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Optimal pension design under stochastic fertility: the role of capital mobility and education financing

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Optimal pension design under stochastic fertility: the role of capital mobility and education financing

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Abstract

This study investigates the optimal design of an unfunded pension system. Convex combinations between a fixed contribution rate and a fixed benefit rate are considered. The objective is to maximize the expected *ex-ante* welfare under stochastic fertility. The model is a three-period CGE framework where the design for the education system and the effect on factor prices are accounted for. The effects on factor prices depend on the degree of capital mobility. Having a fixed benefit rate for the pension system seems optimal in most cases, while the opposite only holds for extreme cases of capital mobility and education financing.

Key Words: Pension schemes, demography, social security, education, fertility.

JEL classification: J13, H55, H52

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1 Introduction

Several countries have re-designed their pay-as-you-go (PAYG) pension systems towards a *fixed contribution rate* (FC scheme). In the literature there is, however, no consensus which design of PAYG that should be preferred. From an *ex ante* perspective, Thøgersen (1998) finds that the *fixed contribution rate* (FC scheme) is preferred over the *fixed replacement rate* (FR scheme). Wagener (2003), however, finds that it is ambiguous which one that is preferred from an *ex ante* perspective; while he finds that the FR scheme is preferred from an *ex post* perspective. Both these studies analyze the effects for a small open economy that exogenously faces stochastic factor prices.[1]

Changes in the demographic structure has been the driving force for recent reforms of PAYG pension systems, particulary changes to the old age dependency ratio. Bohn (2001) investigates how the FR and the FC schemes differ in response to demographic changes. The effect on factor prices is accounted for via changes in the capital labor ratio that occur in a closed economy. He finds that a large cohort faces less favorable factor prices compared to surrounding generations. To compensate for this, the pension system should be of FR type, which implies that the benefit from the reduction in old age dependency ratio accrues to the large cohort. Smith (1982) also finds that having a PAYG pension scheme that counters the factor price effects is preferred from an *ex ante* perspective.

What has received less attention is that fertility fluctuations are the major source for the changing demographic structure.[2] These fluctuations lead first to changes in the young age dependency ratio, and later to changes in the old age dependency ratio. Such changes will affect another important intergenerational transfer system, namely the education system. An education system that operates on the PAYG principle, which is often the case, can also have different designs. In this case the two extreme points would be to either have a fixed contribution rate or a *fixed benefit rate* (FB scheme). The FB scheme is similar to the FR scheme, both have a benefit rule that is insensitive to the dependency ratios. The difference is that the FR scheme relates benefits to previous wages, while the FB scheme guaranties a share of current wages. Just as different pension schemes lead to specific risk-sharing properties, will different education schemes yield a specific outcome.

When analyzing the risk-sharing properties of different pension schemes with respect to fertility fluctuations it is necessary to account for the effects

[1] Thøgersen (1998) only deals with stochastic wages.

[2] As noted by Cutler et al. (1990).

in the education system, and to account for the effects on factor prices. In Žamac (2005) pure designs for the education system and the pension system were analyzed. It was shown that when there are no effects on the factor prices - as is the case in a small open economy - the education system and the pension system should be of opposite type. If the education system operates according to the FB scheme then the pension system should be of FC type, and vice versa. When the analysis was conducted for the closed economy, then it emerged that the pension system should not be of FC type, irrespective of the design of the education system. This since the effect from capital dilution had to be accounted for. This study only investigated the extreme cases: no effect on factor prices vs. fully closed economy, pure FB vs. pure FC education and pension systems. As pointed out by Wagener (2004) pure systems seldom exist, moreover he found that convex combinations between a FC scheme and a FR pension scheme may be preferred, and not the extremes. Further, it is important to allow for intermediate cases with respect to factor prices, and not just the extreme cases, fully closed or fully open small economy.

Here the Žamac (2005) analysis is extended using the Wagener (2004) convexity approach. Convex combinations between the pure FC and the pure FB schemes are analyzed. This is applied for both the education system and the pension system. The second extension consists of allowing for varying impact on factor prices. The aim is to find the pension combination that yields the highest expected *ex ante* welfare, according to a standard utilitarian welfare function, contingent on the education financing system and the degree of capital mobility. The degree of capital mobility attempts to capture varying effect that fertility changes might have on factor prices.

The results are that a pension scheme with a fixed contribution rate can only be motivated for a small (almost) fully open economy which has a (almost) pure fixed benefit rate for the education system. A pension scheme with pure fixed benefit rate is however optimal in most cases.

The remainder of this paper is organized as follows. Section 2 presents the framework. In section 3, the model is calibrated and the steady state results are presented. Section 4 presents the results while section 5 contains some final remarks.

2 The model

The framework consists of a three period overlapping generations model, with four main components: individuals who maximize their lifetime utility, firms which maximize their profit, an international capital market, and the intergenerational transfer systems for education and pension. There is also an standard intergenerational welfare function, similar to the one applied in Boadway et al. (1991), which will be used to evaluate optimal pension design.

2.1 Modelling the transfers

The OLG model consists of one period when young, one period when working, and one period when retired. The young receive contributions from the working population via the education system, and the retired receive contributions from the working population via the pension system. For the systems to be pure intergenerational transfers it is necessary that the budgets are balanced in each period. Assuming a period-by-period balanced budget for each system separately, makes it possible to state the transfers in period t as:[3]

$$b_{E,t}N_t = d_{E,t}N_{t-1}, \tag{1}$$

$$b_{P,t}N_{t-2} = d_{P,t}N_{t-1}, \tag{2}$$

where $b_{E,t}$ denotes the per child benefit from the education system, $d_{E,t}$ is the contribution per worker to the education system, $b_{P,t}$ is the benefit per retired from the pension system, and $d_{P,t}$ denotes the contribution per worker to the pension system. These are indexed with subscript t to denote that the transfer occurs in period t. The size of each generation is denoted by N, where the subscript t indicates in which period the generation is born. In period t the number of children is N_t, while the number of workers is N_{t-1}, and the number of retirees is N_{t-2}.

Suppose that each worker in period t has n_t children. Then the young age dependency ratio in period t, N_t/N_{t-1}, is denoted n_t and hence the old age dependency ratio, N_{t-2}/N_{t-1}, equals n_{t-1}^{-1}. From the balanced budget restrictions in equations (1) and (2) one can immediately see the impact of changes in the dependency ratios. Demographic changes will either change the received benefits or the contributions, or both.

Above the contributions and benefits where not related to the level of income in society. In a world with growing income over time it would not make sense to have fixed benefits/contributions over time. It is reasonable to relate the benefits/contributions to the income, where income refers to the mean income of the working generation.

[3]The assumption regarding two separate systems is mainly based on the fact the existing social security programs have a very weak connection with the education system, if any. If the period-by-period balanced budget assumption was loosened, then there would be other financing opportunities for an open economy.

Let $\tilde{w}_t$ denote the mean labor income of the workers in period t, and let $\tau_{E,t}$ and $\tau_{P,t}$ denote the *contribution rate* devoted for financing the education and the pension system, respectively. The contribution from the workers, $d_{i,t}$, where $i = E, P$, can then be stated as:

$$d_{i,t} = \tilde{w}_t \tau_{i,t}. \tag{3}$$

The received benefits, $b_{i,t}$, can also be related to the income level of the working population according to:

$$b_{i,t} = \tilde{w}_t \gamma_{i,t}, \tag{4}$$

where $\gamma_{i,t}$ are the *benefit rates* in the transfer systems. The benefit rates are the fraction of active workers income that each child/retired receives.[4]

The period-by-period balanced budget constraints for the two transfer systems can then be rewritten as:

$$\gamma_{E,t} = \tau_{E,t}/n_t, \tag{5}$$

$$\gamma_{P,t} = \tau_{P,t} n_{t-1}. \tag{6}$$

Changes in the dependency ratios must affect either the contribution rate or the benefit rate, or both. By inserting equations (5) and (6) into equations (3) and (4) it is clear that the benefits/contributions will not only depend on demographic changes but also on how income changes.

2.1.1 Different schemes

The various intergenerational transfer schemes differ in how the benefits and the contributions respond to changes in demography and income. The difference between the schemes can be understood from the balanced budget restrictions.

From equations (5) and (6) two simple schemes emerge. Either the benefit rate is fixed, $\gamma_{i,t} = \gamma_i$, or the contribution rate is fixed, $\tau_{i,t} = \tau_i$. These schemes will simply be referred to as *fixed benefit rate, FB,* and *fixed contribution rate, FC.* These are the two extreme cases. To allow for convex

[4]The term benefit rate is not to be confused with the term *replacement rate*. The benefit rate is an theoretical abstraction and is also used in Lindbeck (2000), though not using the same term. In the pension literature it sometimes occurs that the replacement rate refers to the fraction of current income (what is referred to as the benefit rate in this paper). This is, however, conceptually obscure since the benefits of the present pensioners does not replace the wages of present workers. Augustinovics (1999), among others, has also pointed at this misuse i the literature.

combinations between the extreme cases, the same approach as in Wagener (2004) will be used.[5] The benefit formula for the education and the pension system are stated as:

$$b_{E,t} = \tilde{w}_t \gamma_E (\phi_E + (1 - \phi_E) n / n_t), \tag{7}$$

$$b_{P,t} = \tilde{w}_t \gamma_P (\phi_P + (1 - \phi_P) n_{t-1} / n), \tag{8}$$

where n is the steady state population growth and $\phi_i \in (0, 1)$ indicates under which scheme the systems operate. The extreme cases are $\phi_i = 0$, which corresponds to a pure FC scheme, and $\phi_i = 1$, which corresponds to a pure FB scheme. The intermediate cases when $\phi_i \in [0, 1]$ indicates specific convex combinations between the extreme cases.

The extreme points are quite natural for the education system, but for the pension system a note is warranted. Usually the *fixed replacement rate, FR* scheme is used as the opposite of the FC scheme. In this case the benefits received in the pension system are related to *previous* income instead of current income, i.e. the income from one's own active life. There are three reasons why the FB scheme is used in this paper instead of the FR scheme. First, the rate of return in the pension system is determined by two factors, the population growth and the productivity growth. The FR scheme differs from the FC in both respects, while the FB scheme only differs with respect to the population growth. Since this paper analyzes the effects from the population growth I wish to isolate this effect. Second, as it was shown in Žamac (2005) there are only slight differences in results between the FR scheme and the FB scheme, while the former is somewhat more complicated to analyze. Third, since the vantage point used in this paper is *ex ante*, there will be no insurance gain in the FR scheme which occurs if the *ex post* approach is used. This since from an *ex ante* perspective one's own wage is unknown, and thus the FR scheme cannot offer a certain benefit.[6]

2.2 Individuals

Individuals live for three periods. During young age, children invest all their time (one unit) in human capital accumulation, from which they all receive the same utility. Children's time input is combined with education benefits,

[5]There are some differences compared to Wagener (2004). Since the PAYG pension is a political process in his modell, it means that the pension system is not *a priori* fixed. Also he models the convex combination between the FC scheme and the FR scheme

[6]Although not covered in this analysis, it can also be noted that when the FR scheme is in place there are no incentives for the workers to invest in the coming generations human capital.

provided by the workers, to develop their human capital which will be used when working. Any difference in the per child education benefit will thus not affect the utility in the first period of life, but will instead alter the human capital. In the next period, when working, all supply inelastically their effective labor, the product of their one unit of time and their human capital, to firms and receive wage income. A fraction of this wage income will finance the education and pension systems; the remaining part will be divided between savings and consumption. In the third and final period, individuals are retired and consume their own savings and income from the pension system.

Since all generations gain the same utility when young this period is suppressed. The lifetime utility of an individual, belonging to generation $t-1$, is assumed to be additively separable according to:

$$U_{t-1} = \ln c_{w,t} + \beta \ln c_{r,t+1}, \tag{9}$$

where β is the subjective discount factor and thus a measure of the individual's impatience to consume. Consumption per worker in period t is denoted with $c_{w,t}$, while consumption per retired in period t is denoted by $c_{r,t}$.

Denote by h_t the human capital for generation $t-1$ while at work. This is a product of the benefits from the education system in period $t-1$, i.e.:

$$h_t = b_{E,t-1}^{\sigma}, \tag{10}$$

where $\sigma \in (0,1]$ measures the elasticity of scale in the production of human capital. The human capital determines the effective labor supply for each individual in period t. The individuals take their human capital, wages, the interest rate, the tax rate, and the benefits in the pension system, as given. Their only decision variable is savings, which they choose as to maximize the lifetime utility, according to equation (9), subject to the following budget constraints:

$$c_{w,t} = (1-\tau_t)\, w_t h_t - s_t, \tag{11}$$

$$c_{r,t+1} = R_{t+1} s_t + b_{P,t+1}, \tag{12}$$

where s_t denotes the per worker savings in period t, w_t is the wage for one unit of effective labor, and R_{t+1} denotes the gross interest rate on savings between period t and $t+1$. Further, τ_t denotes the total tax rate used in the financing of the education and the pension systems, that is $\tau_t = \tau_{P,t} + \tau_{E,t}$.

The individuals can use their savings either for investment in domestic firms or to lent to the rest of the world (or borrow). So we have:

$$s_t = i_t + d_t, \tag{13}$$

where i_t is the investments made in domestic firms, and d_t is the amount borrowed from the rest of the world, which may be negative.

Maximizing the objective function (9) under the constraints (11) and (12) yields the familiar intertemporal Euler equation:

$$c_{r,t+1} = \beta R_{t+1} c_{w,t}. \tag{14}$$

2.3 Production

The aggregate production function in the economy is assumed to be of Cobb-Douglas type and homogeneous of degree 1. Production is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where L_t is the aggregate effective labor, i.e. $L_t = h_t N_{t-1}$, K_t is the aggregate capital stock in the beginning of period t, and A is a scaling parameter. The capital stock K_t depreciates fully during the production process. Defining production in terms of output per worker yields:

$$y_t = Ak_t^{\alpha} h_t^{1-\alpha}, \tag{15}$$

where $y_t = Y_t/N_{t-1}$, and $k_t = K_t/N_{t-1}$.

The prices of the factor inputs are obtained from the firms maximization problem, and since perfect competitive factor markets are assumed these prices equal their marginal product, that is:

$$R_t = A\alpha k_t^{\alpha-1} h_t^{1-\alpha}, \tag{16}$$

$$w_t = A(1-\alpha) k_t^{\alpha} h_t^{-\alpha}, \tag{17}$$

where R_t is the price on physical capital, and w_t is the price per unit of human capital, both in period t.

2.4 Capital mobility

Capital mobility aims to capture the varying effects that demographic changes might have on factor prices. The extreme cases are zero capital mobility, i.e. $d \equiv 0$, and full capital mobility such that d can fluctuate freely as to keep the rate of return on capital constant. These cases correspond to the the closed economy and the small perfectly open economy, respectively. By allowing for imperfect capital mobility it will be possible to analyze the intermediate cases between a small perfectly open economy and the closed economy. This is of interest since not many economies are considered to be closed and there is also ample evidence that the financial integration is imperfect (e.g. Obstfeld and Rogoff (2000)), implying that not many economies belong to the other extreme case.

There are many possible ways to model imperfect capital mobility, or more correctly why capital flows will be less then required to keep the interest rate constant. In this paper limited capital mobility will be modelled in terms of risk-premium, similar to the specification used in Schmitt-Grohé and Uribe (2003). The interest rate is given by:

$$R_t = R^w - \rho \frac{(1-\phi_K)}{\phi_K} d_{t-1}, \tag{18}$$

where R^w is the constant world interest rate, $\rho > 0$ is a scale parameter, and $\phi_K \in (0,1)$ will be a measure of capital mobility, or degree of openness. When $\phi_K = 1$ then there is no risk-premium and d can vary freely as to keep the marginal return on investment constant, i.e. equal with the world interest rate, R^w. If $\phi_K = 0$ then the risk-premium will be infinite which will lead to zero capital mobility, i.e. $d = 0$. In this case the interest rate will be determined by the marginal return on investments, according to equation (16).

2.5 Equilibrium

Given the initial capital stock, $k_0 > 0$, the initial human capital stock, $h_0 > 0$, and the population growth, $\{n_t\}_{t=0}^{\infty}$, a competitive equilibrium for this economy is a sequence of: prices $\{w_t, R_t\}_{t=0}^{\infty}$, allocations $\{c_{w,t}, c_{r,t}, i_t, d_t\}_{t=0}^{\infty}$, human and physical capital stocks $\{k_t, h_t\}_{t=0}^{\infty}$, and benefit rates and tax rates $\{\gamma_{E,t}, \gamma_{P,t}, \tau_{E,t}, \tau_{P,t}\}_{t=0}^{\infty}$, such that the individuals maximize their utility, firms maximize their profits, the budgets of the transfer systems are balanced, and that markets clear. Where the market clearing condition can be reduced to:

$$k_{t+1} = i_t / n_t, \tag{19}$$

and i_t has to be such that equation (13) holds.

Individual saving decisions fully characterize the equilibrium, since they define the equilibrium trajectory for $\{k_t\}_{t=0}^{\infty}$ via eq. (19) and (13). Eqs. (11)-(14) and (16)-(19) yield the following saving function in equilibrium:

$$s_t = \frac{\beta\alpha\,(1-\tau_t)\,w_t h_t}{\lambda_t}, \tag{20}$$

where $\lambda_t = \alpha\,(1+\beta) + (1-\alpha)\,\gamma_{P,t+1}/n_t$. Saving is a fraction of the disposable income and independent of the interest rate in the economy; this result arises because the utility function has an intertemporal elasticity of substitution equal to unity.

The savings are divided between investments, i_t, and capital transactions with rest of the world, d_t, in such a way as to equalize the marginal return on investments with the interest rate; that is, in such a way that equation (16) equals equation (18).

2.6 The intergenerational welfare function

To obtain a compact measure of how all generations are affected by a fertility shock, welfare is defined as:

$$W = E\left[\sum_{t=1}^{T} \psi_t (U_t - U_{ss})\right], \tag{21}$$

where U_{ss} is the lifetime utility in steady state. This is a pure utilitarian welfare function, implying neutrality towards the inequality in the distribution of utility.[7]

There are different views on how the per capita lifetime utility of generation t should be weighted. The question is if the utility should be weighted by the generation size, and whether the utility of future generations should be discounted. It seems more or less necessary to account for the generation size, otherwise there would be an unequal treatment of individuals belonging to generations of different size. A social discount rate will be included and the weighting factor will be the following:

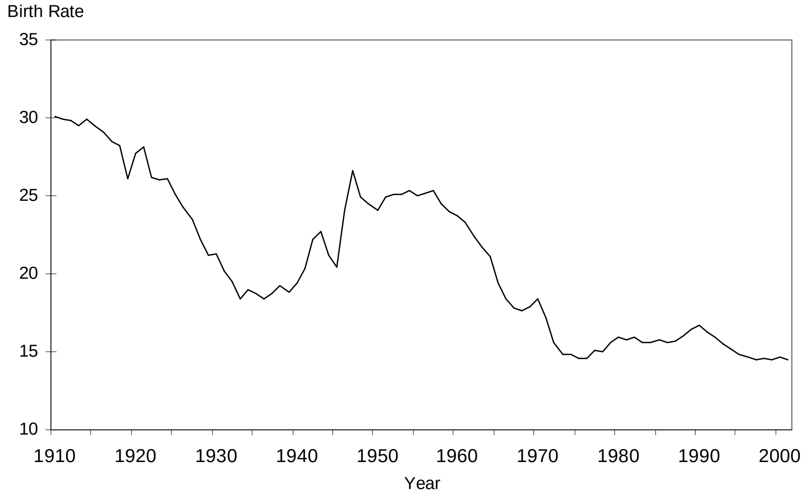
$$\psi_t / \psi_{t-1} = \beta_s n_t, \tag{22}$$

where β_s is the social discount rate. In the benchmark simulation the social discount rate will be set equal to the individuals discount factor, i.e. $\beta_s = \beta$. The formulation allows for varying the social discounting as long as $\beta_s \in (0, 1/n]$. If there is population growth then the discount rate should not exceed the inverse of the population growth; if it does, then the future generations would get an ever increasing impact on the welfare function, due to their larger number.[8]

[7]Choosing a general utilitarian welfare function with aversion towards inequality between generations utility would strengthen the results obtained later in the paper.

[8]See for instance Blanchet and Kessler (1991) and Boadway et al. (1991) for a short comment concerning the weighting problem.

Figure 1: Birth Rate per 1000 inhabitants for the U.S. between 1910 to 2001.



Source: National Vital Statistics Reports 51, no. 2 (2002).

3 Simulation and calibration

3.1 Fertility rate

What will be considered is an one period disturbance in the fertility rate. It is, however, uncertain if the disturbance, shock, will be positive or negative. This is an *ex ante* analysis similar to the one applied in Ball and Mankiw (2001).

The fertility rate can be stated as $n_{t+j} = n \ \forall j \neq 0$ and $n_t = n(1 + x)$, where x is either a positive or negative disturbance. To get an estimate of n, and the magnitude of the potential disturbances, x, consider the U.S. experience. In figure 1 the birth rates per 1000 inhabitants for the U.S. between the period 1910 to 2001 are presented.

Regarding the steady state gross population growth n this will be set to 1.3, based on the annual average for the U.S. between 1910-2001, presented in figure 1.[9] The size of the disturbance does not matter and it will be set to 20 percent, i.e. $x = \{-0.2, 0.2\}$.[10] Changes in these numbers do not alter the qualitative results.

The demographic structure used in the simulation can be stated as $n_{t+j} = 1.3 \ \forall j \neq 0$ and $n_t = \{1.04, 1.56\}$.

[9]The annual average is approximately 1.01, which implies that per period $n = 1.01^{27}$, since one period corresponds to 27 years.

[10]The size of the shock is well within reason if one considers the fluctuations in figure 1.

3.2 Preferences

Regarding preferences, β is the standard measure of the individual's impatience to consume. Using the one year estimate from Auerbach and Kotlikoff (1987) of 0.98 translates to $\beta = 0.6$, since every period represents about 27 years.

3.3 Production

There are two parameters in the production function that need to be calibrated, α and A. The share of capital income in the national product, α, is set to one third. The scale parameter A can be chosen freely since it will not alter the relative outcome in any significant way. To make the results between this study and Žamac (2005) comparable, A will be set to 21.6. This value was chosen in Žamac (2005) to fit the empirical growth rate of U.S., here this is not an issue since only the stationary case is considered.

The parameter ρ in equation (18) will be chosen such that the interest rate deviates half the distance from the world interest rate compared to the autarky interest rate when the economy is half open, i.e. $\phi_K = 0.5$. Thus when $\phi_K = 0.5$, the interest rate will be an average between the fully open and the fully closed economy. The world interest rate is assumed to be equal to the steady state interest rate in autarky, implying that $d = 0$, in steady state.

3.4 Human capital

The production function of human capital has only one exogenous parameter, σ, but this parameter is the most difficult to calibrate. Previous studies by Chakrabarti et al. (1993) and Pecchenino and Utendorf (1999) that use the same production function for human capital have calibrated σ to 0.6 and 1, respectively. Here a compromise between these two values will be used, such that $\sigma = 0.8$.

3.5 Intergenerational transfers

Calibrating the education and the pension system amounts to calibrating the benefit rates in steady state, $\gamma_{i,ss}$. For the pension system it is possible to use the existing PAYG pension systems as a guideline. According to the Social Security Office of the Chief Actuary the current benefit ratio, i.e. benefit to the average wage ratio in the same period, is 0.42. In reality, however, the ratio between working years and years of retirement is almost 2, while in this three period model it is 1. For this reason the benefit rate in the pension system is chosen such that $\gamma_{P,ss} = 0.2$.

To obtain the benefit rate in the education system it is possible to use estimates of the GDP share devoted to education. For the U.S. the GDP share for primary and secondary school spending has been approximately 4 percent during the last three decades and the GDP share for higher education is close to 3 percent.[11] The total share of GDP spent on formal education thus amounts to 7 percent.

Besides the formal education and the children's own time input there is a large amount of leisure time, mainly parental time, invested in educating children. Leibowitz (1974) estimates that 131.6 minutes per day of an average couple's non-sleeping time is spent on educational care. This would imply that 6.9 percent of an individuals non-sleeping time is spent on educational care of children.

Letting expenditures on education be a composite of formal education and home education implies that $\tau_E = 0.12$. As for the pension system the model's time ratio between education and working time is not realistic. If one were to use the real contribution rate then the benefit rate would be underestimated severely. Once again a compromise is made such that the contribution rate is higher and the benefit rate is lower than in real life. The following contribution rate will be used $\tau_{E,ss} = 0.16$, which implies that the benefit rate is $\gamma_{E,ss} = 0.12$.

Table 1: Calibrated values for the exogenous parameters.

Parameter		Value
Time preference	β	0.6
Share of capital income	α	1/3
Efficiency in human capital production	σ	0.80
Steady state benefit rate in the pension system	γ_P	0.20
Steady state benefit rate in the education system	γ_E	0.12
Population gross growth rate	n	1.3
Fertility shock	x	± 0.2
Total factor productivity	A	21.6

3.6 Steady state

Before the model is used to study the effects of fertility changes, it is useful to report the steady state values for some key variables, according to the

[11] See Rangazas (2002) p. 947.

calibration in table 1. To see how the model fits stylized facts the last three

Table 2: Steady state values according to calibration in table 1.

Output per worker	y	1821
Cons. per worker	c_w	571
Cons. per retired	c_r	1037
Wage rate per effective unit of labor	w	22
Gross interest rate for capital	R	3.02
Saving rate	S/Y	3.4%
Capital output ratio	k/y	0.11

variables from table 2 are of most interest. The magnitude of the interest rate is quite realistic when adjusting for the time length in the model.[12] The saving ratio in life-cycle models with no bequests has notorious difficulties to fit empirical facts.[13] As for other similar models the saving rate is considerably below the comparable U.S. rate, which is around 6.7 percent. This should not cause large problems as long as the capital output ratio is within reasonable range. From table 2 the capital output ratio is 0.11, which on annual basis becomes 3. This is slightly higher than the comparable U.S. ratio, which is about 2.5, but still within reason.

4 Results

The results presented in figure 2 show the optimal convex combination for the pension system, ϕ_P, for each educational system and degree of openness, that is ϕ_E and ϕ_K respectively. In the simulation the space [0,1] was evaluated at 51 points, whith equal interval of 0.02. This was done for all ϕ_j where $j = P, E, K$.

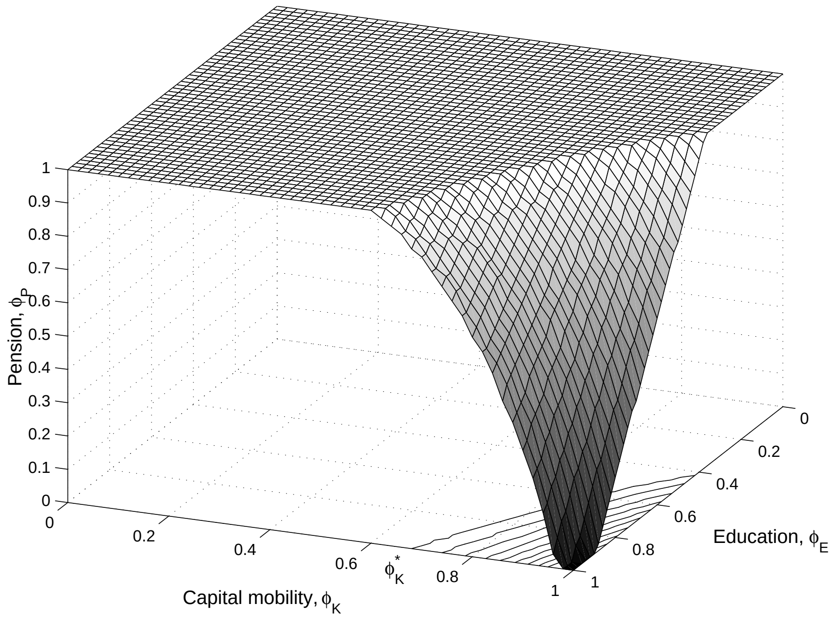
What emerges is that the pure FB pension scheme (i.e. $\phi_P = 1$), is preferred in most cases. Of 2601 evaluated combinations for ϕ_E and ϕ_K the pure FB pension scheme yields the highest welfare for 2228 of these combinations; that is in almost 86 percent of the combinations.

The opposite, i.e. the pure FC pension scheme, is only preferred in 9 of the 2601 possible combinations, which is below 1 percent. For the pure FC scheme to be preferred both $\phi_E \approx 1$ and $\phi_K \approx 1$ has to hold. This means

[12] The reported interest rate is the compounded interest rate over 27 years, which on annual basis becomes 4.2%.

[13] See Kotlikoff and Summers (1981).

Figure 2: Optimal pension combination for different education combinations and degrees of capital mobility.



that the factor prices should (almost) not be affected by the fertility changes and that the education system should be of the (almost) pure FB scheme.

The asymmetry between the pure FB and pure FC is noteworthy. If having to choose ϕ_P without any knowledge of the state, i.e. equal probability for each state, then one would choose the pure FB. For an economy that knows it's specific state, i.e. ϕ_K and ϕ_E, with certainty the choice of ϕ_P is not dependent on what ϕ_P that is preferred in the other states. That is, the choice of ϕ_P cannot be based on the fact that pure FB is preferred in 86 percent of the states. Leaving the discussion of which state might be plausible for the next section, let me give some intuition for the asymmetry in figure 2.

The intuition behind the results can be understood as follows. Consider a baby boom shock in one period only, i.e. $n_{t+j} = n \forall j \neq 0$ and $n_t > n$. This will imply a burden in the education system, and who bears this burden will be a distributional matter between the baby boom generation (the generation born in period t) and their parent generation (the generation born in period $t-1$). If the education system is a pure FB scheme, then every child in period t will obtain the same educational spending per child, as before. This will, however, imply that the tax rate needs to increase, which will burden the parent generation. If the education system on the other hand was of pure FC type, then the parent generation will still pay the same tax rate, but the boom generation will obtain less educational spending per child.

The effects in the pension system are similar but of opposite sign, i.e. instead of a burden the baby boom creates a gain. This since there will be many workers compared to the retirees in period $t+1$, i.e. a temporary decrease in the old age dependency. The allocation of this gain will also be a distributional matter between the baby boom generation and the parent generation. The effects in the education system and the pension system are present irrespective of the degree of openness.

Consider first the fully open economy, $\phi_K = 1$, no effects on factor prices. The fertility will in this case only effect the education system and the pension system, as described above. There are two effects of opposite sign which both are a distributional matter between same two generations. If the education system and the pension system are of opposite type then the same generation will receive both the burden and the gain. Otherwise, one generation will obtain the gain while the other will obtain the burden. For this reason, when the education system is of pure FB the optimal pension design is a pure FC scheme, and when the education system is of pure FC then the optimal pension design is pure FB.

Consider now the other extreme for degree of openness, a fully closed economy, i.e. $\phi_K = 0$. In this case, fertility changes will, besides the affect on the education system and pension system, also affect factor prices. When the baby boom generation reaches productive age there will be capital dilution. This since the savings by the parent generation are not sufficient to equip the baby boom generation with the same capital per worker as before. The capital dilution will burden the baby boom through factor prices. They will receive a low wage and a low interest rate on their savings. The parent generation on the other hand will receive a beneficiary interest rate. This motivates that the baby boom generation should acquire the pension gain instead of the parent generation. The baby boom generation will acquire the pension gain if the pension system is of FB type. As shown in previous studies, the factor price effect alone will dominate the gain in the pension system (e.g. Blomquist and Wijkander (1994); Bohn (2001)). That is, the pension gain can only alleviate some of the burden from the factor price effect that the baby boom generation suffers from. Thus, when there is no capital mobility the pension system should be of FB irrespective of the education system.

The asymmetry in figure 2 stems from the fact that there are two negative effects while only one positive, after a baby boom shock. Since the baby boom generation will obtain the negative factor price effect this motivates the pension system of FB type. Gradually as the factor price effect decreases with higher degrees of capital mobility, it becomes relevant which type of education system that is in existence. It becomes important to account for the design of education system when capital mobility is above ϕ_K^* in figure 2. When capital mobility is close to it's maximum, $\phi_K \approx 1$, then the factor price effect has diminished to almost zero, and all that matters for the choice of pension design is which type of education system that is in place.

5 Discussion

When choosing a pension design that aims to stabilize the effects from fertility fluctuations, it is vital to account for the affect on factor prices and the education system. If the affect on factor prices and the education system is unknown, then the pension design according to the pure FB scheme seems to be preferred. The pure FC scheme seems motivated only when both the factor prices are (totally) unaffected and when the education system is a (almost) pure FB scheme.

The question of interest, which remains unanswered, is how factor prices and the education system *de facto* respond to fertility fluctuations. It could very well be so that they respond in such a manner as to motivate the pure FC scheme for the pension system. There are several reasons to doubt this, however.

Consider first how factor prices might respond to fertility fluctuations. The factor prices are most likely only insensitive to fertility fluctuations for small countries. For non-small countries it seems hard to motivate the pure FC scheme. Moreover, as pointed out by Miles (1999), there are two main reasons why it would be misleading to assume exogenous interest rates for even the smallest country. First, most of domestic investments are financed through domestic savings, as noted in the seminal paper by Feldstein and Horioka (1980). What explains this relationship is a debated issue, and in this paper it was modelled in terms of risk-premium. The specific modelling is not central, and the main results should hold with other explanations as well, for instance transport-costs and the like (e.g. Obstfeld and Rogoff (2000)).

The second reason why it would be wrong to assume that factor prices are unaltered is that most of the financially integrated countries are experiencing similar demographic changes. Even if the economy is small and perfectly open there is no constant world interest rate. Moreover, factor prices will move with fertility fluctuations. The interpretation of ϕ_K would in this case be a measure of the positive correlation between domestic fertility movements with the world fertility movements. The modelling of ϕ_K in this paper is, however, difficult to interpret if the correlation is negative.

For the reasons mentioned above, it is thus unlikely that any country has the extreme $\phi_K = 1$, if we apply the broader view of what ϕ_K might capture. The opposite extreme is however, plausible for some countries.

If one believes that $\phi_K = 1$ might apply for some economies, the pure FC scheme can only be motivated under certain educational financing systems. The education system needs to behave very much like a pure FB scheme, for the pure FC pension scheme to be motivated. It is likely that the education system does not behave like the extremes. This since the benefit formula in the education system is not stated explicitly. Meaning that there is no specified long-term intergenerational contract, as for the pension system. Since there is no explicit contract it is likely that both the benefit rate and the contribution rate are adjusted through a political process. Implying that changes to the young age dependency affects both the parents and the children.

Further, it is important to recognize that the education system mainly is based on the PAYG principle. For this reason the long-term intergenerational contract in the education system should be made explicit. If aiming to give equal opportunities to education, across generations, then introducing an explicit educational long-term contract according to the FB scheme might be warranted. If this would be the case then the FC pension scheme might be preferred for a small economy that is able to eliminate the effect on factor prices via international capital markets.

My belief is that ϕ_K and ϕ_E, for most economies hitherto, are such that the pure FC scheme is not motivated with respect to fertility fluctuations. Based on this, it is not possible to say if the move towards the FC scheme that several countries have adopted is bad or not. Here only the effects from fertility fluctuations where analyzed. There are many other important aspects that have been left out from the analysis, for instance uncertainties about the mortality rate, productivity, and the like. The stylized model did not capture the effects that a varying tax rate, which occurs for the FB scheme, could have on the labor supply. Has this made the analysis biased towards the FB scheme? This depends on if the income or the substitution effect dominates, and by which magnitude. Other fruitful future work is to make the fertility endogenous. This would probably lead to inclusion of altruism into the model which would give a new perspective to the analysis.

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