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Gerhard Glomm, Juergen Jung,
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CESifo
Poschingerstr. 5, 81679 Munich, Germany
Phone: +49 (89) 9224-1410 - Fax: +49 (89) 9224-1409
E-mail: office@CESifo.de
Internet: http://www.cesifo.de

Public Pensions and Capital Accumulation: The Case of Brazil

Gerhard Glomm, Juergen Jung, Changmin Lee and Chung Tran*

Indiana University - Bloomington

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Abstract

We investigate the effects of indexed public pension payments in an OLG setup with two distinct types of agents, private sector employees and public sector employees. The two types of agents have distinct wage incomes, distinct savings rates and also distinct tax rates depending on the type of employment. We analyse the effects of varying the generosity in public pension payments on capital accumulation. We find sizeable effects on steady state income which mostly come from alternative uses of public funds.

1 Introduction

Brazil runs two very different pension systems for the public and the private sector. The public sector system is very generous. "Integrability" ensures that pension payments are 100% of the highest income of a public sector employee. "Parity" makes sure that these pension payments are indexed to wages paid to current public sector employees. Overall, the public sector pension system accounts for 50% of all retirement payments, whereas public sector retirees only account for 5% of all retirees in Brazil. The average contribution rate of public sector employees towards their pension fund is 11%.

In the private sector the contribution rates are much higher, roughly 27% (7.6% employees contribution and 20% employer contribution) in the manufacturing and service sector. In the agricultural (rural) sector contribution rates are somewhat lower and range around 16%. The average pension paid to private sector retirees amounts from 70% to 80% of their (average?) wage income.[1] According to Souza et al. (2004) the deficit of the

*Corresponding Author: Gerhard Glomm, Department of Economics, Wylie Hall, Indiana University, Bloomington, IN, 47405, phone: (812) 855-7256, e-mail: gglomm@indiana.edu

[1] See Bonturi (2002) for more detailed information about the Brazilian pension system.

pension system amounts to roughly 4.5% of GDP, 3.5% is caused by the public sector, the remaining 1% comes from the private sector.

The generosity of the public sector pension system has led to concerns about its sustainability. These concerns inspired the original bill of the Constitutional Amendment 40 (Lula Reform 2003) which had two main objectives. First, it aimed at reducing the huge deficit in the civil sector pension system. Second, it aimed at making the public system more similar to the private sector system to improve equity. The changes that were actually approved fell short of the original goals and mainly affect future public servants.[2]

This paper studies the effects of public sector pension reforms on capital accumulation. We do this using an OLG framework, which is described in detail in section 2, in which the government hires workers and invests in a public capital to provide services to households and firms. These services are made available free of charge. We can think of these as being services flowing from the stock of roads and highways.

The government also finances public expenditures on education and social security payments to the private sector workers. In our model financing generous public sector pensions implies the opportunity cost of lower public expenditures on public education and/or on public capital accumulation. We focus on reduction of public sector pensions. The extra resources freed up by cutting public sector pensions can be used to (*i*) increase private sector pension, (*ii*) increase public education expenditure, or (*iii*) increase investment in the public capital stock.

Section 3 contains the definition of competitive equilibrium. In section 4 we solve the model. In section 5 we calibrate the model to Brazil and in section 6 we conduct policy experiments. In all of the policy experiments conducted we focus on steady state outcomes. We find that steady state income is completely unaffected by shifting from public to private sector pensions. This is due to homogeneity of the utility function assumed here. Decreasing generosity of public sector pensions and increasing either public education expenditures or investment in public capital has sizeable effects on steady state income. We find that the direct effects of public pension reform through influencing savings are small. However, using the resources that become available through reduction in public pensions on public investment in infrastructure or on public education has large effects on steady state income. We conduct extensive sensitivity analysis for these policy reform experiments. Section 7 concludes.

2 The Model

There is a large number of individuals who live for two periods in an OLG set-up. Each period accounts for roughly 30 years. For reasons of simplicity we abstract from population growth and normalize the size of the population to one. A fraction N^r of the population is working in the private sector and a fraction N^u is working in the public sector. We therefore get

$$N^r + N^u = 1.$$

[2] Compare Souza et al. (2004) for further details of the pension reform in Brazil.

Each group has its own pension scheme that differs in contribution rates and also in benefit payments.

Agents value two different types of goods, a privately provided good and a publicly provided good. The utility function of a member of generation t is

$$\begin{aligned} & u\left(c_t, G_t, c_{t+1}, G_{t+1}\right) \\ = & \frac{1}{1-\sigma}\left[\left(c_t^{\rho}+\Theta G_t^{\rho}\right)^{\frac{1}{\rho}}\right]^{1-\sigma}+\beta \frac{1}{1-\sigma}\left[\left(c_{t+1}^{\rho}+\Theta G_{t+1}^{\rho}\right)^{\frac{1}{\rho}}\right]^{1-\sigma}, \end{aligned}$$

where c_i is consumption of the private good, and G is a pure public good provided by the government in the two respective periods $i = 1, 2$. We also assume $\rho \leq 1$ and $\sigma > 0$.

The privately supplied good is produced from three inputs, the publicly provided service G_t, the private capital stock K_t and effective labor (human capital) in the private sector $H_t^r = H_t N_t^r$ according to the production function

$$Y_t = A G_t^{\alpha_1} K_t^{\alpha_2} \left(H_t^r\right)^{\alpha_3},$$

where $\alpha_i \in (0,1)$ for $i = 1, 2, 3$, $\alpha_2 + \alpha_3 = 1$, and $A > 0$. We can think of the (pure) public good as roads or highways. In many countries roads are made available to both firms and households at zero price. If the services of roads are made available to firms at a zero price, firms only hire capital and labor. The condition $\alpha_2 + \alpha_3 = 1$ then ensures constant returns to scale in the two hired factors and zero profits.

Human capital is produced according to

$$h_{t+1} = B E_t^{\gamma_1} h_t^{\gamma_2}, \tag{1}$$

where E_t is public education expenditure, h_t is the parental human capital, $B > 0$, $\gamma_1, \gamma_2 \in (0,1)$, and $\gamma_1 + \gamma_2 = 1$. The government uses effective labor (human capital) $H_t^u = H_t N_t^u = H_t\left(1 - N_t^r\right)$ and public capital K_t^G to produce services (a pure public good) according to

$$G_t = Y_t^G = Z\left[\left(K_t^G\right)^{\eta} + \left(H_t^u\right)^{\eta}\right]^{1/\eta}, \tag{2}$$

where $Z > 0$ and $\eta \leq 1$. Public capital evolves according to

$$K_{t+1}^G = \left(1 - \delta_{K^G}\right) K_t^G + I_t^G. \tag{3}$$

The government collects a tax on labor income of workers in the public sector at τ_{Lt}^u and an ear marked tax τ_{Lt}^{ssu} that goes towards financing social security. Workers in the private sector pay the tax rates τ_{Lt}^r and τ_{Lt}^{ssr}. In addition, capital income is taxed at rate τ_{Kt}. The stock of debt that the government can issue in period t is B_t. In period t the government faces the following expenditures (where we will express expenditures for government program i as fixed share $\Delta_{i,t}$ of output Y_t):

1. public education expenditures

$$E_t = \Delta_{E,t} Y_t, \tag{4}$$

2. investments in public capital

$$I_t^G = \Delta_{G,t} Y_t, \tag{5}$$

3. transfer payments to the old who were employed in the private sector

$$T_t^r = \Delta_{T^r,t} Y_t, \tag{6}$$

4. wage payments of the current public sector employees $w_t^u H_t N_t^u$,

5. payments of public debt $(1 + r_t) B_t$ and

6. pensions of last period's public sector employees $\Psi w_t^u H_t N_{t-1}^u$.

Public pensions are indexed to this period's public sector wages, where $w_t^u H_t$ is an individual public employee's wage income. The total wage bill of the public sector in a given period is $w_t^u H_t N_t^u$. Since $w_t^u H_t$ is the average wage of an individual agent in a period (which is roughly 30 years long), the question arises what fraction of this current wage is paid out to retirees. In order to capture different levels of generosity of a pension system we express the amount of pensions paid to public sector retirees as

$$T_t^u = \Psi w_t^u H_t N_{t-1}^u, \tag{7}$$

where $\Psi > 0$. If $\Psi \in (0, 1)$ then pensions paid are only a fraction of the current average wage. The larger Ψ becomes the more generous the public pension system becomes. As $\Psi > 1$ the pensions paid are actually higher than current average wages.[3] In order to calculate the total amount of public pensions paid to retired public sector employees we multiply the individual wage of a current civil servant $w_t^u H_t$ by the number of public sector retirees (the public employees of the previous period) N_{t-1}. The government budget constraint can be written as

$$\begin{aligned} & (1 + r_t) B_t + \Delta_{E,t} Y_t + \Delta_{G,t} Y_t + \overbrace{\Delta_{T^r,t} Y_t}^{\text{private pension } T^r} + \overbrace{w_t^u H_t N_t^u}^{\text{public wages}} + \overbrace{\Psi w_t^u H_t N_{t-1}^u}^{\text{public pension } T^u} \\ = \; & B_{t+1} + \left(\tau_{L,t}^{ssu} + \tau_{L,t}^u\right) w_t^u H_t N_t^u + \left(\tau_{L,t}^{ssr} + \tau_{L,t}^{ssrf} + \tau_{L,t}^r\right) w_t^r H_t N_t^r + \tau_{K,t} r_t K_t. \end{aligned} \tag{8}$$

We assume that government behavior is exogenous, that is labor taxes $\tau_{Lt}^u, \tau_{L,t}^r, \tau_{L,t}^{ssu}, \tau_{L,t}^{ssr}$, a contribution rate to social security paid by the firm $\tau_{L,t}^{ssf}$, capital taxes $\tau_{K,t}$, the fraction spent on education $\Delta_{E,t}$, the fraction spent on increasing the public capital stock $\Delta_{G,t}$, the fraction that goes to retired private sector employees $\Delta_{T,t}$, the parameter of generosity of the public pension system Ψ are all exogenous.

[3] Since wages in the data are rising with age and in the model wages are constant over the entire period, we will use values of Ψ around 1.5 to capture "integrability".

3 Equilibrium

3.1 Household Problem

We can now state the household problem as

$$\max_{c_t^j, c_{t+1}^j, i_{t+1}^j} \frac{1}{1-\sigma}\left[\left(\left(c_t^j\right)^{\rho}+\Theta G_t^{\rho}\right)^{\frac{1}{\rho}}\right]^{1-\sigma}+\beta\frac{1}{1-\sigma}\left[\left(\left(c_{t+1}^j\right)^{\rho}+\Theta G_{t+1}^{\rho}\right)^{\frac{1}{\rho}}\right]^{1-\sigma} \tag{9}$$

$$s.t.$$

$$\begin{aligned} c_t^j + i_t^j &\leq \mathcal{I}_t^j \\ c_{t+1}^j &\leq R_{t+1} i_t^j + \frac{T_{t+1}^j}{N_t^j} \end{aligned} \tag{10}$$

where, $j = u$ if it is a public sector worker, $j = r$ if it is a private sector worker, $i_t = k_{t+1} + b_{t+1}$ is the agent's savings in form of physical capital or government bonds,

$$\mathcal{I}_t^j = \left(1 - \tau_{Lt}^{ssj} - \tau_{Lt}^j\right) w_t^j h_t \tag{11}$$

is after-tax wage income of agent j when young, R_{t+1} is the gross rate of return on investments, and $T_{t+1}^j$is a government transfer received when old.[4] Household j takes the level of the public good G_t as given.

3.2 Firm Problem

Public services are made available to all firms at a zero price. Hence firms choose physical capital and effective labor to

$$\max_{(H_t^r, K_t)} F\left(G_t, K_t, H_t^r\right) - \left(1 + \tau_t^{ssrf}\right) w_t^r H_t^r - r_t^k K_t.$$

The firm's FOCs are

$$r_t^k = \alpha_2 A G_t^{\alpha_1} K_t^{\alpha_2 - 1} \left(H_t^r\right)^{\alpha_3}, \tag{12}$$

$$w_t^r = \alpha_3 A G_t^{\alpha_1} K_t^{\alpha_2} \left(H_t^r\right)^{\alpha_3 - 1}. \tag{13}$$

3.3 Definition of Equilibrium

Given the government policy $\left\{\tau_{Lt}^r, \tau_{Lt}^u, \tau_{Lt}^{ssr}, \tau_{Lt}^{ssu}, \tau_{Lt}^{ssrf}, \tau_{Kt}, \Delta_{E,t}, \Delta_{K^G,t}, \Delta_{T,t}, w_t^u, N_t^u, \Psi\right\}_{t=0}^{\infty}$, a competitive equilibrium is a collection of sequences of decisions of privately employed

[4] The wage of an agent of group j is $w_t^j h_t$. We assume that human capital in the public and private sector is the same, only the fraction employed will differ, so that in the aggregate we will have $h_t = H_t$ and the fraction employed by the private sector is $H_t N_t^r$ and the fraction employed by the public sector is $H_t N_t^u$.

(individual) households $\left\{c_t^r, c_{t+1}^r, k_{t+1}^r, b_{t+1}^r, h_{t+1}^r\right\}_{t=0}^{\infty}$, sequences of decisions of publicly employed (individual) households $\left\{c_t^u, c_{t+1}^u, k_{t+1}^u, b_{t+1}^u, h_{t+1}^u\right\}_{t=0}^{\infty}$, sequences of aggregate stocks of private physical capital and private human capital $\{K_t, H_t^r\}_{t=0}^{\infty}$, sequences of aggregate stocks of public physical capital and public human capital $\left\{K_t^G, H_t^u\right\}_{t=0}^{\infty}$, sequences of factor prices $\left\{w_t^r, r_{t+1}^k, r_{t+1}^b\right\}_{t=0}^{\infty}$ such that

(*i*) the sequence $\left\{c_t^r, c_{t+1}^r, k_{t+1}^r, b_{t+1}^r, h_{t+1}^r\right\}_{t=0}^{\infty}$ solves the maximization problem of the privately employed household (9) with $j = r$ and the sequence $\left\{c_t^u, c_{t+1}^u, k_{t+1}^u, b_{t+1}^u, h_{t+1}^u\right\}_{t=0}^{\infty}$ solves the maximization problem of the publicly employed household (9) with $j = u$;

(*ii*) factor prices are determined by

$$r_{t+1}^k = \alpha_2 \frac{Y_{t+1}}{K_{t+1}}, \tag{14}$$

$$w_t^r = \frac{\alpha_3}{\left(1+\tau_t^{ssrf}\right)} \frac{Y_t}{H_t^r} = \frac{\alpha_3}{\left(1+\tau_t^{ssrf}\right)} \frac{Y_t}{\left(1-N_t^u\right) H_t}, \tag{15}$$

$$R_t = r_t^b = \left(1-\tau_t^k\right) r_t^k + 1 - \delta^k,$$

(*iii*) capital markets clear, so that aggregate capital stocks are given by

$$\begin{aligned} I_t &= i_t^r(1-N_t^u) + i_t^u N_t^u = K_{t+1} + B_{t+1}, \\ H_t &= H_t(1-N_t^u) + H_t N_t^u = H_t^r + H_t^u, \end{aligned}$$

(*iv*) commodity markets clear

$$\begin{aligned} C_{t-1}^r + C_t^r + C_{t-1}^u + C_t^u + K_{t+1} + I_t^G + E_t &= Y_t, \\ G_t &= Y_t^G, \end{aligned}$$

(*v*) government expenditures are

$$\begin{aligned} E_t &= \Delta_{E,t} Y_t, \\ I_t^G &= \Delta_{K^G,t} Y_t, \\ T_t^r &= \Delta_{T^r,t} Y_t, \text{ and} \\ T_t^u &= \Psi w_t^u H_t N_{t-1}^u; \end{aligned}$$

(*vi*) and the government budget constraint (8) holds.

4 Solving the Model

We assume that the government indexes public worker wages to private worker wages as following

$$w_t^u = \xi w_t^r. \tag{16}$$

We typically restrict ξ to be sufficient large so that we can assume that the government can directly set the fraction of the workforce N_t^u it wants to employ. Then total human capital employed by the public sector is $H_t^u = H_t N_t^u$. All other workers $(1 - N_t^u)$ will work in the private sector, that is $H_t^r = H_t N_t^r = H_t (1 - N_t^u)$. We will justify this assumption with the assumption that lifetime income from working in the public sector has to exceed lifetime income from working in the private sector, then agents would always prefer to work for the government. Appendix A contains a formal presentation of this argument.

Households can invest in two possible assets, physical capital and government issued bonds. In equilibrium both assets have to pay the same rate of return due to non-arbitrage conditions. If we denote $R_{t+1}^k = (1 - \tau_{Kt+1}) r_{t+1}^k + 1 - \delta$ as the after-tax return on capital investment and $R_{t+1}^b = \left(1 + r_{t+1}^b\right)$ as the net return on bonds, we get

$$(1 - \tau_{Kt+1}) r_{t+1}^k + 1 - \delta = 1 + r_{t+1}^b = R_{t+1}.$$

If we assume full depreciation, $\delta = 1$ this becomes

$$(1 - \tau_{Kt+1}) r_{t+1}^k = R_{t+1}. \tag{17}$$

After substituting the budget constraints into the utility function we get the following maximization problem for the households:

$$\max_{\left\{i_t^j, s_t^{hj}\right\}} \left\{ \begin{array}{c} \frac{1}{1-\sigma} \left[\left(\left(\mathcal{I}_t^j - i_t^j \right)^\rho + \Theta G_t^\rho \right)^{\frac{1}{\rho}} \right]^{1-\sigma} \\ + \beta \frac{1}{1-\sigma} \left[\left(\left(R_{t+1} i_t^j + T_{t+1}^j \right)^\rho + \Theta G_{t+1}^\rho \right)^{\frac{1}{\rho}} \right]^{1-\sigma} \end{array} \right\} \tag{18}$$

and the first order condition is

$$\begin{aligned} & \left[\left(\mathcal{I}_t^j - i_t^j \right)^\rho + \Theta G_t^\rho \right]^{\frac{1-\sigma}{\rho} - 1} \left(\mathcal{I}_t^j - i_t^j \right)^{\rho - 1} \\ = \;& \beta R_{t+1} \left[\left(R_{t+1} i_t^j + \frac{T_{t+1}^j}{N^j} \right)^\rho + \Theta G_{t+1}^\rho \right]^{\frac{1-\sigma}{\rho} - 1} \left(R_{t+1} i_t^j + \frac{T_{t+1}^j}{N^j} \right)^{\rho - 1}. \end{aligned}$$

We cannot get any closed form solution for i_t^j unless we make some more assumptions about parameters ρ and σ.

4.1 Case 1: Cobb-Douglas across periods ($\sigma = 1$) and perfect substitution within periods ($\rho = 1$)

In this case we obtain

$$R_{t+1} i_t^j + \frac{T_{t+1}^j}{N^j} + \Theta G_{t+1} = \beta R_{t+1} \left(\mathcal{I}_t^j - i_t^j + \Theta G_t \right),$$

which we solve for

$$i_t^j = \frac{\beta R_{t+1}\left(\mathcal{I}_t^j + \Theta G_t\right) - \frac{T_{t+1}^j}{N^j} - \Theta G_{t+1}}{R_{t+1}\left(1+\beta\right)}. \tag{19}$$

We can also express the decision rules for consumption as

$$\begin{aligned} c_t^j &= \frac{R_{t+1}\mathcal{I}_t^j - \beta R_{t+1}\Theta G_t + \frac{T_{t+1}^j}{N^j} + \Theta G_{t+1}}{R_{t+1}\left(1+\beta\right)} \text{ and} \\ c_{t+1}^j &= \frac{\beta R_{t+1}\left(\mathcal{I}_t^j + \Theta G_t\right) + \beta\frac{T_{t+1}^j}{N^j} - \Theta G_{t+1}}{1+\beta}. \end{aligned}$$

We impose the steady state. From (1) we get an expression for human capital which is $H = \left(BE^{\gamma_1}\right)^{\frac{1}{1-\gamma_2}}$. Then substituting government policy for education expenditure (4) into this expression we have

$$\bar{H}\left(Y\right) \equiv H = \left(B\Delta_E^{\gamma_1}\right)^{\frac{1}{1-\gamma_2}} Y^{\frac{\gamma_1}{1-\gamma_2}}. \tag{20}$$

Since at steady state $K_{t+1}^G = K_t^G = K^G$, and using (5) in the law of motion for capital (3) we have

$$K^G = \frac{\Delta_G}{\delta_{K^G}} Y. \tag{21}$$

We use (20) and (21) in the production function for the public good (2) and get

$$\bar{G}\left(Y\right) \equiv G = Z\left[\left(\frac{\Delta_G}{\delta_{K^G}}Y\right)^{\eta} + \left(N^u\left(B\Delta_E^{\gamma_1}\right)^{\frac{1}{1-\gamma_2}} Y^{\frac{\gamma_1}{1-\gamma_2}}\right)^{\eta}\right]^{1/\eta}, \tag{22}$$

which expresses the output of the public good G as a function of the output of the private good Y. The steady state interest rate can be expressed from the firm's first order condition (14) as

$$R = \frac{\left(1-\tau_K\right)\alpha_2}{K} Y. \tag{23}$$

We can now rewrite the expression for private investment is

$$i^r = \frac{\beta R\mathcal{I}^r + \Theta\bar{G}\left(Y\right)\left(\beta R - 1\right) - \frac{T^r}{N^r}}{R\left(1+\beta\right)}.$$

We substitute for $\mathcal{I}^r$ and using $N^r = \left(1-N^u\right)$ we have[5]

$$i^r = \frac{\beta R\left(1-\tau_L^{ss}-\tau_L\right)\frac{\alpha_3}{\left(1+\tau_t^{ssrf}\right)\left(1-N^u\right)}Y + \Theta\bar{G}\left(Y\right)\left(\beta R-1\right) - \frac{T^r}{\left(1-N^u\right)}}{R\left(1+\beta\right)}.$$

[5] We use the firm's first order condition (15) to express private wages as: $w_t^r H_t = \frac{\alpha_3 Y_t}{\left(1+\tau_t^{ssrf}\right)H_t^r} H_t = \frac{\alpha_3 Y_t}{\left(1+\tau_t^{ssrf}\right)H_t\left(1-N_t^u\right)} H_t = \frac{\alpha_3}{\left(1+\tau_t^{ssrf}\right)\left(1-N_t^u\right)} Y_t$.

We simplify this to

$$i^r = \Gamma_0 Y + \Gamma_1 \bar{G}(Y) - \Gamma_2 \frac{K}{Y} \bar{G}(Y) - \Gamma_3 \Delta_{T^r} K,$$

where $\Gamma_0 = \frac{\beta\left(1-\tau_L^{ssr}-\tau_L\right)\alpha_3}{(1+\beta)\left(1+\tau_t^{ssrf}\right)(1-N^u)}$, $\Gamma_1 = \frac{\Theta\beta}{1+\beta}$, $\Gamma_2 = \frac{\Theta}{(1+\beta)(1-\tau_K)\alpha_2}$ and $\Gamma_3 = \frac{1}{(1+\beta)(1-N^u)\alpha_2(1-\tau_K)}$.

Public investment becomes[6]

$$i^u = \frac{\beta R\left(1-\tau_L^{ss}-\tau_L\right)\xi \frac{\alpha_3}{\left(1+\tau_t^{ssrf}\right)(1-N^u)} Y + \Theta \bar{G}(Y)(\beta R - 1) - \frac{T^u}{N^u}}{R(1+\beta)}$$

which we simplify to

$$i^u = \Gamma_0 \xi Y + \Gamma_1 \bar{G}(Y) - \Gamma_2 \bar{G}(Y) \frac{K}{Y} - \Gamma_3 \Psi \xi \frac{\alpha_3}{\left(1+\tau_t^{ssrf}\right)} K.$$

Aggregate investment then is

$$K = N^u i^u + (1-N^u) i^r,$$

which becomes

$$K = \left[N^u \xi + (1-N^u)\right] \Gamma_0 Y + \Gamma_1 \bar{G}(Y) - \Gamma_2 \bar{G}(Y) \frac{K}{Y} - \left[\Psi \xi \frac{\alpha_3}{\left(1+\tau_t^{ssrf}\right)} N^u + (1-N^u) \Delta_{T^r}\right] \Gamma_3 K,$$

and finally we express K out of this relationship as

$$\bar{K}(Y) \equiv K = \frac{\left[N^u \xi + (1-N^u)\right] \Gamma_0 Y + \Gamma_1 \bar{G}(Y)}{1 + \Gamma_2 \frac{\bar{G}(Y)}{Y} + \left[\Psi \xi \frac{\alpha_3}{\left(1+\tau_t^{ssrf}\right)} N^u + (1-N^u) \Delta_{T^r}\right] \Gamma_3}. \tag{24}$$

Next we can use the production function and substitute $(20), (22)$, and (24) so that we have

$$Y = A\left[\bar{G}(Y)\right]^{\alpha_1}\left[\bar{K}(Y)\right]^{\alpha_2}\left[\bar{H}(Y)(1-N^u)\right]^{\alpha_3}. \tag{25}$$

The six steady state variables H, K^g, G, K, Y, R, are determined the by six equations $(20), (21), (22), (24), (25), (23)$.

[6] We use the firm's first order condition (15) to express public wages as: $w_t^u H_t = \xi w_t^r H_t = \xi \frac{\alpha_3 Y_t}{\left(1+\tau_t^{ssrf}\right) H_t^r} H_t = \frac{\xi \alpha_3 Y_t}{\left(1+\tau_t^{ssrf}\right) H_t \left(1-N_t^u\right)} H_t = \frac{\xi \alpha_3}{\left(1+\tau_t^{ssrf}\right)\left(1-N_t^u\right)} Y_t$. In addition we replace $\frac{T^u}{N^u}$ using (7) and the firm's first order condition (15) and get $\frac{T^u}{N^u} = \Psi w_t^u H_t = \Psi \xi w_t^r H_t = \Psi \xi \frac{\alpha_3 Y_t}{\left(1+\tau_t^{ssrf}\right) H_t^r} H_t = \Psi \frac{\xi \alpha_3 Y_t}{\left(1+\tau_t^{ssrf}\right) H_t \left(1-N_t^u\right)} H_t = \Psi \xi \frac{\alpha_3}{\left(1+\tau_t^{ssrf}\right)\left(1-N_t^u\right)} Y_t$.

4.2 Case 2: Cobb-Douglas within period ($\rho = 0$)

When $\rho \to 0$ then expression (9) reduces to a Cobb-Douglas form and after substituting the budget constraint this problem becomes

$$\max_{\{i_t^j\}} \left\{ \frac{1}{1-\sigma} \left[\Theta \left(\mathcal{I}_t^j - i_t^j \right)^{\theta} G_t^{1-\theta} \right]^{1-\sigma} + \beta \frac{1}{1-\sigma} \left[\Theta \left(R_{t+1} i_t^j + \frac{T_{t+1}^j}{N_t^j} \right)^{\theta} G_{t+1}^{1-\theta} \right]^{1-\sigma} \right\}.$$

We can now derive first order conditions and get

$$\left(\mathcal{I}_t^j - i_t^j \right)^{\theta(1-\sigma)-1} = \beta R_{t+1} \left(R_{t+1} i_t^j + \frac{T_{t+1}^j}{N_t^j} \right)^{\theta(1-\sigma)-1} \left(\frac{G_{t+1}}{G_t} \right)^{(1-\theta)(1-\sigma)}.$$

The optimal decision rules for savings and consumption are

$$\begin{aligned} i_t^j &= \frac{\mathcal{I}_t^j - \mathcal{C}_t \frac{T_{t+1}^j}{N_t^j}}{1 + \mathcal{C}_t R_{t+1}}, \\ c_t^j &= \frac{\mathcal{C}_t R_{t+1} \mathcal{I}_t^j + \mathcal{C}_t \frac{T_{t+1}^j}{N_t^j}}{1 + \mathcal{C}_t R_{t+1}}, \\ c_{t+1}^j &= \frac{R_{t+1} \mathcal{I}_t^j + \frac{T_{t+1}^j}{N_t^j}}{1 + \mathcal{C}_t R_{t+1}}, \end{aligned}$$

where, $\mathcal{C}_t = \left[\beta R_{t+1} \left(\frac{G_{t+1}}{G_t} \right)^{(1-\theta)(1-\sigma)} \right]^{\frac{1}{\theta(1-\sigma)-1}}$. We now impose the steady state. The expressions for private and public investment become

$$\begin{aligned} i^r &= \frac{\left(1 - \tau_L^{ssr} - \tau_L\right) \frac{\alpha_3}{\left(1+\tau_t^{ssrf}\right)(1-N^u)} Y - \mathcal{C} \frac{\Delta_{T^r}}{1-N^u} Y}{1 + \mathcal{C} R}, \\ i^u &= \frac{\left(1 - \tau_L^{ssu} - \tau_L^u\right) \xi \frac{\alpha_3}{\left(1+\tau_t^{ssrf}\right)(1-N^u)} Y - \mathcal{C} \Psi \xi \frac{\alpha_3}{\left(1+\tau_t^{ssrf}\right)(1-N^u)} Y}{1 + \mathcal{C} R}. \end{aligned}$$

We now combine private and public investment and get an expression for aggregate capital

$$\begin{aligned} K &= N^u i^u + (1 - N^u) i^r \\ &= \frac{Y}{1 + \mathcal{C} R} \left\{ \left(\frac{N^u}{1 - N^u} \right) \frac{\xi \alpha_3}{1 + \tau_t^{ssrf}} \left[\left(1 - \tau_L^{ssu} - \tau_L^u\right) - \mathcal{C} \Psi \right] + \left(1 - \tau_L^{ssr} - \tau_L^r\right) \frac{\alpha_3}{1 + \tau_t^{ssrf}} - \mathcal{C} \Delta_{T^r} \right\}. \end{aligned}$$

We next use the expression $R = \alpha_2 (1 - \tau_K) \frac{Y}{K}$ from the firm's first order condition (14) and replace the left hand side to get

$$\alpha_2 (1 - \tau_K) \frac{1}{R} = \frac{1}{1 + \mathcal{C} R} \left\{ \begin{array}{c} \frac{N^u}{1-N^u} \xi \frac{\alpha_3}{1+\tau_t^{ssrf}} \left[\left(1 - \tau_L^{ssu} - \tau_L^u\right) - \mathcal{C} \Psi \right] \\ + \left(1 - \tau_L^{ssr} - \tau_L^r\right) \frac{\alpha_3}{1+\tau_t^{ssrf}} - \mathcal{C} \Delta_{T^r} \end{array} \right\}, \qquad (26)$$

where we know that $\mathcal{C} = [\beta R]^{\frac{1}{\theta(1-\sigma)-1}}$ is a function of steady state R. Then we solve this equation for R. We can now calculate the remaining steady state values. Given R we have

$$\bar{K}(Y,R) = K = \frac{(1-\tau_K)\,\alpha_2}{R} Y. \tag{27}$$

From (1) we get an expression for human capital which is $H = (BE^{\gamma_1})^{\frac{1}{1-\gamma_2}}$. Then substituting government policy for education expenditure (4) into this expression we have

$$\bar{H}(Y) \equiv H = \left(B\Delta_E^{\gamma_1}\right)^{\frac{1}{1-\gamma_2}} Y^{\frac{\gamma_1}{1-\gamma_2}}. \tag{28}$$

Since at steady state $K_{t+1}^G = K_t^G = K^G$, and using (5) in the law of motion for capital (3) we have

$$K^G = \frac{\Delta_G}{\delta_{K^G}} Y. \tag{29}$$

We use (28) and (29) in the production function for the public good (2) and get

$$\bar{G}(Y) \equiv G = Z\left[\left(\frac{\Delta_G}{\delta_{K^G}} Y\right)^{\eta} + \left(N^u \left(B\Delta_E^{\gamma_1}\right)^{\frac{1}{1-\gamma_2}} Y^{\frac{\gamma_1}{1-\gamma_2}}\right)^{\eta}\right]^{1/\eta}, \tag{30}$$

that expresses the output of the public good G as a function of the output of the private good Y. Then the steady state output is given by

$$Y = A\left[\bar{G}(Y)\right]^{\alpha_1}\left[\bar{K}(Y,R)\right]^{\alpha_2}\left[\bar{H}(Y)(1-N^u)\right]^{\alpha_3}. \tag{31}$$

The six steady state variables H, K^g, G, K, Y, R are determined by the six equations $(26), (28), (29), (30), (27), (31)$.

4.3 Case 3: Introduction of Government Debt with Cobb-Douglas within period ($\rho = 0$) Utility Function

Introducing government bonds does not change the household's first order conditions. However, when aggregating over all households we have to include bonds as the additional asset, so that total bonds and capital is

$$\begin{aligned} K + B &= N^u i^u + (1-N^u)\, i^r \\ &= \frac{Y}{1+\mathcal{C}R}\left\{\begin{array}{c} \left(\frac{N^u}{1-N^u}\right)\frac{\xi\alpha_3}{1+\tau_t^{ssrf}}\left[(1-\tau_L^{ssu}-\tau_L^u)-\mathcal{C}\Psi\right] \\ +(1-\tau_L^{ssr}-\tau_L^r)\frac{\alpha_3}{1+\tau_t^{ssrf}} - \mathcal{C}\Delta_{T^r} \end{array}\right\}. \end{aligned}$$

We can express $K = \alpha_2(1-\tau_K)\frac{Y}{R}$ and $B = \Delta_B Y$ where Δ_B is the debt level set exogenously by the government. Making the substitutions we get

$$\Delta_B + \alpha_2(1-\tau_K)\frac{1}{R} = \frac{1}{1+\mathcal{C}R}\left\{\begin{array}{c} \frac{N^u}{1-N^u}\xi\frac{\alpha_3}{1+\tau_t^{ssrf}}\left[(1-\tau_L^{ssu}-\tau_L^u)-\mathcal{C}\Psi\right] \\ +(1-\tau_L^{ssr}-\tau_L^r)\frac{\alpha_3}{1+\tau_t^{ssrf}} - \mathcal{C}\Delta_{T^r} \end{array}\right\}.$$

where we know that $\mathcal{C} = [\beta R]^{\frac{1}{\theta(1-\sigma)-1}}$ is a function of steady state R. Simplifying and assuming that $\tau_L^u = \tau_L^r$ we get

$$\Delta_B + \alpha_2 (1 - \tau_K) \frac{1}{R} = \frac{1}{1 + \mathcal{C}R} \left\{ \begin{array}{c} \frac{\alpha_3}{1+\tau_t^{ssrf}} \left(1 - \tau_L^{ssu} - \tau_L^u\right) \left(1 + \frac{N^u}{1-N^u}\xi\right) \\ - \left(\frac{N^u}{1-N^u}\xi \frac{\alpha_3}{1+\tau_t^{ssrf}} \Psi + \Delta_{T^r}\right) \mathcal{C} \end{array} \right\}. \tag{32}$$

Next we use the government budget constraint in the steady state

$$\begin{array}{c} \frac{\alpha_3}{1+\tau^{ssrf}} \tau_L \left(\xi \frac{N^u}{1-N^u} + 1\right) + \tau_K \alpha_2 \\ = (R-1)\, \Delta_B + [\Delta_E + \Delta_G] + \xi \frac{\alpha_3}{1+\tau^{ssrf}} \frac{N^u}{1-N^u} + \Delta_{T^r} \\ + \Psi \xi \frac{\alpha_3}{1+\tau^{ssrf}} \frac{N^u}{1-N^u} - \frac{\alpha_3}{1+\tau^{ssrf}} \left(\xi \frac{N_t^u}{1-N^u} \tau_L^{ssu} + \tau_L^{ssr} + \tau_L^{ssf}\right). \end{array} \tag{33}$$

These two equations determine steady state R and one endogenized government choice variable. We can now calculate the remaining six steady state variables H, K^g, G, K, Y using expressions $(28), (29), (30), (27), (31)$.

5 Data and Calibration

In this section we calibrate the model to the economy of Brazil. The government budget constraint becomes

$$\begin{aligned} &\overbrace{\Delta_{B,t} Y_t}^{B_{t+1} \text{ new debt}} + \overbrace{\left[\frac{\alpha_3}{1+\tau_t^{ssrf}} \left(\tau_{L,t}^u \xi \frac{N_t^u}{1-N_t^u} + \tau_{L,t}^r\right) + \tau_{K,t}\alpha_2\right] Y_t}^{\text{tax revenue excl. ear market social sec. contribution rates (27\%)}} \\ &= \overbrace{R_t \Delta_{B,t-1} Y_{t-1}}^{R_t B_t \text{ debt service}} + \overbrace{[\Delta_{E,t} + \Delta_{G,t}]}^{\text{Education + Investments (5\%)}} Y_t + \overbrace{\xi \frac{\alpha_3}{1+\tau_t^{ssrf}} \frac{N_t^u}{1-N_t^u}}^{\text{public wages (10\%)}} Y_t \\ &+ \overbrace{\left[\overbrace{\Delta_{T^r,t}}^{\text{private pension } T^r} + \overbrace{\Psi \xi \frac{\alpha_3}{1+\tau_t^{ssrf}} \frac{N_t^u}{1-N_t^u}}^{\text{public pension } T^u} - \overbrace{\frac{\alpha_3}{1+\tau_t^{ssrf}} \left(\xi \frac{N_t^u}{1-N_t^u} \tau_{L,t}^{ssu} + \tau_{L,t}^{ssr} + \tau_{L,t}^{ssf}\right)}^{\text{contribution rate to pension}}\right]}^{\text{net pension balance (+5.5\%)}} Y_t. \end{aligned}$$

Next, in table 1, we list the specific public policy parameters we use for the calibration exercise. The top panel in table 1 contains data on government expenditures, the second panel contains data on tax rates, while the third panel contains data on the relative size of the public and private labor force.

In table 2 we show the preference and technology parameters we use. The preference parameters are perhaps non controversial. Note that for the parameters for the consumption goods technology we are imposing constant returns to scale in the two private factors. Note also that capital's share of 0.5 is large relative to the estimates reported in Gollin (2002), but this relatively large parameter value is consistent with estimates in Elias (1992) and with values used by Barro and Sala-i-Martin (2004).

The value for the elasticity of output with respect to infrastructure capital, α_1 lies between estimates by Holtz-Eakin (1994) and Ai and Cassou (1995).

For the parameter η in the government technology we use a value of 0.5 as a benchmark, but we will use other parameter values in our sensitivity analysis. We use a value of 0.1 for the learning elasticity with respect to public expenditure. This is consistent with an estimate by Card and Krueger (1992) and a value used by Rangazas (2000). The productivity parameters A, B, Z are chosen so that for the benchmark, output is equal to 100.

6 Policy Experiments

The fist set of results reported here are for case 1 from section 4.1, where the utility function is Cobb-Douglas across periods and public and private goods are perfect substitutes. We assume that bond holdings are equal zero. For the following policy experiments we set $\tau^u_{L,t} = \tau^r_{L,t}$, so that the government budget constraint reduces to

$$\begin{aligned} & \tfrac{\alpha_3}{1+\tau_t^{ssrf}} \tau_{L,t} \left(\xi \tfrac{N_t^u}{1-N_t^u} + 1 \right) + \tau_{K,t} \alpha_2 \\ = & [\Delta_{E,t} + \Delta_{G,t}] + \xi \tfrac{\alpha_3}{1+\tau_t^{ssrf}} \tfrac{N_t^u}{1-N_t^u} + \Delta_{T^r,t} + \Psi \xi \tfrac{\alpha_3}{1+\tau_t^{ssrf}} \tfrac{N_t^u}{1-N_t^u} - \tfrac{\alpha_3}{1+\tau_t^{ssrf}} \left(\xi \tfrac{N_t^u}{1-N_t^u} \tau_{L,t}^{ssu} + \tau_{L,t}^{ssr} + \tau_{L,t}^{ssf} \right). \end{aligned}$$

6.1 Public Pensions (Ψ) vs. Private Pensions (Δ_{T^r})

In this policy experiment we use the extra revenue from making public sector pensions less generous to make private sector social security payments more generous. We do this in such a way that government's share of GDP remains constant. The results from this experiment are illustrated in figure 1.

The effect of shifting public funds from public pensions to private pensions on steady state income is nil. This result is not that surprising since this policy is just a reshuffling of expenditures in the government budget constraint and public and private sector have the same propensity to save.

The dashed line indicates the direct effect of making public sector pensions less generous without using the extra funds on the private pensions. This direct effect of reducing generosity of public pensions on output, total saving and savings by sector is positive.

First, public sector workers have a bigger incentive to save as their pensions are reduced. The increase in savings by public sector employees causes the interest rate to decrease which causes private sector savings to fall and aggregate output to increase which, through the wage rate, causes private sector savings to rise. The second effect dominates the first effect.

6.2 Public Pensions (Φ) vs. Education (Δ_E)

In the second experiment we use the extra government revenue from making public pensions less generous to finance extra public education expenditures. According to figure 2, this policy reform raises steady state income. The intuition is clear: Decreasing Ψ increases public sector savings, which in turn increases steady state capital and output. This direct effect indicated by the dashed line is small.

Using the extra revenue to fund higher education increases the steady state level of human capital, hence the rate of return on saving, the capital stock and steady state GDP. This effect is large. Reducing Ψ to 1.3 increases steady state GDP by more than 5%.

Of course the size of these effects depends upon the technology parameters, especially on the size of γ_1, the elasticity of learning output with respect to public expenditures. We summarize the results of this sensitivity analysis in table 3 where we allow γ_1 to vary from 0.05 to 0.15. The effects on steady state income from reducing Ψ from $\Psi = 1.5$ to 1.3 vary from 3% to 10%.

In table 4 we illustrate how shifting public funds from public sector pensions to education depends upon η, the (inverse of the) elasticity of substitution in the public production function. These results can be potentially interesting since changing public education funding changes effective labor in public production. We see from table 4 that out results are relatively robust to sizeable changes in η.

6.3 Public Pensions (Ψ) vs. Public Investment (Δ_{K^G})

In the third experiment the extra revenue from cutting public sector pensions is used to invest in public sector capital. The results are illustrated in figure 3. These results and their intuition are similar to those from the previous experiment.

In table 5 we show how sensitive the results are with respect to changes in α_1, the elasticity of output with respect to public capital. We allow α_1 to vary from 0.05 to 0.15. For this range of parameter values reducing Ψ from 1.5 to 1.3 increases steady state output by between 6 or 21%.

In table 6 we again compare how shifting public funds from public sector pensions into public sector capital depending on η. The effects on steady state income of using the extra revenue from public sector pensions for investment in infrastructure are quite sensitive to changes in the elasticity of substitution parameter η.

6.4 Comparison of Policies

The question arises whether the extra revenue from decreasing public sector pensions is more beneficially allocated to public capital investment or to public education. The answer to this question naturally depends upon the productivity parameters γ_1 and η.

Tables 7 and 8 show the relative steady state output effect from using the extra revenue for education rather than infrastructure. If $\gamma_1 = 0.15$ and $\eta = 0.5$, for example, then cutting Ψ from 1.5 to 1.3 and allocating the surplus to public education increases steady state output 6.04% less than allocating the surplus to public investment.

It is interesting that basically for the whole range of $\gamma_1 \in [0.05, 0.15]$ investment in public capital dominates investment in education. The same holds true for the range of $\alpha_1 \in [0.05, 0.15]$ in table 8.

Once we change the underlying elasticity parameter η to -0.25 we get a range for γ_1 and α_1 where investment in education will dominate investment in public capital (compare tables 9 and 10). This is due to the fact that a negative η changes the relation of public sector investments and investments in education from being substitutes to becoming complements. Increases in γ_1 will now not only increase output through the direct channel of increases in human capital (via increased productivity of educational expenses) but also through the indirect channel of increases in output of the public good. The complementarity enhances the effectiveness of public education versus investments into the public capital.

6.4.1 Case 2

Finally, we perform the same set of three policy reforms for the case analyzed in section 4.2, where the utility function within periods is Cobb-Douglas. As before shifting public pension funds from public sector pensions to private sector pensions has no effect on steady state income (see figure 4).

In figure 5 we show how using the extra funds from lower public pension generosity for public education influences steady state output. As is evident from figure 5, reducing generosity Ψ from 1.5 to 1.3 increases output by 5%. The effect here is roughly the same magnitude as in the figure 2.

The results in figure 6 are analogous in nature and in magnitude to those in figure 3.

6.4.2 Case 3

When we allow the government to issue bonds, the government budget constraint becomes

$$\begin{aligned} &\Delta_{B,t} + \tfrac{\alpha_3}{1+\tau_t^{ssrf}}\tau_{L,t}\left(\xi\tfrac{N_t^u}{1-N_t^u}+1\right)+\tau_{K,t}\alpha_2 \\ &= R_t\Delta_{B,t-1}\tfrac{Y_{t-1}}{Y_t} + [\Delta_{E,t}+\Delta_{G,t}] + \xi\tfrac{\alpha_3}{1+\tau_t^{ssrf}}\tfrac{N_t^u}{1-N_t^u} + \Delta_{T^r,t} \\ &+\Psi\xi\tfrac{\alpha_3}{1+\tau_t^{ssrf}}\tfrac{N_t^u}{1-N_t^u} - \tfrac{\alpha_3}{1+\tau_t^{ssrf}}\left(\xi\tfrac{N_t^u}{1-N_t^u}\tau_{L,t}^{ssu}+\tau_{L,t}^{ssr}+\tau_{L,t}^{ssf}\right). \end{aligned}$$

We impose steady state and get

$$\begin{aligned} &\tfrac{\alpha_3}{1+\tau^{ssrf}}\tau_L\left(\xi\tfrac{N_t^u}{1-N_t^u}+1\right)+\tau_K\alpha_2 \\ &= (R-1)\,\Delta_B + [\Delta_E+\Delta_G] + \xi\tfrac{\alpha_3}{1+\tau^{ssrf}}\tfrac{N^u}{1-N^u} + \Delta_{T^r} \\ &+\Psi\xi\tfrac{\alpha_3}{1+\tau^{ssrf}}\tfrac{N^u}{1-N^u} - \tfrac{\alpha_3}{1+\tau^{ssrf}}\left(\xi\tfrac{N^u}{1-N^u}\tau_L^{ssu}+\tau_L^{ssr}+\tau_L^{ssf}\right). \end{aligned}$$

We can interpret $(R-1)\,\Delta_B$ as the interest level on outstanding debt (government bonds) that the government has to service in the steady state. We perform the following policy experiment: Decrease the generosity of the public sector pensions, i.e. Ψ

goes down. At the same time we let Δ_B adjust to clear the government budget constraint holding all other government choice variables fixed. Then we see that Ψ and Δ_B are negatively related.

When government cuts public pensions its revenue goes up. Government therefore has more funds available to service the interest payments of a higher debt level. With lower payments into public pensions, the government can sustain a higher debt level in the steady state.

On the other hand if government would increase the generosity of public pensions, its steady state revenue goes down. Therefore, government cannot afford high interest payments on outstanding debt and it therefore has to reduce the amount of debt (compare figure 7).

7 Conclusion

In this paper we have used an overlapping generations model to assess the effects of public pension reform on capital accumulation. We have calibrated the model to Brazil. We found (*i*) The direct effects of pension reform through savings of public sector employees are small. (*ii*) Shifting government funds from public to private sector pensions leaves steady state GDP unaffected. (*iii*) The indirect effects of reduction of public pensions by freeing resources for public education or investment in public capital are large.

In this paper we have concentrated on one particular channel of how public sector pension reform might influence capital accumulation. Other channels might be: (*i*) The generosity of public sector pensions influences workers' retirement decisions, which in turn has an effect on GDP. (*ii*) The generosity of public sector pensions relative to pensions in the private sector will influence how workers will be allocated across both sectors, which in turn will influence GDP. This would require the introduction of heterogenous agents who make idiosyncratic investment choices into their human capital. This extended framework would allow us to investigate changes in the quality of the public sector labor force, given a specific worker compensation package (wages plus pension plan).

In our model the publicly produced service was made available to all firms and households at a zero price. While this might be a useful assumption for the provision of infrastructure like roads and highways, it clearly does not cover all relevant cases. When governments produce goods like telecommunication services or electricity, they typically charge for these goods/services. Prices charged need not bear any particular relationship to marginal or average costs. This will impact the government budget constraint.

Finally, we restricted our analysis on steady state equilibria. A deeper analysis of policy reform will require emphasis on transition paths from one policy regime to another. We leave these issues for future research.

8 Appendix

If lifetime income (wages plus other compensation packages) from working in the public sector exceeds lifetime income of the private sector, then agents would always prefer to work for the government. The government then just sets N^u directly. From the agent's budget constraint we can derive her lifetime budget constraint as

$$R_{t+1}c_t^j + c_{t+1}^j = R_{t+1}\mathcal{I}_t^j + \frac{T_{t+1}^j}{N_t^j}.$$

The agent prefers (or is indifferent) to work in the public sector if her life-time income from public sector employment is larger (larger equal) than private sector life-time income, that is

$$R_{t+1}\mathcal{I}_t^u + T_{t+1}^u \geq R_{t+1}\mathcal{I}_t^r + \frac{T_{t+1}^r}{N^{r_t}},$$

or reformulated this becomes

$$R_{t+1}(\mathcal{I}_t^u - \mathcal{I}_t^r) \geq \frac{T_{t+1}^r}{N_t^r} - \frac{T_{t+1}^u}{N_t^u}.$$

Once we plug in the expression for income as well as transfers we get

$$R_{t+1}w_t^r H_t[(1-\tau_{Lt}^{ssu} - \tau_{Lt}^u)\xi - (1-\tau_{Lt}^{ssr} - \tau_{Lt}^r)] \geq \frac{\Delta_{T^r,t+1}Y_{t+1}}{N_t^r} - \Psi w_{t+1}^u H_{t+1}.$$

Using the first order conditions of the firms we have

$$R_{t+1}\overbrace{\frac{\alpha_3 Y_t}{\left(1+\tau_t^{ssrf}\right)(1-N_t^u)}}^{w_t^r H_t}[(1-\tau_{Lt}^{ssu} - \tau_{Lt}^u)\xi - (1-\tau_{Lt}^{ssr} - \tau_{Lt}^r)] \tag{34}$$

$$\geq \left(\frac{\Delta_{T^r,t+1}}{1-N_t^u} - \frac{\Psi\xi\alpha_3}{\left(1+\tau_{t+1}^{ssrf}\right)(1-N_{t+1}^u)}\right)Y_{t+1},$$

which is a "public sector participation" condition. As long as this weak inequality holds, agents are willing to work for the government. In the steady state this is

$$R\frac{\alpha_3}{(1+\tau^{ssrf})}\left[(1-\tau_L^{ssu} - \tau_L^u)\xi - (1-\tau_L^{ssr} - \tau_L^r)\right] - \left(\Delta_{T^r} - \frac{\Psi\xi\alpha_3}{(1+\tau^{ssrf})}\right) \geq 0.$$

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Table 1: Government Policy Parameters

	Variables	
Δ_E	Public education (in % of GDP)[7]	4%
Δ_G	Investment in public good (in % of GDP)[8]	1%
Δ_{T^r}	Transfers to old in private sector (in % of GDP)[9]	6.6%
$w_t^u H_t N_t^u$	wages to current public sector employees (in % of GDP)[10]	3.5%
$\Psi w_t^u H_t N_{t-1}^u$	pension payments to public sector retirees (in % of GDP)[11]	5%
ξ	public wages as a fraction of private wages	1[12]
Ψ	indexation parameter (generosity of public pensions)[13]	1.5
τ_L^{ssu}	social security contribution rate of public sector employees[14]	11%
τ_L^{ssr}	social security contribution rate of private sector employees	11%
τ_L^{ssrf}	social security contribution rate of private sector employers[15]	10%
τ_K	capital tax rate[16]	8.29%
τ_L^r	labor tax rate private sector, net of social security[17]	8.29%
τ_L^u	labor tax rate public sector, net of social security	8.29%
N^u	fraction of public sector employees[18]	6%
N^r	fraction of private sector employees	94%

[7] See *Brazil: Selected Issues and Statistical Appendix* (2001) table 4.1 on page 78.

[8] See Calderon, Easterly and Serven (2003) table 4.1 on page 78.

[9] Source: Ministerio de Previdencia e Assistencia Social of Brazil.

[10] We found a number of ways to calculate this number which are all higher than the one we chose. Calculated from 11% of wage income equals 1.7% (of GDP) contribution rate to social security. Then 100% of wages is $\frac{1.7\%*100\%}{11\%} = 15.4\%$ of GDP. An alternative source (Ministerio de Previdencia e Assistencia Social) states a contribution rate to social security of public sector workers of 0.5%, which would result in $\frac{0.5\%*100\%}{11\%} = 4.5\%$ of GDP. From IMF data we get a rough estimate of 11%.

[11] Compare (Souza et al., 2004, p. 3) who report 4.7%.

[12] We consider this a useful benchmark.

[13] Due to *integrality* the value of public pension benefits is equal to the last and highest wage received (compare (Souza et al., 2004, p. 2)). Since one period covers roughly 20 to 30 years in this model and $w_t^u H_t N_{t-1}^u$ is the mean income over these 30 years, we use $\Psi = 1.5$ to account for the high earnings in the last years relative to average earnings overa all years.

[14] Compare (Souza et al., 2004, p. 5).

[15] Compare (Souza et al., 2004, p. 5) who report 20%.

[16] Adjusted endogenously to clear government budget constraint.

[17] Adjusted endogenously to clear government budget constraint. From household income we have $\mathcal{I} = (1 - \tau_L^r - \tau_L^{ssr})\, w^r h^r$. Replacing w^r with the firm's factor condition we have $\mathcal{I} = (1 - \tau_L^r - \tau_L^{ssr})\, \frac{\alpha_3}{1+\tau^{ssrf}} Y$ or simpler $\mathcal{I} = (1 - \tau_{Labor}^r)\, \alpha_3 Y$ where $\tau_{Labor}^r = \frac{1-\tau_L^r-\tau_L^{ssr}}{1+\tau^{ssrf}}$ is the effective tax rate on private sector labor income and τ_L^r, τ_L^{ssr} and τ^{ssrf} are the policy variables.

[18] Calculated as $\frac{5.2Mio.}{84.7Mio.} = 6\%$. Source: Social Security Ministry of Brazil.

Parameters	
Preference Parameters	$\sigma = 1.5$
	$\Theta = 0.05$
	$\theta = 0.5$
	$\beta = 0.995$
Technology Parameters:	
Consumption Goods:	$A = 24.7256$
	$\alpha_1 = 0.1$
	$\alpha_2 = 0.5$
	$\alpha_1 = 0.4$ (0.5)
Government Production	
	$Z = 1$
	$\eta = 0.5$
Education Production	
	$B = 1$
	$\gamma_1 = 0.1$
	$\gamma_2 = 0.5$

Table 2: Preference and Technology Parameters

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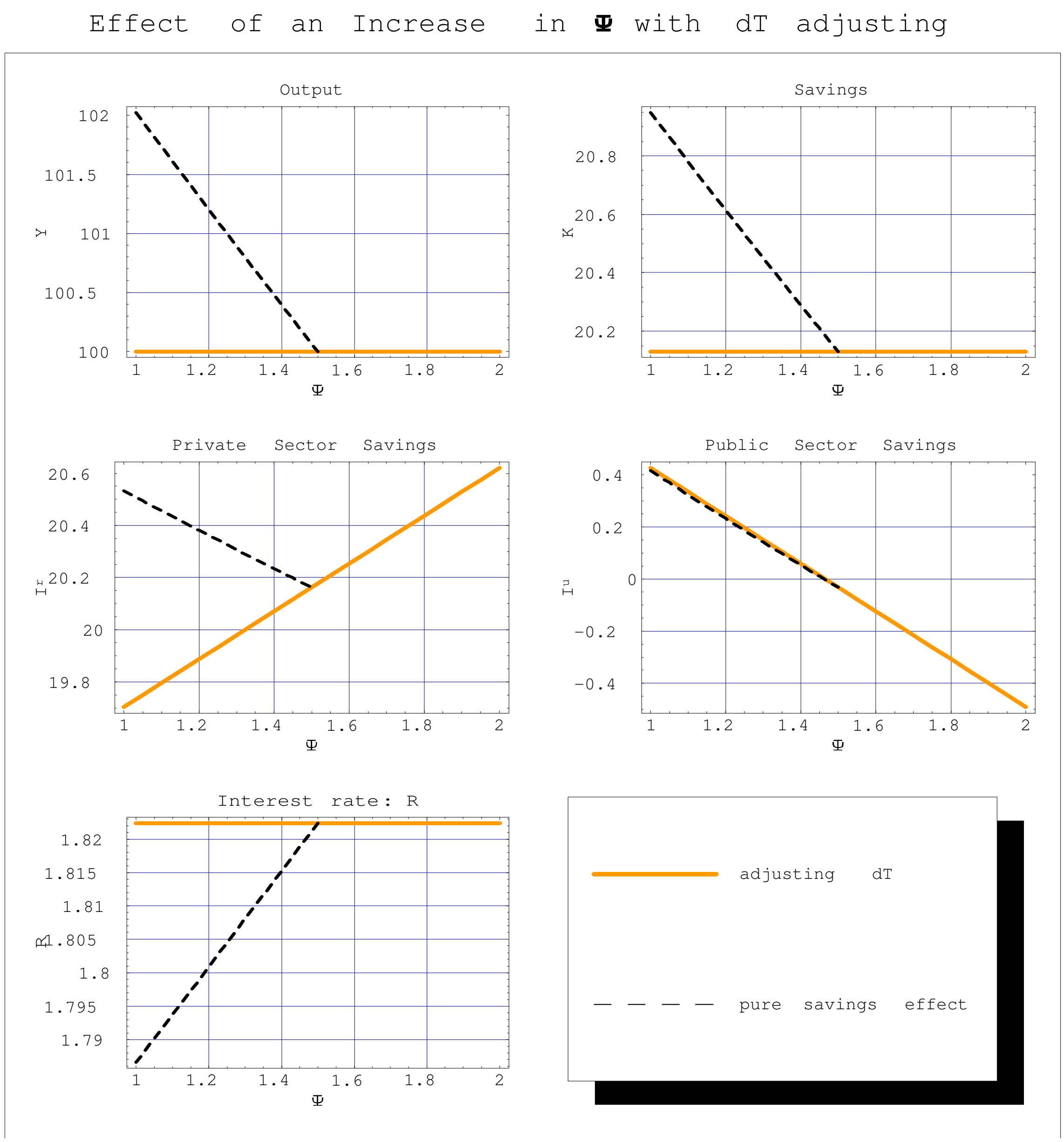
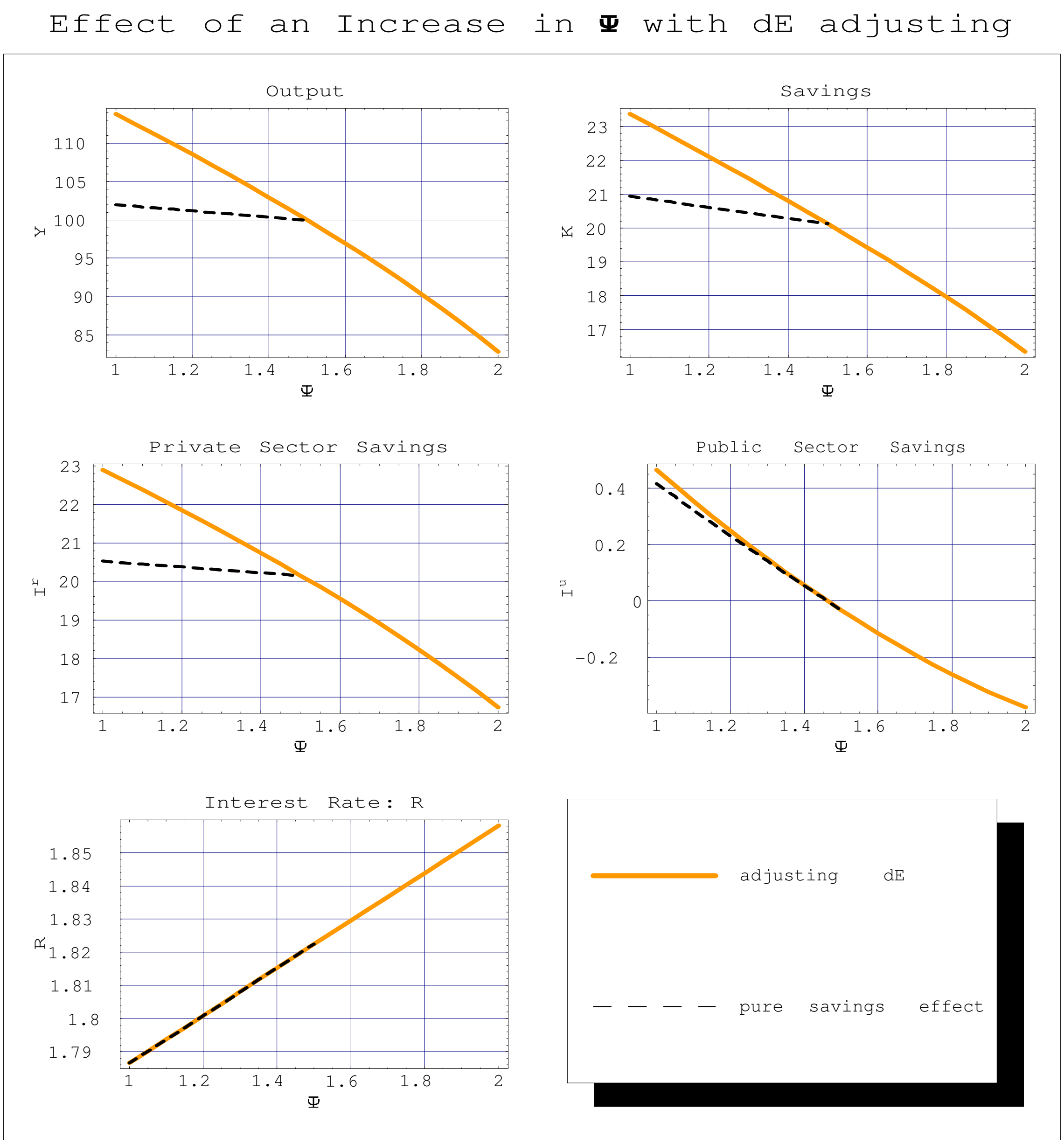
.



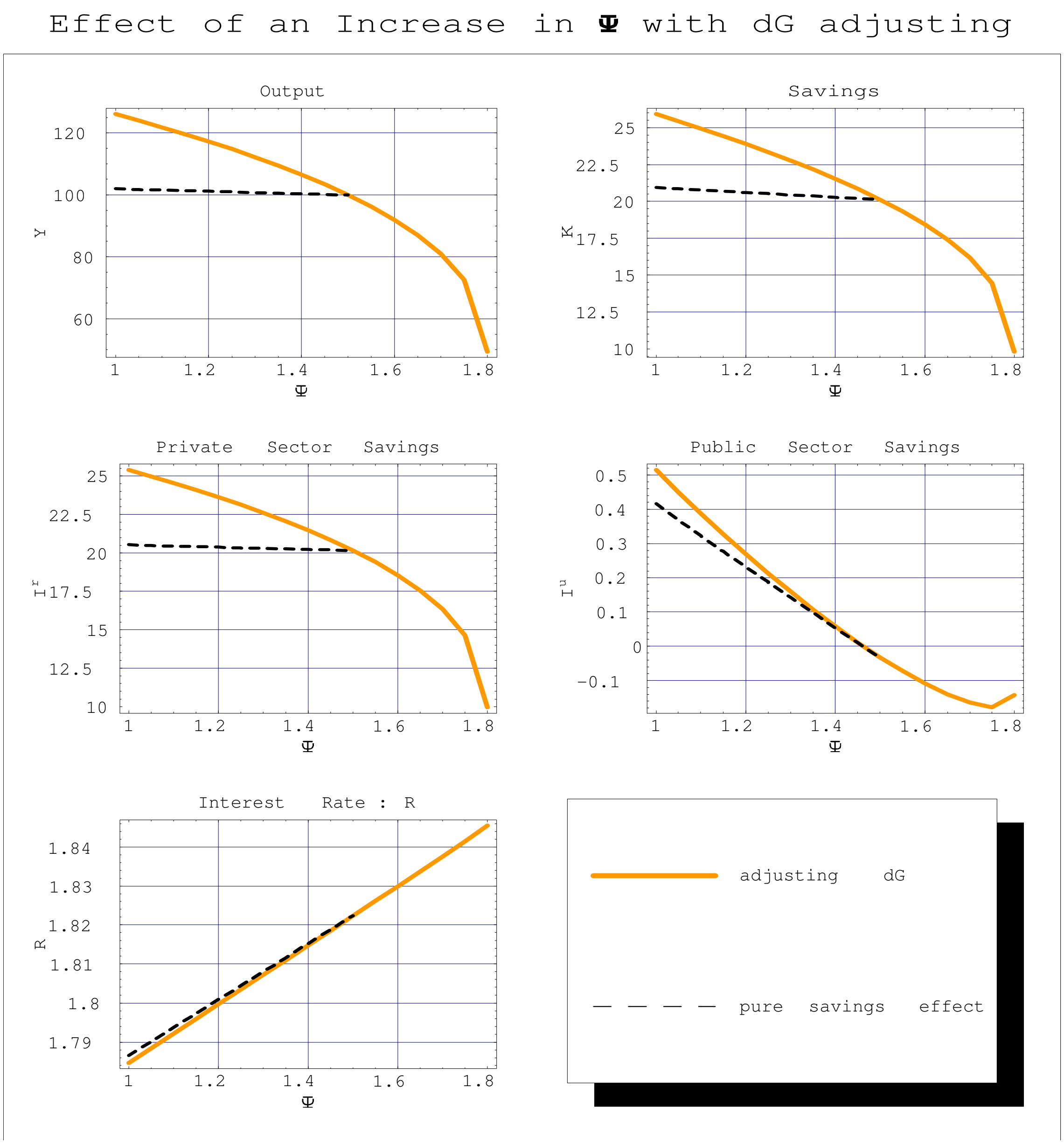
Effect of an Increase in Ψ with dT adjusting
Output
102
101.5
Y
101
100.5
100
1
1.2
1.4
1.6
1.8
2
Ψ
Savings
20.8
20.6
K
20.4
20.2
Private Sector Savings
20.6
20.4
20.2
20
19.8
Public Sector Savings
0.4
0.2
0
-0.2
-0.4
Interest rate: R
1.82
1.815
1.81
1.805
1.8
1.795
1.79
R
adjusting dT
pure savings effect

Figure 1:



Effect of an Increase in Ψ with dE adjusting
Output
Savings
Private Sector Savings
Public Sector Savings
Interest Rate: R
adjusting dE
pure savings effect

Figure 2:



Effect of an Increase in Ψ with dG adjusting
Output
Y
120
100
80
60
1
1.2
1.4
1.6
1.8
Ψ
Savings
K
25
22.5
20
17.5
15
12.5
10
Private Sector Savings
I^r
Public Sector Savings
I^u
0.5
0.4
0.3
0.2
0.1
0
-0.1
Interest Rate : R
R
1.84
1.83
1.82
1.81
1.8
1.79
adjusting dG
pure savings effect

Figure 3:

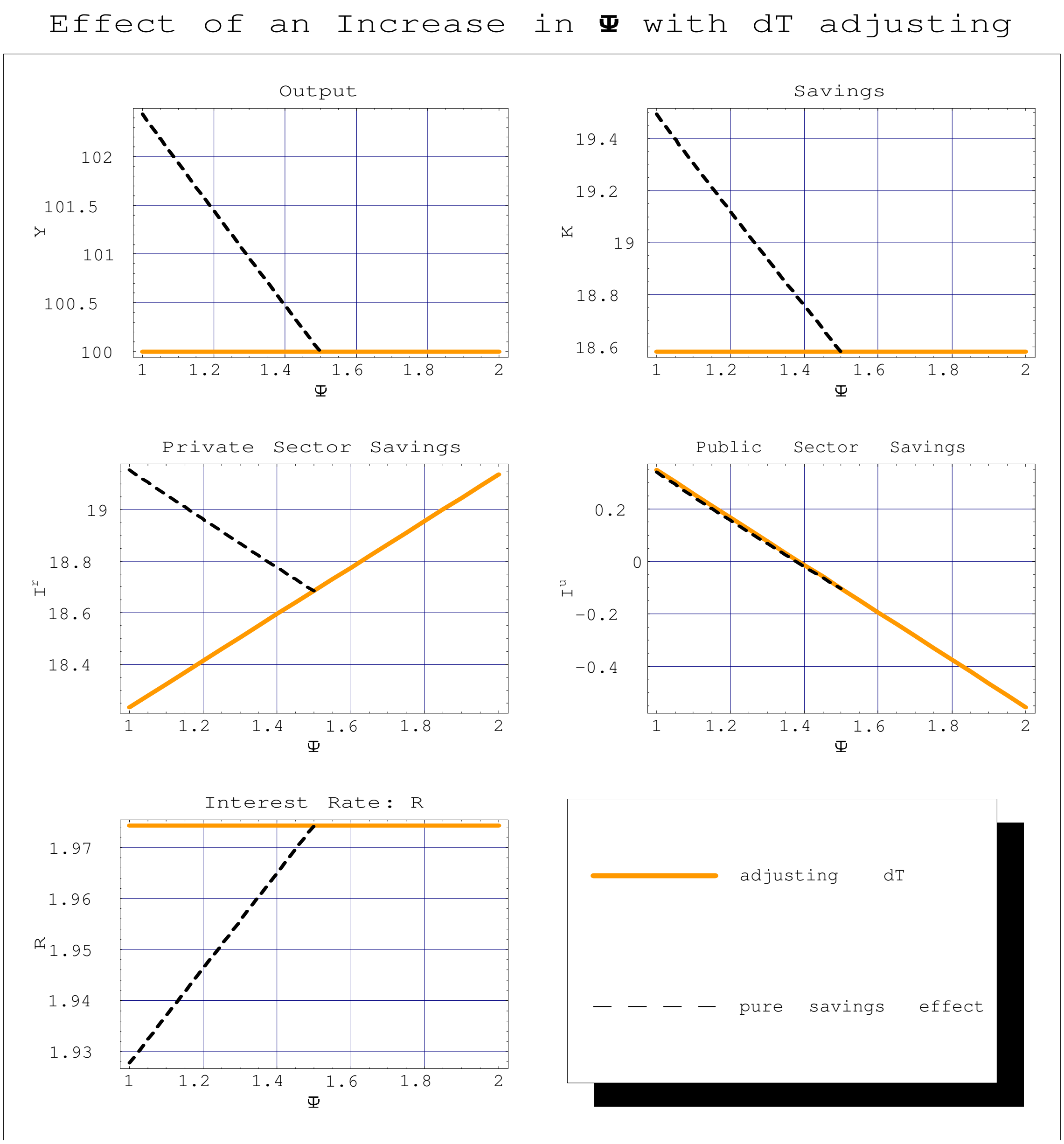
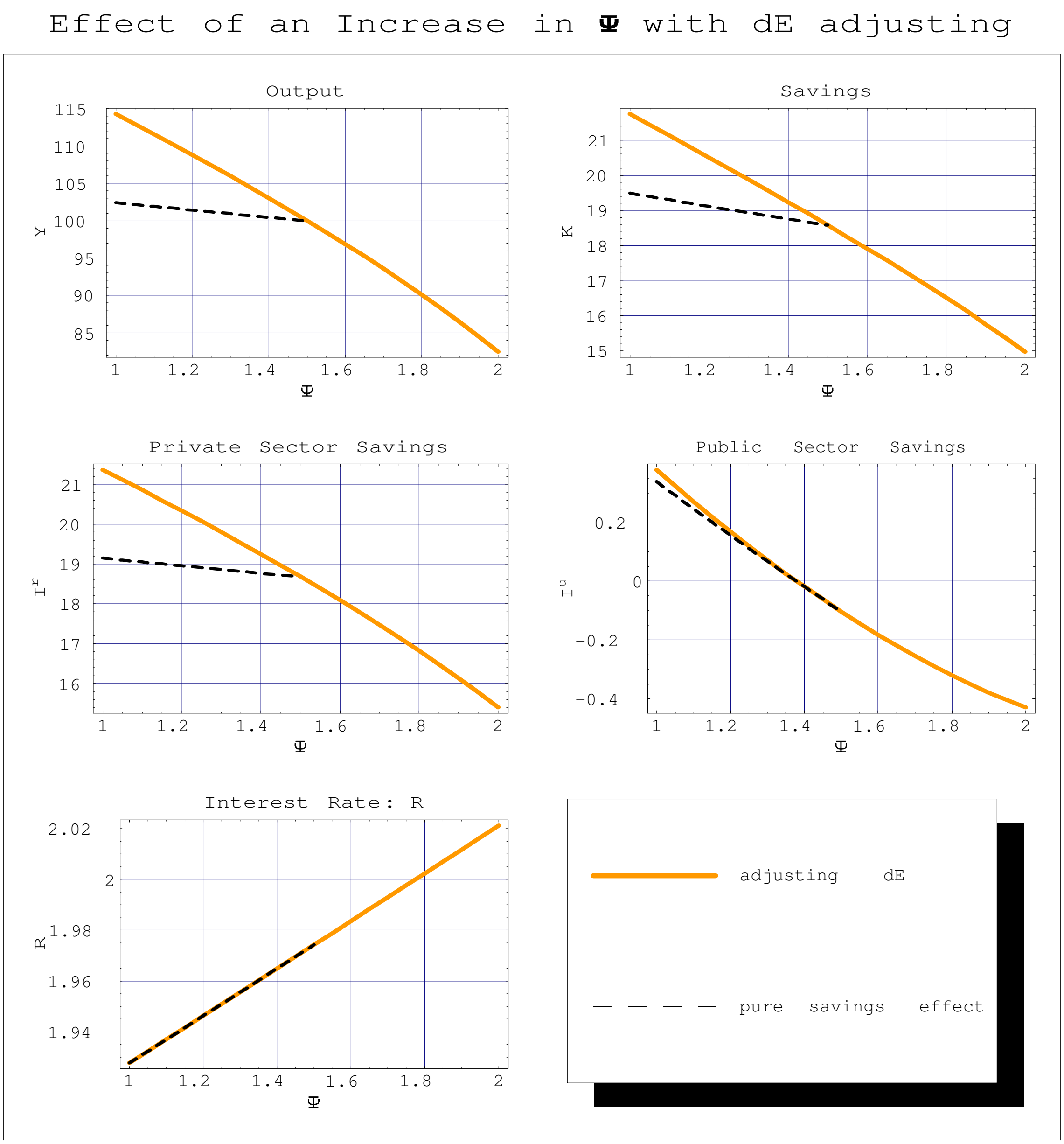

Effect of an Increase in Ψ with dT adjusting
Output
Y
102
101.5
101
100.5
100
1
1.2
1.4
1.6
1.8
2
Ψ
Savings
K
19.4
19.2
19
18.8
18.6
Private Sector Savings
19
18.8
18.6
18.4
Public Sector Savings
0.2
0
-0.2
-0.4
Interest Rate: R
R
1.97
1.96
1.95
1.94
1.93
adjusting dT
pure savings effect

Figure 4:



Effect of an Increase in Ψ with dE adjusting
Output
Y
115
110
105
100
95
90
85
1
1.2
1.4
1.6
1.8
2
Ψ
Savings
K
21
20
19
18
17
16
15
Private Sector Savings
I^r
Public Sector Savings
I^u
0.2
0
-0.2
-0.4
Interest Rate: R
R
2.02
2
1.98
1.96
1.94
adjusting dE
pure savings effect

Figure 5:

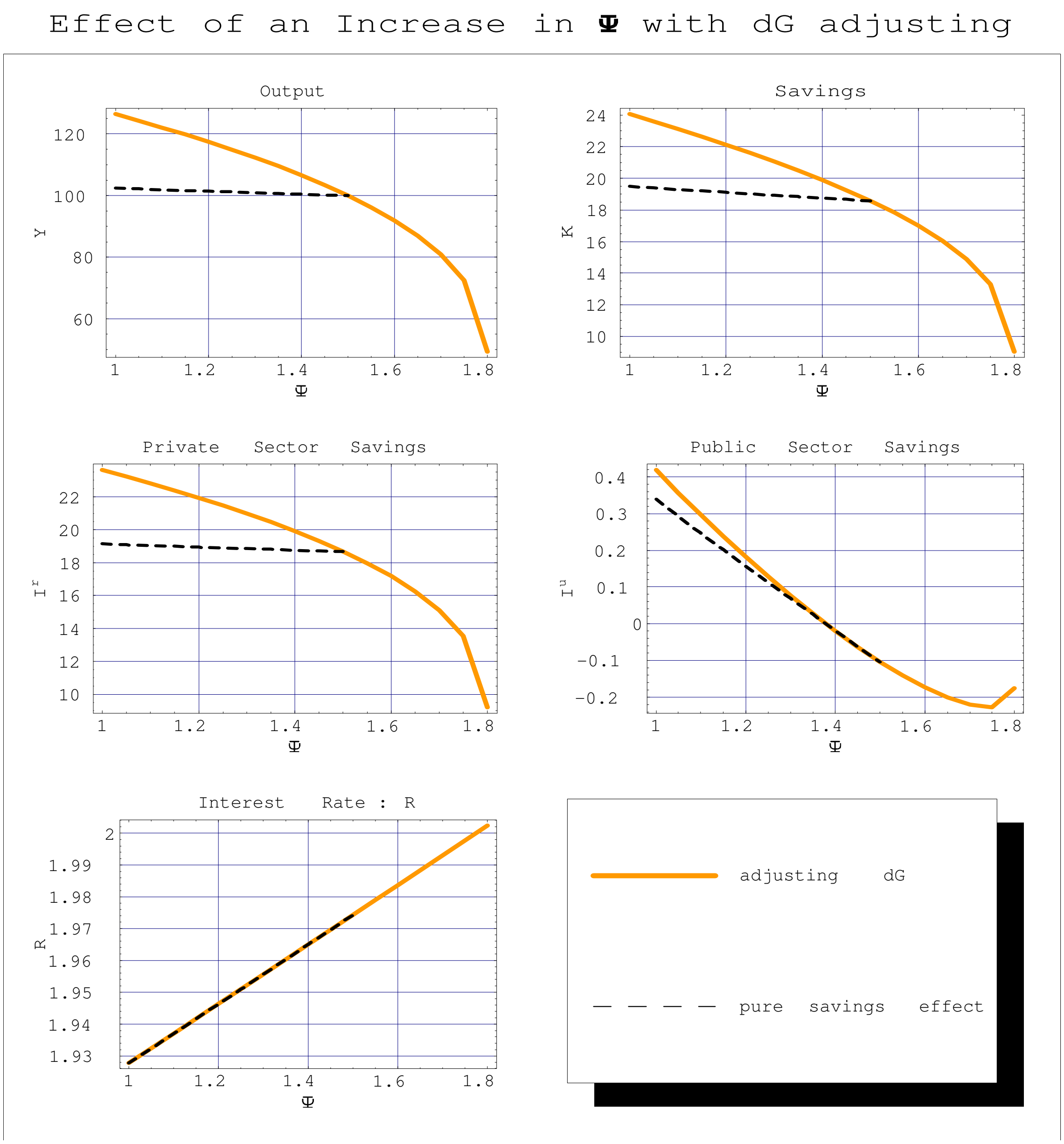

Effect of an Increase in Ψ with dG adjusting
Output
Savings
Private Sector Savings
Public Sector Savings
Interest Rate : R
adjusting dG
pure savings effect

Figure 6:

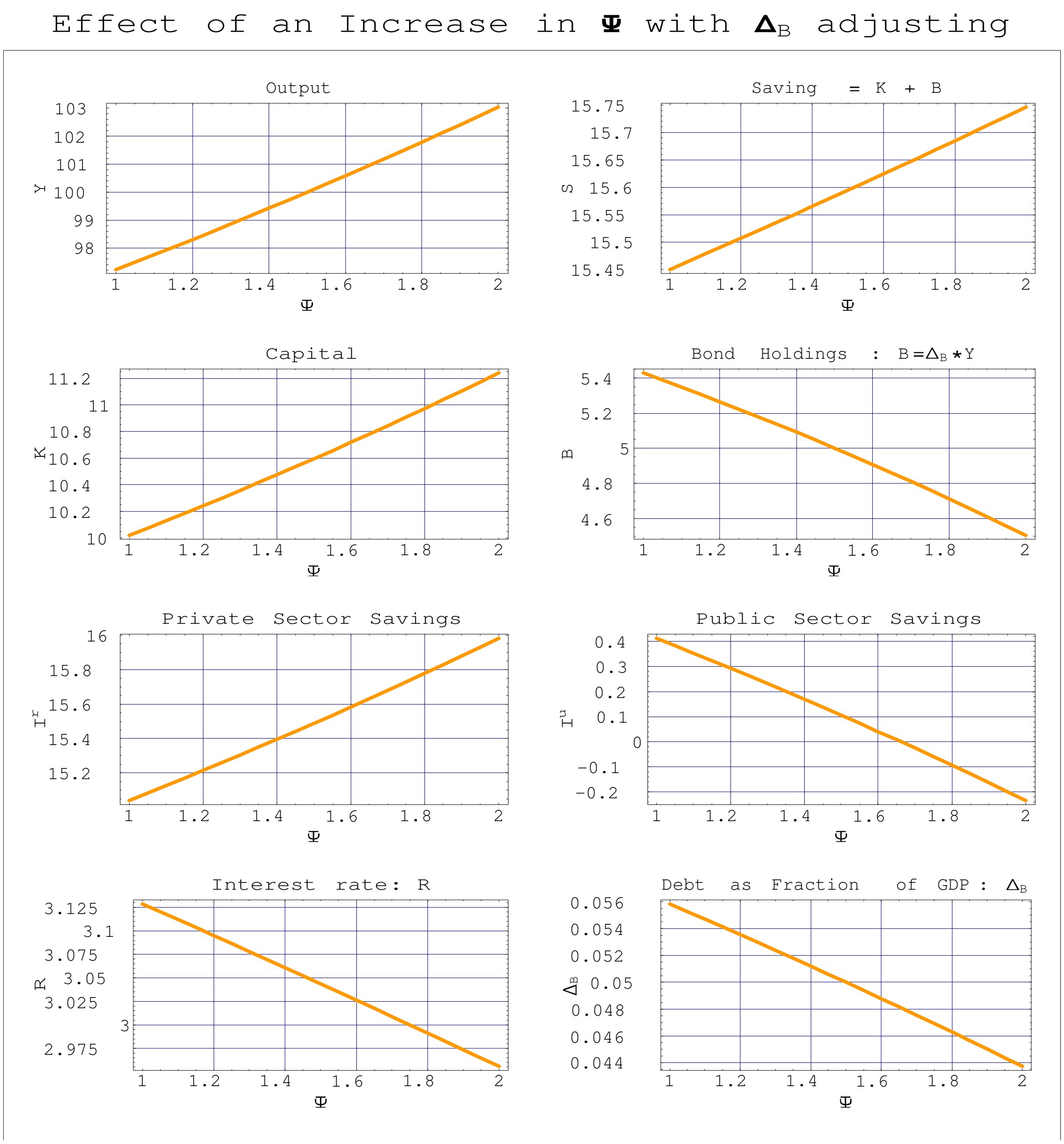

Effect of an Increase in Ψ with Δ_B adjusting
Output
Y
103
102
101
100
99
98
1 1.2 1.4 1.6 1.8 2
Ψ
Saving = K + B
S
15.75
15.7
15.65
15.6
15.55
15.5
15.45
1 1.2 1.4 1.6 1.8 2
Ψ
Capital
K
11.2
11
10.8
10.6
10.4
10.2
10
1 1.2 1.4 1.6 1.8 2
Ψ
Bond Holdings : B = Δ_B * Y
B
5.4
5.2
5
4.8
4.6
1 1.2 1.4 1.6 1.8 2
Ψ
Private Sector Savings
I^r
16
15.8
15.6
15.4
15.2
1 1.2 1.4 1.6 1.8 2
Ψ
Public Sector Savings
I^u
0.4
0.3
0.2
0.1
0
-0.1
-0.2
1 1.2 1.4 1.6 1.8 2
Ψ
Interest rate: R
R
3.125
3.1
3.075
3.05
3.025
3
2.975
1 1.2 1.4 1.6 1.8 2
Ψ
Debt as Fraction of GDP : Δ_B
Δ_B
0.056
0.054
0.052
0.05
0.048
0.046
0.044
1 1.2 1.4 1.6 1.8 2
Ψ

Figure 7:

Change in Ψ with ΔE adjusting

γ_1					Ψ						
	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
0.05	107.00	105.00	104.00	103.00	101.00	100.00	98.50	96.90	95.20	93.40	91.40
0.06	108.00	106.00	105.00	103.00	102.00	100.00	98.20	96.30	94.30	92.20	89.80
0.07	109.00	107.00	106.00	104.00	102.00	100.00	97.90	95.70	93.40	90.90	88.20
0.08	111.00	109.00	107.00	104.00	102.00	100.00	97.60	95.10	92.40	89.50	86.50
0.09	112.00	110.00	108.00	105.00	103.00	100.00	97.30	94.40	91.40	88.20	84.70
0.10	114.00	111.00	109.00	106.00	103.00	100.00	96.90	93.70	90.30	86.70	82.80
0.11	116.00	113.00	110.00	107.00	103.00	100.00	96.60	93.00	89.20	85.20	80.90
0.12	117.00	114.00	111.00	107.00	104.00	100.00	96.20	92.20	88.00	83.60	78.90
0.13	120.00	116.00	112.00	108.00	104.00	100.00	95.70	91.30	86.70	81.90	76.80
0.14	122.00	118.00	113.00	109.00	105.00	100.00	95.30	90.40	85.40	80.20	74.60
0.15	124.00	120.00	115.00	110.00	105.00	100.00	94.80	89.50	84.00	78.30	72.40

Table 3:

Change in Ψ with ΔE adjusting

η					Ψ						
	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
-1.00	113.00	111.00	108.00	106.00	103.00	100.00	97.00	93.90	90.60	87.10	83.40
-0.75	113.00	111.00	108.00	106.00	103.00	100.00	97.00	93.90	90.60	87.10	83.30
-0.50	113.00	111.00	108.00	106.00	103.00	100.00	97.00	93.90	90.60	87.10	83.30
-0.25	113.00	111.00	108.00	106.00	103.00	100.00	97.00	93.80	90.50	87.00	83.20
0.00	114.00	111.00	108.00	106.00	103.00	100.00	97.00	93.80	90.50	86.90	83.10
0.25	114.00	111.00	108.00	106.00	103.00	100.00	97.00	93.80	90.40	86.80	82.90
0.50	114.00	111.00	109.00	106.00	103.00	100.00	96.90	93.70	90.30	86.70	82.80

Table 4:

Change in Ψ with ΔG adjusting

α_1					Ψ			
	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
0.05	112.00	110.00	108.00	106.00	103.00	100.00	96.10	90.50	70.80
0.06	114.00	112.00	110.00	107.00	104.00	100.00	95.30	88.60	66.00
0.07	117.00	114.00	111.00	108.00	104.00	100.00	94.50	86.80	61.50
0.08	120.00	117.00	113.00	109.00	105.00	100.00	93.70	84.80	57.20
0.09	123.00	119.00	115.00	111.00	106.00	100.00	92.90	82.90	53.20
0.10	126.00	122.00	117.00	112.00	107.00	100.00	92.00	80.90	49.40
0.11	129.00	125.00	119.00	114.00	107.00	100.00	91.10	78.90	45.80
0.12	133.00	128.00	122.00	115.00	108.00	100.00	90.10	76.90	42.40
0.13	137.00	131.00	124.00	117.00	109.00	100.00	89.10	74.80	39.20
0.14	142.00	135.00	127.00	119.00	110.00	100.00	88.10	72.70	36.20
0.15	147.00	139.00	130.00	121.00	111.00	100.00	87.10	70.60	33.40

Table 5:

Change in Ψ with ΔG adjusting

η					Ψ				
	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
-1.00	103.00	103.00	102.00	102.00	101.00	100.00	98.60	95.20	6.94
-0.75	105.00	104.00	103.00	102.00	101.00	100.00	97.90	93.60	7.15
-0.50	107.00	106.00	105.00	103.00	102.00	100.00	97.10	91.50	7.72
-0.25	110.00	109.00	107.00	105.00	103.00	100.00	95.90	89.00	9.89
0.00	115.00	112.00	110.00	107.00	104.00	100.00	94.90	87.10	22.70
0.25	121.00	118.00	114.00	110.00	105.00	100.00	93.20	83.40	39.30
0.50	126.00	122.00	117.00	112.00	107.00	100.00	92.00	80.90	49.40

Table 6:

Absolute Difference : ΔE-ΔG, (η=0.5)

γ_1						Ψ		
	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
0.05	-10.90	-8.72	-6.30	-3.46	-0.00	4.51	11.20	33.80
0.06	-10.90	-8.74	-6.33	-3.49	-0.00	4.59	11.50	35.00
0.07	-10.80	-8.75	-6.36	-3.52	-0.00	4.67	11.80	36.30
0.08	-10.80	-8.75	-6.38	-3.55	-0.00	4.76	12.10	37.70
0.09	-10.70	-8.72	-6.39	-3.57	-0.00	4.85	12.40	39.20
0.10	-10.60	-8.68	-6.40	-3.59	-0.00	4.95	12.80	40.90
0.11	-10.40	-8.59	-6.38	-3.61	-0.00	5.06	13.20	42.80
0.12	-10.20	-8.47	-6.35	-3.63	-0.00	5.17	13.60	44.90
0.13	-9.85	-8.29	-6.28	-3.63	-0.00	5.29	14.10	47.30
0.14	-9.41	-8.04	-6.18	-3.62	-0.00	5.43	14.60	49.90
0.15	-8.81	-7.70	-6.04	-3.60	-0.00	5.57	15.30	52.90

Table 7:

Absolute Difference : ΔE-ΔG, (η=0.5)

α_1						Ψ			
	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
0.05	0.24	-0.11	-0.37	-0.50	-0.42	-8.44×10^{-6}	1.06	3.59	19.00
0.06	-1.78	-1.82	-1.74	-1.49	-0.96	-0.00	1.74	5.27	23.60
0.07	-4.00	-3.70	-3.24	-2.56	-1.54	-0.00	2.47	7.02	28.10
0.08	-6.47	-5.77	-4.88	-3.72	-2.17	-0.00	3.24	8.85	32.40
0.09	-9.20	-8.06	-6.69	-5.00	-2.85	-0.00	4.07	10.80	36.70
0.10	-12.20	-10.60	-8.68	-6.40	-3.59	-0.00	4.95	12.80	40.90
0.11	-15.60	-13.40	-10.90	-7.93	-4.41	-0.00	5.89	14.90	45.10
0.12	-19.40	-16.50	-13.30	-9.62	-5.29	-0.00	6.90	17.10	49.20
0.13	-23.70	-20.00	-16.00	-11.50	-6.27	-0.00	7.98	19.50	53.20
0.14	-28.50	-24.00	-19.00	-13.60	-7.34	-0.00	9.15	21.90	57.20
0.15	-33.90	-28.40	-22.40	-15.90	-8.52	-0.00	10.40	24.60	61.20

Table 8:

Absolute Difference : ΔE−ΔG, (η=−0.25)

Υ_1	Ψ							
	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
0.05	−0.76	−0.69	−0.57	−0.36	-5.90×10^{-6}	0.65	2.06	25.70
0.06	−0.52	−0.51	−0.45	−0.30	-5.18×10^{-6}	0.59	1.97	25.80
0.07	−0.27	−0.32	−0.32	−0.24	-4.42×10^{-6}	0.54	1.88	25.90
0.08	0.01	−0.11	−0.18	−0.17	-3.60×10^{-6}	0.48	1.78	26.10
0.09	0.32	0.11	−0.04	−0.10	-2.71×10^{-6}	0.41	1.68	26.20
0.10	0.65	0.36	0.13	−0.02	-1.76×10^{-6}	0.34	1.56	26.30
0.11	1.02	0.63	0.30	0.07	-7.27×10^{-7}	0.27	1.44	26.30
0.12	1.42	0.93	0.50	0.16	3.92×10^{-7}	0.19	1.32	26.40
0.13	1.86	1.25	0.71	0.26	1.61×10^{-6}	0.10	1.18	26.50
0.14	2.35	1.62	0.94	0.38	2.94×10^{-6}	0.01	1.03	26.60
0.15	2.90	2.02	1.20	0.50	4.39×10^{-6}	−0.10	0.87	26.60

Table 9:

Absolute Difference : ΔE−ΔG, (η=−0.25)

α_1	Ψ								
	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
0.05	3.62	2.85	2.08	1.33	0.62	6.83×10^{-6}	−0.40	−0.14	30.00
0.06	2.97	2.30	1.65	1.02	0.46	4.60×10^{-6}	−0.20	0.33	30.80
0.07	2.38	1.81	1.26	0.75	0.31	2.64×10^{-6}	−0.03	0.74	30.60
0.08	1.85	1.38	0.92	0.52	0.19	9.46×10^{-7}	0.12	1.07	29.60
0.09	1.39	0.99	0.62	0.31	0.08	-5.16×10^{-7}	0.24	1.35	28.10
0.10	0.98	0.65	0.36	0.13	−0.02	-1.76×10^{-6}	0.34	1.56	26.30
0.11	0.61	0.35	0.13	−0.03	−0.10	-2.80×10^{-6}	0.43	1.73	24.20
0.12	0.30	0.10	−0.06	−0.16	−0.16	-3.66×10^{-6}	0.50	1.85	22.10
0.13	0.03	−0.12	−0.23	−0.27	−0.22	-4.34×10^{-6}	0.55	1.94	20.00
0.14	−0.20	−0.30	−0.37	−0.36	−0.26	-4.88×10^{-6}	0.58	1.98	17.90
0.15	−0.39	−0.46	−0.48	−0.44	−0.30	-5.28×10^{-6}	0.61	2.00	15.90

Table 10: