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Sustaining Social Security

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Abstract

This paper analyzes the sustainability of intergenerational transfers in politico-economic equilibrium. Embedding electoral competition for the votes of old and young households in the standard Diamond (1965) OLG model, we find that intergenerational transfers naturally arise in a Markov perfect equilibrium, even in the absence of altruism, commitment, or trigger strategies. Not internalizing the negative effects of transfers for future generations, the political process partially resolves the distributive conflict between old and young voters by shifting some of the cost of social security to the unborn. As a consequence, transfers in politico-economic equilibrium are higher than what is socially optimal. Standard functional form assumptions yield closed-form solutions for the politico-economic equilibrium as well as the equilibrium supported by the Ramsey policy. The model predicts population ageing to lead to larger social security systems, but eventually lower benefits per retiree. Under realistic parameter values, it predicts a social-security tax rate of 12.2 percent, as compared to a Ramsey tax rate of 4.6 percent. Closed-form solutions for the case with endogenous labor supply, tax distortions, and multiple policy instruments prove the results to be robust.

KEYWORDS: Social security; intergenerational transfers; Markov perfect equilibrium; probabilistic voting; saving; labor supply.

JEL CLASSIFICATION CODE: E62, H55.

1 Introduction

Many countries sustain pay-as-you-go financed social security systems with large intergenerational transfers. Notwithstanding political promises, these transfers are not written in stone.

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Benefit levels, contribution rates, retirement age and many other parameters of the social security system are politically determined and may, in principle, be altered in the regular legislative process. Naturally, this triggers questions about the determinants of political support for social security: Why do democratic societies choose to sustain large intergenerational transfers, even if these involve net contributions by the majority of voters? And, what are the effects of fundamental demographic and economic changes on the political support for, and the size of, these transfers?

To answer these and related questions, we introduce political choice of social-security transfers in the Diamond (1965) overlapping-generations framework with production and capital accumulation. Households are assumed to be non-altruistic. As consumers, they are price takers. As voters, they rationally take into account how policies affect prices and future political choices; furthermore, they are not bound by past political decisions. The politico-economic equilibrium therefore features subgame-perfect tax and transfer choices supporting a competitive equilibrium.

In the context of similar environments, models in the literature generally focus on the motivation of young, working-age voters to support intergenerational transfers. In these models, political choices reflect the preferences of a tax-paying median voter who supports social security out of fear of otherwise being punished with lower future benefits, due to a link between current contributions and future benefits that is mostly assumed to operate through trigger strategies.¹ While usefully highlighting the connection between current and future political choices, the exclusive emphasis on “expectational” forces driving the political support in these models has several shortcomings: Equilibrium policy functions are typically not unique, thereby rendering it difficult to confront the models with the data; positive intergenerational transfers do not arise in the limit of a finite-horizon economy; and, in the case of trigger strategies, equilibria are not robust to small deviations from the assumption of infinite memory.² More importantly, these models imply an increase in the share of retirees to have no direct effect on the size of social security as long as it does not change the identity of the median voter; this stands in contrast to both empirical evidence and conventional wisdom which holds that influence of the elderly is central to the political sustainability of intergenerational transfers.

In contrast, the framework proposed here emphasizes the political influence of young *and old* voters. More specifically, we model electoral competition under the assumption of probabilistic voting rather than a pivotal median voter. Reflecting the presence of uncertainty in the electoral process and inducing a continuous mapping from candidates’ electoral platforms to their vote shares, the probabilistic-voting assumption allows us to capture gradual differences in the support for social security, even in a stark two-period-lived overlapping-generations environment. In equilibrium, vote-seeking candidates aim at maximizing the average welfare of all voters, not only the median voter. This introduces a fundamental source of political support for intergenerational redistribution that interacts with the expectational forces stressed in the literature. The probabilistic-voting assumption has an additional important advantage: In contrast to the median-voter setup, it can easily handle multi-dimensional policy spaces. This enables us to investigate the political support for social-security transfers if these transfers compete with *other* policy instruments.

¹See Cooley and Soares (1999), Boldrin and Rustichini (2000) or Rangel (2003). Forni (2005) presents a model where anticipated punishments are a function of the capital stock, rather than the history of previous political choices.

²See Bhaskar (1998) who shows that informational constraints in overlapping generations games with a strictly dominant action for the old imply that the unique pure strategy equilibrium is in Markov strategies.

We restrict the attention to Markov perfect equilibria where policy choices are only a function of the natural state variables (in our case, the capital stock per worker). Due to the fundamental redistributive motive, positive transfers arise even in the absence of commitment, altruism, trigger strategies, capital overaccumulation, and an infinite time horizon. Moreover, these positive transfers correspond to the unique equilibrium policy function in the limit of the finite horizon economy. Political competition pushes intergenerational transfers to a point where the consumption ratio of old and young households exceeds the relative political influence, per capita, of the two groups. Raising the consumption of old relative to young households increases the expected vote share of political candidates, because the opposition by young voters against intergenerational transfers is weakened by the fact that such transfers depress capital accumulation, thereby affecting future returns and policy choices and indirectly reducing the net cost of transfers to the young.³

The transfers sustained in politico-economic equilibrium are *higher* than those implemented under the Ramsey policy maximizing the discounted sum of the welfare of current and future generations. “Excess redistribution” arises because electoral competition leads the political process to internalize the positive general equilibrium and policy effects of lower capital accumulation for current voters, but not the negative consequences borne by later cohorts—the distributive conflicts between old and young voters are partly resolved by shifting some of the costs of the transfers to future generations. The government implementing the Ramsey policy, in contrast, internalizes both the positive and the negative implications of lower capital accumulation. It pursues social security policy only to the extent that the implied intergenerational redistribution increases the weighted average of the welfare of all current and future generations (taking tax distortions into account). Moreover, the Ramsey policy implements exactly the same transfers as those sustained in a hypothetical politico-economic equilibrium with symmetric political weights, where young voters neither account for the general equilibrium benefits of depressed capital accumulation nor the effect on future political choices (“double-myopic” equilibrium).

These results follow under general conditions, namely constant returns to scale and equality of households’ and the government’s time discount factors. Once particular functional form assumptions are imposed (logarithmic preferences and Cobb-Douglas technology), the model yields closed-form solutions. This stands in sharp contrast to much of the literature which must resort to numerical characterizations. For standard parameter values and symmetric political weights, our analytical results predict a steady-state social-security tax rate of 12.2 percent, very close to the actual tax rate in the U.S. If young voters did not account for the general equilibrium and policy effects of decreased capital accumulation, this equilibrium tax rate would drop to 4.6 percent, which also represents the Ramsey tax rate. More generally, the model predicts the introduction and extension of social security programs in response to lower population growth rates or higher labor shares, a hump-shaped relationship between the population growth rate and social-security benefits per retiree, higher consumption levels of old households than young households, and stronger support for social security in closed economies. These predictions are broadly consistent with existing empirical evidence.

When we enrich the economic environment relative to existing models in the literature by introducing endogenous labor supply and tax distortions, equilibrium can still be characterized in closed form. Modeling the effects of policy on the labor market is important for several reasons. First, because tax distortions constitute a first-order consideration in their own right in the context of redistributive fiscal policy, and second, because the presence of tax distortions introduces

³Kotlikoff and Rosenthal (1990) model the incentive of the young as an interest group to monopolize the supply of capital. They assume commitment and do not model the political process.

other mechanisms than social-security transfers to manipulate equilibrium prices and quantities. Modeling tax distortions therefore permits us to investigate whether intergenerational transfers are sustained even if they compete with other policy instruments and thus, whether a central driving force emphasized in the previous literature is robust. (Such a robustness check was generally not possible in the previous literature, since the median-voter setup is ill-suited for the analysis of multi-dimensional political preference aggregation.) Introducing a second form of transfers, paid to the *young*, we find that tax distortions in combination with this additional transfer make it possible to affect young households' savings and labor supply without having to resort to intergenerational transfers. Nevertheless, social security continues to be sustained in politico-economic equilibrium. In fact, the equilibrium tax rate funding social-security transfers (but not total transfers) turns out to be identical to that in the more basic environment. The Ramsey government, in contrast, implements a lower social-security tax rate than when taxes are non-distorting.

Our work extends a growing literature on dynamic politico-economic equilibrium, with voters sequentially choosing their preferred policies under rational expectations about the effects on future equilibrium outcomes (see, for example, Krusell, Quadrini and Ríos-Rull, 1997; Hassler, Rodríguez Mora, Storesletten and Zilibotti, 2003). As mentioned above, it also relates to an extensive literature on the sources of political support for intergenerational transfers. In contrast to most models in that literature, our approach does not rely on altruism, commitment, or expected punishments (as imposed, respectively, by Hansson and Stuart (1989) and Tabellini (1990); Cukierman and Meltzer (1989), Conesa and Krueger (1999), and Persson and Tabellini (2002); and Bohn (1999), Cooley and Soares (1999), Boldrin and Rustichini (2000), Rangel (2003), and Forni (2005)), nor does it restrict policy choices to be binary or population growth to be sufficiently high (to render the economy dynamically inefficient), as do some previous models. More generally, our approach stresses the gradual resolution of the fundamental distributive conflict between old and young voters, in addition to the expectational forces shaping this conflict, in particular those relating to general equilibrium effects as stressed by Cooley and Soares (1999), Boldrin and Rustichini (2000) and others. The probabilistic-voting assumption that is central to analyzing this gradual resolution only appears to have been employed in the social security context by Grossman and Helpman (1998). However, their model does not feature any economic decisions and therefore, no interaction between the political and the economic sphere.⁴

The remainder of the paper is structured as follows: Section 2 presents the model, derives the equilibrium allocation under probabilistic voting and the Ramsey policy, and confronts implications of the model with existing empirical evidence. Section 3 introduces endogenous labor supply, tax distortions, and multiple policy instruments. Section 4 concludes.

2 The Model

We consider an overlapping generations economy inhabited by cohorts of representative agents. Households live two periods. Young households in period t inelastically supply labor at wage w_t and pay a labor income tax levied at rate τ_t . (We relax the assumption of inelastic labor supply later.) Disposable income is allocated to consumption, $c_{1,t}$, and savings, s_t ; the latter yields a gross rate of return, R_{t+1} . The consumption of old households, $c_{2,t+1}$, equals the gross return

⁴After finishing the first draft of this paper, we learned about independent work by Katuscak (2002) who also adopts the probabilistic-voting assumption, but does not endogenize factor returns as we do.

on savings, $s_t R_{t+1}$, plus a pension benefit, b_{t+1} . The population grows at the rate $\nu - 1$, such that the ratio of young to old households is given by $\nu > 0$.

Output is produced using an aggregate production function with constant returns to scale. Per-capita output in period t positively depends on the capital-labor ratio which, in turn, is proportional to the per-capita savings of the cohort born in $t-1$. Factor markets are competitive and factor prices thus correspond to marginal products. The wage and the gross interest rate are given by $w_t = w(s_{t-1})$ and $R_t = R(s_{t-1})$, strictly increasing and decreasing in s_{t-1} , respectively. Conditional on prices and policies (that is, w_t, R_{t+1}, τ_t , and b_{t+1}), the indirect utility function of a young household of cohort t is given by

$$U_t = \max_{s_t} u(c_{1,t}) + \beta u(c_{2,t+1}), \quad (1)$$

subject to the budget constraint described above. The felicity function $u(\cdot)$ is continuously differentiable, strictly increasing and concave, and satisfies $\lim_{c \rightarrow 0} u'(c) = \infty$; $\beta \in (0, 1)$.

The government sector consists of a social security administration running a pay-as-you-go system.⁵ Old age pensions are financed out of the payroll taxes paid by the young: the pay-as-you-go budget constraint of the social security administration reads

$$b_t = \tau_t \nu w_t. \quad (2)$$

At this point, the sole policy instrument of the social security administration is the payroll tax rate, τ_t , imposed on the labor income of the young. This tax rate is determined in the political process (described in more detail below), subject to a non-negativity constraint, $\tau_t \geq 0$.

The timing of events is as follows: At the beginning of period t , the tax rate to be imposed in the current period is determined in the political process. When deciding which candidate to support, voters anticipate how each candidate's policy platform will affect subsequent economic and political decisions. The wage rate and the return on the predetermined savings of the old, together with the tax rate implemented by the winning candidate, determine the consumption of the old and the disposable income of the young. Young households then turn to their role as consumers and choose how much to save.

When deciding (as voters) on τ_t and (as consumers) on s_t , young households form expectations about future benefits, b_{t+1} . In a Markovian equilibrium, these benefits depend on a set of "fundamental" state variables, \mathcal{S}_{t+1} : $b_{t+1} = \nu w(s_t) \tau(\mathcal{S}_{t+1})$. Clearly, s_t is an element of \mathcal{S}_{t+1} , since s_t affects future wages and gross returns and therefore, the incomes of next period's voters. Having said this, we conjecture s_t to be sufficient for \mathcal{S}_{t+1} , i.e., $\tau(\mathcal{S}_{t+1}) = \tau(s_t)$. We will return to this point later when discussing the political institutions in place.⁶

To characterize the politico-economic equilibrium, we proceed by backward induction. We start by analyzing the economic choices subject to given prices and policies, and then consider the preferences over policies (and thus, prices) and their aggregation in the political process.

2.1 Choice of Individual Savings

The optimal savings decision of a young household in cohort t is characterized by the Euler equation

$$u'(c_{1,t}) = \beta R_{t+1} u'(c_{2,t+1}).$$

⁵Introducing a fully funded component of social security is inconsequential, as long as the government does not force households to save more than they would voluntarily save, and investment opportunities are the same for households and the social security administration.

⁶We assume $\tau(\cdot)$ to be single-valued, as is the case in the limit of the finite-horizon economy.

Since households are atomistic they take aggregate savings and thus, next period's return on capital and, through $\tau(s_t)$ and $w(s_t)$, social-security benefits as given. Households only take into account that higher individual savings increase their future financial wealth.

Conditional on $\tau(s_t)$, the Euler equation maps disposable income, $w_t(1 - \tau_t)$, and aggregate savings into the optimal savings of an individual household. We denote this mapping by the function

$$S^i(w_t(1 - \tau_t); s_t, \tau(s_t)).$$

An equilibrium *aggregate* savings function, $S(w_t(1 - \tau_t); \tau(\cdot))$, is defined as a fixed point of the functional equation $S(y; \tau(\cdot)) = S^i(y; S(y; \tau(\cdot)), \tau(S(y; \tau(\cdot)))) \forall y \geq 0$.

2.2 Choice of Tax Rate

To characterize society's choice of program size, we first consider the welfare implications for old and young households in general equilibrium. These welfare implications induce group-specific preferences over policies. In a second step, we consider the aggregation of these preferences through the political process.

Old households prefer as high a tax rate, τ_t , as possible. This follows directly from the fact that b_t increases in τ_t , while $s_{t-1}R_t$ is independent of τ_t and the tax bill for funding the benefits is solely shouldered by the young. For an old household, the welfare effect of a marginal increase in the tax rate is given by

$$u'(c_{2,t})w_t\nu. \tag{3}$$

For young households, a change in the tax rate gives rise to more complex welfare implications. Differentiating U_t with respect to τ_t yields

$$-u'(c_{1,t})w_t + \beta u'(c_{2,t+1}) \left[s_t R'(s_t) + \nu \frac{d(w(s_t)\tau(s_t))}{ds_t} \right] \frac{dS(w_t(1 - \tau_t); \tau(\cdot))}{d\tau_t}. \tag{4}$$

(An envelope argument implies that the indirect welfare effects through changes in the household's savings cancel.) The first, negative term reflects the cost of higher tax payments. The second term reflects the welfare implications due to induced changes in the fundamental state variable: By shifting disposable income from the young generation (with a positive marginal propensity to save) to the old generation (with a propensity equal to zero), an increase in the tax rate reduces aggregate savings. This increases next period's return on savings, with a positive welfare effect for the young, and alters social-security benefits, with welfare effects whose sign is unclear in general. The total "general equilibrium" effect of depressed aggregate savings as reflected in the second term of (4) is thus positive, as long as the subsequent political choice of benefits does not strongly increase with aggregate savings.

The extent to which the group-specific welfare implications of a change in the tax rate govern the actual policy choice by political decision makers depends on the political institutions in place to aggregate old and young voters' preferences. Previous literature has generally adopted the median-voter assumption according to which political decision makers exclusively represent young voters (as long as $\nu > 1$). This assumption implies that the total welfare effect for young voters as given in (4) must exceed zero over some range to sustain positive taxes in equilibrium. In other words, the general equilibrium effect must be positive and outweigh the direct cost of higher taxes, at least for low levels of τ_t . This condition can only be satisfied if either the elasticity of the interest rate with respect to savings is very high, or the effect of higher taxes on the subsequent political choice of benefits is strong and positive. Many authors have assumed

the latter, often introducing artificial state variables that allow to sustain trigger strategies and thus, arbitrarily large elasticities of future benefits with respect to current choices of tax rate.⁷

Median-voter models and their implied focus on expectational forces to sustain intergenerational transfers have several problems. First, the emphasis on expectational forces implies that equilibrium policy functions are typically not unique, thereby rendering it difficult to confront the models with the data. The reliance on expectational forces also implies that the existence of positive equilibrium transfers hinges on the auxiliary assumptions of an infinite horizon and, in the case of trigger strategies, infinite memory (see Bhaskar (1998)). Second, median-voter models have extreme implications. On the one hand, the mapping from candidates' policy platforms into vote shares is discontinuous: A marginal change in the gross population growth rate, ν , around unity triggers a large jump in the equilibrium tax rate. On the other hand, variations in ν outside a neighborhood of unity have no direct effect on the size of social security. These implications are stark, in particular in our setup with only two groups of voters, and stand in contrast to both empirical evidence and conventional wisdom which holds that the influence of the elderly is central to the political sustainability of intergenerational transfers. Finally, the median-voter setup generally does not allow us to analyze multi-dimensional policy spaces. Therefore, median-voter models must remain silent on whether social security constitutes the *optimal* policy instrument to manipulate general equilibrium effects, putting the robustness of findings in the literature into doubt.

For these reasons, and because we consider it to be another useful benchmark, we replace the assumption of a pivotal median voter by the assumption of probabilistic voting. Probabilistic-voting models acknowledge the fact that voters support a party not only for its policy platform, but also for party characteristics like "ideology" that are orthogonal to the fundamental policy dimensions of interest; these characteristics are permanent and cannot be credibly altered in the course of electoral competition. The valuation of these party characteristics differs across voters (even if they agree about the preferred policy platform) and is subject to random aggregate shocks, realized after parties have chosen their platforms. This renders the probability of winning a voter's support as a function of the competing policy platforms continuous, in contrast to the median-voter setup, and allows us to analyze multi-dimensional policy spaces (as is done in Section 3).

In a Nash equilibrium with two parties maximizing their expected vote share, both candidates propose the same policy platform.⁸ This platform maximizes a convex combination of the objective functions of all groups of voters, where the weights reflect the group size and sensitivity of voting behavior to policy changes. Groups caring relatively more about policy than party characteristics have more political influence, since they are more likely to shift their vote from one party to the other in response to small changes in the proposed platform. In equilibrium, these groups of "swing voters" thus tilt policy in their own favor. If all voters are equally responsive to changes in the policy platform, electoral competition implements the utilitarian optimum with respect to voters.

In the context of our model, the probabilistic-voting assumption implies that the welfare of the old receives some weight in the objective function maximized by the political process, even if the median voter is young. This implication seems very realistic. Indeed, it is frequently argued that old voters exert *stronger* political influence per capita than young voters in the

⁷See, for example, Cooley and Soares (1999), Boldrin and Rustichini (2000), or Rangel (2003). Forni (2005), in contrast, restricts the set of state variables to the fundamental state variable. He finds that, over some range, multiple policy functions can exist, whose steep negative slope is sufficient to generate the desired effect.

⁸See Lindbeck and Weibull (1987) and Persson and Tabellini (2000) for discussions of probabilistic voting.

social security context, because intergenerational transfers are a more salient issue for old than for young voters (see, for example, Dixit and Londregan (1996, p. 1144) and Grossman and Helpman (1998, p. 1309)). While this is reassuring, our main results will *not* require the old to exert disproportionate influence.

The policy platform proposed by the candidates solves the program

$$\begin{aligned} \max_{\tau_t \geq 0} W(s_{t-1}, \tau_t; \tau(\cdot)), \\ W(s_{t-1}, \tau_t; \tau(\cdot)) \equiv \omega^2 u(c_{2,t}) + \omega^1 \nu(u(c_{1,t}) + \beta u(c_{2,t+1})) \\ \text{subject to } \begin{cases} s_{t-1} \text{ given,} \\ s_t = S(w_t(1 - \tau_t); \tau(\cdot)), \\ \tau_{t+1} = \tau(s_t), \\ \text{household budget constraints.} \end{cases} \end{aligned}$$

Here, the weights ω^1 and ω^2 reflect the sensitivity of the voting behavior of young and old households, respectively, with respect to changes in a candidate's proposed policy platform. The household budget constraint incorporates the benefit, wage, and return functions. Next period's policy choice as a function of the state is taken as given, reflecting our assumption of Markov equilibrium. An interior optimum for the candidates is characterized by the condition that the weighted sum of (3) and (4) (where the weights are given by ω^2 and ω^1 , respectively) be equal to zero. Note that our earlier assumption according to which s_t is the only element of \mathcal{S}_{t+1} , is indeed consistent.

In a rational expectations equilibrium, the anticipated policy function coincides with the optimal one. A rational expectations equilibrium is thus given by a fixed point $\tau(\cdot)$ of the functional equation $\tau(s_{t-1}) = \arg \max_{\tau_t \geq 0} W(s_{t-1}, \tau_t; \tau(\cdot)) \forall s_{t-1} \geq 0$. This probabilistic voting equilibrium under rational expectations contrasts with the probabilistic voting equilibrium under "double-myopia", where naive voters expect future tax rates and factor prices to be independent of their aggregate savings choice.

It is instructive to compare the politico-economic equilibrium with the allocation implemented by a benevolent government, subject to the same set of technological, institutional, and competitive-equilibrium constraints. Conditional on an intergenerational discount factor $\rho, 0 < \rho < 1$, the program of the benevolent government with commitment—the Ramsey program—is given by

$$\begin{aligned} \max_{\{\tau_i\}_{i=t}^{\infty} \geq 0} G(s_{t-1}, \{\tau_i\}_{i=t}^{\infty}), \\ G(s_{t-1}, \{\tau_i\}_{i=t}^{\infty}) \equiv u(c_{2,t}) + \beta^{-1} \sum_{i=t}^{\infty} \rho^{i-t+1} (u(c_{1,i}) + \beta u(c_{2,i+1})) \\ \text{subject to } \begin{cases} s_{t-1} \text{ given,} \\ s_i = S(w_i(1 - \tau_i); \tau_{i+1}), i \geq t, \\ \text{household budget constraints.} \end{cases} \end{aligned}$$

Access to a commitment technology implies that the Ramsey program involves the choice of a *sequence* of tax rates. This sequence need not be optimal ex post and thus, need not satisfy fixed-point conditions as in the politico-economic equilibrium. More importantly, in contrast to the political process, the benevolent government values the welfare of all households, not only those currently alive (and voting). It takes into account, for example, how a change in the current tax rate affects future wages and thus, the consumption of the next generation. (Absent

distortions, the Ramsey policy supports the social-planner allocation, see below. In general, and in particular in the setup analyzed in Section 3, this is not the case. To avoid confusion, we consistently refer to the benchmark equilibrium as the “Ramsey equilibrium.”)

2.3 Equilibrium

Turning to the characterization of the equilibrium allocation, our objective is twofold. First, we want to demonstrate that the general equilibrium benefits of depressed aggregate savings significantly reduce the opposition of young voters against intergenerational transfers. To show this point, we compare the politico-economic equilibrium under rational expectations to the politico-economic equilibrium under double-myopia. Second, we want to demonstrate that intergenerational transfers in politico-economic equilibrium are too high, relative to the level chosen by a benevolent government.

For analytical purposes, it is useful to first consider the social-planner allocation, i.e., the optimal allocation subject to resource, but not private sector implementability constraints. Letting $f(\cdot)$ denote the economy’s production function in intensive form, the social-planner allocation is characterized by the first-order conditions

$$\begin{aligned}\rho u'(c_{1,t+i}) &= \nu \beta u'(c_{2,t+i}), i \geq 0, \\ u'(c_{1,t+i}) &= \beta f'(s_{t+i}/\nu) u'(c_{2,t+i+1}), i \geq 0.\end{aligned}\tag{5}$$

According to the first condition, the planner’s marginal rate of substitution between the consumption of young and old households, $\rho u'(c_{1,t+i})/(\beta u'(c_{2,t+i}))$, is equal to the corresponding marginal rate of transformation, ν . According to the first and second condition, the planner’s marginal rate of substitution between consumption in two successive periods, $u'(c_{1,t+i})/(\rho u'(c_{1,t+i+1}))$, is equal to the corresponding intertemporal marginal rate of transformation, $f'(s_{t+i}/\nu)/\nu$.

In contrast to the social planner, the Ramsey government is not only bound by the resource constraint but must also satisfy the implementability constraint of the private sector (i.e., $s_{t+i} = S(w_{t+i}(1 - \tau_{t+i}); \tau_{t+i+1})$) and the non-negativity constraint on tax rates. The former constraint is inconsequential because the social planner’s dynamic first-order condition is identical to the household’s Euler equation, which holds under the Ramsey policy.⁹ Conditional on the intergenerational wealth distribution, the savings choices induced by the Ramsey policy therefore conform with the social planner’s investment policy. Moreover, due to their identical marginal rates of substitution between consumption of young and old households, the planner and the government aim at the same wealth distribution across cohorts. An interior Ramsey policy (i.e., a Ramsey policy without binding non-negativity constraints on tax rates) therefore implements the social-planner allocation and thus, is time-consistent. The latter also holds true if the non-negativity constraint does bind.¹⁰

In the remainder of the paper, we assume that the government weighs generations by their size, and discounts the welfare of future generations according to the household’s discount factor, i.e., we let $\rho \equiv \beta\nu$ (implying $\beta\nu < 1$). The Ramsey policy therefore aims at equalizing per-capita consumption of old and young households at any point in time. We summarize the preceding discussion as follows:

Proposition 1. Consider the Ramsey equilibrium. Unless otherwise mentioned, suppose that strictly positive intergenerational transfers are sustained.

⁹This follows from the fact that $R_{t+i} = f'(s_{t+i-1}/\nu)$ in a competitive equilibrium.

¹⁰Appendices A.1 and A.2 offer a more formal discussion.

- (i) $c_{1,t} = c_{2,t} \forall t$.
- (ii) In steady state, $\beta R = 1$ and the economy is dynamically efficient (since $\beta\nu < 1$).
- (iii) The Ramsey policy implements the same allocation as the social planner, and is therefore unique. Independently of whether transfers are positive, the Ramsey policy is time-consistent.

In politico-economic equilibrium, political influence enters the picture. The political weights of households currently alive, ω^1 and ω^2 , replace the government's intergenerational weights. Equally importantly, some effects internalized by the benevolent government—in particular, the welfare effects of an induced change in aggregate savings on future generations—are no longer accounted for. In the politico-economic equilibrium with rational expectations, candidates only internalize the direct effect due to the social-security transfer from young to old households, and the indirect welfare effects on the current young due to the change in next period's interest rate and social-security benefits. In the following, we denote these indirect welfare effects (which are proportional to the second term in (4)) by \mathcal{B}_t :

$$\begin{aligned} \mathcal{B}_t &\equiv \frac{dS(w_t(1-\tau_t); \tau(\cdot))}{d\tau_t} \nu \beta \omega^1 u'(c_{2,t+1}) [s_t R'_{t+1} + \nu w'_{t+1} \tau_{t+1} + \nu w_{t+1} \tau'_{t+1}] \\ &= \frac{dS(w_t(1-\tau_t); \tau(\cdot))}{d\tau_t} [w'_{t+1}(\tau_{t+1} - 1) + w_{t+1} \tau'_{t+1}] \nu^2 \beta \omega^1 u'(c_{2,t+1}), \end{aligned}$$

where the second line follows from constant returns to scale. Letting λ_t^W denote the non-negative multiplier on the constraint that taxes be non-negative, we thus find the following first-order condition with respect to τ_t for the politico-economic equilibrium under rational expectations:

$$w_t (\nu \omega^2 u'(c_{2,t}) - \nu \omega^1 u'(c_{1,t})) + \mathcal{B}_t + \lambda_t^W = 0, \quad \lambda_t^W \tau_t = 0. \quad (6)$$

Under double-myopia, none of the indirect welfare effects due to induced changes in aggregate savings is internalized. The first-order condition with respect to τ_t for the politico-economic equilibrium under double-myopia therefore reduces to

$$w_t (\nu \omega^2 u'(c_{2,t}) - \nu \omega^1 u'(c_{1,t})) + \lambda_t^M = 0, \quad \lambda_t^M \tau_t = 0, \quad (7)$$

where λ_t^M denotes the non-negative multiplier on the constraint that taxes be non-negative.

Let $\omega \equiv \omega^2/\omega^1$ denote the relative weight of the old in the political process. Conditions (6) and (7) imply the following result:¹¹

Proposition 2. Consider the politico-economic equilibrium under rational expectations. Suppose that strictly positive intergenerational transfers are sustained and $\mathcal{B}_t > 0$, $\omega = 1$.

- (i) $c_{1,t} < c_{2,t} \forall t$.
- (ii) In steady state, $\beta R > 1$ and the economy is dynamically efficient, savings are lower and the tax rate is higher than in the Ramsey equilibrium.

Consider the politico-economic equilibrium under double-myopia. Suppose that strictly positive intergenerational transfers are sustained and $\omega = 1$.

- (iii) $c_{1,t} = c_{2,t} \forall t$.
- (iv) In steady state, $\beta R = 1$, the economy is dynamically efficient, and the allocation is identical to the Ramsey allocation.

¹¹We assume double-myopic expectations to be self-confirming: Double-myopic voters correctly anticipate next period's factor prices and policy choices, but not the price or policy *functions*.

Results (i) and (ii) follow directly from (6). With the balanced consumption profile implemented in the Ramsey equilibrium, the direct welfare effect of a transfer from young to old households is equal to zero. But $\mathcal{B}_t > 0$ implies that there are additional indirect benefits from higher taxes. In political equilibrium, taxes and the relative consumption of old households are therefore higher than in the Ramsey equilibrium. From the household's Euler equation, $c_1 < c_2$ implies that the steady-state interest rate is higher and savings are lower than in the Ramsey equilibrium.

Results (iii) and (iv) follow directly from (7). With balanced political influence, the political process equalizes the consumption of old and young households, parallel to the Ramsey allocation. Since in steady state, the interest rate is the same in the Ramsey equilibrium and the double-myopic equilibrium, the condition $c_1 = c_2$ and therefore $w(1 - \tau) - s = sR + \nu\tau w$ imply that the two tax rates are also the same. Both in the rational-expectations and the double-myopic equilibrium, relatively stronger political influence of the old ($\omega > 1$) increases the support for social security and the relative consumption of old households.

If lower savings indirectly benefit young voters and higher taxes depress savings, then redistribution from the young to the old beyond the extent under the Ramsey policy ($c_1 = c_2$) constitutes the vote maximizing platform in the electoral competition game with fully rational voters, even if voters have symmetric political influence. Proposing relatively low young-age consumption maximizes the expected vote share of political candidates, because the opposition of young voters against intergenerational transfers is weakened by the fact that such transfers depress capital accumulation, indirectly reducing the net cost of transfers borne by the young by improving their intertemporal terms of trade. The mirror image of the improved terms of trade for young households is lower wages for the next, yet unborn generation. But in contrast to the benevolent government, the political process does not account for this fall in wages (beyond its effect on lower social-security benefits), because it only represents the interests of voters currently alive. "Excessive" social-security transfers, i.e., transfers beyond the level implemented by the Ramsey government, are therefore sustained by shifting some of the cost of these transfers to future generations. With double-myopic voters, in contrast, the political process does not only disregard the negative welfare effects of social security for future generations, but also the positive general equilibrium effects and policy repercussions of concern to young voters. The force sustaining excessive transfers therefore disappears.

If we replaced the probabilistic-voting assumption by the median-voter assumption commonly adopted in the social-security literature (corresponding to $\omega = 0$), the model would replicate the conventional finding that threats of "punishment", for example backed by trigger strategies, are needed to sustain transfers in politico-economic equilibrium. With probabilistic voting and symmetric political influence, in contrast, transfers are not only sustained in the absence of trigger strategies, but they are excessively high. Trigger strategies may thus be needed to *reduce* rather than support intergenerational transfers.

To go further and derive closed-form solutions, we impose functional form assumptions:

Assumption 1. Preferences are logarithmic: $u(c) \equiv \ln(c)$. The production function is of the Cobb-Douglas type: $w(s) \equiv A\alpha(s/\nu)^{1-\alpha}$, $R(s) \equiv A(1 - \alpha)(s/\nu)^{-\alpha}$, $A > 0, 0 < \alpha < 1$.

Here, α denotes the labor share, s/ν the capital-labor ratio, and A the level of productivity.

Under Assumption 1, household savings are given by

$$\begin{aligned}
s_t &= w_t(1 - \tau_t) - \frac{w_t(1 - \tau_t) + \nu w_{t+1}\tau_{t+1}/R_{t+1}}{1 + \beta} \\
&= \frac{A\alpha\beta}{1 + \beta} \left(\frac{s_{t-1}}{\nu}\right)^{1-\alpha} (1 - \tau_t) - \frac{\alpha s_t \tau_{t+1}}{(1 + \beta)(1 - \alpha)}, \\
\Rightarrow s_t &= A\alpha \left(\frac{s_{t-1}}{\nu}\right)^{1-\alpha} (1 - \tau_t) \frac{(1 - \alpha)\beta}{(1 - \alpha)(1 + \beta) + \alpha\tau_{t+1}} \equiv s_{t-1}^{1-\alpha} \cdot z(\tau_t, \tau_{t+1}), \tag{8}
\end{aligned}$$

implying

$$\begin{aligned}
c_{1,t} &= s_{t-1}^{1-\alpha} \frac{A\alpha\nu^{\alpha-1}(1 - \tau_t)(1 - \alpha(1 - \tau_{t+1}))}{(1 - \alpha)(1 + \beta) + \alpha\tau_{t+1}} \equiv s_{t-1}^{1-\alpha} \cdot \gamma(\tau_t, \tau_{t+1}), \\
c_{2,t} &= s_{t-1}^{1-\alpha} A\nu^\alpha(1 - \alpha(1 - \tau_t)) \equiv s_{t-1}^{1-\alpha} \cdot \delta(\tau_t).
\end{aligned}$$

If current and future tax rates are chosen by the benevolent government, these equations represent the equilibrium aggregate savings and consumption functions. In politico-economic equilibrium, in contrast, where future tax rates generally depend on future state variables, the equations represent the equilibrium aggregate savings and consumption *functions* only in the special case of $\tau'(s_t) = 0$. In this case—which will be of particular interest—we also have

$$\mathcal{B}_t = \frac{(1 - \alpha)\alpha\beta(1 - \tau_{t+1})\nu\omega^1}{(1 - \tau_t)(1 - \alpha(1 - \tau_{t+1}))} > 0 \quad \forall \tau_t, \tau_{t+1} \in [0, 1].$$

Note that, in “autarky” (i.e., if all tax rates are set to zero), the ratio $c_{2,t}/c_{1,t}$ is given by

$$\hat{c} \equiv \frac{(1 - \alpha)(1 + \beta)\nu}{\alpha},$$

independent of the capital-labor ratio. Moreover, with a time-invariant tax rate τ , the relative consumption of old and young households equals

$$\frac{c_{2,t}}{c_{1,t}} = \frac{\hat{c} + \nu\tau}{1 - \tau}. \tag{9}$$

Equation (9) implies that a particular time-invariant tax rate,

$$\tau^{\mathcal{G}} \equiv \frac{1 - \hat{c}}{1 + \nu} = \frac{\alpha - (1 - \alpha)(1 + \beta)\nu}{\alpha(1 + \nu)},$$

suffices to equalize the per-capita consumption of old and young households in all periods, and thus to attain the social-planner allocation. If $\hat{c} \leq 1$, this tax rate does not violate the non-negativity constraint of the Ramsey program. Proposition 1 thus implies that $\tau_t = \tau^{\mathcal{G}} \forall t$ constitutes the unique and time-consistent Ramsey policy. If $\hat{c} > 1$, in contrast, such that old households consume more than young households in autarky, setting τ_t equal to $\tau^{\mathcal{G}}$ violates the non-negativity constraint. In that case, the Ramsey policy amounts to keeping all tax rates at their constrained value, $\tau_t = 0 \forall t$.¹²

¹²Under Assumption 1, $G(\cdot)$ is given by

$$\begin{aligned}
G(s_{t-1}, \{\tau_i\}_{i=t}^\infty) &= (\ln[s_{t-1}] + \ln[z(\tau_t, \tau_{t+1})]\nu\beta + \ln[z(\tau_{t+1}, \tau_{t+2})]\nu^2\beta^2 + \dots) \frac{(1 + \nu)(1 - \alpha)}{1 - \nu\beta(1 - \alpha)} \\
&\quad + (\ln[\delta(\tau_t)] + \nu\beta \ln[\delta(\tau_{t+1})] + \dots) \\
&\quad + (\nu \ln[\gamma(\tau_t, \tau_{t+1})] + \nu^2\beta \ln[\gamma(\tau_{t+1}, \tau_{t+2})] + \dots).
\end{aligned}$$

Substituting the definitions of $\gamma(\tau_t, \tau_{t+1})$, $\delta(\tau_t)$, and $z(\tau_t, \tau_{t+1})$, differentiating with respect to $\tau_i, i = t, t + 1, \dots$, and using $\hat{c} > 1$ yields the result.

Proposition 3. Consider the Ramsey equilibrium. Suppose that Assumption 1 holds.

The (unique) Ramsey policy features time-invariant tax rates, $\tau_t = \max(\tau^G, 0) \forall t$. The steady state under the Ramsey policy is globally stable and the transition to steady state is unique.

The result follows from (8), Proposition 1, the above discussion, and Lemma 1 in Appendix A.3 which proves stability. If $\hat{c} < 1$ and therefore taxes are strictly positive, the steady-state level of savings is $s^G \equiv (A(1 - \alpha)\beta)^{1/\alpha}\nu$, which is lower than the autarky level of savings, $s^A \equiv (A\alpha\beta\nu^{\alpha-1}/(1 + \beta))^{1/\alpha}$. If $\hat{c} \geq 1$ and therefore taxes are zero, steady-state savings are at their autarky level, s^A . Given that $\{\tau_t\}$ equals $\{\max(\tau^G, 0)\}$, the complete transition of all endogenous variables is fully characterized by the initial condition, s_{t-1} , and the coefficients $\gamma(\tau_t, \tau_{t+1})$, $\delta(\tau_t)$, and $z(\tau_t, \tau_{t+1})$.

Consider next the politico-economic equilibrium under rational expectations. First, we restrict attention to the case where the candidates face a *flat* tax function (imposed by the winner of the electoral competition game in the next period), given by τ_{t+1} . Differentiating $W(\cdot)$ with respect to τ_t and defining

$$\hat{d} \equiv \frac{(1 - \alpha)(1 + \beta(1 - \alpha))\nu}{\alpha\omega} = \hat{c} \frac{1 + \beta(1 - \alpha)}{(1 + \beta)\omega}$$

then yields¹³

$$\frac{\partial W(s_{t-1}, \tau_t; \tau_{t+1})}{\partial \tau_t} = \frac{\alpha\omega(1 - \hat{d}) - \alpha\tau_t(\omega + \nu(1 + \beta(1 - \alpha)))}{(1 - \alpha(1 - \tau_t))(1 - \tau_t)}.$$

For all $\tau_t \in [0, 1)$, this derivative is strictly negative if $\hat{d} > 1$. If $\hat{d} < 1$, in contrast, this derivative is strictly positive up to the strictly positive tax rate

$$\tau^{\mathcal{W}} \equiv \frac{\alpha\omega(1 - \hat{d})}{\alpha(\omega + \nu(1 + \beta(1 - \alpha)))} = \frac{\alpha\omega - (1 - \alpha)(1 + \beta(1 - \alpha))\nu}{\alpha(\omega + \nu(1 + \beta(1 - \alpha)))}$$

rendering the numerator equal to zero, and negative thereafter. These observations lead to

Proposition 4. Consider the politico-economic equilibrium under rational expectations. Suppose that Assumption 1 holds.

(i) There exists an equilibrium with a flat policy function, $\tau(s) = \max(\tau^{\mathcal{W}}, 0)$. In this equilibrium, the steady state is globally stable and the transition to steady state is unique.

(ii) The flat policy function is the unique equilibrium policy function in the limit of the finite-horizon economy.

Result (i) follows from (8), the properties of $\partial W(\cdot)/\partial \tau_t$, and Lemma 1 in Appendix A.3 which proves stability. A flat policy function is consistent with equilibrium because the tax rate maximizing $W(\cdot)$ is independent of the state variable as long as next period's tax function is also independent of the capital-labor ratio. If $\omega > 1 - \nu\beta(1 - \alpha)$ (and thus, in particular, if $\omega \geq 1$),

¹³Under Assumption 1, $W(\cdot)$ is given by

$$\begin{aligned} W(s_{t-1}, \tau_t; \tau_{t+1}) &= \omega \ln[s_{t-1}^{1-\alpha} \delta(\tau_t)] + \nu \{ \ln[s_{t-1}^{1-\alpha} \gamma(\tau_t, \tau_{t+1})] + \beta \ln[(s_{t-1}^{1-\alpha} z(\tau_t, \tau_{t+1}))^{1-\alpha} \delta(\tau_{t+1})] \} \\ &= \ln[s_{t-1}] [(1 - \alpha)(\omega + \nu) + (1 - \alpha)^2 \nu \beta] + \omega \ln[\delta(\tau_t)] + \nu \ln[\gamma(\tau_t, \tau_{t+1})] + \ln[z(\tau_t, \tau_{t+1})] (1 - \alpha) \nu \beta + \nu \beta \ln[\delta(\tau_{t+1})]. \end{aligned}$$

Substituting the definitions of $\gamma(\tau_t, \tau_{t+1})$, $\delta(\tau_t)$, and $z(\tau_t, \tau_{t+1})$ and differentiating yields the result.

then $\tau^{\mathcal{W}} > \tau^{\mathcal{G}}$. Moreover, if $\hat{d} < 1$, then the policy function is strictly positive and the steady-state level of savings, $s^{\mathcal{W}} \equiv \nu(A(1-\alpha)\beta/(\omega + \beta\nu(1-\alpha)))^{1/\alpha}$, falls short of $s^{\mathcal{G}}$ and s^A . If $\hat{d} \geq 1$, the policy function is given by the autarky policy function, $\tau(s) = 0$, and steady-state savings are at their autarky level, s^A . Positive transfers are sustained in politico-economic equilibrium, but not under the Ramsey policy, if $\hat{d} < 1$ and $\hat{c} > 1$, or $\hat{c} > 1$ and $\omega > \underline{\omega} \equiv \hat{c} \frac{1+\beta(1-\alpha)}{1+\beta}$. Since $\{\tau_t\}$ equals $\{\max(\tau^{\mathcal{W}}, 0)\}$, the complete transition of all endogenous variables is fully characterized.

For the uniqueness result under (ii), consider the final period, T , in the finite-horizon economy. The consumption of old and young households in T is given by

$$\begin{aligned} c_{1,T} &= w_T(1 - \tau_T) = s_{T-1}^{1-\alpha} A \alpha \nu^{\alpha-1} (1 - \tau_T), \\ c_{2,T} &= s_{T-1}^{1-\alpha} A \nu^\alpha (1 - \alpha(1 - \tau_T)), \end{aligned}$$

respectively. If $\omega^1/c_{1,T}$ exceeds $\omega^2/c_{2,T}$ in the absence of transfers, then the equilibrium tax rate is in a corner, $\tau_T = 0$. Otherwise, the tax rate is set to achieve

$$\frac{\omega^1}{c_{1,T}} = \frac{\omega^2}{c_{2,T}} \Rightarrow \tau_T = \frac{\alpha\omega - (1-\alpha)\nu}{\alpha(\omega + \nu)}.$$

It follows that the policy function satisfies $\tau_T(s_{T-1}) = \max(\frac{\alpha\omega - (1-\alpha)\nu}{\alpha(\omega + \nu)}, 0)$ and therefore, that it is flat. Moving to period $T-1$, we know from above that flatness of the policy function in T implies flatness of the policy function in $T-1$, in particular $\tau_{T-1}(s_{T-2}) = \max(\tau^{\mathcal{W}}, 0)$. The same logic applies for all preceding periods.

A parallel argument shows that the policy function in the limit of the finite-horizon economy with myopic voters is also unique. Moreover, from Proposition 2, the steady-state tax rate in this economy is the same as under the Ramsey policy if $\omega = 1$.

Similarly to $\tau^{\mathcal{G}}$, $\tau^{\mathcal{W}}$ increases in α and decreases in β and ν . To better understand this result, it is instructive to return to the first-order condition (6). For $\lambda_t^{\mathcal{W}} = 0$ and under Assumptions 1 and a constant tax rate τ , this condition reduces to

$$\frac{c_{2,t}}{c_{1,t}} - \omega = (1 - \alpha)\beta\nu,$$

where the right-hand side equals $\mathcal{B}_t c_{2,t}/(\nu w_t \omega^1)$ and measures the benefit from depressing capital accumulation (and therefore tilting the consumption profile). This incentive to raise taxes increases in the capital share, households' patience, and the population growth rate. On the other hand, the autarky consumption profile \hat{c} (which enters in $c_{2,t}/c_{1,t}$, see equation (9)) also increases in these three parameters, thereby reducing the incentive to raise taxes. The latter effect dominates.

2.4 Empirical Evidence and Significance

The model entails a variety of predictions which we now confront with available empirical evidence. We start with comparative statics results for the politico-economic equilibrium under rational expectations, and then turn to a calibrated version of the model to assess the quantitative importance of the general equilibrium effects and thus, excessive redistribution.

In a politico-economic equilibrium with positive tax rates, pensions as a share of GDP equal

$$\frac{w\tau^{\mathcal{W}}\nu}{w\nu + Rs} = \alpha\tau^{\mathcal{W}},$$

implying that the pension share is decreasing in ν . This result finds support in the data. Analyzing the rise of the welfare state in a sample of 30 countries during the 1880–1930 period, Lindert (1994) finds a significant positive relationship between the pension share and the share of the elderly. A sample of OECD countries during the 1960s and 1970s (Lindert, 1996) and a panel of 60 countries during the 1960–1998 period (Persson and Tabellini, 2003) produce similar findings. Finally, Boldrin, De Nardi and Jones (2005) report cross-section and time-series evidence of a negative relationship between fertility and social-security transfers (104 countries in 1997, and post-war data for the U.S. and European countries, respectively). Boldrin et al. (2005) interpret this evidence as support for their view that increased social-security transfers caused a fall in fertility. Our model suggests a mechanism with reversed causality.¹⁴

The model also replicates the apparently non-monotone empirical relationship between the share of elderly in the population and public pension payments *per retiree*: For the 1880–1930 period, Lindert (1994) estimates an elasticity of the pension share in GDP with respect to the share of elderly that is larger than unity. The OECD data suggest a hump-shaped relationship between the share of the elderly and public pension payments per retiree (Lindert, 1996), and Mulligan and Sala-i-Martin (2004) conclude that there is no clear relationship.¹⁵ Since population growth rates have declined over time, the model can account for these observations as it predicts an inverse-U shaped relationship between the share of elderly and public pension payments per retiree, corresponding to $w\tau^W\nu$. In other words, population ageing leads to a rise in the pension share of GDP but eventually, a decline of social-security benefits per retiree, see Figure 1.

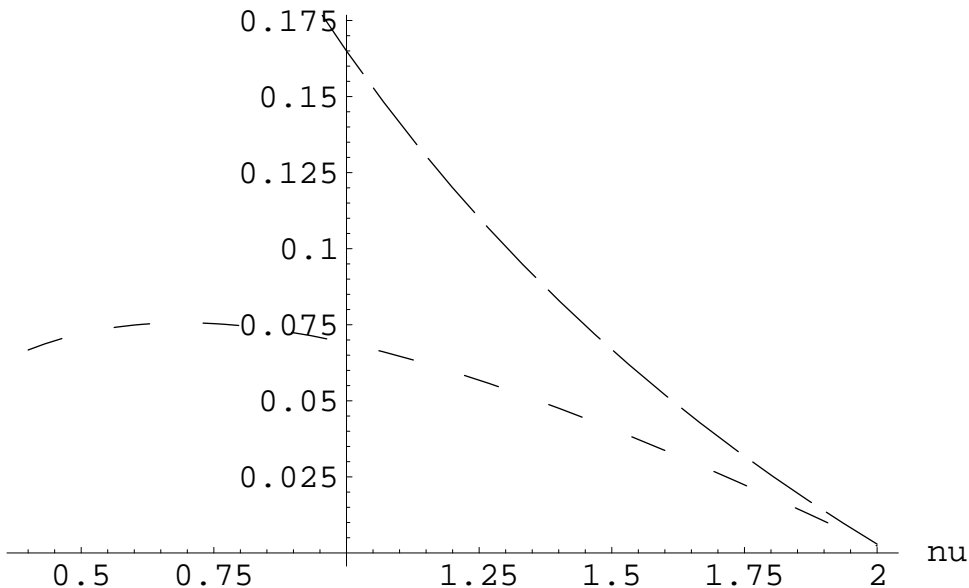


Figure 1: Social-security transfers as share of GDP and per retiree as a function of ν (in order of decreasing length of line segments; α, β, ω at baseline values).

¹⁴Forni’s (2005) model, which adopts the Markov assumption as we do but maintains the traditional median-voter setup, counter-factually predicts higher population growth to result in a higher pension share. This suggests that the probabilistic-voting assumption adopted here is central for the model to be able to replicate the data.

¹⁵Based on data for the United States and 12 European countries over the period 1965–92, Razin, Sadka and Swagel (2002) argue that the dependency ratio is negatively related to per capita transfers.

While the pension share in the model increases in α and decreases in ν , the effect of changes in α or ν on GDP per capita is ambiguous. The model therefore predicts no clear correlation between GDP per capita on the one hand and the pension share of GDP on the other. Indeed, while Lindert (1994; 1996) reports a positive relationship between the pension share and lagged GDP per capita, Persson and Tabellini (2003) do not find any such relationship.¹⁶

Since the political support for social security partly arises from the motivation to affect interest rates, the model predicts a negative relationship between a country’s integration with international capital markets and its level of intergenerational transfers. To the extent that capital- and goods-market integration go hand in hand, this prediction is also borne out in the data, as reported by Persson and Tabellini (2003), who find more trade to be associated with less welfare spending.¹⁷ Similarly, time variation in the openness of the capital account should be reflected in time-varying political support for social security, and the old should oppose capital account liberalization because it reduces the elasticity of the interest rate with respect to domestic savings and thus, strengthens young voters’ opposition against social security.

Another prediction of the model (under the assumption $\omega \geq 1 - \nu\beta(1 - \alpha)$) is that in societies with social security systems, old households consume more than young households. This prediction is consistent with the empirical findings reviewed by Mulligan and Sala-i-Martin (2004) according to which the net income of a typical elderly person is at least as high as the net income of a typical non-elderly person in most developed economies. The discrepancies in consumption or living standards are even larger than those in net incomes.¹⁸

To quantitatively assess the model, we calibrate it.¹⁹ In particular, we impose Assumption 1 and set α to 0.7, ω to unity, and ν to 1.384, the gross growth rate of the U.S. population between 1970 and 2000.²⁰ To calibrate β , we use the following relationship between the model parameters and the steady-state interest rate in politico-economic equilibrium: $\beta = \frac{\omega}{R - \nu(1 - \alpha)}$. Approximating “the” U.S. interest rate by a weighted average of the returns on different asset classes and accounting for the fact that real-world interest rates include a growth component that is absent in our model, we arrive at a value $\beta = 0.97731$ ^{30, 21}

¹⁶A direct test of the model’s prediction of a positive relationship between α and the pension share is difficult. Cross-sectional data on labor shares is likely to be misleading, and the sample of countries for which corrected measures are available is small (see Gollin, 2002).

¹⁷On the other hand, it runs counter to Rodrik’s (1998) finding of a positive relationship between public spending and trade exposure. Rodrik (1998) does not control for the share of the old as do Persson and Tabellini (2003). He suggests that the positive relationship he finds might be due to the public sector providing social insurance against external risk, an aspect not present in our deterministic framework.

¹⁸Alternatively, high relative consumption of old households could arise due to complementarities between consumption and leisure.

¹⁹The general equilibrium effects that remain unaccounted for in politico-economic equilibrium are larger with two-period lived households than with three- or four-period lived households, say. In that sense, the model overemphasizes the impact of general equilibrium effects. Qualitatively, however, the general equilibrium effects are robust—as long as agents have finite lives, the channel is present.

²⁰Piketty and Saez (2003) find α to vary between 0.68 and 0.75 in post-war U.S. data. The population growth rate is reported by the U.S. Census Bureau. We also set A to unity.

²¹Campbell and Viceira (2005) report annualized gross returns for 90-day treasury-bills (1.01518), 5-year treasury-bonds (1.02889), and stocks (1.07829) for the period 1952–2002. We approximate the average return on savings by a weighted average of these returns (1.02970) where the weights are proportional to the relative sizes of “deposits”, “credit market instruments”, and “equity shares at market value, directly held plus indirectly held” in the balance sheets of households and non-profit organizations (Board of Governors of the Federal Reserve System, *Flow of Funds Accounts of the United States: Annual Flows and Outstandings*, several years [we use averages for the period 1955–2002]). In the model, productivity is constant. Relaxing this assumption, we have $A_{t+1} = A_t\gamma_A$ with $\gamma_A > 1$ rather than equal to unity. On a balanced growth path, the interest rate is constant, implying that the capital-labor ratio grows at a gross rate of $\gamma_A^{1/\alpha}$. Using (8) (which is unaffected

Table 1 reports the implied equilibrium values of the steady-state tax rate, capital-labor ratio, factor prices (over thirty years), young and old-age consumption, and life-time welfare.

Table 1: Steady-state equilibrium values

	Politico-economic equilibrium	Ramsey equilibrium
τ	0.1225	0.0457
capital-labor ratio	0.0511	0.0670
R	2.4061	1.9909 ($= \beta^{-1}$)
w	0.2868	0.3111
c_1	0.1810	0.2042
c_2	0.2187	0.2042
U	-2.4728	-2.3868

For the chosen parameter values, $\hat{c} < 1$ (the autarky consumption of young households exceeds the autarky consumption of old households) and $\hat{d} < 1$. Both the Ramsey and the politico-economic equilibrium therefore feature strictly positive tax rates.

The predicted tax rate in politico-economic equilibrium is very close to the actual tax rate of 12.4 percent in the United States (OASDI). Moreover, the tax rate implemented in politico-economic equilibrium is nearly three times as high as in the Ramsey or double-myopic equilibrium. Ceteris paribus, the positive difference between τ^W and τ^G further increases for lower values of α , or higher values of β or ν , see Figure 2. As reported in Table 1, the politico-economic equilibrium supports a lower capital-labor ratio, higher interest rates, lower wages, and a steeper consumption profile than the Ramsey equilibrium. (On an annual basis, the difference between the interest rates is 65 basis points.) In terms of steady-state welfare, the move from the Ramsey equilibrium to the politico-economic equilibrium is equivalent to a permanent reduction in consumption by about 5 percent.

As a robustness check, we numerically solve for the politico-economic equilibrium under the assumption that the intertemporal elasticity of substitution, ε , differs from unity. We find relatively minor changes: For $\varepsilon = 1.5$, the capital-labor ratio increases, the interest rate falls, the tax rate increases slightly, and the equilibrium tax function is slightly upward sloping. For $\varepsilon = 0.5$, the capital-labor ratio falls, the interest rate increases, the tax rate falls slightly, and the equilibrium tax function is slightly downward sloping.

3 Elastic Labor Supply and Multiple Policy Instruments

Previous literature has argued that the sustainability of intergenerational transfers hinges on the general equilibrium effects of depressed capital accumulation. The model of the previous section suggests that general equilibrium effects are important for the *size* of equilibrium transfers, but

by changes in γ_A , as is τ^W) we conclude that, on a balanced growth path, $A_t s_{t-1}^{-\alpha}$ and thus, the interest rate increase by a factor of $\gamma_A^{1/\alpha}$ if the gross growth rate of A increases from unity to γ_A . According to the Bureau of Labor Statistics, multifactor productivity of private businesses grew by a factor of 1.86813 between 1952 and 2002 (<http://www.bls.gov/mfp/home.htm>, series MPU740023 (K)). This implies $\gamma_A = 1.01258$.

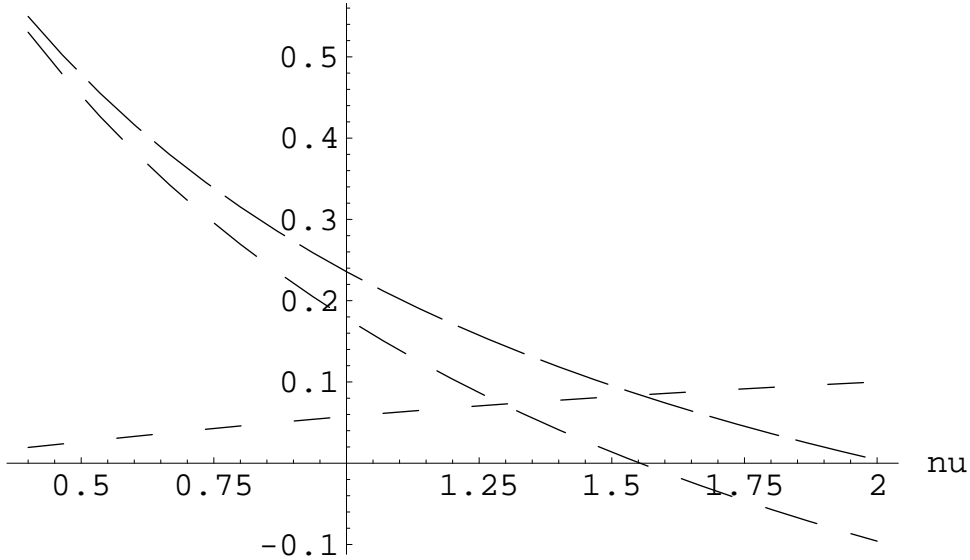


Figure 2: (Unconstrained) $\tau^{\mathcal{W}}$, $\tau^{\mathcal{G}}$, and $\tau^{\mathcal{W}} - \tau^{\mathcal{G}}$ as a function of ν (in order of decreasing length of line segments; α, β, ω at baseline values).

not essential for their existence. Both results raise the question of whether alternative policies exist to depress aggregate savings and, if so, whether social-security transfers continue to be sustained in a more general setup allowing for such alternative policies. The probabilistic-voting assumption allows us to further pursue these questions.

We examine the robustness of the support for social security in an extended framework with endogenous labor supply, tax distortions, and two political transfer choices. More specifically, we extend the model in two directions. First, we introduce an endogenous labor-leisure choice. Labor income taxes therefore depress aggregate savings twofold, by reducing young households' wealth after taxes, as in the model of the previous section, and by distorting young households' propensity to consume goods rather than leisure out of disposable wealth.²² Second, we introduce an additional tax, levied at rate θ_t , on labor income whose revenue is reimbursed to young households. In the previous setup with inelastic labor supply, such a tax-cum-reimbursement would have had no effects. With elastic labor supply, in contrast, the new instrument makes it possible to subsidize leisure consumption and thus, monopolize labor supply and savings of the young *without* having to transfer resources to the old.

For tractability, we assume a young household's felicity function to be separable in consumption and leisure, x_t . The indirect utility function defined in (1) is thus replaced by

$$U_t = \max_{s_t, x_t} u(c_{1,t}) + v(x_t) + \beta u(c_{2,t+1}) \text{ s.t. household budget set,}$$

where $v(\cdot)$ is continuously differentiable, strictly increasing and concave, and satisfies $\lim_{x \rightarrow 0} v'(x) = \infty$. A young household's time endowment is normalized to one. While the tax revenues $w_t(1 - x_t)\tau_t$ continue to fund social security, the additional tax revenues $w_t(1 - x_t)\theta_t$ fund

²²A capital income or savings tax fails to depress capital accumulation in our benchmark economy with logarithmic preferences. If the income effect outweighed the substitution effect, such a tax would even encourage savings.

a lump-sum transfer to young households. The budget constraint of a young household thus reads

$$w_t(1 - x_t)(1 - \tau_t - \theta_t) + T_t = c_{1,t} + s_t,$$

where, in equilibrium, $T_t = w_t(1 - x_t)\theta_t$. Second-period consumption is still given by $c_{2,t+1} = s_t R_{t+1} + b_{t+1}$, where $b_{t+1} = \nu w_{t+1} \tau_{t+1} (1 - x_{t+1})$.²³

Optimal savings and labor supply decisions of a young household are characterized by the first-order conditions

$$\begin{aligned} u'(c_{1,t}) &= \beta u'(c_{2,t+1}) R_{t+1}, \\ u'(c_{1,t}) w_t (1 - \tau_t - \theta_t) &= v'(x_t), \end{aligned}$$

subject to the budget set described earlier. Conditional on $\tau(s_t)$, $\theta(s_t)$, and $\tilde{X}(s_t)$, the household's first-order conditions and budget constraint map s_{t-1} , aggregate savings and leisure as well as the contemporaneous tax rates into the optimal leisure and savings choice of an individual household, $X^i(\cdot)$ and $S^i(\cdot)$, respectively. Equilibrium *aggregate* savings and leisure functions, $S(\cdot)$ and $X(\cdot)$, respectively, are defined as fixed points of the functional equations

$$\begin{aligned} S(s_{t-1}, \tau_t, \theta_t; \tau(\cdot), \theta(\cdot)) &= S^i(s_{t-1}, \tau_t, \theta_t, X(\cdot), S(\cdot); \tau(S(\cdot)), \theta(S(\cdot)), \tilde{X}(S(\cdot))), \\ X(s_{t-1}, \tau_t, \theta_t; \tau(\cdot), \theta(\cdot)) &= X^i(s_{t-1}, \tau_t, \theta_t, X(\cdot), S(\cdot); \tau(S(\cdot)), \theta(S(\cdot)), \tilde{X}(S(\cdot))), \\ \forall s_{t-1} \geq 0, 0 \leq \tau_t, \theta_t, \tau_t + \theta_t &\leq 1. \end{aligned}$$

The modified program of the political candidates reads

$$\begin{aligned} \max_{\tau_t, \theta_t \geq 0} W^\theta(s_{t-1}, \tau_t, \theta_t; \tau(\cdot), \theta(\cdot)), \\ W^\theta(s_{t-1}, \tau_t, \theta_t; \tau(\cdot), \theta(\cdot)) &\equiv \omega^2 u(c_{2,t}) + \omega^1 \nu (u(c_{1,t}) + v(x_t) + \beta u(c_{2,t+1})) \\ \text{subject to } &\begin{cases} s_{t-1} \text{ given,} \\ s_t = S(s_{t-1}, \tau_t, \theta_t; \tau(\cdot), \theta(\cdot)), \\ x_t = X(s_{t-1}, \tau_t, \theta_t; \tau(\cdot), \theta(\cdot)), \\ x_{t+1} = \tilde{X}(s_t), \\ \tau_{t+1} = \tau(s_t), \\ \theta_{t+1} = \theta(s_t), \\ \text{household budget constraints.} \end{cases} \end{aligned}$$

In a rational expectations equilibrium, the anticipated policy functions coincide with the optimal ones and, evaluated at the equilibrium policy functions, the functions $X(\cdot)$ and $\tilde{X}(\cdot)$ are consistent with each other. The modified program of the Ramsey government,

$$\max_{\{\tau_i, \theta_i\}_{i=t}^{\infty} \geq 0} G^\theta(s_{t-1}, \{\tau_i, \theta_i\}_{i=t}^{\infty}),$$

is defined similarly.

Does society still choose to sustain social security in this environment? Or does the option to depress aggregate savings without having to shift resources to the old lead to a collapse of the support for intergenerational transfers? In Appendix A.4, we show that the second alternative can generally be rejected under plausible conditions. Here, we focus directly on the special case of logarithmic preferences in consumption and Cobb-Douglas technology, since this allows us to characterize the politico-economic equilibrium in closed form.

²³To capture the dependency of benefits on individual labor supply, the benefit function can be generalized to $b_{t+1} = \nu w_{t+1} \tau_{t+1} (1 - X_{t+1}) \frac{1-x_t}{1-X_t}$, with $1 - X$ denoting aggregate labor supply. While this affects the household's intratemporal first-order condition, the analysis still goes through without major changes.

Assumption 2. Preferences are logarithmic in consumption: $u(c) \equiv \ln(c)$. The production function is of the Cobb-Douglas type: $w(s_{t-1}, x_t) \equiv A\alpha(s_{t-1}/(\nu(1-x_t)))^{1-\alpha}$, $R(s_{t-1}, x_t) \equiv A(1-\alpha)(s_{t-1}/(\nu(1-x_t)))^{-\alpha}$.

Under Assumption 2, equilibrium savings and consumption choices are given by

$$\begin{aligned} s_t &= s_{t-1}^{1-\alpha}(1-x_t)^\alpha \cdot z(\tau_t, \tau_{t+1}), \\ c_{1,t} &= s_{t-1}^{1-\alpha}(1-x_t)^\alpha \cdot \gamma(\tau_t, \tau_{t+1}), \\ c_{2,t} &= s_{t-1}^{1-\alpha}(1-x_t)^\alpha \cdot \delta(\tau_t), \end{aligned}$$

where functions $z(\cdot)$, $\gamma(\cdot)$, and $\delta(\cdot)$ have been defined earlier, and where τ_{t+1} in politico-economic equilibrium is a function of s_t . Moreover, the household's intratemporal optimality condition yields

$$v'(x_t) = \frac{1 - \tau_t - \theta_t}{(1-x_t)(1-\tau_t) - \frac{\beta(1-\alpha)(1-x_t)(1-\tau_t)}{(1-\alpha)(1+\beta)+\alpha\tau_{t+1}}}$$

and thus, an expression for leisure as a function of τ_t , θ_t , and τ_{t+1} , but not (directly) of s_{t-1} :

$$x_t = x(\tau_t, \theta_t, \tau_{t+1}). \quad (10)$$

$G^\theta(\cdot)$ can therefore be expressed as

$$\begin{aligned} &G^\theta(s_{t-1}, \{\tau_i, \theta_i\}_{i=t}^\infty) \\ &= G(s_{t-1}, \{\tau_i\}_{i=t}^\infty) + (\ln[1-x_t] + \ln[1-x_{t+1}]\nu\beta + \ln[1-x_{t+2}]\nu^2\beta^2 + \dots) \frac{(1+\nu)\alpha}{1-\nu\beta(1-\alpha)} \\ &\quad + (\nu v(x_t) + \nu^2\beta v(x_{t+1}) + \nu^3\beta^2 v(x_{t+2}) + \dots) \text{ subject to (10)} \\ &= G(s_{t-1}, \{\tau_i\}_{i=t}^\infty) + h(x_t) + h(x_{t+1})\nu\beta + h(x_{t+2})\nu^2\beta^2 + \dots \text{ subject to (10)}, \end{aligned}$$

where the function $G(\cdot)$ has been defined earlier and $h(x) \equiv \nu v(x) + \ln(1-x) \frac{(1+\nu)\alpha}{1-\nu\beta(1-\alpha)}$. The first-order optimality conditions include

$$\begin{aligned} \frac{\partial G(\cdot)}{\partial \tau_i} + h'(x_i)(\nu\beta)^{i-t} \frac{\partial x_i}{\partial \tau_i} + h'(x_{i-1})(\nu\beta)^{i-t-1} \frac{\partial x_{i-1}}{\partial \tau_i} + \zeta_i &= 0, \quad i > t, \\ h'(x_i)(\nu\beta)^{i-t} \frac{\partial x_i}{\partial \theta_i} + \chi_i &= 0, \quad i > t, \end{aligned}$$

where ζ_i and χ_i denote the non-negative multipliers associated with the non-negativity constraints on tax rates. These conditions imply that the steady-state Ramsey social-security tax rate is *lower* in the new environment, $\tau \leq \max(0, \tau^G)$.²⁴ Absent non-negativity constraints on tax rates, the government would impose the *same* social-security tax rate as in the main model and eliminate the distortions caused by a non-zero social-security tax rate by setting θ_t equal to

²⁴To see this, denote the tax rates that solve the Ramsey program in steady state by τ and θ , respectively, and first assume that $\tau, \theta > 0$ such that $\zeta = \chi = 0$. This implies (since $\partial x_i / \partial \theta_i \neq 0$) that $h'(x) = 0$, $\tau = \tau^G$ and thus, $h'(x(\tau^G, \theta, \tau^G)) = 0 \Rightarrow \theta = \frac{\nu(1+\beta)(1-\alpha)-\alpha}{\alpha(1+\nu)} = -\tau^G$, yielding a contradiction. At least one of the two tax rates must therefore be zero. If the social-security tax rate is zero, the result immediately follows. Otherwise, $\theta = 0$ and $\tau > 0$. The second first-order condition then implies $h'(x_t) \leq 0$ (since $\frac{\partial x_i(\tau_i, 0, \tau_{i+1})}{\partial \theta_i} > 0$), and the first condition implies $\frac{\partial G(\cdot)}{\partial \tau_i} + h'(x_{i-1})(\nu\beta)^{i-t-1} \frac{\partial x_{i-1}}{\partial \tau_i} = 0$ (since $\frac{\partial x_i(\tau_i, 0, \tau_{i+1})}{\partial \tau_i} = 0$). Since $\frac{\partial x_i(\tau_i, 0, \tau_{i+1})}{\partial \tau_{i+1}} > 0$, we conclude that $\frac{\partial G(\cdot)}{\partial \tau_i} \geq 0$ and thus, due to monotonicity of $\frac{\partial G(\cdot)}{\partial \tau_i}$, that $\tau \leq \max(0, \tau^G)$.

$-\tau_t$. The non-negativity constraints render such a strategy unfeasible. Balancing its allocative and distributive goals, the government therefore reduces the social-security tax rate.

The politically determined social-security tax rate, in contrast, does not change in the new environment. To see this, consider $W^\theta(s_{t-1}, \tau_t, \theta_t; \tau_{t+1}, \theta_{t+1})$, the objective of a vote-maximizing candidate that anticipates *flat* future policy functions and thus, future labor-supply choices independent of the capital-labor ratio. Under Assumption 2, $W^\theta(\cdot)$ is given by

$$\begin{aligned} & W^\theta(s_{t-1}, \tau_t, \theta_t; \tau_{t+1}, \theta_{t+1}) \\ &= W(s_{t-1}, \tau_t; \tau_{t+1}) + \ln[1 - x_t]\alpha(\omega + \nu + (1 - \alpha)\beta\nu) + \nu v(x_t) + \ln[1 - x_{t+1}]\alpha\nu\beta \\ & \quad \text{subject to (10)} \\ &= W(s_{t-1}, \tau_t; \tau_{t+1}) + g(x_t) + \ln[1 - x_{t+1}]\alpha\nu\beta \text{ subject to (10),} \end{aligned}$$

where the function $W(\cdot)$ has been defined earlier and $g(x) \equiv \nu v(x) + \ln(1 - x)\alpha(\omega + \nu + (1 - \alpha)\beta\nu)$. An optimum satisfies

$$\begin{aligned} \frac{\partial W(\cdot)}{\partial \tau_t} + g'(x_t) \frac{\partial x_t}{\partial \tau_t} &\leq 0, \\ g'(x_t) \frac{\partial x_t}{\partial \theta_t} &\leq 0, \end{aligned}$$

with equalities if the solution is interior. If the optimal tax rate $\theta_t > 0$, then the optimal policy features the *same* τ_t as in the main model, $\max(0, \tau^W)$, since $\partial x_t / \partial \theta_t > 0$ implies $g'(x_t) = 0$. The optimal θ_t is then pinned down by the condition $g'(x(\max(0, \tau^W), \theta_t, \max(0, \tau^W))) = 0$.²⁵ Alternatively, if the optimal tax rate $\theta_t = 0$, then $\frac{\partial x_t(\tau_t, 0, \tau_{t+1})}{\partial \tau_t} = 0$ and the first condition once more implies the same choice of social-security tax rate as in the main model. As a consequence, the same flat policy function $\tau(\cdot)$ as in the main model arises, implying that $\theta(\cdot)$ is flat as well and labor-supply is independent of the state variable, confirming the initial guess. Rather than seeing θ_t as a complementary policy instrument to neutralize the distortions caused by the social-security tax (as the Ramsey government does, although it is unable to implement the desired policy mix due to the non-negativity constraints), the political process chooses θ_t to maximize the welfare of current voters; the induced negative welfare effects on future generations again remain unaccounted for. Our benchmark parameter values imply an equilibrium tax rate $\theta^W = 0.066$ for an arbitrary choice of preferences $v(x)$. We summarize these findings as follows:

Proposition 5. Suppose that Assumption 2 holds.

- (i) The steady-state social-security tax rate implemented by the Ramsey government is lower than in the main model.
- (ii) There exists a politico-economic equilibrium with flat policy functions. The social-security tax rate in that equilibrium is the same as in the main model, $\tau(s) = \max(\tau^W, 0)$.
- (iii) The flat policy functions are the unique equilibrium policy functions in the limit of the finite-horizon economy.

For the uniqueness result under (iii), consider the final period, T , in the finite-horizon economy. The consumption of old and young households in T is given by

$$\begin{aligned} c_{1,T} &= w_T(1 - x_T)(1 - \tau_T) = s_{T-1}^{1-\alpha} A(1 - x_T)^\alpha \alpha \nu^{\alpha-1} (1 - \tau_T), \\ c_{2,T} &= s_{T-1}^{1-\alpha} A(1 - x_T)^\alpha \nu^\alpha (1 - \alpha(1 - \tau_T)), \end{aligned}$$

²⁵Since $g'(x)$ is decreasing in x , the condition is “stable”; low θ_t and thus x_t imply that $dg/dx > 0$, pushing x_t and thus θ_t upwards, and vice versa.

respectively. The optimal choices for τ_T and θ_T satisfy

$$\begin{aligned} -\frac{\nu}{1-\tau_T} + \frac{\omega\alpha}{1-\alpha+\alpha\tau_T} + \left(-\frac{\nu\alpha}{1-x_T} - \frac{\omega\alpha}{1-x_T} + v'(x_T)\right) \frac{\partial x_T}{\partial \tau_T} &\leq 0, \\ \left(-\frac{\nu\alpha}{1-x_T} - \frac{\omega\alpha}{1-x_T} + v'(x_T)\right) \frac{\partial x_T}{\partial \theta_T} &\leq 0, \end{aligned}$$

with equality for an interior solution. These conditions, together with the household's intratemporal optimality condition

$$v'(x_T) = \frac{1-\tau_T-\theta_T}{(1-x_T)(1-\tau_T)},$$

imply that the tax rates at time T and thus, also the labor supply, are independent of the capital-labor ratio. Furthermore, the term $\left(-\frac{\nu\alpha}{1-x_T} - \frac{\omega\alpha}{1-x_T} + v'(x_T)\right) \frac{\partial x_T}{\partial \tau_T}$ in the first-order condition with respect to τ_T vanishes for the reasons discussed earlier. Therefore, the same τ_T as in the main model results, $\tau_T(s_{T-1}) = \max\left(\frac{\alpha\omega-(1-\alpha)\nu}{\alpha(\omega+\nu)}, 0\right)$, and the optimal θ_T satisfies $\theta_T(s_{T-1}) = \max((1-\tau_T)(1-\alpha(\nu+\omega)), 0)$. Having established that the policy functions in period T are flat, the arguments employed before imply the same for all earlier periods.

Returning to the question motivating this section, we first conclude, that society sustains social security even in an environment with endogenous labor supply, distorting taxes, and alternative policy instruments. In that sense, the political support for social-security transfers is robust. Second, we conclude that the benevolent government, but not the political process, reduces intergenerational transfers in response to the presence of tax distortions. Excessive redistribution in politico-economic equilibrium is therefore even more pronounced than in the main model.

4 Conclusion

We have argued that the political support for intergenerational transfers reflects the interests of all voters rather than a young median voter alone. The micropolitical foundation for that view—probabilistic voting, i.e., a non-deterministic relationship between candidates' platforms and citizens' voting behavior—is natural and has realistic implications. Introducing the probabilistic-voting assumption in the standard Diamond (1965) model preserves that model's tractability and delivers intuitive and novel results in a strikingly transparent fashion.

It is often argued in the policy discussion that cuts in social-security benefits in response to population ageing herald the dismantling of pay-as-you-go social security systems. According to the model, in contrast, the size of social security programs is positively related to the old-age dependency ratio, even if pensions per retiree as a function of the old-age dependency ratio are hump-shaped. These model predictions are consistent with the data. Therefore, we have reason to expect population ageing to go hand-in-hand both with further increases in the size of social security systems and reductions of benefits per retiree.

Normative implications of the model also accord well with frequently expressed notions in the social-security debate, according to which intergenerational transfers are unfairly high, due to a lack of political representation of future generations. While the political process in our model is "inclusive" in the sense of representing the interests of all voters, it is not sufficiently inclusive from a broader social welfare point of view that also accounts for the interests of future cohorts. In effect, political competition in the model partially resolves the conflict between old and young voters by shifting some of the cost of the social security system to future generations.

As a consequence, the social security system is too large relative to a system balancing the interests of all generations.

Since the model is very tractable, it lends itself to a variety of interesting extensions. We have discussed one, central extension with endogenous labor supply, tax distortions, and multiple policy instruments. Another extension, due to Song (2005), features intragenerational heterogeneity and analyzes the interaction between social-security transfers and wealth inequality. Other possible extensions might analyze the interaction between social security policy and various other policies or household choices. Examples of the former include public education, publicly provided intergenerational risk sharing, or immigration restrictions; examples of the latter include fertility or portfolio choices.

In ongoing work, we relax the assumption of a balanced government budget and allow for government debt, introducing political deficit and default choices. We show the politico-economic equilibrium to sustain arbitrary combinations of government debt, social-security transfers, and taxes as long as the policy mix is equivalent in terms of the allocation it supports to the transfer policies characterized in the current paper. In other words, the economic equivalence between social security and debt-plus-tax policies extends to the political sphere, and the results of the paper therefore apply under much more general conditions.

A Appendix

A.1 Relationship Between Social Planner and Ramsey Allocation

Let $n_{t+i} \equiv \rho^{t+i-1}(\nu\beta u'(c_{2,t+i}) - \rho u'(c_{1,t+i}))$ denote the net social benefit of transferring one unit of resources in period $t+i$ from young to old households. From (5), the planner's optimal policy is to set $n_{t+i} = 0, i \geq 0$.

Let I_{t+i} denote the effect on the objective function of a marginal increase in savings in period $t+i$. Under the Ramsey policy, I_{t+i} equals²⁶

$$I_{t+i}^{\mathcal{G}} = w'_{t+i+1}(\tau_{t+i+1} - 1)n_{t+i+1} + w'_{t+i+1} \frac{dS_{t+i+1}}{dw_{t+i+1}} w'_{t+i+2}(\tau_{t+i+2} - 1)n_{t+i+2} + \dots$$

Higher savings (i) raise the wage and thus transfers in the next period and (ii) reduce the interest rate. The corresponding welfare effects are (i) $\beta\rho^{t+i}\nu w'_{t+i+1}\tau_{t+i+1}u'(c_{2,t+i+1})$ on account of the old, $\rho^{t+i+1}w'_{t+i+1}(1-\tau_{t+i+1})u'(c_{1,t+i+1})$ on account of the young, and (ii) $\beta\rho^{t+i}s_{t+i}R'_{t+i+1}u'(c_{2,t+i+1})$ on account of the old.²⁷ The initial increase in savings is also propagated over time through higher wages and savings, and thus causes parallel welfare effects in subsequent periods. Using the constant-returns-to-scale property $R'(s)s + w'(s)\nu = 0$, the expression for $I_{t+i}^{\mathcal{G}}$ results.

With $I_{t+i}^{\mathcal{G}}$ thus representing the shadow value of a marginal increase in savings, the Ramsey policy satisfies the following first-order condition with respect to $\tau_{t+i}, i \geq 0$:

$$w_{t+i}n_{t+i} + \frac{dS_{t+i}}{d\tau_{t+i}} I_{t+i}^{\mathcal{G}} + \frac{\partial S_{t+i-1}}{\partial \tau_{t+i}} w'_{t+i} \left[(\tau_{t+i} - 1)n_{t+i} + \frac{dS_{t+i}}{dw_{t+i}} I_{t+i}^{\mathcal{G}} \right] + \lambda_{t+i}^{\mathcal{G}} = 0, \quad (11)$$

$$\lambda_{t+i}^{\mathcal{G}} \tau_{t+i} = 0,$$

where $\lambda_{t+i}^{\mathcal{G}}$ denotes the non-negative multiplier on the constraint that τ_{t+i} be non-negative.²⁸ The first term on the left-hand side represents the direct welfare gain and loss for old and young households, respectively, due to higher transfers. The second term represents the welfare effects caused by the adjustment in savings resulting from higher taxes (and thus, lower disposable income). The third term represents the welfare effects caused by the adjustment in the savings in the *preceding* period, resulting from higher current taxes (and thus transfers).²⁹ Since $n_{t+i+s} = 0, s \geq 1$, implies that $I_{t+i}^{\mathcal{G}} = 0$, the *distribution* of consumption implemented by the social planner, $n_{t+i+s} = 0, s \geq 0$, satisfies the Ramsey first-order condition as long as the non-negativity constraint on tax rates does not bind.³⁰ Moreover, since the savings choices induced by the Ramsey policy conform with the social planner's investment policy, the consumption *levels* of old and young households in the social-planner allocation and an interior Ramsey equilibrium also coincide.

²⁶We use the short-hand notation S_t to denote $S(w_t(1-\tau_t); \tau_{t+1})$. w and R denote wage and return as functions of aggregate savings. A prime denotes the first derivative.

²⁷There is no direct welfare effect from induced changes in savings, since savings choices are privately optimal.

²⁸We do not impose an upper bound on the tax rate. Since $\lim_{c \rightarrow 0} u'(c) = \infty$, however, and since, in equilibrium, households cannot borrow against future benefits (the capital stock must be non-negative), tax rates will always be lower than unity in equilibrium.

²⁹The first-order condition with respect to the tax rate in the initial period, τ_t , does not feature the third term in (11), since s_{t-1} is predetermined. The implications for time-consistency are discussed below.

³⁰This would even be the case if the savings choices under the Ramsey policy conflicted with the social planner's investment policy, since the shadow value of a marginal increase in savings in an interior Ramsey equilibrium equals zero.

An interior Ramsey policy therefore implements the social-planner allocation. By implication, it is necessarily time-consistent. This can also be seen from (11): If $n_{t+i} = 0, i \geq 0$, and therefore $I_{t+i}^{\tilde{G}} = 0$, the terms in square brackets in (11) add up to zero, and the potential source of time inconsistency—the effect of a change in tax rate on savings in the preceding period—disappears.

A.2 Time-Consistency of Ramsey Policy

The first-order condition of a benevolent government without commitment differs from (11) in two respects: First, since the government takes future tax choices to be functions of the state, it only chooses the contemporaneous tax rate, τ_t . Second, since changes in savings do not only affect future wages and returns, but also future tax rates, the expression for I_t is replaced by³¹

$$\begin{aligned} I_t^{\tilde{G}} &= [w'_{t+1}(\tau_{t+1} - 1) + w_{t+1}\tau'_{t+1}] n_{t+1} + \\ &\quad [w'_{t+2}(\tau_{t+2} - 1) + w_{t+2}\tau'_{t+2}] \left(w'_{t+1} \frac{dS_{t+1}}{dw_{t+1}} + \tau'_{t+1} \frac{dS_{t+1}}{d\tau_{t+1}} \right) n_{t+2} + \dots \\ &= [w'_{t+1}(\tau_{t+1} - 1) + w_{t+1}\tau'_{t+1}] n_{t+1} + \\ &\quad [w'_{t+2}(\tau_{t+2} - 1) + w_{t+2}\tau'_{t+2}] \frac{dS_{t+1}}{d\tau_{t+1}} \left(\tau'_{t+1} + w'_{t+1} \frac{\tau'_{t+1} - 1}{w_{t+1}} \right) n_{t+2} + \dots \end{aligned}$$

The first-order condition with respect to τ_t reads

$$w_t n_t + \frac{dS_t}{d\tau_t} I_t^{\tilde{G}} + \lambda_t^{\tilde{G}} = 0, \quad \lambda_t^{\tilde{G}} \tau_t = 0, \quad (12)$$

where $\lambda_t^{\tilde{G}}$ denotes the non-negative multiplier on the constraint that τ_t be non-negative. Since $n_{t+i} = 0, i \geq 1$, implies that $I_t^{\tilde{G}} = 0$, and since the implementability constraints under the Ramsey policy and the policy without commitment are identical (conditional on tax rates), (12) confirms that an interior Ramsey policy is time-consistent.

To see that the Ramsey policy is time-consistent even if the non-negativity constraint on tax rates is binding, consider the program of the benevolent government without commitment. Combining conditions (12) in t and $t + 1$ and exploiting the relationship between $I_t^{\tilde{G}}$ and $I_{t+1}^{\tilde{G}}$ yields, after some manipulations,

$$w_t n_t + \lambda_t^{\tilde{G}} - \frac{dS_t}{d\tau_t} \lambda_{t+1}^{\tilde{G}} \left[\tau'_{t+1} + \frac{w'_{t+1}(\tau_{t+1} - 1)}{w_{t+1}} \right] = 0, \quad \lambda_t^{\tilde{G}} \tau_t = 0, \quad \lambda_{t+1}^{\tilde{G}} \tau_{t+1} = 0.$$

If the policy function is differentiable, then $\lambda_{t+1}^{\tilde{G}} \tau'_{t+1} = 0$ (since taxes are non-negative) and the third term in the left-hand equation is weakly negative. The equation can therefore be satisfied under the autarky allocation $n_t < 0$ and a binding non-negativity constraint ($\tau_t = 0, \lambda_t^{\tilde{G}} > 0$). A strictly positive τ_t , in contrast, necessarily violates the condition, because an increase in τ_t from zero renders n_t further negative, but implies that $\lambda_t^{\tilde{G}} = 0$. We conclude that the benevolent government without commitment chooses $\tau_t = 0 \forall t$ and therefore, that the Ramsey policy is time-consistent.

³¹ S_t now serves as short-hand notation for $S(w_t(1 - \tau_t); \tilde{\tau}(\cdot))$, i.e., the savings function of households that anticipate future governments to implement the tax policy $\tilde{\tau}(\cdot)$.

A.3 Lemma

Lemma 1. Under Assumption 1, and for a continuously differentiable policy function, there is at least one non-trivial steady state. Furthermore, if $\tau'(s) = 0$ at a steady state, then this steady state is stable with monotone dynamics.

Proof. Under Assumption 1, the law of motion for aggregate savings is implicitly given by (8). Differentiability of the policy function implies that s_t is a continuous function of s_{t-1} , with the slope given by

$$\frac{ds_t}{ds_{t-1}} = \frac{\beta}{1 + \beta} \frac{A\alpha}{\nu^{1-\alpha}} \frac{s_{t-1}^{-\alpha} ((1-\alpha)(1-\tau(s_{t-1})) - s_{t-1}\tau'(s_{t-1}))}{1 + \frac{\alpha}{1-\alpha} \frac{\tau(s_t)}{1+\beta} + s_t \frac{\alpha}{1-\alpha} \frac{\tau'(s_t)}{1+\beta}}. \quad (13)$$

The first-order condition for the choice of tax rate can be written as

$$\omega \frac{c_{1,t}}{c_{2,t}} = 1 + \frac{dS_t}{d\tau_t} \alpha \left(\frac{1 - \tau(s_t) - \frac{s_t}{1-\alpha} \tau'(s_t)}{w_t} \right) - \lambda_t + \mu_t = 1 - \mathcal{B}_t \frac{c_{1,t}}{\nu w_t \omega^1} - \lambda_t + \mu_t, \quad (14)$$

where \mathcal{B}_t denotes the indirect welfare effects and λ_t, μ_t denote the non-negative multipliers associated with the constraints that tax rates are non-negative and smaller than unity, respectively. From (8),

$$\frac{dS_t}{d\tau_t} = - \frac{w_t \beta (1 - \alpha)}{(1 - \alpha)(1 + \beta) + \alpha \tau(s_t) + \alpha s_t \tau'(s_t)}. \quad (15)$$

Finally, from (8) and the individual budget constraints,

$$\frac{c_{1,t}}{c_{2,t}} = \frac{1 - \tau_t}{\nu \left(\tau_t + \frac{1-\alpha}{\alpha} \right)} \left(1 - \frac{(1 - \alpha)\beta}{(1 - \alpha)(1 + \beta) + \alpha \tau(s_t)} \right). \quad (16)$$

First, we will show that there exists at least one non-trivial steady state. Continuity of $s_t(s_{t-1})$ implies that $s_{t-1} \rightarrow 0 \Rightarrow s_t \rightarrow 0$; differentiability and boundedness of the policy function imply $\lim_{s_{t-1} \rightarrow 0} s_{t-1} \tau'(s_{t-1}) = \lim_{s_t \rightarrow 0} s_t \tau'(s_t) = 0$. Since $s_{t-1}^{-\alpha}$ approaches infinity as $s_{t-1} \rightarrow 0$, $\lim_{s_{t-1} \rightarrow 0} \frac{ds_t}{ds_{t-1}}$ is larger than unity iff $\lim_{s_{t-1} \rightarrow 0} \tau(s_{t-1}) = \tau(0) < 1$ (see (13)). This condition is necessarily satisfied because the alternative possibility, $\tau(0) = 1$, leads to a contradiction. (On the one hand, if $\tau(0) = 1$, then, from (14), $\lim_{s_{t-1} \rightarrow 0} \omega \frac{c_{1,t}}{c_{2,t}} \geq 1$. On the other hand, from (16), $\lim_{s_{t-1} \rightarrow 0} \frac{c_{1,t}}{c_{2,t}} = 0$.) From (8), $\lim_{s_{t-1} \rightarrow \infty} \frac{s_t}{s_{t-1}} = 0$. It follows that there exists at least one non-trivial steady state.

Evaluating (13) at a steady state, we find

$$\left. \frac{ds_t}{ds_{t-1}} \right|_{\text{steady state}} = \frac{\beta}{1 + \beta} \frac{A\alpha}{\nu^{1-\alpha}} \frac{(1 - \alpha) \nu^{1-\alpha} \frac{1+\beta}{A\alpha\beta} \left(1 + \frac{\alpha}{1-\alpha} \frac{\tau(s)}{1+\beta} \right) - s^{1-\alpha} \tau'(s)}{1 + \frac{\alpha}{1-\alpha} \frac{\tau(s)}{1+\beta} + s \frac{\alpha}{1-\alpha} \frac{\tau'(s)}{1+\beta}}. \quad (17)$$

If the policy function is flat at the steady state, $\tau'(s) = 0$, then $0 < \left. \frac{ds_t}{ds_{t-1}} \right|_{\text{steady state}} = 1 - \alpha < 1$, implying stability of the steady state and monotone dynamics. \square

A.4 Elastic Labor Supply and Tax Distortions

For convenience, we treat T_t and the sum of the two tax rates, $\sigma_t \equiv \tau_t + \theta_t$, rather than τ_t and θ_t as the independent political choice variables. Old age benefits and the composition of tax

rates are implicitly defined. Otherwise, the objective function pursued by the political parties, the implementability constraints, and the conditions for a rational expectations equilibrium are identical to those in the text.

Consider the effect of a marginal increase in T_t , which consists of three parts:

- i. The direct welfare effect due to lower transfers from young to old households,

$$-\nu(\omega u'(c_{2,t}) - u'(c_{1,t})).$$

- ii. The welfare effect on young voters due to lower aggregate savings. This effect parallels the general equilibrium and policy effects in the main model and equals³²

$$\frac{\partial S_t}{\partial T_t} \nu \beta u'(c_{2,t+1}) \left\{ s_t R'(s_t) + \frac{\nu d[w(s_t)(1 - \tilde{X}(s_t))\sigma(s_t) - T(s_t)]}{ds_t} \right\}.$$

- iii. The welfare effects on young and old voters due to changes in the labor supply. These effects, which did not arise in the main model, equal

$$\frac{\partial X_t}{\partial T_t} \nu \left\{ -\omega u'(c_{2,t}) w_t \sigma_t - \frac{\partial w(s_{t-1}, x_t)}{\partial x_t} (1 - x_t)(1 - \sigma_t)(\omega u'(c_{2,t}) - u'(c_{1,t})) \right\},$$

where we use the household's intratemporal optimality condition as well as the constant returns to scale property. The terms in curly brackets represent the loss for old households from lower social-security benefits (due to lower labor supply), and the general equilibrium welfare effects on young and old households. Higher wages (due to lower labor supply) benefit young households and also old households (due to the effect on pensions) but, at the same time, old households suffer from lower returns on their savings. Due to constant returns to scale, this latter effect is proportional to the change in wages.³³

(The effects of a marginal increase in σ_t closely resemble the terms above. The term in i. is multiplied by $-w_t(1 - x_t)$, and in the expressions in ii. and iii., the derivatives with respect to T_t are replaced by the derivatives with respect to σ_t .)

The second of the above three effects is negative if conditions parallel to those relevant for Proposition 2 are satisfied. In particular, the effect is negative if aggregate savings increase in the lump-sum subsidy T_t , and if depressing aggregate savings is beneficial for the young. The first of the three effects is negative if $\omega u'(c_{2,t}) - u'(c_{1,t})$ is positive, i.e., if old households consume relatively little. Finally, the third effect is negative if $\omega u'(c_{2,t}) - u'(c_{1,t})$ is positive and leisure is a normal good on the aggregate level, i.e., if an increase in T_t reduces young households' labor supply. In sum, this suggests that starting from an autarky allocation satisfying $\omega u'(c_{2,t}) > u'(c_{1,t})$ (a sufficient condition for which is, $c_{2,t} \leq c_{1,t}$ in autarky and $\omega \geq 1$), the political process does *not* introduce transfers to the young but *does* sustain transfers to the old if $dX/d\sigma \leq 0$ and $dS/d\sigma \leq 0$. Even if the young might prefer the distorting labor income tax to fund a lump-sum transfer to themselves, the vote maximizing policy is to fund social security. If, due to social security payments to the old, the politico-economic equilibrium features relatively high old-age consumption ($\omega u'(c_{2,t}) < u'(c_{1,t})$), then optimal T_t might differ from zero, as found in the text under Assumption 2.

³² S_t and X_t denote $S(s_{t-1}, \sigma_t, T_t; \sigma(\cdot), T(\cdot))$ and $X(s_{t-1}, \sigma_t, T_t; \sigma(\cdot), T(\cdot))$, respectively.

³³The welfare loss for young households from lower consumption (due to lower labor supply) and the welfare gain from higher leisure consumption exactly offset each other.

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