Taxation and Long-Run Growth

Cecilia García Peñalosa
CNRS and GREQAM

Lecture 1
Taxation and growth: some basic results

The first lecture provides the basic framework for our subsequent analysis. We will first examine the basic Ramsey growth model, and examine the impact of different types of taxation on the steady state. The basic result, obtained by Chamley, is that capital incomes should not be taxed in the long-run. The second part of the lecture will consider two endogenous growth models, one with government infrastructure and one with human capital. These models combine the negative effect of taxes examined by Chamley with a positive one through the factor that generates growth.

References

Chamley, C. 1986. "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives", *Econometrica.*

Barro, R. 1990. "Government Spending in a Simple Model of Endogenous Growth", *Journal of Political Economy.*

Rebelo, S. 1991. "Long-Run Policy Analysis and Long-Run Growth ", *Journal of Political Economy*.

Taxation and long-run growth

- Traditional view
  - income taxation is harmful for growth
  - capital income should not be taxed in the long-run

- These results imply a trade-off between growth and equality

- Endogenous growth literature
  - emphasis on externalities and market imperfections
  - central role for the government

- What is the role of taxation in the new growth literature?

Lecture plan

1. Taxation and growth: some basic results

- Chamley result
  - no capital income tax in the long-run
- Endogenous growth
  - infrastructure: optimal tax rate
  - human capital: taxation has no effect

2. Should capital income be taxed?

- The AK model
- Second-best results
  - human capital
  - presence of an informal sector

3. Redistribution and growth

- The AK model:
  - imperfect credit markets
- The Schumpeterian growth model
  - redistribution as social insurance

Taxation of Capital Income with Infinite Lives
Chamley (1986)

Introduce taxation in the Ramsey-Cass-Koopmans model

Firms

Output

$$Y_t = F(K_t; L_t)$$

in per capita terms

$$y_t = f(k_t; l_t)$$

where l_t is the individual labour supply, which we are going to assume is inelastic and equal to one

Perfect competition

$$r_t = f'(k_t)$$
$$w_t = f(k_t) - f'(k_t)k$$

Consumers

Utility

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

hence

$$U = \int_0^{\infty} \frac{c_t^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

Government

Taxes

$$\bar{r}_t = (1 - \tau_K) r_t$$
$$\bar{w}_t = (1 - \tau_W) w_t$$

Budget constraint

$$\begin{aligned} \dot{b}_t &= \bar{r}_t b_t + G - \tau_K r_t k_t - \tau_W w_t \\ &= \bar{r}_t b_t + G - (r_t - \bar{r}_t) k_t - (w_t - \bar{w}_t) \\ &= \bar{r}_t b_t + \bar{r}_t k_t + \bar{w}_t - f(k_t; l_t) + G \end{aligned}$$

The individual's problem

$$\max \int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt$$

$$\text{s.t.} \quad \dot{a} = \bar{r}a + \bar{w} - c$$

with $a = b + k$

The present value Hamiltonian is

$$H(c; u; k; h; \lambda; \mu; t) = \frac{c^{1-\sigma}-1}{1-\sigma} + \lambda(\bar{r}a + \bar{w} - c)$$

f.o.c.:

$$\frac{\partial H}{\partial c} = 0 \quad \Rightarrow \quad c^{-\sigma} = \lambda$$

$$\dot{\lambda} = \lambda\rho - \frac{\partial H}{\partial a} \quad \Rightarrow \quad \dot{\lambda} = \lambda\rho - \lambda\bar{r}$$

Hence

$$\frac{\dot{c}}{c} = \frac{\bar{r} - \rho}{\sigma}$$

The social planner's problem

$$\max \int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt$$

s.t.

$$\dot{k} = f(k) - c - G$$

$$\dot{b} = \bar{r}b + G - (r - \bar{r})k - (w - \bar{w})$$

$$\dot{\lambda} = \lambda(\rho - \bar{r})$$

$$\lambda = u_c$$

$$\bar{r} \geq 0$$

From $\lambda = u_c$ we obtain consumption as a function of λ

$$c(\lambda)$$

Hence, the indirect utility function

$$v(\lambda)$$

The present value Hamiltonian is

$$
\begin{aligned}
H = {} & v(\lambda) + \mu_1 \lambda (\rho - \bar{r}) \\
& + \mu_2 (f(k) - c - G) \\
& + \mu_3 (\bar{r} b + G - (r - \bar{r}) k - (w - \bar{w})) \\
& + \mu_4 \bar{r}
\end{aligned}
$$

f.o.c.:

$$\mu_1' = \rho \mu_1 - \frac{\partial H}{\partial \lambda} \tag{1}$$

$$\mu_2' = \rho \mu_2 - \frac{\partial H}{\partial k} \tag{2}$$

$$\mu_3' = \rho \mu_3 - \frac{\partial H}{\partial b} \quad \Rightarrow \quad \mu_3' = \mu_3 (\rho - \bar{r}) \tag{3}$$

$$\frac{\partial H}{\partial \bar{r}} = -\mu_1 \lambda + \mu_3 (b + k) + \mu_4 = 0 \tag{4}$$

Differentiating (4) and using (1)-(3)

$$\dot\mu_4 = \mu_4\rho + \frac{c}{\sigma}z \qquad (5)$$

where

$$z \equiv \lambda - \mu_2 + \sigma\mu_3$$

and

$$\dot z = z(\rho - \bar r) + (\mu_3 - \mu_2)(\bar r - r) \qquad (6)$$

Characterisation of the optimum

- Initially $\bar r \geq 0$ binding $\Rightarrow$ maximal taxation
  Then
  $$\dot\lambda = \lambda(\rho - \bar r) \quad \Rightarrow \quad \dot\lambda = \lambda\rho$$
  and the marginal utility of consumption grows at a constant rate
- But the marginal utility cannot reach ∞
  Hence $\bar r \geq 0$ not binding at t^*
- Then $\bar r \geq 0$ not binding $\Rightarrow$ zero tax

  If $\bar r \geq 0$ not binding $\Rightarrow$ $\mu_4 = 0$
  From (5) $\Rightarrow$ $z = 0$
  From (6) and since $\mu_3 \neq \mu_2$ $\Rightarrow$ $\bar r - r$

Endogenous Growth: Infrastructure

Barro (1990)

Assumption: government expenditures affect the productivity of privately owned factors

Production function:

$$Y_t = AK_t^{1-\alpha}(\gamma_t L_t)^{\alpha}$$

where $0 < \alpha < 1$

Then

$$y = Ak^{1-\alpha}\gamma^{\alpha}$$

Public expenditure is financed by a proportional tax on income, τ

Government –> balanced budget

$$\gamma = \tau Y$$

Then

$$\gamma = \tau A k^{1-\alpha}\gamma^{\alpha} L \quad \Rightarrow \quad \frac{\gamma}{k} = (\tau A L)^{1/(1-\alpha)}$$

Individuals

$$\max \int_0^\infty \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

s.t. $\dot{k}_t = (1-\tau)(rk + w) - c_t$

Euler equation

$$\frac{\dot{c}}{c} = \frac{(1-\tau)r - \rho}{\sigma}$$

Firms

Competitive economy

$$r = (1-\alpha)A(\gamma/k)^{\alpha}$$

$$w = \alpha A(\gamma/k)^{\alpha} k$$

Then

$$\frac{\dot{c}}{c} = \frac{(1-\tau)A(1-\alpha)(\gamma/k)^{\alpha} - \rho}{\sigma}$$

Recall

$$\frac{\gamma}{k} = (\tau A L)^{1/(1-\alpha)}$$

Equilibrium growth rate

$$g = \frac{(1-\alpha)A^{\frac{1}{1-\alpha}}(1-\tau)(\tau L)^{\frac{\alpha}{1-\alpha}} - \rho}{\sigma}$$

Why do we get a constant growth rate if there are CRS?

$$\begin{aligned} y_t &= A\left(\frac{\gamma}{k}\right)^{\alpha} k \\ &= A(\tau AL)^{\alpha/(1-\alpha)}k \end{aligned}$$

The maximum rate of growth in a competitive economy

$$\begin{aligned} \max_{\tau} g \quad &\Rightarrow \quad \max_{\tau}\ (1-\tau)\tau^{\frac{\alpha}{1-\alpha}} \\ &\Rightarrow \quad \tau^{*} = \alpha \end{aligned}$$

then

$$g = \frac{(1-\alpha)^2\alpha^{\frac{\alpha}{1-\alpha}}(AL^{\alpha})^{\frac{1}{1-\alpha}} - \rho}{\sigma}$$

Endogenous growth: human capital

Lucas (1988)

Accumulation of human capital: “time”

Output: $F(K; L^e)$, where $L^e = uhL$

$u \in [0; 1]$

Per capita output

$$y = f(k; h; u) = Ak^{\beta}(uh)^{1-\beta}$$

Human capital accumulation

$$\dot{h} = \phi(u; h) = zh(1 - u)$$

Then

$$\max \int_0^{\infty} \frac{c^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} dt$$

subject to

$$\dot{k} = (1 - \tau)Ak^{\beta}(uh)^{1-\beta} - c$$

$$\dot{h} = z(1 - u)h$$

The Hamiltonian is

$$H = \frac{c^{1-\sigma}-1}{1-\sigma}e^{-\rho t} + \lambda((1-\tau)f(k,h,u) - c) + \mu\psi(h,u)$$

f.o.c.

$$\frac{\partial H}{\partial c} = 0 \quad \Rightarrow \quad c^{-\sigma}e^{-\rho t} = \lambda \tag{1}$$

$$\frac{\partial H}{\partial k} = -\dot{\lambda} \quad \Rightarrow \quad -\dot{\lambda} = \lambda(1-\tau)f_k \tag{2}$$

$$\frac{\partial H}{\partial u} = 0 \quad \Rightarrow \quad \lambda(1-\tau)f_u + \mu\psi_u = 0 \tag{3}$$

$$\frac{\partial H}{\partial h} = -\dot{\mu} \quad \Rightarrow \quad -\dot{\mu} = \lambda(1-\tau)f_h + \mu\psi_h \tag{4}$$

From (1)

$$\frac{\dot{c}}{c} = \frac{-\dot{\lambda}/\lambda - \rho}{\sigma}$$

from (2)

$$\frac{\dot{c}}{c} = \frac{(1-\tau)A\beta(uh)^{1-\beta}k^{\beta-1} - \rho}{\sigma}$$

We need to determine the human to physical capital ratio, uh/k

We can rewrite (3) as

$$\frac{\lambda}{\mu} = \frac{z}{(1-\tau)(1-\beta)A}\left(\frac{uh}{k}\right)^{\beta}$$

In steady state, $g_c = g_k = g_h$ hence

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\mu}}{\mu}$$

Consider now the shadow price of human capital in (4)

$$\begin{aligned} -\frac{\dot{\mu}}{\mu} &= \frac{\lambda}{\mu}(1-\tau)f_h + \phi_h \\ &= \frac{\lambda}{\mu}(1-\tau)f_u\frac{u}{h} - \mu\phi_u\frac{1-u}{h} \\ &= -\frac{\phi_u}{h} \end{aligned}$$

Recall

$$\dot{h} = z(1 - u)h$$

Then

$$\frac{\dot{1}}{1} = -\frac{\phi_u}{h} = -z \Rightarrow \frac{\dot{\lambda}}{\lambda} = -z$$

so that

$$g = \frac{z - \rho}{\sigma}$$

In fact what we have is that preference parameters determine the accumulation of human capital, with

$$1 - u = \frac{z - \rho}{z\sigma}$$

The rate of growth is then simply the rate of accumulation of human capital

$$g = \frac{\dot{h}}{h} = z(1 - u)$$

Since τ does not affect human capital accumulation, it does not affect growth

Revisiting Barro

Now government uses two taxes

$$\gamma = \tau_K r k + \tau_w w$$

Then

$$\max \int_0^\infty \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

s.t. $$\dot{k} = (1 - \tau_K) r k + (1 - \tau_w) w - c$$

and

$$\frac{\dot{c}}{c} = \frac{(1 - \tau_K) A (1 - \alpha)(\gamma / k)^{\alpha} - \rho}{\sigma}$$

Government's budget constraint

$$\gamma = \tau_K (1 - \alpha) k^{1-\alpha} \gamma^{\alpha} L + \tau_w \alpha k^{1-\alpha} \gamma^{\alpha} L$$

Hence

$$\frac{\gamma}{k} = [((1 - \alpha)\tau_K + \alpha \tau_w) A L)]^{1/(1-\alpha)}$$

Substituting

$$g = \frac{\varphi(\tau_K;\tau_w)(1-\alpha)(AL^{\alpha})^{\frac{1}{1-\alpha}} - \rho}{\sigma}$$

where

$$\varphi(\tau_K;\tau_w) = (1-\tau_K)((1-\alpha)\tau_K + \alpha\tau_w)^{\frac{\alpha}{1-\alpha}}$$

What are the optimal tax rates?

Labour tax

$$\frac{dg}{d\tau_w} \overset{sign}{=} \frac{\alpha^2}{1-\alpha}((1-\alpha)\tau_K + \alpha\tau_w)^{\frac{\alpha}{1-\alpha}-1} > 0$$

which implies that growth is maximized at

$$\tau_w^* = 1$$

Capital tax

$$\frac{dg}{d\tau_K} \overset{sign}{=} [\alpha(1-\tau_w) - \tau_K]$$

at $\tau_w^* = 1$, we have

$$\tau_K^* = 0$$

Revisiting Lucas

Rebelo, 1991

Human capital technology

$$\dot{h} = ((1-\phi)k)^{\beta}((1-u)h)^{1-\beta}$$

where $\phi, u \in [0;1]$

Output is now

$$y = A(\phi k)^{\beta}(uh)^{1-\beta}$$

Accumulation of physical capital

$$\dot{k} = y - c$$

Two decisions:

- how much to consume
- what proportion of the two types of capital to devote to the production of human capital and final output

Preferences: as before

Taxation: proportional tax on all income, τ

Static optimality conditions

$$(1-\tau)A\left(\frac{uh}{\phi k}\right)^{1-\beta} = q\left(\frac{(1-u)h}{(1-\phi)k}\right)^{1-\beta}$$

$$(1-\tau)A\left(\frac{\phi k}{uh}\right)^{\beta} = q\left(\frac{(1-\phi)k}{(1-u)h}\right)^{\beta}$$

where q denote the price of a unit of human capital in terms of physical capital

These two equations together yield

$$\frac{\phi k}{uh} = \frac{(1-\phi)k}{(1-u)h}$$

Dynamic optimality condition

Net rate of return to capital in final-good production

$$\bar{r} = (1-\tau)\beta A(\phi k)^{\beta-1}(uh)^{1-\beta}$$

Return to $1/q$

$$r^h = (1-\beta)((1-\phi)k)^{\beta}((1-u)h)^{-\beta} + \frac{\dot{q}}{q}$$

Optimum

$$\bar{r} = r^h$$

In the steady state, q is constant, hence

$$(1-\tau)\beta A\left(\frac{\phi k}{uh}\right)^{\beta-1} = (1-\beta)\left(\frac{(1-\phi)k}{(1-u)h}\right)^{\beta}$$

Equilibrium

Two optimality conditions

$$\frac{\phi k}{uh} = \frac{(1-\phi)k}{(1-u)h}$$

$$(1-\tau)A\beta\left(\frac{\phi k}{uh}\right)^{\beta-1} = (1-\beta)\left(\frac{(1-\phi)k}{(1-u)h}\right)^{\beta}$$

Together they yield the capital-labour intensities

$$\frac{\phi k}{uh} = \frac{(1-\phi)k}{(1-u)h} = (1-\tau)\frac{A\beta}{1-\beta}$$

Recall

$$\bar{r} = (1-\tau)A\beta\left(\frac{uh}{\phi k}\right)^{1-\beta}$$

hence

$$\bar{r} = (1-\tau)^{\beta}(1-\beta)^{1-\beta}(A\beta)^{\beta}$$

The growth rate

The rate of growth of consumption is given by

$$g = \frac{\bar{r} - \rho}{\sigma}$$

hence

$$g = \frac{(1 - \tau)^{\beta}(1 - \beta)^{1-\beta}(A\beta)^{\beta} - \rho}{\sigma}$$

and taxation reduces growth

Note that the impact is smaller than the direct effect

$$\bar{r} = (1 - \tau)r$$

Reason: the tax on income leads to a reduction in the capital/labour intensity in both sectors, as

$$\frac{\phi k}{uh} = \frac{(1 - \phi)k}{(1 - u)h} = (1 - \tau)\frac{A\beta}{1 - \beta}$$

and this increases the gross interest rate, partly offsetting the effect of the tax