectations resulting in multiple equilibria. However, if the *government commits ex-ante* to an optimal bailout before the private sector forms expectations and make their investment optimal decisions, it can serve as a strategic device to implement a single and more efficient equilibrium. It is efficient in the sense that there will be lower (or none) devaluation as well as lesser detrimental of the fundamentals. Only a smaller bailout will be also necessary because of lesser financial distress. In view of this, the government should generally have incentives to pre-commit to making such bailouts. The case of explicit ex-ante commitment to bailing out has never been considered before and this is perhaps one of our main contributions to policy analysis: To analyse how committing ex-ante to bail out can influence in the desirable manner the private sector's behaviour.

Our analysis is based on the assumption that there are aggregate shocks and that inside liquidity (Holmstrøm and Tirole (2001)) is not properly allocated within the private sector.[2]

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[1] Liquidity provision in Cooper and Corbae (2002) and related papers is usually modelled as exogenous variables and not derived from principles of utility maximisation.
[2] Tirole (2002) defines *inside liquidity* as the liquidity firms and financial institutions can obtain by selling the securities that they usually hold in other firms and financial institutions.

We here argue that *outside liquidity* will be then needed and here the Lender-of-last-resort (LOLR) will have a function.[3] Entrepreneurs, who need to borrow in foreign currency, will be only rescued if a bad shock occurs since when a bad shock occurs, it causes more uncertainty about more subsequent devaluations and financial losses that finally lead to deep deterioration of the economic fundamentals. Committing to ameliorate the private sector's financial status will make devaluation less necessary and the private sector will rationally anticipate this. We will have an optimising government that takes into account the moral hazard effects that bailouts and less devaluation can have on the private sector.

We will consider the moral hazard effects caused by implicit and explicit government guarantees that have been emphasised in certain related literature. Such effects are said to lead to over borrowing and over lending, excessive current account deficits, and socially irresponsible forms of both public and private behaviour. See for example Dooley (1997), Corsetti, Pesanti and Roubini (1999), Bordo and Schwartz (2000), Schneider and Tornell (2000), Eichenbaum, Burnside, and Rebello (2000, 2002), among others. We do not either disregard the argument made by Caballero and Krishnamurthy (2003) that the reason why firms take so much foreign currency debt is because the government implicitly insure them against currency risks when it defends at any cost the fixed exchange rate regime. Thus, our optimal policy regarding bailout and exchange rate will be based on the trade-off between large deterioration of the fundamentals and the moral hazard effects on the private sector. [4]

We should keep in mind that emerging economies are usually characterised by large volatility of the exchange rate even though inflation has in the later years diminished

[3] We do not here emphasise on whether such outside insurance should be created by the existence of domestic outside liquidity or foreign outside liquidity. For our purpose, we do not distinguish between these two. See Tirole (2002) for further discussion.

[4] We do not focus on the argument of Burnside et al. (2000) that government guarantees completely eliminates the incentives to hedge the risk of devaluation in light of Eichengreen and Haussman (1999)'s Original Sin Hypothesis that has emphasized on the incompleteness in financial markets. The private sector may then want to hedge their exposures, specially the ones denominated in foreign currency, but they are unable to do so. Assuming that there is unrestricted and non-distorted supply of hedges is equivalent to assuming that the country can borrow abroad in its own currency. This is unfortunately unrealistic at this point.

substantially. Still the policymakers in these economies face many credibility problems so that an expansionary monetary policy to promote recovery from a financial crisis is likely to lead to higher inflation and depreciation of the national currency. Consequently, the international financial sector has very few incentives to sign contracts denominated in the domestic currency and foreign lending, to a large extent, usually takes the form of foreign currency loans. These economies may lose control over foreign claims on the economy because they are denominated in foreign currency. In such circumstances, an optimal provision of liquidity may play an important role for shoring up balance sheets and ameliorate the effects of a financial crisis, should it occur.

Mishkin (1995), Freixas, Parigi and Rochet (1998) and Freixas (2001) indicate that the existence of a LOLR is typically justified by the possibility of a systemic financial crisis, defined as a crisis where the standard mechanisms and financial channels stop working, thus impairing the well-functioning of the payment side of economic transactions. Bolton (2003) argues that bailouts both in the domestic and sovereign context, even when only a bankruptcy mechanism is in place, serve if anything an important economic role in overcoming liquidity crises and contagion.

We analyse if bailouts are counterproductive, by *taking into account (not assuming) the moral hazard effects of the bailout policy* (determined by welfare maximising agents). In contrast to previous literature, we here provide a complete characterisation of an (scheme or rule) optimal bailout policy and of the optimality of a bailout policy, both when the government commits and when it does not commit ex-ante.

In the related literature, governments are only concerned with satisfying the budget constraint when they decide (ex-post) to bail out and they do so by means of seignorage. The private sector agents will rationally anticipate a collapse of the government's finances given that the government's present value of the deficit is expected to increase due to permanent rise in future transfer payments (i.e. bailouts). We believe that only when liquidity transfers are determined optimally (by welfare maximising agents), one can demonstrate formally the consequences of bailing out. Also, in contrast to previous work in the subject, the government is here concerned with minimising the social costs of both financial and currency crises (i.e. on unemployment), the costs of bailing out, and the moral hazards that bailing out would imply. Finally, the government always finds the optimal bailout conditional *only* on a stochastic bad shock occurring. All this issues have not been considered in the previous relevant literature.

This paper again will study the interrelationship between exchange rate crises and debt crises. The model here shows that crises may be both belief-driven and fundamentals-based attacks. The model is presented as a four-stage sequential game in which the players are the government and the private sector. By private sector we mean entrepreneurs. The sequence of our game is as follows. Information about the probabilistic distribution of a forthcoming (good or bad) shock is given in the *first stage*. This shock will be realised in the fourth stage and will affect the exchange rate, private-sector debt and unemployment. In this *first stage*, the government announces that it will pursue a fixed exchange rate regime and it may or may not already at this stage commit to an optimal bailout only if a bad shock occurs.[5] It will also incur certain (fixed, constant and not further explained) costs when the fixed exchange rate regime is abandoned. In the *second stage*, the private-sector agents know that there could be devaluation with certain probability, and in addition undertake risky investment activities. The private sector borrows in foreign currency but generate income in domestic currency (producing mostly non-tradable goods). If a bad shock occurs, the net returns of investment become negative and devaluation will make them even more negative. *In stage 3*, the government may react by either keeping the fixed exchange rate regime or letting the

[5] This policy does not need to be interpreted as strictly as a fixed exchange rate regime, but it can be more realistically thought of as having an exchange rate target.

exchange rate to float freely.[6] Keeping the fixed exchange rate regime while there are expectations of devaluation during this stage implies costs in the interim period to the government, and to the economy in the form of adverse shifts in the economic fundamentals. The regime can however be abandoned in this stage 3 before it does further significant damage to the economy. This outcome, where a self-fulfilling expectations lead necessarily to devaluation, resembles the Krugman (1979) where a deterioration of the fundamentals (in addition to the market's anticipation of a financial crisis) plays an important role in triggering crises. In the *final stage 4,* the shock realises conditioning the government's decisions on two things: the optimal bailout, if it has not committed already in stage 1; and whether or not to abandon the fixed-rate regime (given that the fixed rate has been retained up to this stage). We will present a systematic backward induction analysis of the continuation of sub-game perfect equilibria, where we first look at equilibria starting at stage 4, then at stage 3 and so on.

We show that when *no commitment* has been made to a specific bailout, the model may yield multiple (2 or 3) equilibria. One of the equilibria involves always retaining the fixed peg *when there are not devaluation expectations* even in the event of a bad shock and cumulative investment losses. *When devaluation is however expected*, two more equilibria are possible. One involves abandoning the peg only in the worst state, and the last one that can occur in stage 3 involves always abandoning the peg. The last equilibrium is most likely reached when there is a sufficiently high prior probability that a bad shock will occur and/or if shocks are expected to be quite serious when they do occur.

It turns out, as mentioned, that a *commitment* to a bailout (*only if a bad shock occurs*) in stage 1, expressly announced before the market forms expectations, has very important implications for the outcome of this sequential game. We show that it is possible to obtain

[6] One way to view how the setting of the free-floating exchange rate in this stage 3 takes place is assuming that the government uses (and this is common knowledge to everybody already at stage 1) the forward market. In stage 4, if the government devalues, the exchange rate will be rather determine optimally depending on the size of the shock and the final financial conditions in the private sector. We will come back to this below.

non-devaluation or a smaller devaluation than in the case when there is no commitment. Committing to bail out guarantees a less drastic impact of the outstanding debt on the economic fundamentals. The private sector takes this into account when forming expectations: A commitment will reduce the private sector debt so that there will be a more limited adverse effects on the fundamentals. Devaluation (or large devaluation) is to be regarded by the private sector as less likely to take place. It is true that reducing (if not eliminating) devaluation expectations will influence the private sector's incentives to insure themselves against international credit constraint and sudden stops, but notice that such effect as mentioned is internalized in the government' optimization problem. Our most important result is that with ex-ante commitment to provide liquidity in the bad state, an endogenously smaller devaluation and bailout are anticipated because the returns to investment (and the economic fundamentals) are likely to improve even though investment will increase somewhat.

The paper is organised as follows: The next section highlights some contributions to the related literature. Section 3 presents an overview of the stages of the sequential game; section 4, presents the representative investor's problem, while section 5 presents the government's problem. Section 6 aims to show the optimal solutions at stage 4 in the absence of ex-ante commitments. Section 7 shows the optimal decisions that can be taken in stage 3, while section 8 presents the optimal solutions when the government decides to commit to a bailout at stage 1. Section 9 presents a numerical example to the non-commitment and commitment cases and finally, section 10 presents a conclusion.

2. Additional related literature

As noted, this paper proposes a new theoretical approach to several different strands of literature, in particular to the "first-" and "second-generation" models of currency crises, and the twin crises models. It also presents certain characteristics of both models. Henderson and

Salant (1978), Krugman (1979) and Flood and Garber (1984) represent the "first-generation" models, in which devaluation entails no reputational cost but a currency crisis arises as a necessary consequence of adverse fundamentals. Our model also presents an "endogenous-policy" model of currency crises such as the ones pioneered by Obstfeld (1986, 1994). The government rationally chooses – on the basis of their assessment of costs and benefits in terms of social welfare - whether or not to maintain a fixed exchange rate regime. A crisis is driven by self-validating shifts in expectations where multiple equilibria are possible is obtained when the government does not commit ex-ante to any bailout.

There is one feature that makes our model different from the "second-generation" models. In these "second-generation" models, private sector expectations are assumed to be formed at the "ex ante" stage, while the government's decision to retain or abandon the fixed exchange rate regime is made "ex post", after the realisation of some stochastic variable. Our approach, in contrast to these models, is one in which the government's own possible actions at the "ex ante" stage are also considered, which they ought to be if "ex ante" and "ex post" are separated in real time.

Morris and Shin (1998) have extended the basic results of Obstfeld's (1994) model to the case with less than common beliefs about the actual game played. By assuming that agents in the economy may not have common knowledge of the underlying fundamentals, they show that such uncertainty about the beliefs of others regarding the fundamentals may yield a single course of action leading to uniqueness of equilibrium. Extensions of the present model incorporating uncertainty about beliefs should, however, be pursued in future work.

With respect to the twin crisis, Allen and Gale (2000) find that although banking crises typically precede currency crises, the common cause of both is usually a fall in asset values due to a recession or a weak economy. Recent empirical work has also attempted to study empirically the most likely causes of currency crises and their relationship with financial markets, see (IMF (1998), Kaminsky and Reinhart (1999), Glick and Hutchison (1999), Eichengreen and Rose (1998) and Rossi (1999)). The consensus appears to be that, at least in some cases, financial crises may have "caused" exchange rate crises; in many cases, however, the twin phenomena are likely to be symptoms of underlying weaknesses of the economy, which may be manifested in various ways.[7]

Chang and Velasco (1998) consider the interaction between bank fragility, exchange rate and monetary regimes on the basis of the Diamond and Dybvig (1983) model of bank runs. They find that maintaining both a fixed exchange rate peg and stabilizing the banking sector may become mutually incompatible objectives. In their set-up, there are multiple equilibria, with the crisis brought on by a pure shift in expectations. Allen and Gale (2000) argue that it is problematic to obtain multiple equilibria because both the selection between the good and the bad equilibrium is not modelled, and the "sunspots" equilibria are unrelated to the real economy.

Chari and Kehoe (2000) take another approach by modelling investors as having herd-like behaviour. In each period a signal about the profitability of the risky project reaches the economy and is privately observed by the investors. If the signals lead investors to be sufficiently optimistic, they choose to forgo the opportunity to acquire information and they all immediately continue investing. If investors become sufficiently pessimistic they all invest in their home economy and capital flows to the emerging economy dry up completely.

Aghion et al. (2000) present a model that analyses what they define as "triple" crises, which involve currency, banking and output crises where multiple equilibria in the foreign exchange market are possible. In their dynamic model, prices are rigid in the short run and currency depreciation leads to an increase in the foreign currency debt repayment obligations of the firms that will consequently have reduced profits. Lower profits will in turn mean lower net

[7] See also Miller (1996), who considers the opposite type of causation, from a currency attack to a banking crisis.

worth, leading to less investment and lower output in the next period. Finally, there has been work to analyse financial crises using what is by now known as the Balance Sheet Approach, (Calvo et al. (2004), Allen et al. (2002)). This approach focuses, not on flow variables but on the examination of stock variables in a country's sectorial balance sheets and its aggregate balance sheet (assets and liabilities). A financial crisis will then occur when there is a plunge in demand for financial assets on one or more sectors: creditors may lose confidence in the ability of a country (either on its government, its banking system, corporations or all of them together) to earn foreign exchange to service the debt. It can then become difficult to attract new financing or roll over existing short-term liabilities (e.g. sudden stops). Massive outflows of capital, a sharp depreciation of the exchange rate, and finally a deep recession reduces further the investors' willingness to hold a country's accumulated stock of financial assets.

To our knowledge, this paper presents the first sequential-game model integrating the issues of exchange rate crisis and financial crisis where the government's optimal policy will be largely based on the trade-off between large deterioration of the fundamentals and the moral hazard effects on the private sector. As such, it represents an advance.

3. Stages of the sequential-game theoretical model

The four stages are represented by figure 1.

Stage 1

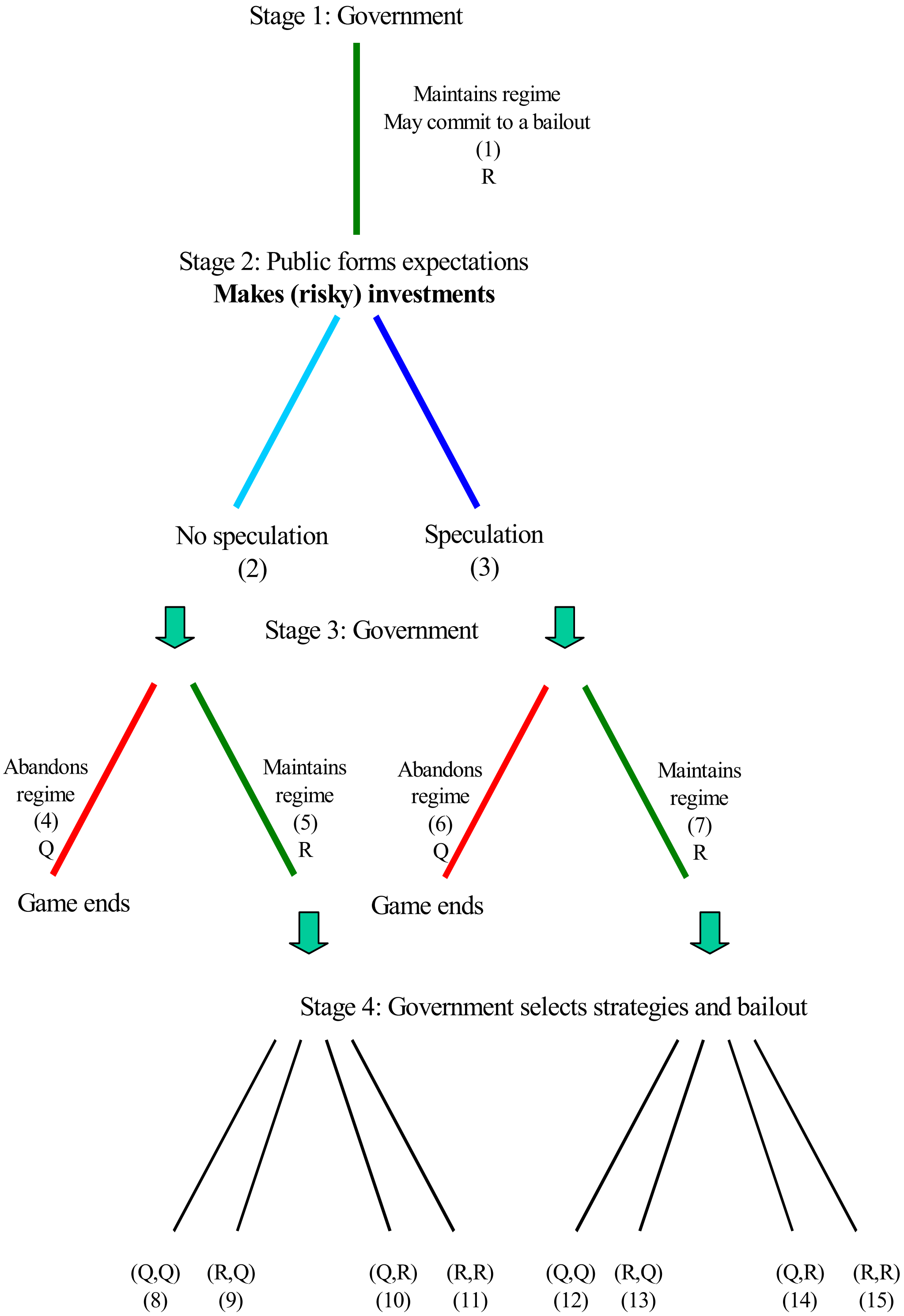
All the agents in the economy receive information about the statistical distribution of a shock that will occur in the future, in the last stage of the game, affecting the exchange rate, unemployment and the net returns on investments made by the private sector. A bad shock, s_1, will occur with probability π, while the good shock, s_2, will occur with probability $(1-\pi)$. We assume that s_2 takes the value of zero. That is, if s_2 occurs, this shock will have no effect on the economic fundamentals. At this stage, the government announces that it will be pursuing a fixed exchange rate regime and may or may not commit to bailing out part of the outstanding debt of the private sector.

Stage 2

If it is assumed that the regime is maintained until stage 4:

- The private sector forms expectations about the exchange rate that will prevail at stage 4 in basis on the information they get on the statistical distribution of the shock. They will attach a probability π that the exchange rate with a bad shock will be $x(s_1) > \bar{x}$ (e.g. they have devaluation expectations), but with probability $(1-\pi)$ the exchange rate will take a value $x(s_2) = \bar{x}$ (e.g. there are not devaluation expectations).
- The private sector makes (risky) investments to generate revenues in domestic currency. We assume that this sector needs foreign currency for investment, and they will usually need to borrow from abroad. They will then convert all their possible revenues denominated in domestic currency to acquire foreign currency, both to import some investment goods that are inputs to production, and to pay their foreign debt. It is well known that developing countries are not only net borrowers in international markets (Caballero and Krishnamurthy (2003)) but are also relatively closed economies (Calvo et al. (2003)) which limit them to obtain foreign currency through exports. The final net return to investment depends on the level of investment, the shock and on the exchange rate that prevail at stage 4. Here, both the return and marginal return at the level of K that maximizes expected returns will be negative *only* when a bad shock occurs. The bad shock is associated, among other things with higher borrowing costs (either abroad and/or in the national market, but mostly due to devaluations), extremely credit constraint conditions in the national or international market (i.e. usual credit market imperfections or the sudden stops of Calvo (1998)), and/or simply low productivity.

Figure 1: Game tree describing possible strategies in the sequential game



Q = Abandon the fixed exchange rate regime; R = Maintain the fixed exchange regime.
(m,n), m=R or Q if s_1 occurs; and n=R or Q if s_2 occurs.

We then concentrate on both the economic performance, and currency risk, usually associated with large financial currency-denomination mismatches.

Stage 3

Only the government moves. It may fight to continue keeping fixed exchange regime in spite of expectations of devaluation, or it may give up this regime already at this stage. We find that when there are not devaluation expectations, the equilibrium strategy of the government involves retaining the fixed exchange rate regime at this stage. However, as soon as there are such expectations, the government incurs certain costs that may or may not lead it to abandon the regime already at this stage. All depends on how costly it becomes to defend the fixed exchange rate regime until stage 4 while there are devaluation expectations. These costs take the form of first, expected adverse shifts in unemployment (or output) because the government follows a tight monetary policy to keep the exchange rate fixed by increasing interest rates to defend the fixed exchange rate regime. Calvo and Reinhart (2002) have characterized such situation as “fear to float” because the government recognizes that the cost of allowing a depreciation during crises is too high. Devaluation affects not only output but may also leave the sector highly indebted because of their high leverage in terms of foreign currency. The other additional costs are the expected loss of credibility (since devaluation is highly expected).

Note that all possible sub-game equilibria at this stage are very much dependent on the possible strategies and outcomes in stage 4. For this reason we solve this sequential game model for all its possible sub-game equilibria, using backward induction.

Stage 4

A stochastic shock, either s_1 or s_2, occurs affecting employment and the net returns on investment. Given the realised shock, the government minimises its loss function by choosing the optimal value of the exchange rate and the proportion of the private liabilities of outstanding stock if s_1 occurs and no ex-ante commitment to bailing out has been made in stage 1.[8]

A few additional introductory remarks are in order. First, we assume that whenever the government abandons the fixed exchange rate before stage 4, we say that the game ”ends”. This applies to the game of setting the exchange rate free to float (i.e. equal to the forward rate, for example) independent of the government’s bailout policy, which is always determined at stage 4. If the fixed exchange regime is abandoned at stage 4 (which will usually occur in the event of a bad shock), the exchange rate of equilibrium will be determined simultaneously with the optimal bailout, which are both obtained when the government minimises its loss function. Secondly, there will be several possible end nodes for the game or sub-games equilibria (provided that the game progresses to that stage) but only few of these can be reached. In other words, only some sub-game equilibria are possible after applying the method of backward induction. We will come back to these issues in detail below.

4. The private sector’s problem

By the private sector we mean the entrepreneurs who participate in stage 2 of the sequential game. They expect either that the exchange rate will, with certain probability, float freely or that the fixed exchange rate regime will be maintain. Also at stage 2, they also decide on the level of investment, which is financed by borrowing abroad. The type of expectations that the private sector has on the exchange rate will influence the level of risky investment that they will make. Here, a large dangerous liability in foreign currency can be created if devaluation expectations are at work at the same time, especially because these entrepreneurs finance their

[8] See Freixas (2000) for a description of the different ways of providing liquidity guarantees and managing crises.

investment with debt denominated in foreign currency and generate income in domestic currency (mostly producing non-tradable goods). At the same time, losses in investment are most likely damaging because it will precipitate further not only currency crisis and mismatches but also financial crises. It is not a topic of this paper the issue of debt restructuring of any type and the optimal choice over liability denomination (see Caballero and Krishnamurthy (2001, 2003) on that).

K will denote private-sector investment and $R_i(K,x(s_i))$ the net gross return on these investments (after paying the foreign debt), in the final stage, 4, when the shock i (s_i, i=1,2) is realised. To simplify notation, we will write instead just $R(K, x(s_i))$.

The private sector maximises expected *net* return with respect to the amount invested, taking into consideration that a fraction ϕ of the *net* losses will be bailed out by the government *only* in state 1. The expected net return function, E[NR], is as follows:

$$E[NR] = \pi\,(1-\phi(s_1))\,[R(K, x(s_1)))] + (1-\pi)\,[R(K, x(s_2))], \qquad (1)$$

where $x(s_i)$ is the exchange rate (number of domestic currency units per unit of foreign currency) in state s_i (i=1,2); $\bar{x}$ is the (log) fixed exchange rate level and it is normalised it to be equal one; and $\phi(s_1)$ takes values between 0 and 1 and $\phi(s_2)$ equals zero. The absolute value of $\phi(s_1)R(K,x(s_1))$ is the total bailout in state 1. $(1-\phi(s_1))R(K,x(s_1))$ is then the net loss of the private sector. A bad shock then reduces the investor's expected return (1).

Maximising (1) with respect to K yields the following first-order condition, given that $\bar{x}=1$:

$$\frac{R_K(K,x(s_1))}{R_K(K,x(s_2))} = -\frac{(1-\pi)}{\pi(1-\phi)} \qquad (2)$$

In (2), the left-hand side is the ratio between the marginal net return on investment in the bad state and the good state. At the optimal solution (equilibrium level) for K and $x(s_i)$, say K* and $x(s_i)^*$ respectively, we assume the following:

When *s_1 occurs*: $R(K^*, x(s_1)^*) < 0$; $R_K(K^*, x(s_1)^*) < 0$; $R_{KK}(K^*, x(s_1)^*) < 0$.
When *s_2 occurs*: $R(K^*, x(s_2)^*) \geq 0$; $R_K(K^*, x(s_2)^*) \geq 0$; $R_{KK}(K^*, x(s_2)^*) < 0$;

Thus, (2) will hold true if the marginal net return at equilibrium is positive in state 2, while negative in the "bad" state, state 1.

The effects of the exchange rate and the bailout on K can be seen from the following partial derivatives:

$$\frac{\partial K}{\partial \phi} = \frac{\pi R_K(K, x(s_1))}{(1-\pi) R_{KK}(K, x(s_2)) + \pi(1-\phi) R_{KK}(K, x(s_1))} > 0 \,; \tag{3}$$

if $R_K(K,x(s_1)) < 0$; $R_{KK}(K,x(s_1)) < 0$ and $R_{KK}(K,x(s_2)) < 0$; and

$$\frac{\partial K}{\partial x(s_1)} = \frac{-\pi (R_{Kx_1}(K, x(s_1))(1-\phi)}{(1-\pi) R_{KK}(K, x(s_2)) + \pi(1-\phi) R_{KK}(K, x(s_1))} < 0 \,; \tag{4}$$

if $R_{Kx1}(K, x(s_1)) < 0$; $R_{KK}(K,x(s_1)) < 0$ and $R_{KK}(K,x(s_2)) < 0$

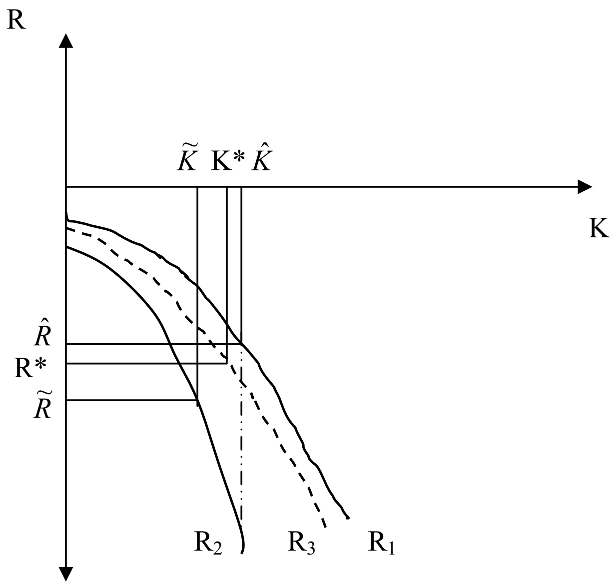
(3) indicates that the anticipation of a greater fraction of bailouts, ϕ, in the "bad" future state (1), all else being equal, thus leads to higher risky investment, and consequently to more foreign debt, in stage 2. Thus, safety net evidently causes moral hazard problems as many others have found, as for example Eichenbaum, Burnside, and Rebello (2000, 2002).[9]

While (4) shows that an increase in $x(s_1)$, i.e. a stage 4 devaluation which is rationally anticipated by the private sector in stage 2, makes them incur less debt if and only if R_{Kx1} is negative. Given that at the optimal solution (equilibrium level) for investment K, the marginal return to K is negative in the bad state (R_K is negative), it is reasonable to expect that a devaluation will make such marginal return even more negative, since investment is less productive as borrowing in foreign currency becomes more expensive. This implies that the private sector may have incentives to insure themselves against large leverage in foreign currency and sudden stops. They may react by borrowing less in the international markets.

[9] Note also that we are not modelling how different types of investment may be chosen, e.g. in terms of their riskiness, in response, for example, to ϕ. We will take up this issue in future research.

Caballero and Krishnamurthy (2003) argue that the reason why firms take so much foreign currency debt is because the government implicitly insure them against currency risks when it defends at any cost the fixed exchange rate regime. Thus, given that in the bad state $R(K^*, x(s_1)^*) < 0$, and $R_K(K^*, x(s_1)^*) < 0$, K may increase with prospects of a smaller (or none) devaluation if also $R_{Kx1}(K^*, x(s_1)^*) < 0$.[10] The reason for that is that the marginal return to investment K will be less negative with smaller or none devaluation. The return to investment, the marginal return to investment, and how the exchange rate affect both of these two in the bad state, will determine investment, and how profitable is for the entrepreneurs to insure themselves from currency risks. We can illustrate all of these in figure 2.

Figure 2: Net returns to investment in the bad state



First of all notice in figure 2 that in the bad state for given devaluation, entrepreneurs would chose $\hat{K}$. An anticipated further devaluation will shift the net return to investment curve from

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[10] If it were the case that $R(K^*, x(s_1)^*) > 0$; $R_K(K^*, x(s_1)^*) > 0$; and $R_{Kx1}(K^*, x(s_1)^*) > 0$, K will not necessarily decrease, it can in fact increase. $R_{Kx1}(K^*, x(s_1)^*) > 0$ must mean that will be when these entrepreneurs are exporters, and devaluations will increase their returns to investment. We will move away from this case since the case of $R_{Kx1}(K^*, x(s_1)^*) < 0$ is more relevant for emerging economies, since Calvo (1998) has argued that exports have not been playing a large role in economies as Argentina.

R_1 to R_2 making $\hat{K}$ even less profitable. A new level of investment $\tilde{K}$ will be chosen along the curve R_2 to achieve a net return to investment $\tilde{R}$.[11]

Thus, when devaluation expectations are born in anticipation to a bad shock, expectations of larger negative net returns on investment will arise simultaneously leading to a worsening of the economic fundamentals. A second round follows validating expectations that the time for devaluation is ripe given the large deterioration of the economy, and so deepening the impact of the initial devaluation expectations on investment returns and on the economic fundamentals, as we modelled below, unless investment decreases substantially. Note that this low level of investment, $\tilde{K}$, may be suboptimal if a good shock occurs.

We will also consider the case in which the government commits ex-ante in stage 1 to provide liquidity in the bad state before expectations about the relevant variables are formed. The government will then take into account the moral hazard effects indicated in (3) and (4) while it minimises its loss function. We analyze if this ex-ante commitment to provide liquidity affect expectations on the exchange rate in the desired way, and how changing these expectations can affect the private sector's investment. That is, will ex-ante commitments to bailing out that reduces (if not eliminate) devaluation expectations and consequently influence the private sector's incentives to insure themselves against international credit constraint and sudden stops? In figure 2, we illustrate such possibility. As a result of an ex-ante commitment to provide liquidity in the bad state, an endogenously *smaller devaluation* is anticipated; the *returns to investment will improve* even though investment will increase somewhat. These two effects will be reflected in the shift from R_2 to R_3. Indeed, will investment increase from $\tilde{K}$ to K*, and give a net return equal to R*. Ameliorating the bad effects of the bad shock on the

[11] One could think that $\tilde{R}$ can be greater than $\hat{R}$ which would mean that with a devaluation entrepreneurs either must have decreased investment substantially, they are better of with a devaluation (i.e. entrepreneurs are exporters and they borrow little in foreign currency) or both. We think that assuming that $\tilde{R}$ is smaller than $\hat{R}$ is more realistic since in emerging economies entrepreneurs are net borrowers in foreign currency.

returns to investment may reduce devaluation expectations in response to better prospective returns and fundamentals.

We will compare this ex-ante commitment case of liquidity provision with the case in which the government does not commit but decide optimally on it only at stage 4 after a bad shock occurs.

5. The government's problem

We assume that it is costly for the government to have high levels of unemployment and bailouts. These variables will enter the government's loss function at the stages of the game in which they are relevant. Costs of bailing out, independent of the sources for financing it, has not been modelled in the previous and related literature. Most of the time bailouts are financed by seignorage, and this is assumed to be costless. We here consider such costs to be of importance if the government needs to evaluate the effectiveness and optimality of bailing out. We think that even when an international institution gives liquidity provision, the government should internalise it and consider it costly. After all, the country in question should pay back such financial assistance, and/or such granted resources can be used in some other productive manner.

We assume that the unemployment rate in stage 4 after a shock s_i has occurred is determined as follows:[12]

$$u_4(s_i) \equiv u(x(s_i)) = -\alpha_1[x(s_i) - \bar{x}] + u_n + \alpha_2 r + s_i - \alpha_3[(1-\phi(s_i))R(K, x(s_i))]; \qquad (5)$$

In (5), u_n is its natural rate level, and the first term implies that currency devaluation rate ex-post (occurring after the shock realises) in stage 4 reduces unemployment.[13] Private-sector debt problems also have negative consequences for employment in state 1 because debts lead to insolvency and bankruptcies and consequently job losses. The unemployment gap increases

[12] Note that $\phi(s_2)=0$.

with the outstanding debt that is not being bailed out. Moreover, the government may use high interest rates to eliminate or decrease devaluation expectations for fear to float (Calvo and Reinhart (2002)), but tighten monetary policy may slow down economic activity yielding higher unemployment rates. Specification as (5) catches up the idea of Aghion et al. (2000) who model the "triple" crises. In stage 3 however, while the fixed exchange rate regime is defended, the unemployment at this stage 3 is assumed to be equal to:

$$u_3 - u_n = \alpha_2 g(Ex - \bar{x}); \qquad (6)$$

which of course in absence of devaluation expectations, unemployment remains unaffected. If the exchange rate were abandoned at this stage 3, the free-floating exchange rate obeys, for example, the forward market.

We now need to specify the government's loss functions to be minimised at stages 3 and 4. Recall that the shock of type 2, s_2, always equal zero; and all parameters are greater than zero. E represents the conditional expectations operator while C represents the costs of abandoning the fixed exchange regime, for example through loss of credibility. λ is a dummy variable that takes the value of one when the fixed-rate policy is abandoned, and zero otherwise.

At stage 3, given that the fixed exchange rate is retained until then, the government will decide whether or not it is optimal to maintain such exchange rate regime. For that purpose, it will minimise the following expected loss function:[14]

$$V_3(s_i, Ex, x(s_i), \phi(s_i)) = a_1(u_3 - u_n)^2 + a_2 E(u_4 - u_n)^2 - E\phi(s_1)R(K, x(s_1)) + EC; \qquad (7)$$

Since $\phi(s_2) = 0$ and $E[\phi(s_2)]=0$. (7) includes the costs of excess unemployment at this 3[rd] stage as a result of devaluation expectations, plus the expected unemployment costs and the expected costs of bailing out in stage 4, given that a bad shock may occur at stage 4.

The loss function ***in stage 4*** is:

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[13] The devaluation rate is formally $(x(s_1)- \bar{x})/\bar{x}$, to simplify, we eliminate the denominator since $\bar{x}$ =1.

[14] Note that we still write $\bar{x}$ even though it equals 1.

$$V_4(s_i, Ex, x(s_i), \phi(s_1) = a_2(u_4 - u_n)^2 - \phi(s_1)R(K, x(s_1)) + \lambda C \quad (8)$$

(8) includes the costs of unemployment and bailing out at this 4th stage.

The following relationship is assumed:

$$r = g(Ex - \bar{x}); \quad (9)$$

where g is a positive constant. Ex-$\bar{x}$ represents the magnitude of the devaluation expectations that the government may try to fight against in stages 3 and 4 by increasing interest rates. This is recognized in the literature as a typical reaction of the government for their fear to float (Calvo and Reinhart (2002)).

Now, as we pointed out before, the possible sub-game equilibria at each state requires that we solve this sequential game using the backward induction method. Let us then present first the sub-game equilibria for stage 4.

6. State contingent bailout policy: No ex-ante bailout commitments

6.1 Stage 4

The following propositions show the different optimal decisions made by the government on the exchange rate and bailout with and without expectations of devaluation. Keep in mind that in the absence of ex-ante commitments, the government takes as given the entrepreneurs decisions. These on the other hand will also take as given the sub-game equilibrium for the exchange rate and bailout that will yield at stage 4. *Agents (and we) in this economy will solve the possible set of sub-game equilibria by the method of backward induction.*

Proposition 1: *With no devaluation expectations* (Ex=$\bar{x}$), a *rational expectations equilibrium can only be compatible with the end node (11), in figure 1, and a level for the risky investment equal to* $\tilde{K}$ (to be obtained from (2)). *Node (11) corresponds to the decision of maintaining the fixed exchange rate when either shock takes place. That is:*

$$x(s_1) = \bar{x};$$
$$x(s_2) = \bar{x}$$

Proof: See Appendix A.

In short, nodes (8), (9) and (10) must be counted out because if the government is certain to abandon the regime when either shock occurs, the private sector will always anticipate this and will instead expect devaluation. If there is a bad shock there will be some unemployment gap and negative returns to investment even though there are not devaluation expectations. The optimal proportion of debt that will be bailed out can be obtained by minimising the following government's loss function:

$$V_4(s_1, Ex = \bar{x}, x(s_1) = x(s_2) = \bar{x}, \tilde{\phi}, \tilde{K}, C) = a_2[s_1 - \alpha_3(1-\tilde{\phi})R_1(\tilde{K}, \tilde{x}(s_1))]^2 - \tilde{\phi}(s_1)R_1(\tilde{K}, x(s_1)) + C \quad (10)$$

The optimal bailout is:[15]

$$\tilde{\phi} = 1 - \frac{2a_2 s_1 \alpha_3 - 1}{2a_2 \alpha_3^2 R(\tilde{K}, x(s_1))} \quad (11)$$

Proposition 2: *With devaluation expectations, ($Ex > \bar{x}$), a rational expectations equilibrium is compatible only with end node (14), in figure 1, a level of risky investment equal to* $\hat{K}$ (to be obtained from (2), *and an optimal bailout that is larger than when there are not devaluation expectations. Such equilibrium corresponds to abandoning the fixed rate if a "bad" shock takes place but maintaining it if a "good" shock occurs. This implies that:*

$$x(s_1) > \bar{x};$$
$$x(s_2) = \bar{x}$$

Proof: See Appendix A.

Here the government cannot abandon the fixed exchange rate regime if also the good shock occurs. If so, as we will demonstrate later, it will be rather optimal to abandon the regime

[15] Note first that there will not be any bailout if: $0 \leq \frac{2a_2 s_1 \alpha_3 - 1}{2a_2 \alpha_3^2 R(K, x(s_1))} < 1$

already at stage 3. Also, if the regime is never abandoned, this will be anticipated by the private sector and their optimal decisions regarding investment will depend rather on that there will be never devaluation, so that they will have no reasons to expect devaluation. Proposition 1 and 2 indicate that these sub-game equilibria are self-fulfilling equilibria.

With a bad shock (and devaluation expectations), there will be some unemployment gap and an outstanding debt.

Now, the expectations in the foreign exchange market are determined as follows:

$$Ex = \pi x_1 + (1-\pi)x_2 \,; \tag{12a}$$

and given that the government sets $x_2 = \overline{x}$, we will have that:

$$Ex - \overline{x} = \pi(x_1 - \overline{x}) \tag{12b}$$

The optimal bailout is here obtained by minimising:

$$V_4(s_1, Ex > \overline{x}, x(s_1) > \overline{x}, x(s_2) = \overline{x}, \hat{\phi}, \hat{K}, C) = a_2[-\alpha_1(\hat{x}(s_1) - \overline{x}) + \alpha_2 g\pi(\hat{x}(s_1) - \overline{x}) + s_1$$

$$-\alpha_3(1-\hat{\phi}(s_1))R_1(\hat{K}, \hat{x}(s_1))]^2 - \hat{\phi}(s_1)R_1(\hat{K}, \hat{x}(s_1)) + C \tag{13}$$

Assuming $\overline{x}$ =1, the optimal bailout, $\hat{\phi}$, when the government devalues in state 1 is: [16]

$$\hat{\phi} = 1 - \frac{2a_2\alpha_3(\hat{x}(s_1) - \overline{x})(\alpha_2 g\pi - \alpha_1) + 2a_2\alpha_3 s_1 - 1}{2a_2\alpha_3^2 R(\hat{K}, \hat{x}(s_1))} \tag{14}$$

It is here easy to notice that $\hat{\phi} > \tilde{\phi}$. The optimal exchange rate, $\hat{x}(s_1)$, will be then:

$$\hat{x}(s_1) = \overline{x} + \frac{\alpha_3(1-\hat{\phi})R(\hat{K}, \hat{x}(s_1)) - s_1}{\alpha_2 g\pi - \alpha_1} + \tag{15}$$

$$\frac{\hat{\phi} R_{x_1}(\hat{K}, \hat{x}(s_1))}{2a_2[\alpha_2 g\pi - \alpha_1 - \alpha_3(1-\hat{\phi})R(\hat{K}, \hat{x}(s_1))][\alpha_2 g\pi - \alpha_1]}$$

[16] Note that there will not be any bailout if: $0 \leq \frac{2a_2\alpha_3 s_1 - 1 + \left[2a_2\alpha_3(\hat{x}(s_1) - \overline{x})(\alpha_2 g\pi - \alpha_1)\right]}{2a_2\alpha_3^2 \hat{x}(s_1) R(\hat{K}, \hat{x}(s_1))} < 1$

If we concentrate in the first part of (15), we can see that there will be a devaluation if this itself has stronger effects than devaluation expectations ($\alpha_1 > \alpha_2 g\pi$) on unemployment, otherwise, it is not obvious that it will be optimal for the government to devalue. Now it we look at the whole expression (15), to devalue we need to have R_{x1} negative, which is uncontroversial because the marginal return with respect to the exchange rate will be negative when there is devaluation since borrowing become more expensive. Finally, it will be necessary that the investment losses in the bad state are sufficiently large, $R(\hat{K}, \hat{x}(s_1))$, to warrant a devaluation. In such case we will have that $\alpha_2 g\pi - \alpha_1 - \alpha_3(1-\hat{\phi})R(\hat{K}, \hat{x}(s_1)) > 0$. Again, note that the larger the outstanding debt, the larger the unemployment, and when fundamentals deteriorates the government will have more difficulties to maintain the fixed exchange rate regime.

6.2 Stage 3

We will below present some propositions that will rationalize the possible sub-game equilibria at this third stage. To find out such equilibria, we use the method of backward induction and take into account the solutions presented in Propositions 1 and 2. Note that at this stage, the government can make decisions about the exchange rate while decisions about bailouts are not made until stage 4. At the outset it is also important to be aware that when there are no devaluation expectations, it will never be optimal to abandon the regime in stage 3. The government might as well wait until stage 4 to make decisions about the exchange rate and bailouts. A different issue will be when $Ex > \bar{x}$, as we will explain below.

From Proposition 2 we know that in the presence of devaluation expectations, and a "bad" shock, the rational expectations equilibrium for the bailout is $\hat{\phi}$ (equation (14)), for the exchange rate will be $\hat{x}_1(s_1)$ (equation (15)), while the optimal investments equals $\hat{K}$.

The government's loss function in stage 3 as described in (7) will take the following specific functional form after using (5), (6) and (9):

$$V_3(s_i, Ex > \overline{x}, x(s_i), \hat{\phi}, \hat{K}) = a_1(\alpha_2 g)^2 (Ex - \overline{x})^2 + a_2\{\pi[u_4(s_1) - u_n]^2 + (1-\pi)[u_4(s_2) - u_n]^2\} - \pi\hat{\phi}(s_1)R(\hat{K}, \hat{x}(s_1)) + \pi C; \tag{16}$$

Now, when there are devaluation expectations, the government can make either of the following decisions:

i) To abandon the fixed exchange regime immediately after the private sector's expectations have formed, that is at the very beginning of stage 3. This decision will cost the government C.

ii) To maintain the exchange rate regime, bearing all the costs of defending it, and wait until stage 4 to make a decision on the exchange rate and bailing out. Such a decision implies that the government considers it worthwhile to wait until the shock to the economy occurs and to abandon the regime only in the bad state, i.e. $x(s_1) > \overline{x}$ and $x(s_2) = \overline{x}$, as Proposition 2 suggests. If the government chooses such option, its loss function (16) after using (12) becomes:

$$V_3(Ex > \overline{x}, \hat{x}(s_1) > \overline{x}, x(s_2) = \overline{x}, \hat{\phi}, \hat{K}) = a_1(\alpha_2 g\pi)^2(\hat{x}(s_1) - \overline{x})^2 + a_2\pi[(-\alpha_1(\hat{x}(s_1) - \overline{x}) + \alpha_2 g\pi(\hat{x}(s_1) - \overline{x}) + s_1 - \alpha_3(1 - \hat{\phi}(s_1))R(\hat{K}, \hat{x}(s_1))]^2 - \pi\hat{\phi}(s_1)R(\hat{K}, \hat{x}(s_1)) + \pi C \tag{17}$$

Note that one of the differences between (17) and (13) (the government's loss function in the fourth stage) is that (17) now includes both the shifts during stage 3, and the expected shifts in the fundamentals in stage 4. The latter occurs when at stage 3, devaluation and net investment losses are anticipated to occur in stage 4.

iii) The costs of staving off speculative attacks and defending the exchange rate regime from, during and after stage 3 become too high. In such a case the fixed exchange rate regime will be abandoned already at stage 3. All agents will also anticipate already at stage 3, that it will not be affordable to keep the fixed exchange rate regime in stage 4 regardless of whether a "bad" or "good" shock is expected to occur in that stage 4, so that $x(s_1)>\bar{x}$ and $x(s_2)> \bar{x}$. The government's loss function (16) at stage 3 will become:[17]

$$V_3(Ex > \bar{x}, \hat{x}(s_1) > \bar{x}, x(s_2) > \bar{x}, \hat{\phi}, \hat{K}) = a_1(\alpha_2 g)^2 (Ex - \bar{x})^2 + a_2\{\pi[-\alpha_1(\hat{x}(s_1) - \bar{x}) +$$

$$\alpha_2 g\pi(Ex - \bar{x}) + s_1 - \alpha_3(1-\phi(s_1))R(\hat{K}, \hat{x}(s_1))]^2 + (1-\pi)[-\alpha_1(x(s_2) - \bar{x}) \tag{18}$$

$$+\alpha_2 g(Ex - \bar{x})]^2\} - \pi\hat{\phi}(s_1)R(\hat{K}, \hat{x}(s_1)) + \pi C$$

In order to choose optimally between decisions (i), (ii) or (iii), it is necessary to evaluate each of the loss functions above, say (17) and (18) against the one (equation (13)) that results from maintaining the exchange rate in stage 4 where there are devaluation expectations but such expectations are self-fulfilled only when the bad shock occurs.

Proposition 3: *When there are devaluation expectations, it will never be optimal to abandon the exchange rate regime at the very start of stage 3 if the probability that a bad shock will occur, π, is very small.*

Proof: If the government abandons the regime just after the end of stage 2, it will incur in a cost equal to C, the loss of credibility. To wait until stage 4 and to decide on the optimal devaluation and bailout in case a bad shock occurs, it implies for the government to evaluate the losses derive from its possible loss function at stage 4 (equations (13)) against C. Note

[17] Note that we now do not use (11) but rather $Ex = \pi x(s_1) + (1-\pi)x(s_2)$ since it is expected rationally that

from (13) that the government will face certain costs only if a bad shock occurs, and the latter will occur only with certain probability. Having this probability small enough, it will not be optimal to abandon the exchange rate regime even though the costs of maintaining the regime until stage 4 are significant.

Proposition 4: *When there are devaluation expectations, the government will stave off the speculative attacks during stage 3 and wait until stage 4 to make decisions on the exchange rate and bailout if the following condition is fulfilled:*

$$a_1(\alpha_2 g)^2 \pi^2 (\hat{x}(s_1) - \overline{x})^2 > (1-\pi)\{V_4[s_1, Ex > \overline{x}, \hat{x}(s_1) > \overline{x}, x(s_2) = \overline{x}, \hat{\phi}, \hat{K}, C] \quad (19)$$

Note that the expression on the left-hand side represents the costs of staving off speculative attacks in terms of higher unemployment during stage 3. The expression in the right-hand side is the probability that a "good shock" will occur times the total cost of maintaining the exchange rate regime until stage 4 when there are devaluation expectations.[18]

Proof. (19) is obtained by comparing (13) and (17).

Proposition 5: *When there are devaluation expectations, it will be optimal for the government to abandon the exchange rate regime already in stage 3 if the following condition is fulfilled:*

$$a_1(\alpha_2 g)^2 \pi^2 (\hat{x}(s_1) - \overline{x})^2 < (1-\pi)\{V_4[s_1, Ex > \overline{x}, \hat{x}(s_1) > \overline{x}, x(s_2) = \overline{x}, \hat{\phi}, \hat{K}, C] + \\ a_2[-\alpha_1(x(s_2) - \overline{x}) + \alpha_2 g(Ex - \overline{x})]^2\} \quad (20)$$

In contrast to (18), the right-hand side now also includes the costs, in terms of higher unemployment, for planning to devalue also in both states.[19]

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x(s_2)> $\overline{x}$.

[18] Notice that: $a_1(\alpha_2 g)^2 \pi^2 (\hat{x}(s_1) - \overline{x})^2 = a_1\{\alpha_2 g(Ex - \overline{x})\}^2 = a_1(u_3 - u_n)^2$.

[19] Notice that: $(1-\pi)a_2\{\alpha_1(x(s_2) - \overline{x}) + \alpha_2 g(Ex - \overline{x})\}^2 = (1-\pi)a_2\{u_4(s_2) - \overline{u}\}^2$, since $\hat{x}(s_2) > \overline{x}$.

Proof.

(20) is obtained by comparing (13) and (18).

7. Bailout commitments

We have found so far that that *when the government does not commit* ex-ante to an optimal bailout, the exchange rate and financial crises can be driven by self-fulfilling expectation in which case there will be multiple equilibria. One can reach node (11), node (14) or node (6). When expectations for devaluation are born, not bailout commitments before stage 4 has been made, and it is too costly to devalue already in stage 3, we have found that it will be optimal to devalue in stage 4 only in the bad state (proposition 2) in which case there will be self-fulfilling expectations.

In this section we analyse a situation in which the government commits ex-ante at stage 1 before the market forms expectations to provide liquidity in the bad state. Obviously, committing in stage 1 is only relevant if expectations of devaluation are present in anticipation that a bad shock may occur (with probability π).[20] Here, we would like to address the following question: Can the government, by committing ex-ante to an optimal bailout (i.e. that minimises its loss function) avoid not only the multiple equilibria outcome but also ameliorate devaluation expectations in spite that a bad shock is anticipated?

If the answer to the above question is yes, we can argue that by committing to a certain bailout, the government may avoid the costs of staving off large devaluation expectations, and those associated with devaluation itself in stages 3 and 4. This is easy to recognize by noting the fundamental differences between minimising the loss function (10) when there are not devaluation expectations, and the loss function (13) when there are such expectations.

[20] When there are not devaluation expectations, as we stated in Proposition 1, it will never be optimal to devalue when either shock occurs. From Proposition 2, we also know that with devaluation expectations, it will be optimal to devalue only if a bad shock occurs.

Then, the only reason to commit ex-ante to an optimal bailout, it should be to avoid a large deterioration of the economic fundamentals that could arise as a result of an anticipated devaluation and large negative returns to investment. With better perspectives on the economic fundamentals, the less likely that the public will rationally anticipate a devaluation.

The scenario that the government faces to determine whether or not it is beneficial to commit or not ex-ante to an optimal bailout is the following. In stage 1, the government views that there is a probability (π) that there could be devaluation expectations (and a bad shock). It is only in such case, as mentioned, that it will make sense to commit to any bailout. The definition of the government's loss function in such a case, say V_1, will include the expected costs of defending the fixed exchange rate regime during stages 3 and 4, and the expected outstanding debt from the private sector and bailout expenditures that could incur with probability π because of a bad shock at stage 4. The government will minimise this loss function in order to find the optimal bailout, while the exchange rate will be determined endogenously as a result of the government's and entrepreneurs' reactions to such commitment. Having this established, the loss function is defined as:

$$V_1 = V_1(Ex > \bar{x}, \hat{x}(s_1) > \bar{x}, x(s_2) = \bar{x}, \phi^{COMM}, K^{COMM}); \tag{21}$$

where ϕ^{COMM} will represent the optimal committed bailout while K^{COMM} will be the optimal investment decision of the private sector when taking into account such commitment.

We should now realise that the functional form of (21) needs to be like (17). The main difference is that the government will minimize this loss function in stage 1, before the entrepreneurs form expectations. This implies that the government will take into account how entrepreneurs will respond, regarding their investment and expectations about the exchange rate to such commitment. At the same time entrepreneurs will consider how such commitment to bailing out will affect their net return to investment to thereafter decide on the level of investment, as they should rationally anticipate certain level for the exchange rate that results from the interaction between the government and entrepreneurs. Thus, the optimal bailout (say ϕ^{COMM}) that the government can commit in stage 1 is obtained from the following expression:

$$\frac{dV_3}{d\phi}+\frac{dV_3}{dK}\left[\frac{\partial K}{\partial \phi}+\frac{\partial K}{\partial \hat{x}(s_1)}*\frac{d\hat{x}(s_1)}{d\phi}\right]=0, \tag{22}$$

where:

$$\frac{dV_3}{d\phi}=\frac{\partial V_3(Ex>\bar{x},\hat{x}(s_1)>\bar{x},x(s_2)=\bar{x},\phi,K)}{\partial \hat{x}_1(Ex>\bar{x})}*\frac{d\hat{x}(s_1)}{d\phi}$$

$$+\frac{\partial V_3(Ex>\bar{x},\hat{x}(s_1)>\bar{x},x(s_2)=\bar{x},\phi,K)}{\partial \phi},$$

and:

$$\frac{dV_3}{dK}=\pi(x(s_1)/\bar{x})(R_1')\{2a_2\alpha_3(1-\phi)[(-\alpha_1(x(s_1)-\bar{x})+\alpha_2 g\pi(x(s_1)-\bar{x})+s_1-$$

$$\alpha_3(1-\phi)(x(s_1)/\bar{x})R_1(K)]-\phi\}$$

$\delta K/\delta\phi$ (the moral hazard effect of bailing out on investment (borrowing abroad)) and $\delta K/\delta\hat{x}(s_1)$ can be obtained from (3) and (4), respectively. While $d\hat{x}(s_1)/d\phi$ in turn will be calculated from (15).

Having ($d\hat{x}(s_1)/d\phi$) in (22) indicates that the optimal bailout will be a function of how the exchange rate, $\hat{x}(s_1)$, will be affected by such ex-ante commitment. [21] [22]

That expressions $\delta K/\delta\phi$ and $\delta K/\delta\hat{x}(s_1)$ enter in (22) implies that when the government minimises its loss function, *it also takes into consideration those moral hazard effects of bailing out*. As we explain in section 4, devaluation will necessarily decrease investments

[21] Notice that a government's ex-ante commitments to bailing out will determine the exchange rate, say x^{COMM}, along the function (15). A new exchange rate function under commitment will not be then derived (that will require to minimize (22) with respect to $x(s_1)$).

[22] Recall that when the government has not made any commitment to bail out before stage 4, the optimal exchange rate $\hat{x}(s_1)$, is determined by (15) and the optimal bailout $\hat{\phi}(s_1)$, by(14) at stage 4. Such a solution is

below in the bad state. Now, if ex-ante commitments to provide liquidity affect expectations on the exchange rate in the desired way, the private sector's decisions on investment may be also affected. That is, if ex-ante commitments to bailing out reduce (if not eliminate) devaluation expectations, the private sector may have fewer incentives to insure themselves against international credit constraint and sudden stops. We illustrate this in figure 2, section 4. There, we indicate that as a result of an ex-ante commitment to provide liquidity in the bad state, entrepreneurs will anticipate a lower (or none) devaluation (i.e. the curve of net return to investment shifts to the left, from R_2 to R_3), inducing them to increase investment (i.e. from $\widetilde{K}$ to K*). One may conclude that indeed avoiding a devaluation will the private sector with fewer incentives to insure themselves against currency crises. We here argue that such devaluation can lead to substantial financial losses and deterioration of the economic fundamentals which will only self-fulfil the initial devaluation expectations. People start to panic and their behaviour is likely to become far less predictable, financial markets will become more volatile, existing sudden stops will worsen, and policy mistakes become much more likely in such an environment.

In this respect, a policy that considers a well-planned ex-ante commitment to bailing out can be viewed as a back-up instrument that helps to protect the economy and the government from Open Market Operation mistakes. It should be clear that such policy should be implemented to rescue the entrepreneurs only if a bad shock occurs. There is then also the issue to whether ameliorating (by committing ex-ante to bailing out) the bad effects of the bad shock on the returns to investment may reduce devaluation expectations in response to better prospective returns and fundamentals. Otherwise, investment may decrease excessively in anticipation to a devaluation and become suboptimal if the economy ends up in the good state.

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one in which the government devalues optimally in the most adverse state and when the private sector expects devaluation in the bad state. In the good state there will not be any devaluation so $x(s_2) = \bar{x}$.

In contrast to what previous literature has assumed, moral hazard effects are not only endogenised but they are also taken into account when the government minimises its loss function. The optimal policy regarding the commitment to bailing out is based in the trade-off between the moral hazard effects and the deterioration of the fundamentals.

Note than in the *no-commitment case*, the government decisions do not depend on δK/δϕ and δK/δ$\hat{x}(s_1)$, as one can see from the loss functions (10), (12), (17) and (18). Moral hazard effects are not taken into consideration when there are not commitments. This is at the heart of the literature that is very critical to bailing out. Such critics are based on models where authorities disregard such moral hazard problems. On the other hand, announcing should bailouts should be never given because of the moral hazards that it could involve is not a credible policy. If bailouts are going to be given anyway, one should take into consideration not only how bailouts will affect the private sector's behaviour when determining the optimal bailout but also how to minimise such effect as we suggest it should done by committing ex-ante to bailing out. Without commitments, we first cannot account for the moral hazard effect of bailing out; and secondly, we will not be able to eliminate devaluation expectations that easily trigger deterioration of the fundamentals.

We believe that neither of these effects have been analysed before in the related literature. Our analytical framework seems also appropriate to analyze the twin crises and the effects of bailing out.

To summarise, *an ex-ante commitment to an optimal bailout* (ϕ^{COMM}) in stage 1 only when a bad shock occurs, requires that the government takes into account i) the private sector's returns to investment, ii) the effect of such commitment on the private sector's investment decisions (i.e. moral hazard problems), and iii) the private sector's expectations regarding the future exchange rate in light of the degree of deterioration in the fundamentals due to a bad shock. With no commitment, as propositions 1 and 2 indicate, government decisions on the exchange rate and bailout in stage 4 are made after the private sector has formed expectations on the exchange rate and decided on investment levels. This is not the case when there are commitments.

8. Numerical example

The purpose of a numerical exercise is to compare ϕ^{COMM} with the non-committed bailout ($\hat{\phi}$). It is rather cumbersome to solve analytically for ϕ^{COMM} from the first order condition (22). The following additional results are less abstract but equally interested. We solve for the equilibrium levels of the exchange rate, bailout and investment, for both cases with and without ex-ante commitments to bailing out. For this purpose, we assume an explicit functional form for the returns to investment ($R_i(K)$, i=1,2) and some reasonable values for the parameter of the model. We assume that:

$$R_i(K) = \gamma_i K - \delta_i K^2; \qquad i=1,2$$

which together with the parameter values ($a_1=a_2=0.78$, $\alpha_1=2.2$, $\alpha_2=4$, $\alpha_3=1$, $g=0.7$, $p=0.5$, $s_1=0.25$, $\gamma_1=0.2$, $\delta_1=0.8$, $\gamma_2=2$, $\delta_2=0.5$) gives us that investments are risky and the losses can be large if a bad shock occurs but positive in the good state (when a good shock occurs). These values and the functional forms may seem arbitrary but we made several simulations to test the robustness of those parameter values. Table 1 presents only a handful of them.[23]

Note that when the private sector expects devaluation, and the government wishes to influence these expectations, the equilibrium values for the exchange rate, bailout, and investment with and without commitments should be obtained by *solving simultaneously (14), (15) and (2) for the non-commitment case; and (22), (15) and (2) for the commitment case.* [24]

[23] Not all the simulation results are presented in table 1. The other results are available upon request.
[24] As we mentioned above, agents in this economy uses backward induction when making optimal decisions.

Table 1 shows the numerical results on the equilibrium levels of the exchange rate, investment and bailouts both when the government does not commit (before stage 4) and when it commits (in stage 1) to bail out.

Table 1. Equilibrium values of the exchange rate, bailout and investment*

NON-COMMITMENT CASE		$\hat{\phi}$	$\hat{x}$	$\hat{K}$
	P.V.*	0.33	1.523	1.674
	γ_2=1.5,δ_2=0.35	0.39	1.30	1.674
COMMITMENT CASE		ϕ^{COMM}	x^{COMM}	K^{COMM}
	P.V.*	0.18	1.197	1.712
	γ_2=1.5,δ_2=0.35	0.26	1.02	1.723

* P.V. means parameter values: $\bar{x}$ =1, a_1=a_2=0.78, α_1=2.2, α_2=4, α_3=1, g=0.71, p=0.5, s_1=0.25, γ_1=0.2, δ_1=0.8, γ_2=2, δ_2=0.5 which were mentioned above.

The following conclusions are obtained from this numerical exercise:

1. When the government commits to a specific optimal bailout at stage 1, it will need to devalue less or simply not to devalue. An ex-ante commitment to bailing out can prevent a currency crisis and ameliorate the effects of financial crisis on the fundamentals in spite that bailouts, under commitment, are smaller. There will be higher investments.
2. If there are devaluation expectations, and the fixed exchange rate regime is expected to be abandoned in state 1 but not in state 2, the optimal bailout under commitment will smaller than the optimal bailout with non-commitment. There will be however an increase in the risky investment. Note that an ex-ante commitment may warrant smaller investment losses and deterioration of the fundamentals making a large or any devaluation unnecessary. The private sector takes this into account when forming expectations about the exchange rate and anticipates that devaluation will be smaller

or less likely to occur. A lower (or none devaluation) will lead to higher return to investment than otherwise. As a consequence only a smaller bailout will be necessary.

3. We obtain a unique equilibrium characterised by having less or none devaluation in comparison with the case in which there are not commitments ex-ante. This is in contrast to the work of Diamond and Dybvig (1983), Chang and Velasco (1998), Cooper and Corbae (2002) among others. Cooper and Corbae (2002) also present numerical solutions for their model using specific parameter values. Thus, in contrast to what we have here, these authors are then not able to make predictions about the effects of monetary policy because do not have a theory of equilibrium selection. Our most important result here is that it may be possible to implement not only a unique equilibrium but also an efficient one.
4. The government will commit to a larger bailout and will also devalue more than otherwise if: i) unemployment response to government decisions to devalue becomes larger (i.e. α_1 is larger), and ii) unemployment response to the private sector expectations of devaluation becomes smaller (i.e. α_2 smaller). That is, if the improvement in the macroeconomic fundamentals is larger when devaluation occurs and/or fundamentals are somewhat independent of the market's expectations, the government will have more incentive to devalue, and it will more than self-fulfil expectations in order to affect the fundamentals.
5. If there are larger negative effects of the outstanding private sector debt on the macroeconomic fundamentals, the government may find it optimal to commit to a larger bailout, ϕ^{COMM}.

9. Conclusions

We have studied a model for analysing the interrelationships between exchange rate crises and debt crises. It is shown that crises can be characterised by both belief-driven and fundamentals-driven attacks. The model is presented as a four-stage sequential game where the players are the government and the private sector.

It turns out that when there is no commitment to provide liquidity at the early stage of the game (i.e. before the private sector forms expectations), the model yields multiple equilibria. However with an ex-ante commitment to provide liquidity only in the bad state, the model yields a unique equilibrium. Moreover, the proportion of private sector debt that will be bailed out will be smaller than the proportion (non-committed) that is contingent on the state of the economy. It then seems that just the *commitment* to reduce the private sector's outstanding debt that could also reduce the severe impact on the economic fundamentals, will make devaluation or large devaluation unnecessary. That is, anticipating a less adverse effect on the fundamentals, the private sector will find that to expect large devaluation cannot be rational. The government will be then be avoid a currency crisis by committing ex-ante to bailing out part of the private sector's debt. Thus, ex-ante commitment to optimal bailouts serves as a strategic device for the government when the government has a potential problem of credibility in its exchange rate policy, since it facilitates the implementation of a more efficient equilibrium (without devaluation or with smaller devaluation). This is what we mean by not having "sunspot" equilibrium with commitments. This result becomes unambiguous even after the moral hazard effects of bailing out and smaller (or none) devaluation are taken into consideration. In the model, the government measures these moral hazard problems when it decides on the optimal bailout that it can commit to before the private sector forms expectations and decides on the level of risky investments. Another result is that it is never optimal for the government to allow devaluation to take place when there are no speculative attacks even if a bad shock occurs and the private sector suffers investment losses. Otherwise, with greater shock in the bad state, stronger the government's aversion to unemployment and greater the probability that the economy will end up in the bad state, the government will choose a higher equilibrium bailout level. This holds true whether the government commits ex-ante to a specific bailout or not.

This paper has some weaknesses that need to be taken up in future work. First, private-sector debt is introduced in a somewhat rudimentary manner because we do not consider the actual process of debt formation in the private sector. Nor do we model how different types of investment may be chosen, e.g. in terms of their riskiness, in response to the amount of (committed or non-committed) bailout that the government will give. Nevertheless, in the model presented here, the moral hazard effects of bailing out are balanced against expectational effects and deteriorations of the economic fundamentals. This issue has not theoretically been considered before in the relevant literature and neither the consequences of it when committing to ex-ante bailouts on the relationship between currency and financial crisis. Note also that the case of explicit commitment as such to an *optimal* bailing out has never been considered before.

Thus, in contrast to the existent literature, we analyse in this paper if bailouts are counterproductive, by *taking into account (not assuming) the moral hazard effects of the bailout policy* (determined by welfare maximising agents). We here provide a complete characterisation of an (scheme or rule) optimal bailout policy and of the optimality of a bailout policy, both when the government commits and when it does not commit ex-ante.

The government always finds the optimal bailout conditional *only* on a stochastic bad shock occurring. Finally, the government is also here concerned with minimising the social costs of both financial and currency crises (i.e. on unemployment), and the costs of bailing out. Bailouts, independent of whether a domestic or international Lender of Last Resource finances them, should be considered as costly by the government. These costs should be always considered in order to evaluate the effectiveness and optimality of any bailout policy.

We believe that we are making a contribution to the literature by considering all these issues that we just mentioned.

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APPENDIX A

Proof of Proposition 1:

i) Nodes (8), (9) and (10) cannot be possible equilibria. If the government is certain to abandon the regime when either shock occurs, the private sector will always anticipate this and will always expect devaluation triggering speculative attacks.

ii) When there are not devaluation expectations, the end nodes (8) and (9) in figure 1 must be counted out: It will not be optimal to devalue in state 2. In state 2, the shock and the outstanding debt are zero. There will not be any bailout and unemployment gap. Devaluing will be more costly for the government because:

The loss associated with maintaining a fixed exchange rate ($x(s_2)=\bar{x}$) is zero:

$$V_4(s_2, Ex=\bar{x}, x(s_2)=\bar{x})=0, \tag{A.1}$$

while the loss when devaluing in state 2 will be C:

$$V_4(s_2, Ex=\bar{x}, x(s_2)> \bar{x})=C. \tag{A.2}$$

iii) If there are not devaluation expectations and the loss of credibility, C, is sufficiently large, node (10) will never be reached: It will not be optimal to devalue when a bad shock occurs. We can see this by comparing the following loss functions:

$$V_4(s_1, Ex=\bar{x}, x(s_1)=\bar{x}, \tilde{\phi}, \tilde{K}) = a_2[s_1 - \alpha_3(1-\tilde{\phi})R_1(\tilde{K})]^2 - \tilde{\phi}R_1(\tilde{K}), \text{ and} \tag{A.3}$$

$$\begin{aligned} V_4(s_1, Ex=\bar{x}, \hat{x}(s_1)>\bar{x}, \hat{\phi}, \tilde{K}, C) = a_2[&-\alpha_1(\hat{x}(s_1)-\bar{x}) + s_1 \\ &-\alpha_3(1-\hat{\phi})(\hat{x}(s_i)/\bar{x})R_1(\tilde{K})]^2 - \hat{\phi}(\hat{x}(s_i)/\bar{x})R_1(\tilde{K}) + C \end{aligned} \tag{A.4}$$

If the government decides instead to devalue in state 1 (setting say $\hat{x}(s_1)>\bar{x}$), it takes as given $\tilde{K}$ which the private sector's investment decision when it does not expect a devaluation. Under such conditions, the optimal proportion of debt that will be bailed out, say ($\hat{\phi}$) is obtained by minimising (A.4) with respect to ϕ:

$$V_4(s_1, Ex = \bar{x}, \hat{x}(s_1) > \bar{x}, \hat{\phi}, \tilde{K}, C) = a_2[-\alpha_3(\hat{x}(s_1) - \bar{x}) + s_1$$

$$-\alpha_3(1-\hat{\phi})(\hat{x}(s_i)/\bar{x})R_1(\tilde{K})]^2 - \hat{\phi}(\hat{x}(s_i)/\bar{x})R_1(\tilde{K}) + C \tag{A.5}$$

The optimal ϕ will be:

$$\hat{\phi} = 1 - \frac{2a_2\alpha_3 s_1 - 1 - \left[2a_2\alpha_3\alpha_1(\hat{x}(s_1) - \bar{x})\right]}{2a_2\alpha_3^2(\hat{x}(s_1)/\bar{x})R_1(\tilde{K})} \tag{A.6}$$

The optimal devaluation $\tilde{x}(s_1)$ will be:

$$\hat{x}(s_1) = \frac{2a_2\alpha_1^2\bar{x} + 2a_2\alpha_1\bar{x}(1-\hat{\phi})R_1(\tilde{K}) + 2a_2 s_1\alpha_3(1-\hat{\phi})R_1(\tilde{K})}{2a_2[-\alpha_1 - (1-\hat{\phi})R_1(\tilde{K})]^2} + \frac{\phi R_1(\tilde{K}) + 2a_2 s_1\alpha_3}{2a_2[-\alpha_1 - (1-\phi)R_1(\tilde{K})]^2} \tag{A.7}$$

Note also here that there will not be a bailout if:

$$R_1(\tilde{K}) \geq \frac{2a_2\alpha_3 s_1 - \left[1 + 2a_2\alpha_3\alpha_1(\hat{x}(s_1) - \bar{x})\right]}{2a_2\alpha_3^2(\hat{x}(s_1)/\bar{x})}$$

Therefore, *the only possible reachable node is then (11)* where it is optimal for the government not to devalue in either state.

Proof of Proposition 2:

i) Node (12) in figure 1 will never be reached because if the regime will be definitely abandoned in both states, the government will find it optimal to abandon the regime already in stage 3, reaching instead node (6). In such case, the costs of defending the exchange rate regime in stage 4 will be avoided.

ii) When a “good” shock (state 2) occurs and there are speculative attacks, it will never be optimal to devalue.[25] The structure of payoffs in the game is such that end nodes

[25] Note that by assumption, the shock and the outstanding debt are zero in state 2, making a bailout unnecessary, however, there will be an unemployment gap caused solely by the devaluation expectations.

(12) and (13)) in figure 1 must be also counted out because when $x(s_2)=\bar{x}$, the loss resulting from keeping the fixed exchange rate is:

$$V_4(s_2, Ex>\bar{x}, x(s_2)=\bar{x})=a_2\,[\alpha_2 r]^2 \tag{A.8}$$

while the loss when devaluing will be:

$$V_4(s_2, Ex>\bar{x}, x(s_2)>\bar{x}, C)= a_2\,[-\alpha_1(x_2-\bar{x}) +\alpha_2 r]^2 +C. \tag{A.9}$$

It becomes evident that devaluing will be more costly for the government than not doing so.

iii) Node (15) can never be reached either, because if the government is certain to maintain the regime in both states, the private sector will not have devaluation expectations or anticipate a devaluation.

Note also that if the government considers not to devalue, it will take $\hat{K}$ (the investment level chosen when the private sector decides to speculate in the foreign exchange market) as given but it will need to determine another bailout, say $\breve{\phi}$, by minimising the following loss function:

$$V_4(s_1, Ex=\bar{x}, x(s_1)=\bar{x}, \breve{\phi}, \hat{K}) = a_2[s_1-\alpha_3(1-\breve{\phi})R_1(\hat{K})]^2-\breve{\phi}R_1(\hat{K}) \tag{A.10}$$

Note however that equation (16) equals equation (11), for the following: If the private sector expects that the government will never devalue, it will not speculate and chose a risky investment equal to $\tilde{K}$ and not $\hat{K}$. Consequently, the optimal bailout will again be $\tilde{\phi}$ ($\breve{\phi}$ must be then equal to $\tilde{\phi}$). The government will rationally never maintain the fixed exchange rate regime in the event of speculative attacks and a "bad" shock.

Thus, the only possible reachable node is then (14) where the government devalues only in the bad state or when a bad shock occurs.