Stabilizing Rational Speculation with a Taylor Rule?

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Abstract

We analyze the contribution of speculation to exchange rate volatility using different assumptions regarding speculation strategies and monetary policy rules.

We take the DORNBUSCH (1976) model as the starting point and adopt a slight modification of the money demand specification. With a money supply rule, rational speculation dampens the overshooting of the exchange rate following a money supply shock, compared with speculation based on static expectations.

Then, we replace the LM condition by a TAYLOR rule. The resulting "DORNBUSCH-TAYLOR" model generates a unique saddle point solution even under 'strict' inflation targeting, if speculation is based on rational expectations. Under 'flexible' inflation targeting, exchange rate overshooting induced by a monetary policy shock is less pronounced under rational speculation than under static speculation. FOREX market equilibrium doesn't exist at all if speculation is static and monetary policy adopts 'strict' inflation targeting.

1. Introduction

What is the contribution of speculation to the volatility of exchange rates when speculative activity is based on rational expectations? Is exchange rate volatility dampened or is it magnified by rational speculation? Moreover, does the contribution of speculation to exchange rate volatility depend on the specific monetary policy strategy?

In the present paper, we try to address these questions within a simple macroeconomic framework. It consists of a convenient combination of the MUNDELL (1963) and DORNBUSCH (1976) models. In both models, the monetary policy instrument is the money supply. Price adjustment is completely absent in the MUNDELL model, while DORNBUSCH incorporates a simple Phillips curve describing gradual price adjustment over time. In both models, equilibrium in the foreign exchange market requires the validity of the uncovered interest parity; in this condition, the expectational term vanishes in the MUNDELL model reflecting static expectations, while DORNBUSCH builds on rational (or at least semi-rational) expectations.

As an additional – rather curious – modification, DORNBUSCH – contrary to MUNDELL – does not make use of the conventional money demand specification, but replaces actual output by potential output as the transactions variable. As a consequence, actual output and money supply are completely disconnected, with severe implications for the role of speculation in foreign exchange markets.

In order to concentrate on the contribution of rational speculation to exchange rate volatility, in what follows we apply the common macroeconomic MUNDELL framework, augmented by the PHILLIPS curve specification used by DORNBUSCH, and analyze the differences implied by the two expectations hypotheses. In a second step, we replace the LM condition by a TAYLOR-type monetary policy rule, we consider the “strict” as well as the “flexible” inflation targeting version of the interest rate rule.

The plan of the paper is as follows. In Section 2, monetary policy is described with a conventional *LM* condition, where the Central Bank uses money supply as its instrument. Within this framework, we compare market reactions to an unexpected monetary policy shock under static and under rational speculation. In Sections 3 and 4, we replace the *LM* condition by a Taylor-type interest rate rule with a price level target. Within this framework, the monetary policy shock is represented by a change of the target price level. Existence and uniqueness of a rational expectations equilibrium is shown in Section 3, while the implications of static expectations are analyzed in Section 4. Section 5 gives a brief summary of the results.

2. ‘Overshooting’ and 'stabilizing speculation' with a money supply instrument

The basic approach for the analysis of exchange rate behaviour under a money supply regime is DORNBUSCH (1976). DORNBUSCH’s theoretical framework is, in turn, very similar to MUNDELL (1963). Both papers describe the perspective of a small open economy with fully integrated financial markets and limited price flexibility. The main improvements made by DORNBUSCH are commonly seen to consist of the introduction of a Phillips curve describing sticky price adjustment, and rational exchange rate expectations. MUNDELL, by contrast, assumed an absolutely fixed price level and static exchange rate expectations.

An additional modification adopted by DORNBUSCH and usually not emphasized in the literature (an exception is OBSTFELD and ROGOFF, 2003) concerns the money demand function: DORNBUSCH uses potential output as the transactions variable in the money demand equation instead of actual output.

For the purposes of the present paper, it seems natural to specify a common block of *IS*, *LM* and *AS* conditions as a combination of the MUNDELL and DORNBUSCH models, and to put the specific form of speculative behaviour into the equilibrium condition for the foreign exchange market, that is the uncovered interest parity condition.

The common model block consists of the following three equations,

(1.1) $$y = a + \delta(e + p^* - p) - \sigma i \,,$$

(1.2) $$m - p = y - \lambda i \,,$$

(1.3) $$\dot{p} = \varphi(y - \bar{y}) \,.$$

All variables except the interest rates are measured in logs. The parameters σ, δ, λ and φ are defined as positive

(1.1) is the *IS* condition: in goods market equilibrium, output y equals aggregate demand. Aggregate demand is affected by the real exchange rate ($e + p^* - p$) and the interest rate (i), and a represents autonomous demand.

(1.2) is the *LM* condition with nominal money supply m and aggregate price level p; the income elasticity of money demand is set at unity.

The AS condition (1.3) is a simple version of the Phillips curve as used by DORNBUSCH (1976): price level adjustment in time is proportional to the output gap ($\bar{y}$ denotes potential output).

With perfect capital mobility and substitutability equilibrium in the FOREX market requires the equalization of expected returns in domestic and foreign bonds. The specific form of the uncovered interest parity condition depends on the hypothesis regarding exchange rate expectations.

With static expectations (MUNDELL version), UIP amounts to

(1.4a) $$i = i^*,$$

where i^* is foreign interest rate (exogenous to the small country).

The DORNBUSCH version of UIP with rational expectations can be written as

(1.4b) $$i = i^* + \alpha(\bar{e} - e)$$

Where $\bar{e}$ is the model-consistent long run equilibrium exchange rate, and $-\alpha < 0$ is the stable root of the saddle point dynamic equilibrium system.

Summarizing so far, equations (1.1) to (1.4a) represent the AS-augmented MUNDELL model (M+), while the combination (1.1) to (1.4b) describe a kind of MUNDELL-DORNBUSCH model (MD).

Admittedly, from a modern theoretical point of view, both models exhibit at least two serious shortcomings. First, they are not able to capture steady state inflation. Second, no distinction is made between real and nominal interest rates. The incorporation of these features is postponed to later research. As an interesting step in this direction, with results similar to ours, see PROAÑO (2009).

Now we are able to analyze the models. We assume a permanent increase of nominal money supply to show how different expectation assumptions can alter the models predictions. The comparative statics are shown in Figure 1.

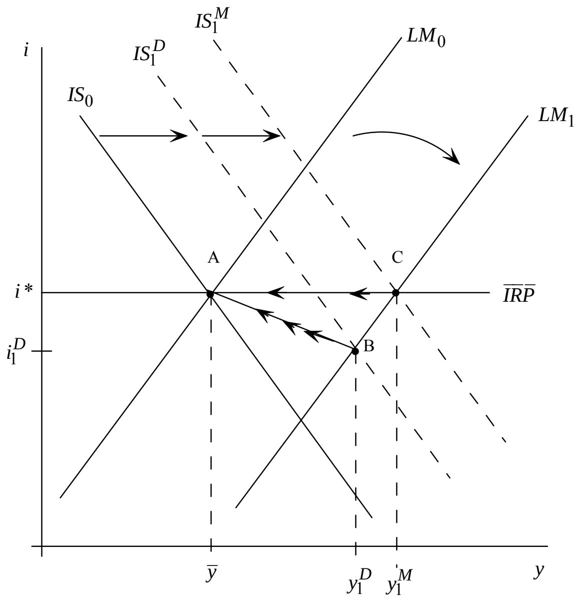

Fig. 1: expansionary monetary shock (MD vs. M+)

In both models, the monetary expansion leads to an outward shift of the *LM* curve because prices adjust slowly and therefore the real money supply increases. The resulting downward pressure to the domestic interest rate induces capital flows towards the foreign currency and to a depreciation of the home currency. This increases the competitiveness of home products, and the *IS* curve shifts to the right.

The main difference between the impact implications of the two models is how far the *IS* curve shifts. The augmented MUNDELL model predicts a shift to IS_1^M, while the DORNBUSCH approach leads to a shift to IS_1^D. While the market participants in the MUNDELL-DORNBUSCH model can anticipate future adjustments of the nominal exchange rate, the static speculators can not. Rational market participants know that future adjustment of prices will ensure economy's return to the old steady state and that this implies downward pressure on the nominal exchange rate. As a result, depreciation in the MD model is less pronounced than in the M+ model, which leads to a smaller *IS* shift.

In Fig. 1, point C shows the impact equilibrium with static expectations, while point B represents the rational expectations solution of the MD model. An important implication of these results is that output volatility in the rational expectations framework is lower as well.

In the following, increasing prices lead to a real appreciation which drives down foreign demand for home produced goods. This shifts the IS curve back to its initial location. The *LM* curve is shifted back because higher prices lower real money supply.

The graphical exposition is confirmed by the explicit multiplier analysis. The long run multipliers of the nominal exchange rate and the price level imply that neutrality of money holds in both models. A change of nominal money supply causes an equi-proportional change of prices and the nominal exchange rate,

$$\frac{d\bar{e}}{dm} = \frac{d\bar{p}}{dm} = 1 .$$

For the impact multipliers, the results are as follows,

$$\frac{de}{dm}(MD) = \frac{1+\alpha(\lambda+\sigma)}{\delta+\alpha(\lambda+\sigma)} > 1 \quad \text{for} \quad 0 < \delta < 1 ,$$

$$\frac{de}{dm}(M+) = \frac{1}{\delta} > 1 \quad \text{for} \quad 0 < \delta < 1 ,$$

$$\frac{de}{dm}(M+) > \frac{de}{dm}(MD) ,$$

Thus, in both cases the condition $0 < \delta < 1$ is necessary and sufficient for overshooting. The condition $\alpha > 0$ is not necessary. This implies that fundamental based speculation is not responsible for overshooting. In contrast, rational behaviour dampens the exchange rate reaction. In this sense, rational speculation stabilizes the economy. Beside that, the multiplier regarding the impact interest rate level in the MD model, which is responsible for the degree of overshooting is:

$$\frac{di}{dm}(MD) = -\frac{\alpha(1-\delta)}{\delta+\alpha(\lambda+\sigma)} \leq 0 \quad \text{for} \quad \alpha \geq 0 \text{ and } 0 < \delta < 1 .$$

A higher multiplier indicates a stronger interest rate reaction and this is equivalent to a smaller exchange rate overshooting.

3. Rational speculation with a TAYLOR rule

How does the DORNBUSCH model behave when monetary policy follows a TAYLOR-type interest rate rule instead of a money supply target? This is the question addressed in the present section. Formally, we replace the *LM* condition (1.2) by an interest rule that does not include the money stock variable.

Additionally, we switch from continuous to discrete time notation. This modification is not only convenient with respect to the numerical calculations we develop below, but also necessary in order to capture the specific dynamic interaction of exchange rate expectations, sticky prices and the interest rate rule.

The resulting model consists of the following four equations.

(2.1) $y_t = a + \delta(e_t + p^* - p_t) - \sigma i_t$,

(2.2) $p_{t+1} = p_t + \varphi(y_t - \bar{y}) + u_t$,

(2.3) $i_t = i^* + E_t e_{t+1} - e_t$,

(2.4) $i_t = i^* + \mu_\pi(p_t - \hat{p}) + \mu_y(y_t - \bar{y})$.

Equations (2.1) and (2.3) represent the *IS* and *UIP* conditions in discrete time, respectively. (2.2) is the ‘strong’ version of the *AS* condition with price stickiness: the contemporaneous price level does not respond at all to the output gap. u_t is an *iid* stochastic supply shock term $\left(u_t \sim N(0,\sigma^2)\right)$.

The novelty in the model is equation (2.4) which represents a specific version of a TAYLOR rule. In order to keep the analogy to the DORNBUSCH original as strong a possible, we postulate that the Central Bank follows a price level target $(\hat{p})$ instead of an inflation target (that is, steady state inflation is zero in this model). In what follows, the monetary policy shock consists of an unexpected change in $\hat{p}$.

3.1 Derivation of the semi-reduced form

The complete semi-reduced form is a four-equations system that gives the equilibrium values of the endogenous y_t, e_t, i_t and p_{t+1} for given values of $E_t e_{t+1}$. In order to reduce the dimensionality of the problem, we derive a ‘condensed’ version of the semi-reduced form by eliminating the interest rate variable.

Eliminating i_t from (2.3) and (2.4) gives

(2.5) $i^* + E_t e_{t+1} - e_t \overset{!}{=} i^* + \mu_\pi(p_t - \hat{p}) + \mu_y(y_t - \bar{y})$

and substituting (2.3) in (2.1) gives

(2.6) $y_t = a + \delta(e_t + p^* - p_t) - \sigma(i^* + E_t e_{t+1} - e_t)$.

The resulting three-equations system (with $E_t e_{t+1}$ expressed as $\hat{e}_{t+1}$) is summarized in CRAMER-style in Table 1.

y_t	e_t	p_{t+1}	$\hat{e}_{t+1}$	p_t	u_t	(eq.)
1	$-(\delta+\sigma)$	0	$-\sigma$	$-\delta$	0	(2.6)
$-\varphi$	0	1	0	1	1	(2.2)
μ_y	1	0	1	$-\mu_\pi$	0	(2.5)

Table 1

The Jacobian to the system is $\Delta = -\left(1+\mu_y(\delta+\sigma)\right) < 0$.

The partial multipliers to this semi-reduced from are reported in the Appendix.

3.2 Model solution

According to the methodology discussed in McCALLUM (1983, 1999), the variables p_t and u_t can be identified as the (minimum) state variables, and the RE[1] solution will satisfy the following linear equations,

(3.1) $p_{t+1} = \beta_0 + \beta_1 p_t + \beta_2 u_t$,

(3.2) $e_t = \lambda_0 + \lambda_1 p_t + \lambda_2 u_t$,

(3.3) $y_t = \eta_0 + \eta_1 p_t + \eta_2 u_t$,

with β_j, λ_j and η_j as unknown coefficients.[2]

A few points are worth to be emphasized at this stage. First, the parameter β_1 plays a pivotal role: for a unique RE equilibrium to exist, the solution has to give exactly one value

[1] RE means Rational expectations

[2] λ_i is not to confuse with the interest rate elasticity of money demand (λ).

$0 < \beta_1 < 1$; second, the impact reaction of the exchange rate following a monetary policy shock $(d\overline{p} = d\hat{p} \neq 0)$ can be written (see Appendix) as:

$$\frac{de}{d\overline{p}} = \frac{d\overline{e}}{d\overline{p}} - \lambda_1 ,$$

thus, 'overshooting' requires $\lambda_1 < 0$.

The implied equilibrium trajectory (see Fig. A1 in the Appendix) in (i, y)-space can be derived from (2.4) in combination with the steady state solution to (3.3),

(3.4) $\quad y - \overline{y} = \eta_1 (p - \overline{p})$.

Inserting (3.4) into (2.4) yields:

$$i - i^* = \left(\frac{\mu_\pi}{\eta_1} + \mu_y \right) (y - \overline{y}), \text{ or}$$

$$\frac{di}{dy} = \frac{\mu_\pi + \eta_1 \mu_y}{\eta_1} =: \tau$$

as the trajectory slope; a unique RE equilibrium requires $\tau < 0$.

At this point, we report only basic starting steps of the identification procedure. The complete algebra can be found in the Appendix.

From (3.2) it follows that

(3.5) $\quad E_t e_{t+1} = \lambda_0 + \lambda_1 E_t p_{t+1}$,

and from (3.2) we have

(3.6) $\quad E_t p_{t+1} = p_{t+1}$

for u_t known in t. Together, we get

(3.7) $\quad E_t e_{t+1} = \lambda_0 + \lambda_1 \beta_0 + \lambda_1 \beta_1 p_1 + \lambda_1 \beta_2 u_t$.

The latter expression can be used to eliminate $E_t e_{t+1}$, where needed.

For λ_1 and η_1, conditional on β_1, we find the solutions

(3.8) $$\lambda_1 = \frac{\mu_\pi - \delta\mu_y}{(1+\sigma\mu_y)\beta_1 - \left(1+\mu_y(\delta+\sigma)\right)}$$

(3.9) $$\eta_1 = -\frac{\mu_\pi(\delta+\sigma) + \delta(1-\lambda_1\beta_1)}{1+\mu_y(\delta+\sigma)}$$

For the pivotal coefficient β_1, we find a characteristic equation of the form

$$\beta_1^2 + a\beta_1 + b = 0\,,$$

with

$$a = \frac{\Delta^2 + \varphi\delta(\mu_\pi - \delta\mu_y) - \left[\varphi(\delta + \mu_\pi(\delta+\sigma)) + \Delta\right](1+\sigma\mu_y)}{\Delta(1+\sigma\mu_y)}$$

$$b = -\frac{\Delta + \varphi(\delta + \mu_\pi(\delta+\sigma))}{1+\sigma\mu_y}$$

and the Jacobian: $\Delta = -(1+\mu_y(\delta+\sigma))$.

We discuss this quadratic equation numerically in the next section. For a unique RE equilibrium to exist, the solving roots have to fulfill $\beta_1^1 < 1$, and $\beta_1^2 > 1$; additionally, a monotonic saddle path requires $\beta_1^1 > 0$.

3.3 Numerical parameter analysis

While we have obtained the identifying restrictions regarding the dynamics of prices, nominal exchange rate and output, it is still questionable if the system is stable for alternative central bank preferences and if adjustment processes are monotonistic. We apply numerical calculations to investigate these problems, when central bank behaviour is altered. We assume the following parameter values, which should be inline with common macroeconomic sense[3]:

$\sigma = 0.3$ $\delta = 0.2$ $\varphi = 0.1$

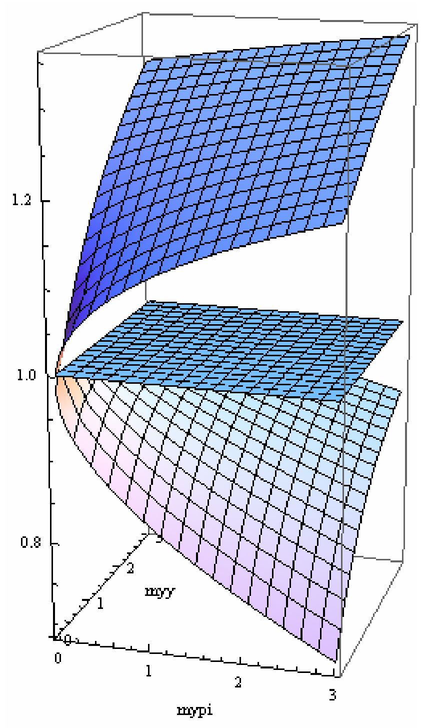

Fig. 2: β_1 for different central banking policies

Figure 2 shows the values of β_1 for different values of μ_π and μ_y between zero and three. We obtain two different real solutions for each combination of μ_π and μ_y. Each pair of solution satisfies $\left|\beta_1^A\right| > 1$ and $-1 < \beta_1^B < 1$. These conditions are needed to ensure a unique solution and monotonistic adjustment.

We have also computed surfaces for λ_1, η_1 and τ. These are documented in the appendix in Figures A1-A3. The values of η_1 and τ are below zero for all choices of μ_π and μ_y between zero and three. There is an area when μ_π is smaller than 0.5 and μ_y becomes very large where λ_1 is not negative, but the policy relevance of an aggressively output stabilizing central bank which doesn't care about price stability seems questionable. For some relevant combinations of central bank preferences the resulting values of β_1, λ_1, η_1 and τ are documented in Tables A1-A3 in he appendix.

[3] Further calculations, which are not reported here have shown, that the model is robust to marginal changes in these parameters. To give an example from the open economy literature: McCallum and Nelson use $\varphi = 0.086$, $\delta = 0.356$ and $\sigma = 0.2$ (McCallum/Nelson 2001).

3.4 Impulse response functions

We have computed impulse responses for two different central bank preference settings. The first set of graph illustrates pure inflation targeting and the second assumes a central bank which incorporates output stabilization. The chosen parameter values are $\mu_y = 0.5$ or zero and $\mu_\pi = 0.5$ in both cases. The other parameters are chosen as in the former section. Figures 3-6 illustrate the adjustment of prices, nominal exchange rate output and nominal interest rate after an exogenous ten percent raise of $\hat{p}$. The blue lines denote strict inflation targeting and the pink lines represent the utilization of output targeting.

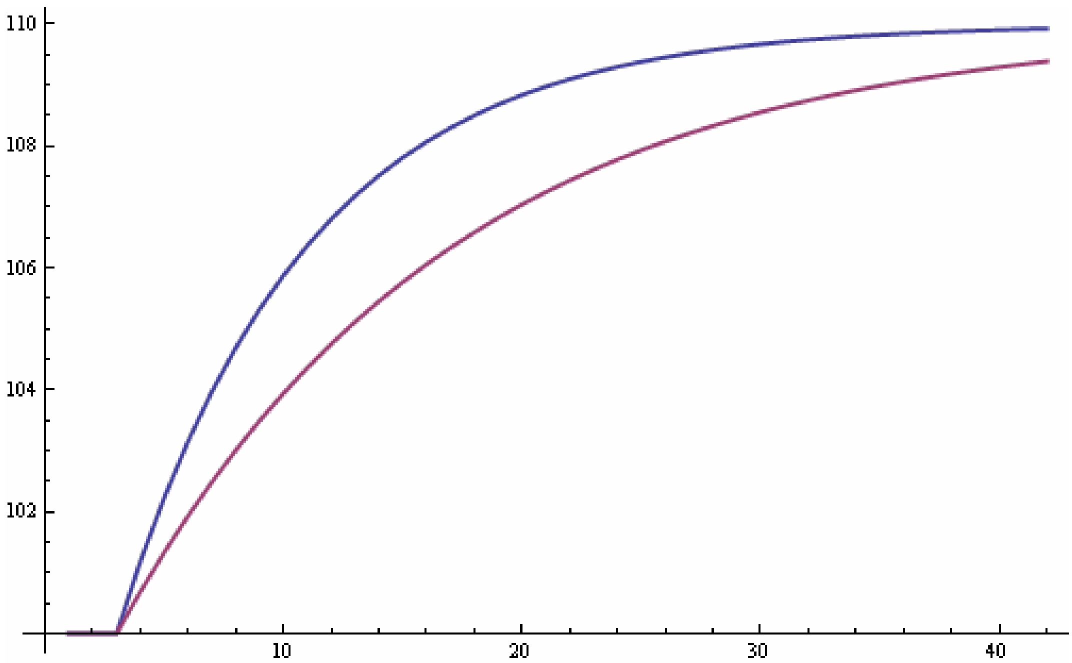

Fig. 3: Prices

The price adjustment under strict inflation targeting runs faster than under the mixed approach.

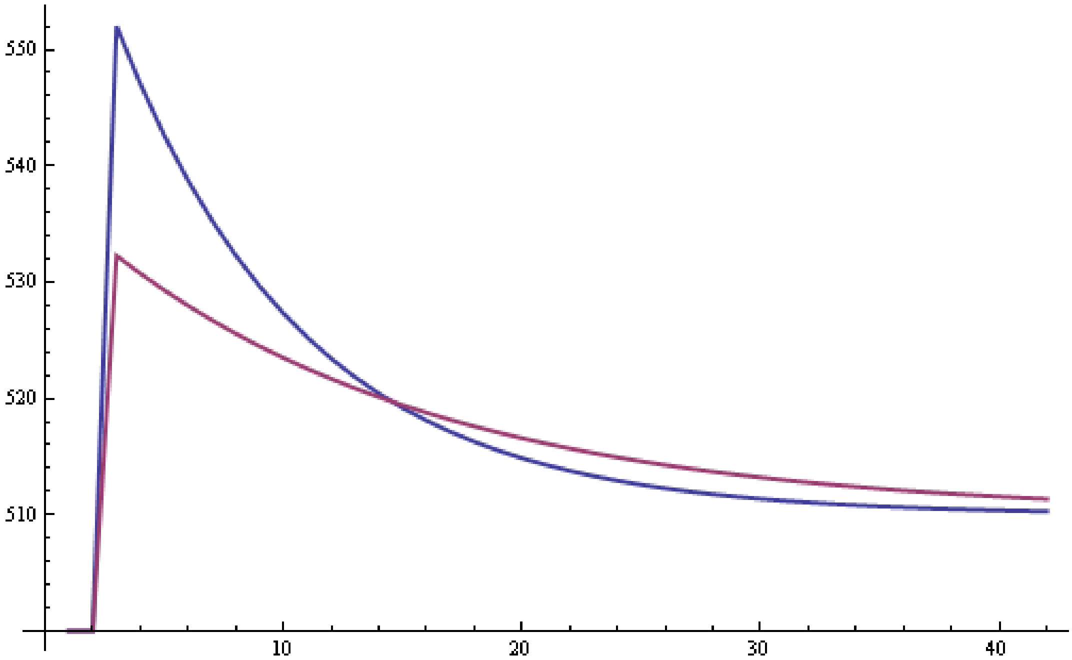

Fig. 4: Exchange rate

There is an exchange rate overshooting in both cases. Strict inflation targeting leads to depreciation of 10 percent at its peak while in the other setting online the depreciation reaches only a little more than six percent. Both experiments yield a small persistent change of the exchange rate.

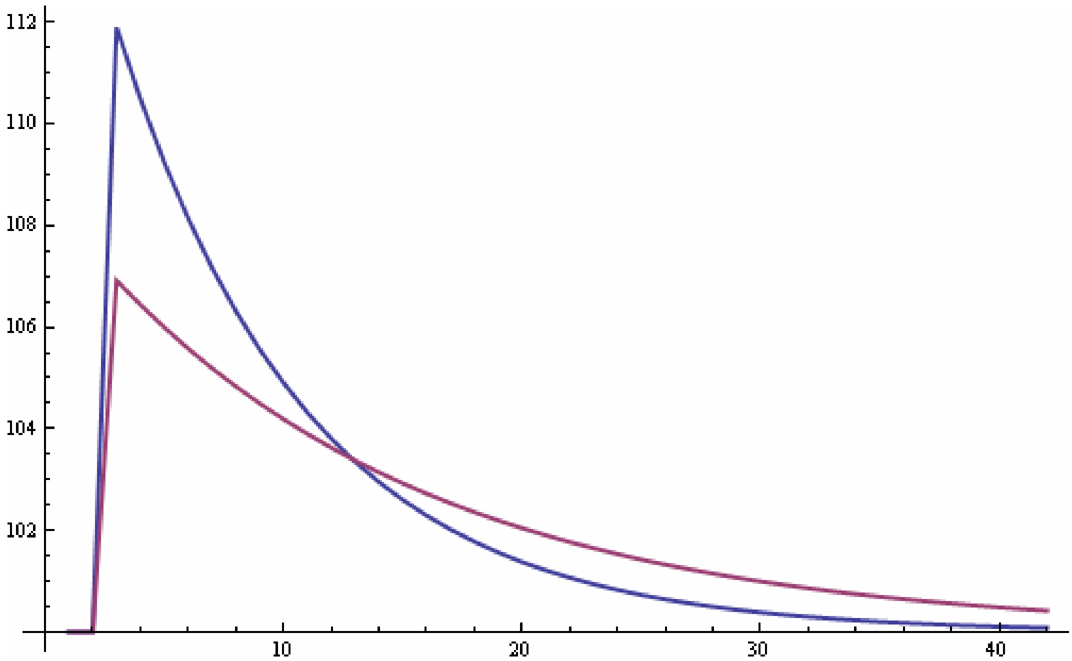

Fig. 5: Output

The shock yields an impact boost on output of twelve respectively seven percent. Output targeting leads to slower adjustment. Because of the Phillips curve restriction there is - in both cases – no room for a persistent change of the economy's output. Not surprisingly output deviation from steady state is smaller when central bank looks for output stabilization.

Figure 6 shows the reaction of the nominal interest rate. The shock is followed by a sharp drop of it. Central bank's interest rate response to the target change is smaller, when output stabilization is taken into account. The interest rate drop under strict inflation targeting is three times higher. The stronger interest rate drop is responsible for the stronger output reaction and the stronger devaluation of the exchange rate as seen in Figure 4 and 5.

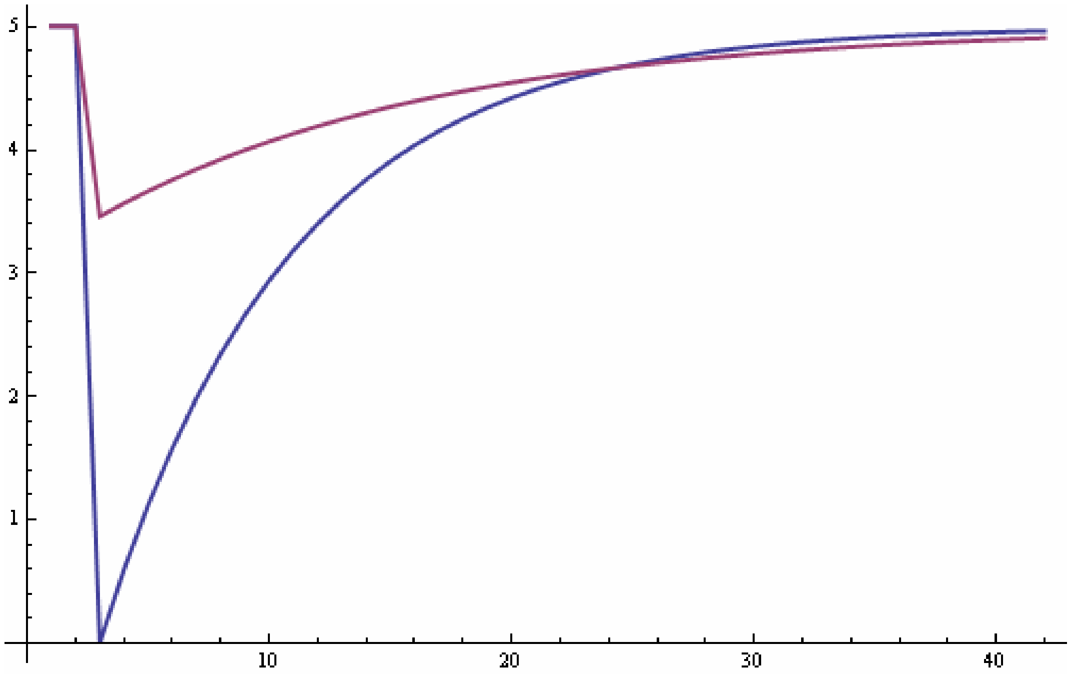

Fig. 6: Nominal Interest rate

The main results from these experiments are, that first disregarding output stabilization leads to higher volatility of the exchange rate after the shock occurs. The second interesting finding is that under strict inflation targeting the adjustment process runs a faster. There seems to be a trade-off between macroeconomic volatility and adjustment speed.

4. Static expectations with a Taylor rule

In discrete time-stochastic notation, static expectations amount to the hypothesis that the exchange rate follows a random walk,

(4.1) $e_t = e_{t-1} + z_t$,

with $z_t \sim N(0, \sigma_z^2)$, and, accordingly

(4.2) $E_t e_{t+1} = e_t$.

This means that FOREX equilibrium requires $i_t = i^*$, and the interest rate rule degenerates to the constraint on p_t and y_t

(4.3) $\mu_y(y_t - \overline{y}) = -\mu_\pi(p_t - \hat{p})$.

The resulting temporary equilibrium system is given in CRAMER-type style in Table 2.

y_t	e_t	p_{t+1}	p_t	$\hat{p}$
1	$-\delta$	0	$-\delta$	0
$-\varphi$	0	1	1	1
μ_y	0	0	$-\mu_\pi$	μ_π

Table 2

The Jacobian to this system is $\Delta_{\text{SE}} = -\delta\mu_y$, indicating that no determinate equilibrium exists for 'strict' inflation targeting $(\mu_y = 0)$.

This feature is also revealed by the exchange rate multiplier for the monetary policy shock,

$$\frac{de}{d\hat{p}} = \frac{\mu_\pi}{\delta\mu_y} > 0,$$

which implies $\lim\limits_{x \to \mu_y} \frac{de}{d\hat{p}} = +\infty$

The economic reasoning behind these results runs as follows. The decision of the Central Bank to raise the target price level $(d\hat{p} > 0)$ leads to a lower interest rate and a FOREX disequilibrium in favour of the foreign currency. The resulting depreciation of the domestic currency has an expansionary effect on output demand. If $\mu_y > 0$, this effect exerts an upward pressure to the domestic interest rate, via the interest rate rule. The price level cannot contribute to the correction in period t, since it is not able to adjust before $t+1$.

With 'strict' inflation targeting $(\mu_y = 0)$, the interest rate does not respond to output expansion, and there is no limiting force to the depreciation of domestic currency.

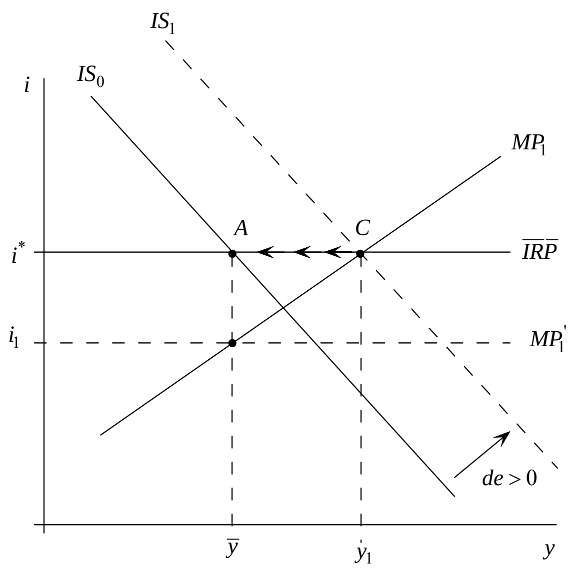

Fig. 7: Static Expectations

The situation with static speculation is illustrated in Fig. 7. The upward sloping line MP_1 represents the 'flexible' inflation targeting rule $(\mu_y > 0)$ after the expansionary monetary policy shock; the depreciation of domestic currency induces an outward shift of the *IS* line (from IS_0 to IS_1), and the impact equilibrium is found in point *C*.

By contrast, the flat line $MP_1^{'}$ represents 'strict' inflation targeting, where the outward shift of the *IS* line does not induce a higher domestic interest rate. In this regime, no determinate equilibrium exists.

As it regards exchange rate volatility, point *C* in Fig. 7 clearly indicates that, with static expectations, overshooting is more pronounced than with rational expectations.

5. Summary

With monetary policy following a money supply strategy, both static and rational speculation generate a well defined equilibrium path. Under both expectational behaviour patterns, a monetary policy shock induces exchange rate overshooting. With static expectations, overshooting is more pronounced than with rational speculation. In this sense, rational expectations reduce exchange rate volatility. The necessary and sufficient condition for overshooting is a sufficiently weak sensitivity of output demand vis-à-vis the exchange rate $(0 < \delta < 1)$. These results apply for a conventional money demand function with actual output as the transactions variable.

The stabilizing property of rational speculation is even confirmed if monetary policy follows a Taylor-type interest rate rule. With 'flexible' inflation targeting $(\mu_y > 0)$, equilibrium is well defined under both expectational schemes. With 'strict' inflation targeting $(\mu_y = 0)$, however, equilibrium remains well defined under rational speculation, while under static expectations no stable equilibrium exists at all. The stabilizing property of rational speculation is stronger when monetary policy adopts 'flexible' inflation targeting than with 'strict' inflation targeting.

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Appendix

Appendix

A. Partial multipliers to the semi-reduced form (Table 1)

Output

$$\frac{\partial y_t}{\partial \hat{e}_{t+1}} = \frac{\delta}{1+\mu_y(\delta+\sigma)} > 0\,,$$

$$\frac{\partial y_t}{\partial p_t} = -\frac{\delta+\mu_\pi(\delta+\sigma)}{1+\mu_y(\delta+\sigma)} < 0\,,$$

$$\frac{\partial y_t}{\partial u_t} = 0\,;$$

Price level

$$\frac{\partial p_{t+1}}{\partial \hat{e}_{t+1}} = \frac{\varphi\delta}{1+\mu_y(\delta+\sigma)} > 0\,,$$

$$\frac{\partial p_{t+1}}{\partial p_t} = \frac{1+\mu_y(\delta+\sigma)-\varphi(\delta+\mu_\pi(\delta+\sigma))}{1+\mu_y(\delta+\sigma)}\,,$$

$$\frac{\partial p_{t+1}}{\partial u_t} = 1\,;$$

Exchange rate

$$\frac{\partial e_t}{\partial \hat{e}_{t+1}} = \frac{1+\sigma\mu_y}{1+\mu_y(\delta+\sigma)} > 0 \quad (\text{and} < 1)\,,$$

$$\frac{\partial e_t}{\partial p_t} = -\frac{\mu_\pi-\delta\mu_y}{1+\mu_y(\delta+\sigma)} < 0 \text{ for } \mu_y < \mu_\pi/\delta)\,,$$

$$\frac{\partial e_t}{\partial u_t} = 0\,.$$

B. UC-MSV approach

We postulate the following linear solutions in the state variables p_t and u_t:

- **Price level**

$$p_{t+1} = \beta_0 + \beta_1 p_t + \beta_2 u_t , \qquad (\beta_j \text{ unknown}) \qquad \text{(B1.1)}$$

implying

$$E_t p_{t+1} = p_{t+1} \qquad \text{(for } u_t \text{ known in } t\text{).} \qquad \text{(B1.2)}$$

Steady state:

From

$$p_{t+1} - \overline{p} = \beta_1 (p_t - \overline{p}) + \beta_2 u_t ,$$

$$p_{t+1} = (1 - \beta_1)\overline{p} + \beta_1 p_t + \beta_2 u_t ,$$

or (identifying restriction, regime specific)

$$\beta_0 = (1 - \beta_1)\hat{p} \qquad \text{(for } \overline{p} = \hat{p}\text{)} . \qquad \text{(B1.3)}$$

- **Exchange rate**

$$e_t = \lambda_0 + \lambda_1 p_t + \lambda_2 u_t \; , \qquad (\lambda_j \text{ unknown}) \qquad \text{(B2.1)}$$

$$E_t e_{t+1} = \lambda_0 + \lambda_1 E_t p_{t+1} ,$$

$$E_t e_{t+1} = \lambda_0 + \lambda_1 \beta_0 + \lambda_1 \beta_1 p_t + \lambda_1 \beta_2 u_t . \qquad \text{(B2.2)}$$

Steady state:

$$e_t - \overline{e} = \lambda_1 (p_t - \overline{p}) + \lambda_2 u_1 ,$$

$$e_t = \overline{e} - \lambda_1 \overline{p} + \lambda_1 p_t + \lambda_2 u_t , \qquad \text{(B2.3)}$$

or (identifying restriction, regime specific)

$$\lambda_0 = \overline{e} - \lambda_1 \hat{p}, \qquad \text{(for } \overline{p} = \hat{p}\text{)}. \qquad \text{(B2.4)}$$

Comment:

From (8.3), the impact multiplier is

$$\frac{de}{d\overline{p}} = \frac{d\overline{e}}{d\overline{p}} - \lambda_1, \qquad \text{(B2.5)}$$

and $> \frac{d\overline{e}}{d\overline{p}}$ for $\lambda_1 < 0$, required for 'overshooting'.

- Output

$$y_t = \eta_0 + \eta_1 p_t + \eta_2 u_t. \qquad (\eta_j \text{ unknown}) \qquad \text{(B3.1)}$$

Steady state:

$$y_t - \overline{y} = \eta_1 (p_t - \overline{p}) + \eta_2 u_t,$$

$$y_t = \overline{y} - \eta_1 \overline{p} + \eta_1 p_t + \eta_2 u_t,$$

or (identifying restriction, regime specific)

$$\eta_0 = \overline{q} - \eta_1 \hat{p} \qquad \text{(for } \overline{p} = \hat{p}\text{)}. \qquad \text{(B3.2)}$$

Identifying η_1:

$$\frac{dy_t}{dp_t} = \frac{\partial y_t}{\partial \hat{e}_{t+1}} \frac{d\hat{e}_{t+1}}{dp_t} + \frac{\partial y_t}{\partial p_t} \overset{!}{=} \eta_1,$$

$$= -\frac{\delta}{\Delta} \lambda_1 \beta_1 + \frac{\delta + \mu_\pi (\delta + \sigma)}{\Delta},$$

$$= \frac{\mu_\pi (\delta + \sigma) + \delta(1 - \lambda_1 \beta_1)}{\Delta} = \eta_1 \quad . \qquad \text{(B3.3)}$$

- Implied (*i*, *y*) - trajectory

$$i - i^* = \mu_\pi (p - \overline{p}) + \mu_y (y - \overline{y}), \tag{2.4}$$

with

$$y - \overline{y} = \eta_1 (p - \overline{p}) \text{, or} \tag{B3.1}$$

$p - \overline{p} = \frac{1}{\eta_1}(y - \overline{y})$, inserted in (2.4) gives

$i - i^* = (\frac{\mu_\pi}{\eta_1} + \mu_y)(y - \overline{y})$, and

$$\frac{di}{dy} = \frac{\mu_\pi + \eta_1 \mu_y}{\eta_1} =: \tau \quad \text{(trajectory slope).} \tag{B3.4}$$

Identifying β_1 and λ_1

Substitution of $\hat{e}_{t+1}$ through (B2.2) into the semi-reduced form using the partial multipliers given in section A gives the identifying restrictions

$$\frac{dp_{t+1}}{dp_t} = \frac{\varphi\left(\delta + \mu_\pi(\delta + \sigma)\right) + \Delta - \varphi\delta\lambda_1\beta_1}{\Delta} \overset{!}{=} \beta_1 \quad , \tag{B4.1}$$

$$\frac{de_t}{dp_t} = \frac{\mu_\pi - \delta\mu_y - (1 + \sigma\mu_y)\lambda_1\beta_1}{\Delta} \overset{!}{=} \lambda_1 . \tag{B4.2}$$

These are two independent equations that can be solved for β_1 and λ_1.

Solutions

- **λ_1 conditional on β_1**

$$\lambda_1 = \frac{\varphi\left(\delta + \mu_\pi(\delta + \sigma)\right) + \Delta(1 - \beta_1)}{\varphi\delta\beta_1}, \qquad \text{and} \tag{B5.1}$$

$$\lambda_1 = \frac{\mu_\pi - \delta\mu_y}{\Delta + (1+\sigma\mu_y)\beta_1}\,. \tag{B5.2}$$

- **Characteristic Equation in β_1**

$\beta_1^2 + a\beta_1 + b = 0$, where

$$a = \frac{\Delta^2 + \varphi\delta(\mu_\pi - \delta\mu_y) - \left[\varphi(\delta + \mu_\pi(\delta+\sigma)) + \Delta\right](1+\sigma\mu_y)}{\Delta(1+\sigma\mu_y)} \tag{B6.1}$$

$$b = -\frac{\Delta + \varphi(\delta + \mu_\pi(\delta+\sigma))}{1+\sigma\mu_y} \tag{B6.2}$$

$$\Delta = -(1 + \mu_y(\delta+\sigma))$$

The solutions to this quadratic problem are discussed numerically. The coefficients β_2, λ_2, η_2 are not calculated.

C. Numerical results

μ_π \ μ_y	0	0,5	1,0
0,5	0.88098	0.93082	0.95520
1,0	0,83139	0,88808	0,92090
1,5	0,79127	0,85299	0,89138

Table A1: Different values of β_1

μ_π \ μ_y	0	0,5	1,0
0,5	-4,20099	-2,22773	-1,16173
1,0	-5,93070	-3,93521	-2,64171
1,5	-7,18639	-5,20322	-3,81008

Table A2: Different values of λ_1

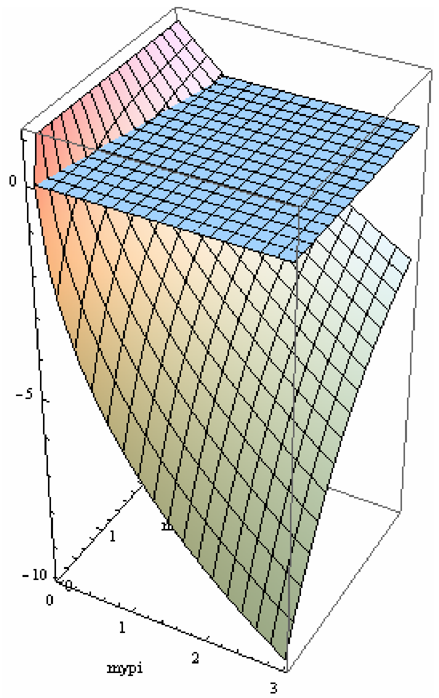

Fig. A1: λ_1 for different central banking policies

μ_π \ μ_y	0	0,5	1,0
0,5	-1,19020	-0,69178	-0,44796
1,0	-1,68614	-1,11917	0,79103
1,5	-2,08728	-1,47012	-1,08616

Table A3: Different values of η_1

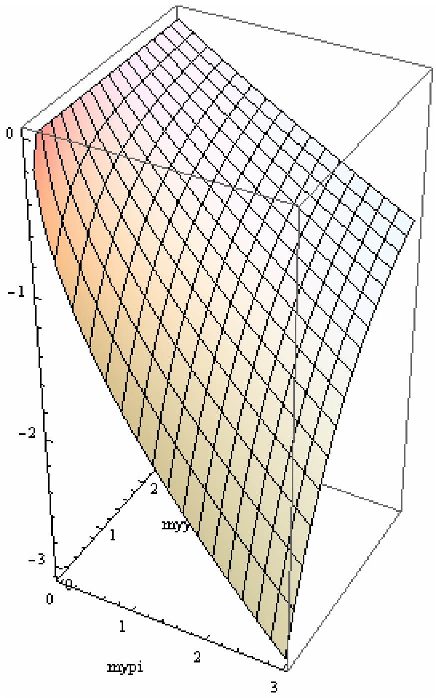

Fig. A1: η_1 for different central banking policies

μ_y / μ_π	0	0,5	1,0
0,5	-0,4200	-,2228	-,1162
1,0	-0,5931	-,3935	-,2642
1,5	-0,7186	-,5203	-,3810

Table A3: Different values of τ

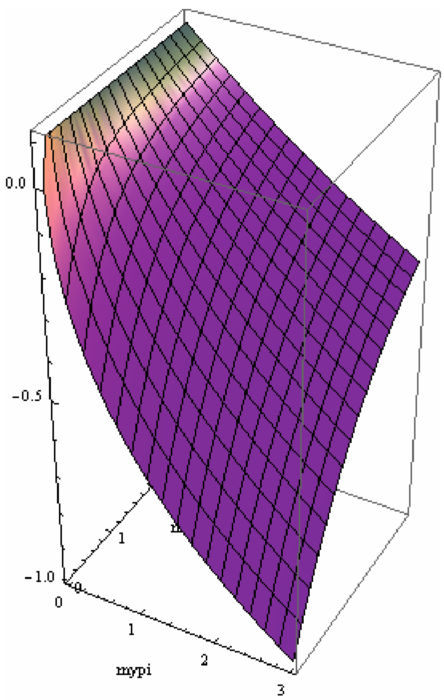

Fig. A1: τ for different central banking policies