



# Taxation and Long-Run Growth

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## Lecture 3 Redistribution and growth

This lecture examines a number of mechanisms through which income redistribution can increase growth. First, we consider a framework with imperfect capital markets. In this setup, redistribution may have both an *opportunity creation* effect and an *incentive* effect that increase investment and foster growth. Second, we exploit the fact that, when incomes are uncertain, redistribution creates insurance, and show that in an R&D-driven growth model (with perfect capital markets) redistribution can accelerate the pace of innovation.

### References

- Aghion, P., E. Caroli and C. García Peñalosa. 1999. "Inequality and Economic Growth: the perspective of the new growth theories", *Journal of Economic Literature*.
- Aghion, P. and P. Howitt. 1998. *Endogenous Growth Theory*, MIT press. Chapter 2.
- C. García Peñalosa and J.-F. Wen. "Optimal Taxation in a Schumpeterian Growth Model", mimeo.

## **The Effects of Inequality on Growth: An $A - K$ Formulation**

Traditional view  $\rightarrow$  *trade-off* between productive efficiency/growth and redistribution

Two main considerations

1. *investment indivisibilities*
2. *incentive* considerations

New approach based on credit market imperfections

### **The opportunity-enhancing effect of redistribution**

Discrete-time version of the  $A - K$  model

One good

Continuum of OLG families,  $i \in [0, 1]$

Intertemporal utility of individual  $i$  born at  $t$

$$U_t^i = \ln c_t^i + \rho \cdot \ln d_t^i$$

Production

$$y_t^i = (k_t^i)^\alpha (A_t)^{1-\alpha}$$

where

$$A_t = \int_0^1 y_{t-1}^i di = y_{t-1}$$

*Inequality:* Individuals differ in their initial endowments of capital

$$w_t^i = \varepsilon_t^i \cdot A_t$$

$\varepsilon_t^i$  is an i.i.d shock with mean one, so that  $\int_0^1 w_t^i di = A_t$

Individual  $i$  can either “consume” her endowment or invest it

*Capital market imperfections:* Credit simply is unavailable

*Redistribution:* ex ante redistribution of endowments

$$w^i = w^i + \beta(A - w^i), \quad 0 < \beta < 1$$

## No credit restrictions

Individuals solve

$$\max_{b^i, k^i} \ln(w^i + \beta(A - w^i) + b^i - k^i) + \rho \ln(y^i - \{z^i\})$$

The first-order conditions are

$$A^{1-\alpha}(k^i)^\alpha - rb^i = \rho r(w^i + \beta(A - w^i) + b^i - k^i)$$

and

$$r = \alpha \frac{A^{1-\alpha}}{k^i}$$

Which yield

$$k = \frac{\rho\alpha}{1 + \rho\alpha} A = s \cdot A$$

The steady-state growth rate is

$$g = \ln \frac{y_t}{y_{t-1}} = \ln \frac{k^\alpha A^{1-\alpha}}{A} = \alpha \ln s$$

Distribution and redistribution have no impact on growth

## No borrowing

Individual  $i$  choose  $k^i$  so as to:

$$\max_{k^i} \ln(w^i - k^i) + \rho \cdot \ln y^i$$

which yields

$$k^i = \frac{\rho\alpha}{1 + \rho\alpha} w^i = s \cdot w^i$$

Then,

$$y = \int_0^1 y^i di = \int_0^1 A^{1-\alpha} s^\alpha (w^i)^\alpha di$$

Hence:

$$g = \alpha[\ln s - \ln A] + \ln \int_0^1 (w^i)^\alpha di$$

*Theorem: Let  $u$  be a concave function. Let  $X$  and  $Y$  be two random variables, such that the expectations  $Eu(X)$  and  $Eu(Y)$  exist and are finite, and such that  $Y$  is obtained from  $X$  through a mean-preserving spread. Then  $Eu(X) \geq Eu(Y)$ .*

More inequality is bad for growth when capital markets are highly imperfect

## No borrowing and redistribution

Individual  $i$ 's problem is

$$\max_{k^i} \ln(w^i + \beta(A - w^i) - k^i) + \rho \cdot \ln y^i$$

which yields

$$k^i = s \cdot ((1 - \beta)w^i + \beta A).$$

As  $\beta$  increases, investments by the poorly endowed will increase whilst investments by the rich will decrease

The “*opportunity creation*” effect dominates

$$y = \int_0^1 y^i di = \int_0^1 A^{1-\alpha} s^\alpha ((1-\beta)w^i + \beta A)^\alpha di$$

and

$$g = \alpha [\ln s - \ln A] + \ln \int_0^1 ((1 - \beta)w^i + \beta A)^\alpha di$$

Using the concavity of the  $z \mapsto z^\alpha$  function

$$\frac{dg}{d\beta} > 0$$

More redistribution is good for growth when capital markets are highly imperfect

## The incentive effects of redistribution

Aghion-Bolton (1997)

Challenge the view that the incentive effect of redistribution is always negative

OLG families, indexed by  $i \in [0, 1]$

Utility of individual  $i$

$$U_t^i = d_t^i - c(e_t^i)$$

where the effort cost is  $c(e^i) = A_t \frac{(e^i)^2}{2}$

As before

$$w_t^i = \varepsilon_t^i \cdot A_t$$

The production technology

1. *fixed* and indivisible capital outlay equal to

$$k_t^i = \varphi \cdot A_t$$

2. the (conditional) output from investment is

$$y_t^i = \begin{cases} \sigma \cdot A_t & \text{with probability } e_t^i \\ 0 & \text{with probability } 1 - e_t^i, \end{cases}$$

Outcomes  $y_t^i$  are i.i.d. across individuals

Individuals with initial endowments  $w_t^i < \varphi A_t$  will borrow  $b_t^i = \varphi A_t - w_t^i$  from wealthy individuals

The source of capital market imperfection will be moral hazard with limited liability

### *Assumptions*

1. efforts  $e^i$  are not observable
2. a borrower's repayment to his/her lenders cannot exceed his/her second period output  $y_t^i$

### *First-Best*

If either (a) or (b) were violated  $\rightarrow$  all agents exert first-best effort

$$e^* = \arg \max_e \{e(\sigma A) - c(e)\} = \sigma$$

Growth rate is unaffected by distribution

$$g = \ln \frac{\int_0^1 y_t^i \cdot e^i di}{y_{t-1}} = \ln \frac{\sigma A_t \cdot \int_0^1 \sigma di}{A_t} = \ln \sigma^2$$

*Imperfect capital markets*

Optimal repayment schedule  $R(w^i)$  is such that

$$\begin{aligned} R(w^i) &= (\varphi A - w^i)\rho && \text{if project succeeds,} \\ &= 0 && \text{if project fails,} \end{aligned}$$

A borrower will choose her effort  $e^i$  to maximize

$$\max_e e(\sigma A - \rho(\varphi \cdot A - w^i)) - A \frac{e^2}{2}$$

$$e^i = \sigma - \rho\varphi + \rho \cdot \frac{w^i}{A} = e(\rho, w^i)$$

The lower a borrower's initial wealth, the *less* effort she will exert

Redistributing wealth towards borrowers will have a *positive* effect on their effort *incentives*

Lump-sum tax  $t^i < w^i - \varphi A$  on the endowment of individuals with  $w^i > \varphi A$ , and redistribute the revenue amongst borrowers

This, will

1. not affect the effort  $e^*$  supplied by the wealthy, as  $w^i - t^i > \varphi A$
2. increase the effort supplied by any subsidized borrower

Unambiguously positive *incentive* effect on growth,  $\mathbb{R}$

$$g = \ln \frac{e^i \cdot \sigma A}{A} = \ln \sigma + \ln \int_0^1 e^i di$$

with efforts  $e^i$  either increasing or remaining constant as a result of redistribution.

# **Income Redistribution in a Schumpeterian Growth Model**

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## **Population**

OLG - two period lives

$N$  individuals in each generation

$L$  unskilled manufacturing

$H$  skilled entrepreneurs  $R_t$

producers  $M_t = H - R_t$

## **Production Technologies**

Final good

$$Y_t = A_t x_t^\theta L^{1-\theta}$$

competitive sector

Intermediate good

$$x_t = M_t$$

may be monopolistic or competitive

## Quality ladder

$$\begin{aligned} A_{t+1} &= \gamma A_t \text{ if an innovation occurs in } t - 1 \\ &= A_t \text{ if no innovation occurs in } t - 1 \\ A_0 &\text{ is given, } \gamma > 1 \end{aligned}$$

## Probabilities

$$\begin{aligned} \Pr(\text{at least one innovator}) &= \lambda R_t^\eta \\ \Pr(\text{no innovator}) &= 1 - \lambda R_t^\eta \end{aligned}$$

where  $0 \leq \eta \leq 1$

$$\begin{aligned} \Pr(\text{patent to } i | \text{innovation}) &= 1/R_t \\ \Pr(\text{no patent to } i | \text{innovation}) &= (R_t - 1)/R_t \end{aligned}$$

Patent: lasts for a *single* period  
then technology is freely available

No storage

## Market Structure, Profits and Wages

Monopoly  $\phi_t = 1$

$$\max_{x_t} \Pi = p(x_t)x_t - w_t^1 x_t$$

Then

$$p_t^1 = \frac{w_t^1}{\theta} > w_t^1$$
$$v_t^1 = (1 - \theta) \frac{Y_t^1}{L}$$
$$w_t^1 = \theta^2 \frac{Y_t^1}{M_t^1}$$
$$\Pi_t = (1 - \theta) \theta Y_t^1$$

Competition  $\phi_t = 0$

$$p_t^0 = w_t^0$$
$$v_t^0 = (1 - \theta) \frac{Y_t^0}{L}$$
$$w_t^0 = \theta \frac{Y_t^0}{M_t^0}$$

## Equilibrium with Risk Neutrality

Utility

$$U(C^j) = C_j^j + C_{j+1}^j$$

Arbitrage

$$U_{mt}^\phi = U_{et}^\phi, \quad \phi = 0, 1$$

that is

$$\frac{\theta^{1+\phi} Y_t^\phi}{M_t^\phi} = \lambda \left( R_t^\phi \right)^{\eta-1} (1-\theta) \theta Y_{t+1}^1$$

Steady state

$$y^\phi = \frac{Y_t^\phi}{A_t} = L^{1-\theta} i M_t^\phi \zeta_\theta$$
$$U_{et}^\phi = A_t u_e^\phi$$

and

$$R_t^\phi = R_{t+1}^\phi = R^\phi$$
$$M_t^\phi = M_{t+1}^\phi = M^\phi$$

$$\frac{\theta^2}{H - R^1} = \lambda^i R^1 \zeta_{\eta-1} (1 - \theta)\theta$$

$$\frac{\theta}{H - R^0} = \lambda^i R_t^0 \zeta_{\eta-1} (1 - \theta)\theta \frac{H - R^1}{H - R^0} \mu \pi_\theta$$

## The Probability of Innovation

Probabilities of innovation in each period

$$Q = \begin{pmatrix} q^1 & 1 - q^1 \\ q^0 & 1 - q^0 \end{pmatrix}$$

where

$$q^1 \equiv \lambda (R^1)^{\zeta_\eta} \quad \text{and} \quad q^0 \equiv \lambda (R^0)^{\zeta_\eta}$$

The steady state probability of innovation  $q$

$$\begin{aligned} q &= \frac{q^0}{1 - q^1 + \frac{q^0}{\lambda (R^0)^{\zeta_\eta}}} \\ &= \frac{q^0}{1 - \lambda (R^1)^\eta + \lambda (R^0)^\eta} \end{aligned}$$

## Economic Growth

$$g_t = \ln \frac{A_{t+1}}{A_t} + \ln \frac{M_{t+1}}{M_t}$$

The average growth rate in the long-run

$$\begin{aligned} g &= E(\ln A_{t+1} - \ln A_t) \\ &= q (R^0)^{\zeta_\eta} \ln \gamma \end{aligned}$$

## Taxation

Optimal linear tax system

$$T(I) = -B + \tau I$$

Balanced budget in each period

$$\begin{aligned} N \cdot B_t^1 &= \tau^1 \Pi_t + w_t^1 M_t^1 + v_t^1 L^{\text{c}} \\ &= \tau^1 Y_t^1 \end{aligned}$$

and

$$\begin{aligned} N \cdot B_t^0 &= \tau^0 w_t^0 M_t^0 + v_t^0 L^{\text{c}} \\ &= \tau^0 Y_t^0 \end{aligned}$$

Three features

1. individuals born at  $t$  receive transfer at end of  $t$
2. different taxes in the two states of the world
3. considers individual uncertainty but ignores aggregate uncertainty

## Risk-Aversion

Utility

$$U(C^j) = C_j^j + C_{j+1}^j, \quad 0 < \alpha < 1$$

Manufacturing workers

$$\begin{aligned} U_{mt}^\phi &= (1 - \tau^\phi)w_{mt}^\phi + B_t^\phi \\ &= \frac{\tilde{A} Y_t^\phi}{N} (1 - \tau^\phi) \frac{\theta^{1+\phi} N}{M_t^\phi} + \tau^\phi \end{aligned}$$

Entrepreneurs

$$\begin{aligned} U_{et}^\phi &= \lambda R_t^\phi C_{\pi t}^\phi \\ &+ (1 - \lambda) R_t^\phi (B_t^\phi)^\alpha \end{aligned}$$

with

$$C_{\pi t}^\phi \equiv (1 - \tau^1)(1 - \theta)\theta Y_{t+1}^1 + \frac{\tau^\phi Y_t^\phi}{N}$$

## Steady state

Equilibrium

$$U_{et}^\phi = U_{mt}^\phi$$

Monopoly

$$\mu \frac{(1 - \tau^1)\theta^2}{H - R^1} + \frac{\tau^1 \Pi_\alpha}{N} = \frac{1 - \lambda^i R^1 \zeta_{\eta-1}}{1 - \lambda^i R^1 \zeta_{\eta-1}} \mu \frac{\tau^1 \Pi_\alpha}{N} + \lambda^i R^1 \zeta_{\eta-1} (1 - \tau^1)(1 - \theta)\theta\gamma + \frac{\tau^1 \Pi_\alpha}{N}$$

Competition

$$\mu \frac{(1 - \tau^0)\theta}{H - R^1} + \frac{\tau^0 \Pi_\alpha}{N} = \frac{1 - \lambda^i R^0 \zeta_{\eta-1}}{1 - \lambda^i R^0 \zeta_{\eta-1}} \mu \frac{\tau^0 \Pi_\alpha}{N} + \lambda^i R^0 \zeta_{\eta-1} (1 - \tau^1)(1 - \theta)\theta\gamma \frac{\mu}{H - R^1} \Pi_\theta + \frac{\tau^0 \Pi_\alpha}{N}$$

## Taxation with Risk Neutrality

With  $\alpha = 1$

$$\frac{(1 - \tau^1)\theta^2}{H - R^1} + \frac{\tau^1}{N} = 1 - \lambda i R^1 \zeta_{\eta-1} \frac{\tau^1}{N} + \lambda i R^1 \zeta_{\eta-1} (1 - \tau^1)(1 - \theta)\theta\gamma + \frac{\tau^1}{N}$$

then

$$\frac{\theta^2}{H - R^1} = \lambda i R^1 \zeta_{\eta-1} (1 - \theta)\theta\gamma$$

and the tax has no effect

Also

$$\frac{(1 - \tau^0)\theta}{H - R^1} = (1 - \tau^1)\lambda i R^0 \zeta_{\eta-1} (1 - \theta)\theta\gamma \frac{H - R^1}{H - R^0}$$

and taxes have no effect as long as  $\tau^0 = \tau^1$

## The Effect of Taxes

$$\mu \frac{(1 - \tau^1)\theta^2}{H - R^1} + \frac{\tau^1 \Pi_\alpha}{N} = 1 - \lambda^i R^1 \zeta_{\eta-1} - \mu \frac{\tau^1 \Pi_\alpha}{N} + \lambda^i R^1 \zeta_{\eta-1} \mu \frac{\tau^1 \Pi_\alpha}{N} (1 - \tau^1)(1 - \theta)\theta\gamma + \frac{\tau^1 \Pi_\alpha}{N}$$

Three effects

- ‘insurance premium’  $\rightarrow$  discourages entrepreneurship

- ‘insurance effect’  $\rightarrow$  encourages entrepreneurship

- ‘redistribution effect’  $\rightarrow$  discourages production

The role of social insurance is crucial: at  $\tau^1 = 0$ , the effect of  $\tau^1$  on the r.h.s. is *infinite*

**Proposition 1:** *The introduction of a small amount of redistribution increases research and hence the long-run probability of innovation.*

**Proposition 2:** *The introduction of a small amount of redistribution can be Pareto efficient.*