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Patent Policies and Global Growth

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Abstract: This paper develops a simple two-country (innovating Home and imitating Foreign), dynamic, general-equilibrium model of endogenous, scale-invariant, Schumpeterian growth and product-cycle trade. Intellectual property protection takes the form of country-specific patents of finite length and patent-enforcement policies. The latter take the form of an instantaneous probability that patent infringement (illegal imitation) does not occur within a particular country. The model generates scale-invariant Schumpeterian growth, international technology transfer, and product-cycle trade based on a Home-Foreign wage gap. We use the model to address the effects of patent-policies on global growth, international technology transfer and global wage-income distribution. Stricter Home (Foreign) patent policies have an ambiguous (positive) effect on long-run growth, deteriorate (improve) the global wage-income distribution, and accelerate (have an ambiguous effect on) the rate of international technology transfer. Harmonization of patent policies improves the global wage-income distribution, but has an ambiguous effect on growth and the rate of international technology transfer. In world economy with harmonized patent policies, stricter patent enforcement policies increase long-run global growth, accelerate the rate of international technology transfer and have no impact on the world wage-income distribution.

JEL Classification: O34, F23, F13.

Key Words: Patents, Schumpeterian Growth, North-South Trade, Imitation, Innovation.

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1. Introduction

The present paper constructs a simple dynamic, general-equilibrium model of scale-invariant, endogenous, global Schumpeterian (R&D-based) growth generated by finite-length patents. Schumpeterian growth is a particular type of economic growth which emerges through endogenous technological progress fueled by the process of creative destruction (Schumpeter 1942). The term "scale-invariant" growth refers to bounded long-run per-capita growth in the presence of exponential expansion of market size - measured by the level of population - caused by the presence of positive population growth. Following the spirit of Romer's (1990) pioneering article, we use the term "endogenous" to long-run growth that can be affected by government actions (i.e., R&D subsidies and various patent policies).

This study makes two novel contributions to the voluminous body of Schumpeterian growth literature: First, it develops a simple model of a global economy experiencing long-run growth populated by two countries (Home and Foreign) with different national patent policies. As such, the model is suited to analyze the effects of changes in national patent policies, including the impact of patent harmonization, on long-run growth, international technology transfer and global wage income distribution. Second, it offers a novel solution to the scale-effects problem of earlier Schumpeterian-growth models by exploring a novel feature of finite-length patents: the generation of natural asymmetries in the flow of knowledge spillovers across industries by partitioning the economy into industries with active patents and industries with inactive patents.

The need for analyzing the effects of national patent policies on global growth serves as the primary motivation for the present paper. This need is highlighted by the intense academic and policy debate that emerged after the signing of the General Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPs Agreement) in the Uruguay Round. This agreement calls for all World Trade Organization (WTO) members to adopt a set of minimum standards on intellectual property rights that are closer to the ones prevailing in advanced countries. Because patent polices in advanced (as opposed to developing) countries are characterized by stricter enforcement and longer patent lengths, the TRIPs agreement has been opposed by advocates of poor countries on several grounds: First, poor countries rely on international technology transfer, which is partially based on imitation of newly developed products, to improve their technological infrastructure and living standards. Second, an intellectual property regime with longer patents would increase the flow of royalties that poor countries pay to innovating firms in advanced countries. Third, poor countries might not be able to afford the additional costs and resources required for stricter enforcement of longer patents. Proponents of TRIPs argue that stronger intellectual property protection, generated by higher levels of patent enforcement and longer patents, would stimulate global economic growth and improve global standards of living. However, a serious academic researcher would scrutinize these arguments through the lenses of general-equilibrium dynamic growth theory which provides the required logical consistency, takes into account the effect of externalities and distortions - that are impossible to measure empirically- and allows the important distinction between level and growth effects.

The primary reason for the provision of patent protection is to permit inventors to earn a return on their R&D efforts and thus to offer the necessary incentives for innovation, which in turn fuels technological progress and economic growth. Patent policies are multidimensional in nature consisting of the duration (length), scope of coverage (breadth), and the degree of legal enforcement. As such they are inherently difficult to analyze within the context of a simple economic model. This analytical difficulty is one reason for the scarcity of good insights on the effects of patent policies on global economic growth. For instance, R&D-based growth models typically abstract from addressing the growth effects of finite-length patents and patent-enforcement policies. These models assume that each innovation is protected by a perfectly-enforceable patent of infinite length.[1]

The analytical framework of the present paper generalizes and complements the work of Dinopoulos, Syropoulos and Gungoraidinoglu (2005), who develop a North-South model of trade and long-run Schumpeterian growth in the presence of perfectly enforceable global patents of finite length. As in Dinopoulos et al. (2005), the present model combines the deterministic R&D sector in Romer's (1990) model with the preferences and industrial structure of the quality-ladders growth model developed by Grossman and Helpman (1991, chapter 4). This combination allows the incorporation of finite patents in a tractable growth model. However, in Dinopoulos at al. (2005), the removal of scale effects generates long-run scale invariant growth that depends only on the length of the global patent. In contrast, in the present model we increase the set of policy parameters that affect the long-run growth rate by using an alternative assumption to remove the scale-effects property and by incorporating a patent enforcement mechanism proposed by Grossman and Lai (2004). Following the latter, we assume that a government can enforce each patent with a country-specific exogenous instantaneous probability. The resulting analytical framework generates long-run global growth that depends on virtually all parameters of the model (not just on the length of the common patent), including country-specific patents and

[1] Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Jones (1995), Segerstrom (1998), Howitt (1999) and Dinopoulos and Syropoulos (2006) among many others adopt the assumption of infinite-length patents.

patent-enforcement policies. Consequently, we are able to address similar questions to those addressed by Grossman and Lai (2004), namely the effects of changes in patent lengths and patent enforcement policies, and the impact of patent-policy harmonization; but instead of analyzing the welfare effects of patent policies, we complement the work of Grossman and Lai (2004) by focusing on the effects of patent policies on long-run, scale-invariant, global Schumpeterian growth.[2]

We model a global economy populated by identical dynastic families with infinite-lived members. The size of each family grows exponentially over time at an exogenous rate which equals the growth rate of world population. Labor is the only factor of production, and households reside either at Home or Foreign. Each member of a typical household supplies one unit of labor and therefore the global labor supply equals the level of population at each instant in time. The taste and production side of our model follows the standard quality-ladders growth framework: There is a continuum of industries of measure one producing final consumption goods whose quality can be improved through intentional R&D investment. We assume that there are two countries in the world, innovating Home and imitating Foreign. We analyze a steady-state equilibrium in which Home enjoys a higher wage than Foreign. This assumption is made for the purpose of interpreting our results in the context of the North-South debate on the desirability of TRIPs. The model admits alternative interpretations as well. For instance, the model applies to United States (Home) interacting with lower-wage Japan (Foreign), or to innovating Northern Europe (Home) engaging in free trade with Southern Europe (Foreign).

Labor can be employed in two activities in each country: manufacturing and R&D. We assume that one unit of labor can produce one unit of output independently of the location of production and the quality level of each final consumption product. By assumption, only Home workers can participate in innovative R&D; and workers in Foreign can engage in imitative R&D. We embed a modified version of Romer's (1990) innovation technology into a quality ladders growth model by assuming that the process of innovation and imitation is deterministic and instantaneous, and that the productivity of labor engaged in R&D declines exponentially over time, or the difficulty of R&D increases over time. The assumption that R&D is becoming more difficult permits the removal of the scale-effects property as will be explained below.

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[2] Grossman and Lai (2004) assume instant depreciation of the utility generated by goods whose patents have expired or not enforced. This assumption results in zero per-capita growth, but enhances the tractability of the welfare analysis. Our model abstracts from welfare analysis due to tractability considerations and complements their analysis by focusing on the effects of patent policies on long-run growth..

Firms at Home hire labor to engage in innovative R&D and discover new higher-quality products. For tractability considerations, we assume that Home firms target products produced under perfect competition at Home or abroad. The firm that discovers a state-of-the-art quality product receives a patent that excludes other firms at Home to copy the product for a finite time period $T > 0$. Following Grossman and Lai (2004), we assume that the government at Home enforces the patent with an instantaneous and exogenous probability $\varepsilon > 0$. In other words, no other Home firm can copy a product whose patent has not expired if the government enforces the patent, and this event occurs with instantaneous probability ε. Therefore, each new product enjoys temporary monopoly power at the Home market for an expected time period equal to $\varepsilon T > 0$. If a patent expires or if it is not enforced the technology becomes common knowledge at Home and the product could be produced under perfect competition. We use the term "generics" to refer to products produced under perfect competition.

We assume that the technology of all products one step below the state-of-the-art quality level in each industry is common knowledge to all firms in the world. However, the transfer of technology of state-of-the-art quality products from Home to Foreign requires imitative R&D. Again maintaining our intention to develop a simple model, we assume that firms at Foreign target only generic Home products (as opposed to products enjoying effective patent protection) to copy their technology by engaging in imitative R&D. The nature of imitative R&D is identical to innovative R&D: We assume that the imitation process is deterministic and instantaneous; and that the difficulty of imitative R&D increases exponentially over time in order to remove the scale-effects property. A firm in Foreign that successfully copies a Home product is awarded a patent by the Foreign government for a finite time period $T^* > 0$, which is enforced with instantaneous probability $\varepsilon^* > 0$. If a patent is enforced, other firms in Foreign cannot legally produce the product. If a patent is not enforced or when it is expired, the technology of the product becomes common knowledge at Foreign and the product is produced under perfect competition. Foreign patent holders engage in limit pricing and drive the producers of generic products at Home out of the market. Because Home firms target only competitively produced products, these Foreign firms enjoy global temporary monopoly power for an expected time period $\varepsilon^* T^* > 0$. Because firms in Foreign enjoy a cost advantage over firms at Home, all state-of-the-art quality products that have been imitated are produced in Foreign by either firms with Foreign patent-protection or by perfectly competitive firms without patent protection. Similarly, Home firms with active patents enjoy global temporary monopoly power for an expected time period $\varepsilon T > 0$.

The above description of the model's product market structure implies that the presence of finite patents conditions the economy-wide degree of competition of the state-of-the-art quality products. At each instant in time, a measure of these products is produced by global Home monopolists enjoying active patent protection, and a measure of generic products is produced by Home firms under perfect competition. Similarly, a measure of products is produced by global Foreign monopolies under Foreign patent protection and a measure of products are produced by Foreign perfectly competitive firms. All products produced at Home are exported to Foreign, and all products produced at Foreign are exported to Home.

In addition to determining the degree of temporary monopoly power, finite patents regulate the flow and degree of knowledge spillovers and offer a novel way of modeling the evolution of innovative (and imitative) R&D difficulty. In general patent policies have ambiguous effects on the evolution of knowledge spillovers: On the one hand, patents enhance the innovation process by making public the technology of patented products. This knowledge can help researchers discover better products that can replace the existing state-of-the-art quality products. On the other hand, patents reduce the flow of knowledge spillovers by excluding other firms from discovering better products, by allowing incumbent firms to engage in patent-infringement litigation, by increasing the probability of royalty payments, etc. In the present paper we assume that the rate of growth of innovative R&D difficulty depends inversely on the measure of Home products produced under perfect competition. Similarly, we assume that the rate of growth of imitative R&D difficulty depends inversely on the measure of Foreign products produced under pure competition. These assumptions capture the notion that patents provide positive incentives for innovation and imitation via temporary monopoly power but restrict the flow of knowledge spillovers to potential entrants relative to industries without patent protection.

The analysis generates several novel findings. The model is consistent with a steady-state wage gab between Home and Foreign workers which results in product-quality cycles. This wage gap increases in the degree of Home patent enforcement, the length of Home patents and the size of quality innovations. It decreases in the degree of Foreign protection of intellectual property (measured by the Foreign-patent length and the enforcement of Foreign patents). Harmonization of patent policies reduces the wage gap and improves global wage-income inequality (Proposition 1).

The steady-state scale-invariant growth of the instantaneous utility is endogenous and depends on virtually all parameters of the model including the Home and Foreign patent lengths, patent enforcement policies, the relative market size of each country and the magnitude of innovations.[3] Stronger Home-patent policies have an ambiguous effect on global growth, whereas stronger Foreign-patent policies increase the global growth rate (Proposition 2). Similar considerations apply to the steady-state rate of international technology transfer, measured by the rate of imitation. Stronger Home-patent policies accelerate the rate of international technology transfer, whereas stronger Foreign-patent policies have an ambiguous impact on the long-run rate of imitation (Proposition 3). Under harmonization (i.e., a global patent regime with a common enforcement policy and same-length patents), a stronger patent-enforcement regime accelerates global growth, increases the rate of international technology transfer and has no effects on the global wage-income inequality (Proposition 4).

The rest of the paper is organized as follows. The next section of the paper describes the building blocks of the model. Section 3 establishes the properties of the steady-state equilibrium and presents the main results, and section 4 offers concluding remarks.

2. The Model

2.1 Consumers

Each of the two countries is populated by a fixed measure of identical dynastic families with infinitely-lived members. We will use star superscripts (upper bars) to denote functions and variables associated with Foreign (world), and the absence of superscripts to refer to Home functions and variables. Denote with L_0 the measure of families residing at Home and with L_0^* the measure of families living at Foreign. We normalize the initial size of each household to unity and assume that its size grows exponentially at the rate $g_L > 0$. Therefore, the level of Home population at time t is $L(t) = L_0 e^{g_L t}$, the level of Foreign population is $L^*(t) = L_0^* e^{g_L t}$, and the level of world population is $\bar{L}(t) = (L_0 + L_0^*)e^{g_L t} = \bar{L}_0 e^{g_L t}$. Each household member supplies one unit of labor to the market, and therefore the supply of labor equals the level of population in each country.

There is a continuum of industries indexed by $\theta \in [0,1]$ producing final consumption goods. The preferences of a typical family are modeled as

[3] In Dinopoulos et al. (2005) we assume that the growth rate of R&D difficulty is an increasing function of the measure of industries with active Home patents. This assumption results in scale-invariant long-run growth that depends only on the length of Home patents, the rate of population growth, and an exogenous parameter governing the intensity of knowledge spillovers.

$$U = \int_0^{\infty} e^{-(\rho - g_L)t} \ln u(t) dt, \tag{1}$$

where $\rho > g_L$ is the subjective discount rate of a representative family. The instantaneous per capita utility function at time *t* is defined by

$$\ln u(t) = \int_0^1 \ln \left[\sum_j \lambda^j q(j,\theta,t) \right] d\theta . \tag{2}$$

Equation (2) defines the standard quality-augmented Cobb-Douglas utility function across all industries $\theta \in [0,1]$. Variable $q(j,\theta,t)$ is the per-capita quantity demanded of a product in industry θ at time *t* that has experience j innovations and has quality level λ^j. Maximization of (2) subject to a static budget constraint generates the following per-capita demand function

$$q(\theta,t) = \frac{c(t)}{p(\theta,t)}, \tag{3}$$

where *c(t)* is per-capita consumption expenditure at time *t* and $p(\theta,t)$ is the lowest quality-adjusted price in industry θ.

Substituting (3) into (2) and the resulting expression into(1), yields a discounted utility function that depends on per-capita consumption expenditure and on time-invariant parameters. Maximization of this discounted utility function subject to the standard intertemporal budget constraint yields the following differential equation:

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho , \tag{4}$$

where $r(t)$ is the instantaneous market interest rate. Equation (4) implies that, in steady-state equilibrium with constant per-capita consumption expenditure, the interest rate must equal to the constant discount rate ($r(t) = \rho$).

2.2 Product Markets

At the steady-state equilibrium, Home firms target industries producing generic products under perfect competition. A successful Home innovator in industry θ becomes a global monopolist for a time period T > 0 and faces a patent-enforcement mechanism $\varepsilon > 0$ and a marginal (average) manufacturing cost equal to the prevailing wage at Home, w. We assume that the technology of all products one step below the state-of-art quality level in each industry's quality ladder is common knowledge in both Home and Foreign. This assumption implies that the instant the state-of-the-art quality product in industry θ is discovered, the foreign competitive fringe can produce the old product in the same industry under perfect competition at a price equal to the wage at Foreign, w^*. We also assume that firms compete in each product market in a Betrand price-competition fashion. By engaging in limit pricing and charging a price $p = \lambda w^*$, the Home monopolist drives out all potential producers of inferior quality products and obtains a flow of global expected profits equal to

$$\pi(t) = [p - w]\bar{q}\varepsilon = [1 - \frac{\omega}{\lambda}]\bar{c}\bar{L}(t)\varepsilon , \tag{5}$$

where $\omega = w / w^* > 1$ is the relative wage at Home, $\bar{q} = \bar{c}\bar{L}(t)/p$ corresponds to the quantity demanded, and $\bar{c} = [cL_0 + c^* L_0^*]/\bar{L}_0$ denotes the global per-capita consumption expenditure. The flow of expected profits $\pi(t)$ provides strong incentives for innovative R&D and must be non negative. It is obvious from (5) that the term in square brackets must be non-negative. A sufficient condition for this requirement is given by

$$\lambda > \omega = \frac{w}{w^*} > 1 , \tag{6}$$

that is, the steady-state value of the relative wage at Home must be greater than unity, but less than the size of each quality improvement $\lambda > 1$. Condition (6) generates product-cycle trade and allows successful Home innovators to replace potential producers of lower quality products.

The market value of an innovation equals the expected discounted value of global monopoly profits and is given by

$$V(t) = \int_0^T \pi(t+s)e^{-r(t)s} ds , \tag{7}$$

where $\pi(t)$ is the flow of expected global profits given by (5), and $r(t)$ is the market interest rate. In the steady-state equilibrium the market interest rate equals the subjective discount rate ρ; and therefore the steady-state value of an innovation (Home patent) can be obtained by substituting (5) into (7) and performing the integration

$$V(t) = \psi(T)\varepsilon[1 - \frac{\omega}{\lambda}]\bar{c}\bar{L}(t) . \tag{8}$$

The first term in the right-hand-side (RHS) of (8) captures the effect of patent length T and the effective discount rate $\rho - g_L$ on expected discounted profits, and is given by

$$\psi(T) = \frac{1 - e^{-(\rho - g_L)T}}{\rho - g_L} . \tag{9}$$

The function $\psi(T) = V(t)/\pi(t)$ equals the expected steady-state price-earnings ratio of a typical innovator and it is increasing monotonically in the patent length T, starting at $\psi(0) = 0$ and approaching $\psi(\infty) = 1/(\rho - g_L)$ as T approaches infinity.

Consider now the pricing behavior of a successful imitator residing in Foreign who is awarded a patent of length T^* enforced by the Foreign government with instantaneous probability ε^*. This firm targets a Home industry producing a generic product under perfect competition and charges a limit price $p^* = w$, which equals the unit cost of its competitors abroad. This price suffices to drive its Foreign rivals who could produce the product of quality one step below the state-of-the- art quality level because $p^* < \lambda w^*$ according to condition (6). These considerations imply that the imitator enjoys an expected flow of global profits equal to

$$\pi^*(t) = [p^* - w^*]\bar{q}\varepsilon^* = [w - w^*]\frac{\bar{c}\bar{L}(t)}{w}\varepsilon^* = [1 - \frac{1}{\omega}]\bar{c}\bar{L}(t)\varepsilon^* \tag{10}$$

where $\omega = w/w^* > 1$ is the relative wage at Home expressed in units of labor in Foreign. The steady-state market value of a Foreign patent is equal to the expected discounted Foreign profits, and it is given by

$$V^*(t) = \int_0^{T^*} \pi^*(t+s)e^{-\rho s}ds = \psi(T^*)\varepsilon^*[1 - \frac{1}{\omega}]\bar{c}\bar{L}(t), \tag{11}$$

where $\psi(T^*) = V^*(t)/\pi^*(t)$ is the expected price-earnings ratio of a successful imitator. The function $\psi(\bullet)$ is defined in (9) and its properties were explained. We conclude this subsection by observing that generic products produced at Home command a price $p = w$, whereas their Foreign counterparts are sold at a lower price $p^* = w^*$.

2.3 Innovation and Imitation

Home firms target industries with inactive patents for further innovation. The innovation process is deterministic and follows the spirit of Jones (1995) version of Romer's (1990) model of endogenous technological progress. We assume that one unit of R&D services requires one unit of labor; in addition we assume that if firm j employs R_j researchers to engage in innovative R&D for a time interval dt, that firm discovers $dA_j = R_j dt / D(t)$ patentable designs that allow that firm to produce a measure of dA_j higher quality products. The term $D(t)$ captures the difficulty of conducting innovative R&D and will be defined bellow. The economy wide rate of innovation is given by $dA = [Rdt]/D(t)$, where

$dA = \sum_j dA_j$ is the aggregate flow of Home patents and $R = \sum_j R_j$ is the economy wide number of researchers devoted to innovative R&D.

The removal of scale effects can be achieved by assuming that the difficulty of R&D is increasing exponentially over time. Following Dinopoulos et al. (2005), we assume that $D(t) = e^{\phi X(t)}$. This specification of D(t) results in a constant steady-state flow of patents, if X(t) is a linear function of time. Substituting D(t) into the expression for dA and dividing both sides by dt yields an equation for the flow of innovations

$$\dot{A}(t) = \mathrm{R}(t)\mathrm{e}^{-\phi X(t)} \quad , \tag{12}$$

where $\dot{A}(t) = dA/dt$ using the standard notation when a dot over a variable denotes its time derivative.

Unlike Dinopoulos et al. (2005), we postulate that the long-run evolution of X(t) is governed by the following differential equation:

$$\dot{X}(t) = \dot{A}(t)n^{-\beta}, \tag{13}$$

where a dot over a variable denotes its time derivative; variable n is the measure of Home c industries producing generic products under perfect competition; and $\beta \in [0,\infty)$ is a parameter that governs the strength of useful knowledge spillovers.

This specification of R&D difficulty captures two novel features of the structure of knowledge spillovers in a simple way. First, it assumes that the difficulty of R&D depends only on the measure of competitive industries located at Home but not on Foreign industries producing generic products. This assumption captures the notion of localized spillovers and enhances the set of parameters that affect long-run growth. Second, it highlights the inherent asymmetry between industries with active patents and industries with inactive patents in regulating the evolution of knowledge spillovers. More precisely, this specification implies that as the measure of competitive industries located at Home increases, the flow of R&D difficulty declines and, as a result, innovative R&D becomes easier. Therefore, we assume that active patents discourage the process of innovation (i.e., they do not provide any useful knowledge spillovers). Notice that if $\beta = 1$ in (13), then the difficulty of R&D is a function of the "average" flow of innovations across all Home industries producing generic products. If $\beta \to \infty$ or if $n \to 0$, then $\dot{X} \to 0$ implying constant difficulty of R&D over time and unbounded long-run growth based on scale effects. The specification of R&D difficulty adopted here differs from Dinopoulos et al (2005), where it was postulated that the R&D

difficulty was an increasing function of the measure of Home industries under active patent protection.[4]

In quality ladders growth models each innovation increases the level of instantaneous utility by a fixed proportion and therefore steady-state bounded growth requires that the flow of innovations $\dot{A}(t)$ is constant over time. Further insights on the long-run flow of innovations can be obtained by observing that the measure of Home industries producing generics $n \in (0,1)$ is bounded and must be constant over time. Therefore, integration of (13) yields

$$X(t) = \int_0^t \dot{A} n^{-\beta} ds = t\dot{A} n^{-\beta}, \tag{14}$$

where we assume that the economy starts at the steady-state equilibrium (otherwise, one has to add an inconsequential constant to the RHS of (14). In addition, observe that (12) can be written as $\dot{A}(t) = [R(t)/\bar{L}(t)]\bar{L}(t)e^{-\phi X(t)}$ where $R(t)/\bar{L}(t)$ is the fraction of world labor devoted to innovative R&D. This fraction is bounded and therefore its steady-state value must be constant over time. Consequently the long-rate of innovation $\dot{A}(t)$ is constant over time only if the term $\bar{L}(t)e^{-\phi X(t)} = \bar{L}_0 e^{tg_L} e^{-\phi X(t)} = \bar{L}_0 e^{t[g_L - \phi \dot{A} n^{-\beta}]}$ is constant at the steady-state equilibrium. Equation (14) was used in the derivation of the last term in square brackets. Consequently, we obtain the steady-state conditions

$$\dot{A} = \frac{g_L}{\phi} n^{\beta}, \tag{15}$$

which is derived by setting the term in square brackets equal to zero, and

$$\bar{L}(t)e^{-\phi X(t)} = \bar{L}_0 . \tag{16}$$

Equation (15) constitutes one of the basic building blocks of patent-based scale invariant innovation. Notice, for instance, that if $\beta = 0$ the R&D difficulty is proportional to A(t) as in Jones (1995) and Segerstrom (1998) and all industries contribute symmetrically to the generation of knowledge spillovers. In this case, the long-run rate of innovation is exogenous and depends only on the population growth rate g_L and parameter ϕ. In addition, as mentioned before, if $\beta \to \infty$ the long-run rate of innovation approaches infinity due to the presence of scale effects.

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[4] Dinopoulos et al. (2005) assume that $\dot{X}(t) = \dot{A}(t)[n_p]^{\beta}$, where n_p is the measure of Home industries with active patents.

Firms in Foreign target Home generic products to copy their technology through imitative R&D investments. The process of imitation is modeled similarly to the process of innovation. One unit of labor produces one unit of imitative R&D services by assumption. In addition, we postulate that if a firm in Foreign hires R_j^* researchers to engage in imitative R&D for an infinitesimal time period dt, that firm manages to copy instantaneously $dA_j^* = [R_j^* dt]/D^*(t)$ designs, where $D^*(t) = e^{\phi X^*(t)}$ is a function that governs the evolution of the difficulty in conducting imitative R&D. As in the case of innovation, the economy-wide rate of imitation is given by $dA^* = R^* e^{-\phi X^*(t)} dt$, where $dA^* = \sum_j A_j^*$ and $R^* = \sum_j R_j^*$ are the aggregate flows of designs copied and labor devoted to imitative R&D. Dividing both sides by dt yields the aggregate imitation rate

$$\dot{A}^*(t) = R^*(t) e^{-\phi X^*(t)} . \tag{17}$$

We assume that the difficulty of conducting imitative R&D increases in the rate of imitation $\dot{A}*(t)$ and declines in the measure of industries in Foreign producing generic products n^* according to

$$\dot{X}*(t) = \frac{dX^*}{dt} = \dot{A}*(t)(n^*)^{-\beta} . \tag{18}$$

Following the same reasoning as in the case of innovation, one can derive the following expressions for the steady-state level of $X^*(t)$ and the long-run imitation rate $\dot{A}*(t)$:

$$X^*(t) = \int_0^t \dot{A}*(n^*)^{-\beta^*} ds = t\dot{A}*(n^*)^{-\beta^*} , \tag{19}$$

$$\dot{A}*(t) = \frac{g_L}{\phi}(n*)^{\beta^*} , \tag{20}$$

where $\beta* \in [0,\infty)$ is a parameter that determines the degree of imitation knowledge spillovers. In addition, the following condition holds in the steady-state equilibrium

$$\overline{L}(t) e^{-\phi X^*(t)} = \overline{L}_0 . \tag{21}$$

2.4 R&D Conditions

We assume that there is free entry in innovating and imitative R&D that drives expected discounted profits of a typical firm down to zero. We derive these two zero-profit conditions by considering the net benefits of engaging in innovative or imitative R&D. A Home firm that hires R_j researchers to conduct R&D for a time interval dt incurs a cost equal to $wR_j dt$.

This firm discovers with certainty $dA_j = R_j e^{-\phi X(t)} dt$ patentable designs which can be used to manufacture new higher quality products. The market value of this discovery is $V(t)dA_j = V(t)[R_j e^{-\phi X(t)} dt]$, where $V(t)$ is the market value of a typical patent defined in (8). The net benefits of innovating R&D are given by $[V(t)e^{-\phi X(t)} - w]R_j dt$, and free entry with positive R&D investment requires that the term is square brackets must be equal to zero. Substituting V(t) from (12) and using the steady-state condition (16) generates the *innovative-R&D condition:*

$$\psi(T)\varepsilon[1-\frac{\omega}{\lambda}]\bar{c}\bar{L}_0 = w. \tag{22}$$

Similar considerations apply to imitative R&D. A Foreign imitator hires R_j^* researchers for a period dt and copies with certainty $dA_j^* = R_j^* e^{-\phi X^*(t)} dt$ state-of-the-art generic products produced at Home. The market value of this action is $V^*(t)R_j^* e^{-\phi X^*(t)} dt$ and the R&D cost $w^* R_j^* dt$, resulting in net benefits equal to $[V^*(t)e^{-\phi X^*(t)} - w^*]R_j^* dt$. Free entry into the activity with $R_j^* > 0$ requires that the term in square brackets is zero. Substituting the value of $V^*(t)$ from (11) and using (21) yields the *imitative-R&D condition*

$$\psi(T^*)\varepsilon^*[1-\frac{1}{\omega}]\bar{c}\bar{L}_0 = w^*. \tag{23}$$

2.5 Labor Markets

Following the standard practice in the theory of Schumpeterian growth, we assume that workers can move instantaneously between manufacturing of final goods and R&D investment activities and that wages are flexible. Therefore, at each instant in time the demand for labor equals its supply in each country. The supply of labor at Home is $L(t) = L_0 e^{g_L t}$ and the demand for labor comes from firms engaged in innovative R&D, firms manufacturing products under active patents, and firms manufacturing generic products. The demand for workers engaged in innovative R&D is $R(t) = \dot{A} e^{\phi X(t)}$ from (12). Each Home monopolist manufactures $q_m = \bar{c}\bar{L}(t)/p_m$ units of output and demands the same amount of labor because one worker produces one unit of output. The monopolist charges a limit price $p_m = \lambda w^*$ and therefore the aggregate demand for manufacturing labor in the sector with active patents is $s\bar{c}\bar{L}(t)/\lambda w^*$, where s is the measure of Home industries with active patents. The demand for each generic product produced at Home is $q_c = c\overline{L}(t)/p_c$, where $p_c = w$ is the competitive price. Because there are n Home generic products the aggregate demand for labor in this sector is $nc\overline{L}(t)/w$. Equalizing the labor demand to its supply yields the *Home full-employment- of-labor* condition

$$L(t) = \dot{A}e^{\phi X(t)} + \frac{sc\overline{L}(t)}{\lambda w^*} + \frac{nc\overline{L}(t)}{w}. \tag{24}$$

Similar considerations apply to the labor market in Foreign. The supply of labor is $L^*(t) = L_0^* e^{g_L t}$ and the demand for labor comes from three sectors: Labor engaged in imitative R&D $R^*(t) = \dot{A}*(t)e^{\phi X^*(t)}$ from (17); labor employed in the monopoly (active- patent) sector which is equal to $s^*c\overline{L}(t)/w$, where s^* is the measure of Foreign firms with active patents; and labor employed by manufacturers of Foreign generic products, which equals $n^*c\overline{L}(t)/w^*$. Setting the supply of Foreign labor equal to its demand yields the *Foreign full-employment-of-labor* condition

$$L^*(t) = \dot{A}*e^{\phi X^*(t)} + \frac{s^*c\overline{L}(t)}{w} + \frac{n^*c\overline{L}(t)}{w^*}. \tag{25}$$

This completes the description of the model's elements.

3. Steady-State Equilibrium

In this section we establish the existence of unique patent-based balanced-growth equilibrium and describe its comparative steady-state properties. We assume, of course that both Home and Foreign governments offer patent policies that guarantee strictly positive effective patents, $\varepsilon T > 0$ and $\varepsilon^* T^* > 0$. Several variables are constant in the steady-state equilibrium: Per-capita consumption expenditures in each country, c and c^*; the instantaneous market interest rate, $r(t) = \rho$; wage rates at Home and Foreign, w and w^*, and the wage gap, ω; the measures of industries at Home and at Foreign with active patents, s and s^*; the measure of industries at Home and at Foreign without patent protection, n and n^*; and the rates of innovation (flow of Home patents) and imitation (flow of Foreign patents), $\dot{A}$ and $\dot{A}*$.

Several variables are growing at the constant population growth rate: The flow of monopoly profits, aggregate resources devoted to manufacturing and R&D in each country, and the market value of patents. In addition, the steady-state utility of each consumer grows at a constant and endogenous growth rate as new higher-quality products replace old ones continually. Let Foreign labor serve as the model's numeraire, that is $w^* = 1$, and therefore $w = \omega > 1$ is the relative wage of Home workers. In the steady-state equilibrium, the flow of patents is constant and the expected length of a typical Home patent is $\varepsilon T > 0$; therefore, the law of large numbers implies that at each instant of time the measure of industries with Home active patents is given by

$$s = \int_0^{\varepsilon T} \dot{A} dt = \varepsilon T \dot{A} \,; \tag{26}$$

similarly, the steady-state measure of industries with Foreign active patents is

$$s^* = \int_0^{\varepsilon^* T^*} \dot{A}^* dt = \varepsilon^* T^* \dot{A}^* . \tag{27}$$

The next step of the analysis is to derive an explicit solution for the steady-state value of the wage gap, by combining the innovative and imitative R&D conditions, (22) and (23)

$$\hat{\omega} = \frac{1+\Phi}{1+\Phi\lambda^{-1}} > 1\,, \tag{28}$$

where a hat over a variable denotes its steady-state equilibrium value, and parameter $\Phi > 0$ captures Home's protection of intellectual property relative to Foreign

$$\Phi = \frac{\varepsilon\psi(T)}{\varepsilon^*\psi(T^*)} = \frac{\varepsilon[1-e^{-(\rho-g_L)T}]}{\varepsilon^*[1-e^{-(\rho-g_L)T^*}]} . \tag{29}$$

The following proposition states the first main result of our paper, by identifying the long-run determinants of global wage income inequality.

Proposition 1: *In the presence of patent-based growth, steady-state global wage-income inequality measured by Home's relative wage* $\hat{\omega} > 1$ *:*

(a) *Worsens in the size of innovations* λ*, Home patent length T, and Home patent enforcement policy* ε *.*

(b) *Improves in Foreign patent length* T^* *and Foreign patent enforcement policy* ε^* *.*

(c) *Improves in the effective subjective discount rate* $(\rho - g_L)$*, if and only if* $T > T^*$*.*

(d) *Improves as a result of patent- policy harmonization that changes* Φ *from* $\Phi \neq 1$ *to* $\Phi = 1$.

Proof: The proof can be established by substituting (29) into (28) and differentiating the resulting expression with respect to the parameter of interest.

Proposition 1 captures the importance of differential patent policies in the determination of global income distribution. The relative wage gap depends on the ability of Home to keep innovating relative to the ability of Foreign to imitate. The stronger is Home's protection of intellectual property the higher is the level of Home monopoly profits which require a higher Home wage according to the innovative-R&D condition. Similarly, the stronger is

Foreign's patent policy, the higher is the flow of imitation profits and the Foreign wage according to the imitative-R&D condition. In addition, notice that harmonization of patent policies that either raise the strength of intellectual property protection in low-wage countries or lowers the strength of intellectual property protection in high-wage countries, improves the global wage-income inequality. Within the context of the model, harmonization of patent policies advocated by the TRIPs agreement improves global wage-income inequality.

Before we establish the existence of unique steady-state equilibrium, it is useful to derive an expression for long-run Schumpeterian growth, which is measured by the rate of growth of each consumer's utility function $u(t)$ defined in (2). This utility function is an appropriate index of each consumer's consumption bundle and takes into account the endogenous evolution of each product's quality. Since the measure of all industries is equal to unity and there is no uncertainty in the process of innovation, A(t) denotes the economy wide number of innovation at time t, and the "average" number of innovations per industry. Therefore the average quality level at time t in industry θ is simply $\lambda^{A(t)}$. The per-capita quantity produced and demanded in a typical industry is $q(\theta,t) = \hat{c} / p(\theta)$, where $\hat{c}$ is the consumer's constant per-capita expenditure at the steady-state equilibrium, and $p(\theta)$ is the prevailing price. The former differs across the two countries and the latter differs across sectors. A measure of s Home industries with active patents charge $p(\theta) = \lambda w^*$, a measure of s^* Foreign industries with patents and a measure of n Home industries without patents charge $p(\theta) = w$, and the remaining measure of n^* Foreign industries charge $p(\theta) = w^*$. Substituting all these expression in (2), yields the steady-state instantaneous utility of typical consumer with consumption expenditure $\hat{c}$

$$\ln u(t) = \int_{s} \ln[\lambda^{A(t)} \frac{\hat{c}}{\lambda w^*}] d\theta + \int_{s^*+n} \ln[\lambda^{A(t)} \frac{\hat{c}}{\lambda w}] d\theta + \int_{n^*} \ln[\lambda^{A(t)} \frac{\hat{c}}{w^*}] d\theta .$$

Perform the integration and collect term to derive the level of the consumer's instantaneous utility at time t

$$\ln u(t) = A(t) \ln \lambda + s \ln[\frac{\hat{c}}{\lambda w^*}] + (s^* + n) \ln[\frac{\hat{c}}{w}] + n^* [\frac{\hat{c}}{w^*}]$$

In the steady-state equilibrium, the only variable which is not constant over time is the number of innovations A(t). Therefore, independently of the per-capita consumption expenditure, the long-run growth of u(t) is proportional to the constant flow of innovations and it is obtained by differentiating the above expression with respect to time

$$g_u = \frac{\dot{u}(t)}{u(t)} = [\ln \lambda] \dot{A} = [\ln \lambda] \frac{g_L}{\phi} n^{\beta} . \tag{30}$$

Equation (15) was used to the derivation of the far RHS of equation (30). It is obvious from the above expression that in the presence of asymmetric knowledge spillovers across industries, captured by a strictly positive value of β , any policy parameter that affects the measure of industries producing generics at Home affects the long-run growth rate. The appendix derives the following equations that determine implicitly the measure of competitive industries at Home and at Foreign:

$$L_0 = \frac{g_L}{\phi}\left[1+\frac{\varepsilon T}{(\lambda-1)}\left(\frac{1}{\varepsilon\psi(T)}+\frac{1}{\varepsilon^*\psi(T^*)}\right)\right]n^{\beta}+\frac{1}{(\lambda-1)}\left(\frac{1}{\varepsilon\psi(T)}+\frac{1}{\varepsilon^*\psi(T^*)}\right)n \tag{31}$$

$$L_0^* = \frac{g_L}{\phi}\left[1+\frac{\varepsilon^* T^*}{(\lambda-1)}\left(\frac{\lambda}{\varepsilon\psi(T)}+\frac{1}{\varepsilon^*\psi(T^*)}\right)\right](n^*)^{\beta^*}+\frac{\lambda}{(\lambda-1)}\left(\frac{1}{\varepsilon\psi(T)}+\frac{1}{\varepsilon^*\psi(T^*)}\right)n^* \tag{32}$$

Equations (30) and (31) determine the long-run properties of patent-based global Schumpeterian growth g_u , and equations (20) and (32) pin down the long-run determinants of the rate of imitation $\dot{A}^*(t)$. The RHS of (31) is an increasing function of the measure of Home industries producing generic products n. As the value of L_0 declines the solution to (31) declines as well and approaches zero as L_0 approaches zero. The same considerations apply to the solution to (32). Therefore for sufficiently low values of the initial levels of Home and Foreign populations, L_0 and L_0^*, there exists unique steady-state values of $\hat{n} \in (0,1)$ and $\hat{n}^* \in (0,1)$ that satisfy equations (31) and (32). Low values of L_0 and L_0^* constitutes a sufficient but hardly a necessary condition for the existence of the unique steady-state equilibrium. Observe that the measure of Home industries producing generic products n depends on virtually all parameters of the model including each country's patent policies. Inspection of (31) reveals that its RHS declines in ϕ, λ, ε^* and T^*, and therefore $\hat{n}$ must increase to restore (31). Consequently, long-run growth is positively related to ϕ, λ, ε^*, T^* and L_0. An increase in ε or T has an ambiguous effect on the RHS of (31), and therefore the growth effects Home's patent policies are in general ambiguous. The following proposition summarizes the above analysis.

Proposition 2: *The model generates unique, scale- invariant, endogenous, patent-based, global Schumpeterian growth* g_u, *with the following properties:*

(a) *Long-run global growth increases in the relative size of the innovating country,* L_0 *; in the parameter that regulates the difficulty of innovative and imitative R&D,* ϕ *; in the size of quality innovations,* λ *; in the degree of imitative-patent enforcement,* ε^* *; and in the length of imitative patents,* T^* *.*

(b) *The growth effects of Home patent policies (a change in patent length* T *, or a change in innovation-based patent enforcement policies* ε *) are ambiguous.*

The economic intuition for the growth ambiguity of Home patent policies is straight forward. On the one hand an increase in the effective patent length εT expands the measure of Home industries with active patents s (see equation (26)) and reduces the measure of industries with inactive patents n. On the other hand, an increase in ε or T raises the profitability of innovative R&D. The innovative R&D condition (22) requires a decline in real per-capita expenditure $\bar{c}/w$ as a result of this change, which implies in turn that the measure of Home industries producing generics needs to expand according to the Home full-employment-of-labor condition (24) for any level of population and any given measure of Home industries with active patents. Which of the two opposite effects dominates depends on the parameters of the model. Changes in Foreign patent policies affect long-run growth only through changes in per-capita consumption expenditure $\bar{c}/w^*$ (see imitative-R&D condition (23)), and therefore an increase in ε^* or/and T^* expands the measure of Home industries producing generics and accelerates long-run growth.

Similar considerations apply to the long-run determinants of the rate of imitation $\dot{A}^* = [g_L/\phi](n^*)^{\beta^*}$ (see equation(20)), where the measure of Foreign industries producing generic products is determined implicitly by (32). The determinants of international technology transfer, captured by the steady-state flow of imitative patents, are identified by the following proposition:

Proposition 3: *The model generates unique, scale- invariant, endogenous rate of international technology transfer, measured by the flow of imitative patents* $\dot{A}^*$*, with the following properties:*

(a) $\dot{A}^*$ *increases in the relative size of the imitating country,* L_0^**; in the parameter that regulates the difficulty of innovative and imitative R&D,* ϕ *; in the size of quality innovations,* λ *; in the degree of enforcement of innovation-based patents,* ε *; and in the length of innovation-based patents,* T *.*

(b) *The effects of Foreign patent policies (a change in patent length* T^* *, or a change in patent enforcement policies* ε^* *) on* $\dot{A}^*$ *are ambiguous.*

As in the case of innovative R&D and growth, Foreign patent policies affect the rate of imitation through two distinct channels. On the one hand, stricter imitative-patent policies increase the expected length of a typical imitative patent $\varepsilon^* T^*$ and the measure of industries in Foreign with active patents. The expansion of the patent-protected sector reduces the measure of industries producing Foreign generics and increases the rate of imitative R&D difficulty. This effect decelerates the rate of international technology transfer. On the other hand, an increase in ε^* or T^* increases the market value of imitative patents, which requires a reduction in real per-capita expenditure (see imitative-R&D condition(23)). The reduction in real consumption expenditure generates a decline in the per-capita demand for manufacturing labor, which must be compensated by an increase in the measure of Foreign industries producing generics. This effect enhances the steady-state rate of imitation. Notice that higher protection of intellectual property at Home does not affect directly the measure of Foreign industries with active patents, and therefore it results in an unambiguous acceleration of the rate of imitation.

We proceed by analyzing the effects of intellectual property protection in a global economy with a TRIPs regime (i.e., a global economy that has adopted a common patent length $T = T^*$ and a common patent enforcement policy $\varepsilon = \varepsilon^*$). In the presence of a common patent policy, it is obvious from (28) and(29) that the wage gap becomes $\omega = 2\lambda/(1+\lambda) > 1$ and depends only on the size of quality innovations. Using T and ε to denote the common patent regime, equations (31) and (32) can be written as

$$L_0 = \frac{g_L}{\phi}\left[1 + \frac{1}{(\lambda-1)}\left(\frac{2T}{\psi(T)}\right)\right] n^{\beta} + \frac{1}{(\lambda-1)}\left(\frac{2}{\varepsilon\psi(T)}\right) n \tag{33}$$

$$L_0^* = \frac{g_L}{\phi}\left[1 + \frac{\lambda}{(\lambda-1)}\left(\frac{2T^*}{\psi(T)}\right)\right] (n^*)^{\beta^*} + \frac{\lambda}{(\lambda-1)}\left(\frac{2}{\varepsilon\psi(T)}\right) n^* . \tag{34}$$

These two equations determine the effects of the model's parameters under a common patent-policy regime. Notice that an increase in the common patent length T has an ambiguous effect on $T/\psi(T)$, and therefore the effect of an increase in the patent length on long-run growth and the rate of imitation is in general ambiguous. However, an increase in the degree of common patent enforcement policy increases both the long-run global growth rate and the rate of imitation. The following proposition summarizes these findings:

Proposition 4: *If the global economy is governed by a common patent-policy regime ($T = T^*$ and $\varepsilon = \varepsilon^*$), then: the global wage income distribution is not affected by changes in the common patent policy; an increase in global patent enforcement increases long-run growth and the rate of international technology transfer; and an increase in the global patent length has an ambiguous effect on long-run growth and the rate of international technology transfer from Home to Foreign.*

4. Concluding Remarks

The present paper developed a two-country (innovative Home and imitating Foreign) general-equilibrium model of product-cycle trade, Schumpeterian growth, and country-specific protection of intellectual property. The model generates patent-based, scale-invariant, endogenous, long-run Schumpeterian growth; product-cycle trade; an endogenous long-run rate of international technology transfer; and an endogenous wage gap between Home and Foreign. We use the model to study the effects of patent policies on long-run growth, the rate of imitation and global income distribution.

An increase in the length or/and stricter enforcement of innovative patents has an ambiguous effect of long-run growth, deteriorates the global wage-income distribution, and accelerates the rate of international technology transfer from Home to Foreign. Similarly, an increase in the length or/and enforcement of imitative patents increases the rate of long-run growth, improves the global wage-income distribution, and has an ambiguous effect on the long-run rate of international technology transfer. Harmonization of patent policies (i.e., a move to identical innovative and imitative patent policies) improves the global wage-income distribution, but has an ambiguous effect on growth and the rate of international technology transfer. In world economy with harmonized patent policies (e.g., a world economy under TRIPs): a stricter patent enforcement policy increases long-run global growth, accelerates the rate of international technology transfer and does not affect the world wage-income distribution; an increase in the international patent length has an ambiguous effect on growth and the international transfer of technology, but does not affect the wage-income distribution.

As always, the above-mentioned results are conditioned on the model's assumptions. The modeling of asymmetric knowledge spillovers between patent-active and patent-inactive industries opened an interesting channel that allows patent policies to affect scale-invariant global growth. Alternative assumptions concerning the structure of knowledge spillovers might reverse some of our findings. The ambiguous effect of Home patent policies on global Schumpeterian growth implies the existence of Home-patent policies that maximize global growth. Similar considerations apply to the effects of Foreign-patent policies on the rate of imitation. Finally, the complexity of the model dictated the absence of welfare analysis. These directions constitute fruitful avenues for future research.

References

Aghion, Phillipe and Peter Howitt, (1992), "A Model of Growth Through Creative Destruction", Econometrica 60. 323-352.

Dinopoulos, Elias, Ali Gungoraidinoglu and Constantinos Syropoulos, (2005), "Patent Protection and Global Schumpeterian Growth", in E. Dinopoulos, P. Krishna, A. Panagariya and K. Wong (eds), *Globalization: Prospects and Problems. Papers in Honor of Jagdish Bhagwati,* Routledge, *forthcoming*.

Dinopoulos, Elias and Constantinos Syropoulos, (2006), "Rent Protection as a Barrier to Innovation and Growth", University of Florida, *mimeo*

Grossman, Gene, and Elhanan Helpman, (1991*), Innovation and Growth in the Global Economy*, The MIT Press.

Grossman, Gene, and Edwin Lai, (2004), "International Protection of Intellectual Property", *American Economic Review* 94,5, 1635-1653.

Howitt, Peter, (1999), "Steady Endogenous Growth with Population and R&D Inputs Growing", *Journal of Political Economy* 107 (4): 715-730.

Jones, Charles, (1995), "Time-Series Tests of Endogenous Growth Models", *Quarterly Journal of Economics*, 110, 495-525.

Romer, Paul, (1990), "Endogenous Technological Change", *Journal of Political Economy* 98, 5: S71-S102.

Segerstrom, Paul, (1998), "Endogenous Growth Without Scale Effects", *American Economic Review* 88: 1290-1310

Appendix

1. Derivation of Equations (31) *and* (32)

Solving the *imitative-R&D condition* (23) yields the following equation for global per-capita consumption expenditure expressed in units of foreign labor $\bar{c}/w^*$:

$$\frac{\bar{c}}{w^*} = \frac{\dfrac{1}{\varepsilon^* \psi(T^*)} + \dfrac{1}{\varepsilon \psi(T)}}{L_0\left(1 - \dfrac{1}{\lambda}\right)} . \tag{35}$$

Similarly, the *innovative-R&D condition* (22) yields the following expression for global per-capita consumption expenditure $\bar{c}/\omega$:

$$\frac{\bar{c}}{\omega} = \frac{\dfrac{\lambda}{\varepsilon \psi(T)} + \dfrac{1}{\varepsilon^* \psi(T^*)}}{\bar{L}_0(\lambda - 1)} . \tag{36}$$

Dividing the Home full-employment condition (24) by the level of global population $\bar{L}(t) = \bar{L}_0 e^{g_L t}$ and using (15), (16) and (26) yields the per-capita Home full-employment condition

$$\frac{L_0}{\bar{L}_0} = \frac{g_L n^\beta}{\phi \bar{L}_0} + \frac{\varepsilon T g_L n^\beta}{\lambda} \frac{\bar{c}}{w^*} + n \frac{\bar{c}}{\omega} . \tag{37}$$

Substituting (35) and (36) into (37), yields equation (31).

Dividing the Foreign full-employment condition (25) by the level of global population and using (20), (21) and (27) yields the per-capita Foreign full-employment condition

$$\frac{L_0^*}{\bar{L}_0} = \frac{g_L (n^*)^\beta}{\phi \bar{L}_0} + \frac{\varepsilon^* T^* g_L (n^*)^\beta}{\phi} \frac{\bar{c}}{\omega} + n^* \frac{\bar{c}}{w^*} . \tag{38}$$

Substituting (35) and (36) into (38) yields equation (32). QED.