# Taxation and Long-Run Growth

Cecilia García Peñalosa
CNRS and GREQAM

# Lecture 2
# Should capital income be taxed?

The most basic endogenous growth setup, the AK model with externalities stemming from the capital stock, calls for *negative* taxation of capital income. Does this mean that Chamley's result is reinforced when we move into the endogenous growth framework? We will see that this is not necessarily the case. We will first examine taxation in the AK model. Then, we will consider the role of an elastic labor supply. Lastly, we will examine second-best results, which show that human capital accumulation or the presence of an informal sector imply that a positive tax on capital income may increase the growth rate.

**References**

Jones, Manuelli, and Rossi. 1997. "On the Optimal Taxation of Capital Income ", *Journal of Economic Theory*.
Turnovsky, S. 2000. "Fiscal Policy, Elastic Labour Supply, and Endogenous Growth", *Journal of Monetary Economics*.
García Peñalosa, and Turnovsky. 2004. "Second-Best Optimal Taxation of Capital and Labour in a Developing Economy", mimeo.

# Should Capital Income be Taxed?

- **Chamley:** basic result
    - $t_K = 0$ in the long-run
- **Endogenous growth**
    - since there are no transitional dynamics, $t_K = 0$ in all periods
    - externality from capital – may imply $t_K < 0$
- **Elastic labor supply:** the role of non-separable preferences
- **Second best:** does $t_K = 0$ still hold when the government is constrained in its use of instruments?
    - human capital
    - informal sector

# Endogenous Growth: The Basic AK Model

There is a mass 1 of firms, indexed by *j*.

Representative firm

$$Y_j = F(AL_j, K_j)$$

$K_j = K$ and $L_j = L$ for all *j*.

Externality : $A = K$

Aggregate output

$$Y = F(KL, K) \equiv Kf(L)$$

Perfect competition in factor markets,

$$r = f(L) - Lf'(L) \qquad \text{and} \qquad w = f'(L)K$$

$L$ constant $\Rightarrow$ $r$ and $w$

$$s_K \equiv \frac{rK}{Y} = 1 - \frac{f'(L)L}{f(L)}$$

all constant

## Consumer Optimization

$$\max \int_0^{\infty} \frac{C^{1-s}}{1-s} e^{-rt} dt, \quad \text{with } -\infty < s < 1$$

s.t.

$$\dot{K} = r(1-t_K)K + w(1-t_W)L - C$$

Growth

$$g = \frac{(1-t_K)(f(L) - Lf'(L)) - r}{s}$$

Welfare

$$W = \frac{1}{1-s} \frac{c^{1-s}}{r - (1-s)g} K_0$$

where $c \equiv C/K$ and

$$c = r(1-t_K) + w(1-t_W)\frac{L}{K} - g$$

## Social planner

Maximise welfare subject to budget constraint

$$\boldsymbol{t}_w s_w + \boldsymbol{t}_K s_K = \boldsymbol{g} \Rightarrow \quad \boldsymbol{t}_K = \frac{\boldsymbol{g} - \boldsymbol{t}_w s_w}{s_K} \text{ with } \boldsymbol{g} \geq 0$$

Maximise

$$\frac{dW / d\boldsymbol{t}_w}{(1 - \boldsymbol{s})W} = \frac{dc / d\boldsymbol{t}_w}{c} + \frac{dg / d\boldsymbol{t}_w}{\boldsymbol{r} - (1 - \boldsymbol{s})g}$$

where

$$\frac{dc}{d\boldsymbol{t}_w} = r\frac{1 - s_K}{s_K} - w\frac{L}{K} - \frac{dg}{d\boldsymbol{t}_w}$$

Then

$$\frac{dW}{d\boldsymbol{t}_w} \stackrel{sign}{=} \frac{dg}{d\boldsymbol{t}_w}$$

maximising welfare and growth is equivalent

# First- Best Optimal Taxation

No revenue requirement: $\boldsymbol{g} = 0$

First-best growth rate

$$g^* = \frac{f(L) - \boldsymbol{r}}{\boldsymbol{s}}$$

Since

$$g = \frac{(1 - \boldsymbol{t}_K) s_K f(L) - \boldsymbol{r}}{\boldsymbol{s}}$$

first-best can be achieved in the competitive economy by

$$\boldsymbol{t}_K^* = -\frac{1 - s_K}{s_K} < 0$$

Budget constraint

$$\boldsymbol{t}_w^* s_w + \boldsymbol{t}_K^* s_K = 0 \quad \Rightarrow \quad \boldsymbol{t}_w^* = 1$$

# Second-Best Optimal Taxation

Government budget constraint

$$gY = \boldsymbol{t}_w wL + \boldsymbol{t}_K rK \qquad \Rightarrow \qquad \boldsymbol{t}_w s_w + \boldsymbol{t}_K s_K = \boldsymbol{g}$$

Recall

$$g = \frac{(1 - \boldsymbol{t}_K) s_K f(L) - \boldsymbol{r}}{\boldsymbol{s}}$$

Maximise growth given $\boldsymbol{g}$

$$\frac{\partial g}{\partial \boldsymbol{t}_K} < 0 \qquad \Rightarrow \qquad \hat{\boldsymbol{t}}_w = 1 \quad \text{and} \quad \hat{\boldsymbol{t}}_K = \frac{\boldsymbol{g} - s_w}{1 - s_w}$$

Chosing $\hat{\boldsymbol{t}}_w$ and $\hat{\boldsymbol{t}}_K$ imply

$$\hat{g} = \frac{f(L)(1 - \boldsymbol{g}) - \boldsymbol{r}}{\boldsymbol{s}}$$

and will also maximise welfare

# Elastic labor supply

Turnovsky, 2000

$$\max \int_0^\infty \frac{1}{1-\boldsymbol{s}} \left( C l^h \right)^{1-s} e^{-rt} dt$$

$$\dot{K} = r(1-\boldsymbol{t}_K)K + w(1-\boldsymbol{t}_W)(1-l) - C(1+\boldsymbol{t}_c)$$

first-order conditions

$$\frac{1}{C}\left( C l^{h} \right)^{1-s} = \boldsymbol{l}(1+\boldsymbol{t}_c)$$

$$\frac{\boldsymbol{h}}{l}\left( C l^{h} \right)^{1-s} = \boldsymbol{l} w(1-\boldsymbol{t}_w)$$

$$\dot{\boldsymbol{l}} = \boldsymbol{l}(\boldsymbol{r} - r(1-\boldsymbol{t}_K))$$

Then

$$g = \frac{(1-\boldsymbol{t}_K)(f(L) - Lf'(L)) - \boldsymbol{r}}{\boldsymbol{s}}$$

$$\frac{C}{K} = \frac{1-\boldsymbol{t}_w}{1+\boldsymbol{t}_C} \frac{f'(L)(1-L)}{\boldsymbol{h}}$$

with $L = 1 - l$

The tax on labor is distortive

However, if consumption taxes are available, the distortion can be offset by choosing

$$\boldsymbol{t}_w = -\boldsymbol{t}_c$$

Then

$$\boldsymbol{t}_K \leq 0$$

and the tax rate on capital should be zero or negative if capital generates an externality

**The taxation of human capital**

Jones, Manuelli and Rossi (1997)

Consider Rebelo's version of the Lucas model

Output

$$y = A(\phi k)^{\beta}(uh)^{1-\beta}$$

where

$$r = \beta A\left(\frac{uh}{\phi k}\right)^{1-\beta}$$

$$w = (1-\beta)A\left(\frac{\phi k}{uh}\right)^{\beta}$$

Human capital technology

$$\dot{h} = ((1-\phi)k)^{\beta}((1-u)h)^{1-\beta}$$

with $\phi, u \in [0;1]$

Consumers

$$\max \int_0^\infty \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

subject to

$$\dot{k} = (1-\tau_k) r \cdot k + (1-\tau_w) w \cdot uh - c$$

$$\dot{h} = ((1-\phi)k)^{\beta}((1-u)h)^{1-\beta}$$

Two decisions:

- how much to consume
- what proportion of the two types of capital to devote to the production of human capital and final output

## Static optimality conditions

$$(1-\tau_k)A\left(\frac{uh}{\phi k}\right)^{1-\beta} = q\left(\frac{(1-u)h}{(1-\phi)k}\right)^{1-\beta}$$

$$(1-\tau_w)A\left(\frac{\phi k}{uh}\right)^{\beta} = q\left(\frac{(1-\phi)k}{(1-u)h}\right)^{\beta}$$

where $q$ denotes the price of a unit of human capital in terms of physical capital

These two equations together yield

$$\frac{\phi k}{uh}\frac{1-\tau_w}{1-\tau_k} = \frac{(1-\phi)k}{(1-u)h}$$

The allocation of factors across sectors depends on the *relative* tax rates

## Dynamic optimality condition

Net rate of return to capital in final-good production

$$\bar{r} = (1 - \tau_k)\beta A(\phi k)^{\beta - 1}(uh)^{1-\beta}$$

Return to $1/q$

$$r^h = (1-\beta)((1-\phi)k)^{\beta}((1-u)h)^{-\beta} + \frac{\dot{q}}{q}$$

Optimum

$$\bar{r} = r^h$$

In the steady state, $q$ is constant, hence

$$(1-\tau_k)\beta A\left(\frac{\phi k}{uh}\right)^{\beta-1} = (1-\beta)\left(\frac{(1-\phi)k}{(1-u)h}\right)^{\beta}$$

The dynamic allocation decision depends on the capital income tax

**Equilibrium**

So

$$\frac{\chi k}{uh}\frac{1-\tau_w}{1-\tau_k} = \frac{(1-\chi)k}{(1-u)h}$$

$$(1-\tau_k)\beta A\left(\frac{\chi k}{uh}\right)^{\beta-1} = (1-\beta)\left(\frac{(1-\chi)k}{(1-u)h}\right)^{\beta}$$

Then

$$\frac{\chi k}{uh} = \frac{A\beta}{1-\beta}\frac{(1-\tau_k)^{1+\beta}}{(1-\tau_w)^{\beta}}$$

hence

$$\bar{r} = (1-\beta)^{1-\beta}(A\beta)^{\beta}(1-\tau_w)^{\beta-\beta^2}(1-\tau_k)^{\beta^2}$$

The rate of growth is

$$g = \frac{\bar{r}-\rho}{\sigma}$$

and $g$ is reduced by both taxes

# Optimal Taxation of Capital and Labor with an Informal Sector

García Peñalosa and Turnovsky (2004)

Formal sector

$$Y_1 = F[L_1 K, K_1]$$

capital intensive

Informal sector

$$Y_2 = Z[L_2 K, K_2]$$

labor intensive

Resource constraints

$$K_1 + K_2 = K$$

$$L_1 + L_2 = 1$$

The aggregate stock of capital yields an externality

Let

$$Y_1 \equiv K_1 f\left(\frac{L_1}{k_1}\right) \qquad \text{with } k_1 \equiv K_1 / K$$

$$Y_2 \equiv K_2 g\left(\frac{L_2}{k_2}\right) \qquad \text{with } k_2 \equiv K_2 / K$$

We can write factor payments as

$$r_1 = r_1\left(\frac{L_1}{k_1}\right) \qquad w_1 = w_1\left(\frac{L_1}{k_1}\right)$$

$$r_2 = r_2\left(\frac{L_2}{k_2}\right) \qquad w_2 = w_2\left(\frac{L_2}{k_2}\right)$$

## Government Policy

Only formal sector can be taxed

$$t_w w_1 L_1 + t_K r_1 K_1 = g Y_1$$

revenue used for lump-sum transfers

$$T = g Y_1$$

## Consumer Optimization

$$\max \int_0^\infty \frac{1}{1-s} C^{1-s} e^{-rt} dt$$

s.t.

$$\dot{K} = \bar{r}_1 K_1 + \bar{w}_1 L_1 + r_2 K_2 + w_2 L_2 + T - C$$

$$L_1 + L_2 = 1$$

$$K_1 + K_2 = K$$

Then

$$g = \frac{\bar{r}_1 - r}{s}$$

# Macroeconomic Equilibrium

*Equilibrium Factor Allocations*

$$(1-\boldsymbol{t}_W)w_1\left(\frac{L_1}{k_1}\right)=w_2\left(\frac{L_2}{k_2}\right)$$

$$(1-\boldsymbol{t}_K)\,r_1\left(\frac{L_1}{k_1}\right)=r_2\left(\frac{L_2}{k_2}\right)$$

$$L_1+L_2=1$$

$$k_1+k_2=1$$

*Equilibrium Growth*

$$g=\frac{(1-\boldsymbol{t}_K)r_1\left(L_1/k_1\right)-\boldsymbol{r}}{\boldsymbol{s}}$$

*Allocation effects of changes in taxes*

For $L_2 / k_2 > L_1 / k_1$

$$\frac{\partial L_1}{\partial \boldsymbol{t}_K} = -\frac{\partial L_2}{\partial \boldsymbol{t}_K} < 0 \qquad \frac{\partial k_1}{\partial \boldsymbol{t}_K} = -\frac{\partial k_2}{\partial \boldsymbol{t}_K} < 0$$

$$\frac{\partial L_1}{\partial \boldsymbol{t}_w} = -\frac{\partial L_2}{\partial \boldsymbol{t}_w} < 0 \qquad \frac{\partial k_1}{\partial \boldsymbol{t}_w} = -\frac{\partial k_2}{\partial \boldsymbol{t}_w} < 0$$

Taxes shift resources to informal sector

Also

$$\frac{\partial L_1 / k_1}{\partial \boldsymbol{t}_K} < 0 \qquad \frac{\partial L_2 / k_2}{\partial \boldsymbol{t}_K} < 0$$

$$\frac{\partial L_1 / k_1}{\partial \boldsymbol{t}_w} < 0 \qquad \frac{\partial L_2 / k_2}{\partial \boldsymbol{t}_w} < 0$$

Taxes make both sectors more capital intensive (as long as formal is more capital intensive)

*Dynamic effects of taxes*

$$\frac{\partial g}{\partial \tau_K} = -\frac{L_2}{k_2}\frac{(1-\tau_w)\alpha f}{\sigma M} = -\frac{\alpha f}{\sigma}\left(1+\frac{L_1}{k_1}\frac{1-\tau_K}{M}\right) < 0$$

$$\frac{\partial g}{\partial \tau_w} = -\frac{L_2}{k_2}\frac{(1-\tau_K)(1-\alpha) f}{\sigma M} < 0$$

where

- $\alpha \equiv 1-\frac{L_1}{k_1}\frac{f'(L_1/k_1)}{f(L_1/k_1)}$ is the share of capital in the formal sector
- $M > 0$ as long as $F(.)$ is more capital intensive than $Z(.)$

## Second-Best Taxation

Growth

$$\max g = \frac{(1-t_K)r_1\left(L_1/k_1\right) - r}{s}$$

$$\text{s.t.} \qquad (1-a)t_w + a t_K = g$$

Then

$$\frac{dg}{dt_w} = -\frac{(1-a)ef}{s}\,\frac{L_2}{k_2}\,\frac{t_K - t_w}{M + (t_w - t_K)L_1/k_1(1-e)}$$

which implies

$$t_K = t_w = g$$

($e$ is the elasticity of substitution in $F(.)$)

Taxes have two distortionary effects

- distort allocation of factors *across* sectors
- change factor intensities *within* each sector

For $\tau_K = \tau_w = \tau$

$$\left. \frac{d(L_2 / k_2)}{d\tau_w} \right|_{\tau_w = \tau_K} = 0$$

$$\left. \frac{d(L_1 / k_1)}{d\tau_w} \right|_{\tau_w = \tau_K} = \frac{1}{1-\tau} \frac{f'}{f''}$$

$$\left. \frac{d((1-\tau_K) r_1)}{d\tau_w} \right|_{\tau_w = \tau_K} = 0$$

Welfare

$$\max \quad W = \frac{c^{1-s}}{(1-\boldsymbol{s})(\boldsymbol{r}-(1-\boldsymbol{s})g)}$$

s.t.

$$(1-\boldsymbol{a})t_w + \boldsymbol{a}t_K = g$$

$$c = k_1 f(L_1/k_1) + k_2 z(L_2/k_2) - g$$

Then

$$\frac{dW/d\boldsymbol{t}_w}{(1-\boldsymbol{s})W} = \frac{dc/d\boldsymbol{t}_w}{c} + \frac{dg/d\boldsymbol{t}_w}{\boldsymbol{r}-(1-\boldsymbol{s})g}$$

For $\boldsymbol{t}_K = \boldsymbol{t}_w$

$$\left.\frac{dW}{d\boldsymbol{t}_w}\right|_{t_w = t_K} \overset{sign}{=} \frac{L_1}{k_1} - \frac{L_2}{k_2} < 0$$

which implies

$$\boldsymbol{t}_w < \boldsymbol{g} < \boldsymbol{t}_K$$

# Infrastructure and the Second-Best Optimum

Two types of government expenditure:

- Redistribution: lump-sum transfer $T = gY_1$
- Infrastructure: $\Phi = fY_1$

Now

$$(1-a)t_w + a t_K = g + f$$

and

$$c = (1-f)k_1 f(L_1 / k_1) + k_2 z(L_2 / k_2) - g$$

As before

$$\left.\frac{dg}{dt_w}\right|_{t_w = t_K} = 0$$

growth is maximized whenever

$$t_K = t_w = t$$

Welfare

$$\max \quad W = \frac{c^{1-s}}{(1-\boldsymbol{s})(\boldsymbol{r}-(1-\boldsymbol{s})g)}$$

s.t. $\quad (1-\boldsymbol{a})t_w + \boldsymbol{a}\boldsymbol{t}_K = g + f$

$$c = (1-\boldsymbol{f})k_1 f(L_1/k_1) + k_2 z(L_2/k_2) - g$$

For $\boldsymbol{t}_K = \boldsymbol{t}_w$

$$\left.\frac{dW}{d\boldsymbol{t}_w}\right|_{t_w = t_K} \overset{sign}{=} (\boldsymbol{f}-\boldsymbol{t})\left(\frac{L_2}{k_2} - \frac{L_1}{k_1}\right)$$

- For $\boldsymbol{g} = 0 \rightarrow \boldsymbol{t}_K = \boldsymbol{t}_w = \boldsymbol{t} = \boldsymbol{f}$

$$\boldsymbol{t}_K = \boldsymbol{t}_w = \boldsymbol{t} = \boldsymbol{f}$$

- For $\boldsymbol{g} > 0 \rightarrow \boldsymbol{t}_K = \boldsymbol{t}_w = \boldsymbol{t} = \boldsymbol{g} + \boldsymbol{f}$

$$\boldsymbol{t}_w < \boldsymbol{g} + \boldsymbol{f} < \boldsymbol{t}_K$$

## Conclusions

- Basic AK model:
    - subsidize capital at $\boldsymbol{t}_K^* = -s_w / s_K$
    - if there is government expenditure, set $\hat{\boldsymbol{t}}_w = 1$ and $\hat{\boldsymbol{t}}_K = (g - s_w)/(1 - s_w)$
- Elastic labor supply
    - $\boldsymbol{t}_K = 0$ if consumption can be taxed
- Impossibility of taxing labor and human capital separately
    - the tax on labor affects growth
- Impossibility of taxing the informal sector
    - taxing labor has both static and dynamics effects
    - maximising the growth : $\boldsymbol{t}_w = \boldsymbol{t}_K$
    - maximising welfare : $\boldsymbol{t}_w < \boldsymbol{t}_K$