Relational Contracts and Their Interactions with Formal Arrangements

—Lecture I—

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...today economists can define their field more broadly, as being about the analysis of incentives in all social institutions.

—Roger B. Myerson, "Nash Equilibrium and the History of Economic Theory," *Journal of Economic Literature* 37 (1999)

Four Channels of Incentives

1. Formal arrangements
2. Informal agreements (“relational contracting”)
3. Market forces
4. “Behavioral” factors

Formal Arrangements

- Explicitly written contracts, ownership, financial structures, etc. → ex ante arrangements
- Enforced by third parties (e.g., courts)
- Contingent only on ex post verifiable performance measures

Relational Contracts

- Informal arrangements sustained by the value of future relationships $\rightarrow$ can be implicit ex ante
- Can be contingent on unverifiable measures
- Must be self-enforcing

In This Lecture Series...

1. The basics of relational contracts
2. The holdup problem in relationships
3. Relational contracts and ownership

I. Relational Contracts

Principal and agent: both risk neutral, living infinitely ($t = 1, 2, \ldots$) with the same discount factor δ

1. At the beginning of each period, they decide whether or not to trade
2. If decide to trade, the agent chooses an action $a_t \in A$
   - $d(a_t)$: the agent's disutility/cost of action
   - Define $\underline{a} = \arg\min_a d(a)$
   - a_t may be observable but unverifiable, or unobservable to the principal (hidden action)
3. Outcome realizes and transfers are made

- x_t: output (observable but unverifiable)
  - $x_t \in X = [\underline{x}, \overline{x}]$
  - Distributed following the probability density function $f(x \mid a)$ under action a
  - Denote $y(a) = E[x \mid a]$
- $W_t = w_t + b_t$: total payment from the principal to the agent
  - w_t: fixed salary enforced by the court
  - b_t: bonus contingent on shared information φ_t $(= \{x_t, a_t\}$ or $\{x_t\})$

- Utility functions
  - Agent: $W_t - d(a_t)$
  - Principal: $x_t - W_t$
- Reservation utilities: $\overline{u}$ (agent), $\overline{\pi}$ (principal), $\overline{s} = \overline{u} + \overline{\pi}$ (total)
- $s(a) = y(a) - d(a)$: expected joint surplus
- Assumption: $\max_a s(a) > \overline{s} \geq \underline{s} \equiv s(\underline{a})$

One-Period Case

- Only the fixed salary is enforced
- Agent thus chooses $\underline{a}$
- No trade

Multi-Period Case

- Relational contract: a complete plan for the relationship

  → For each t and each history up to t, it describes

  (i) the principal's compensation plan

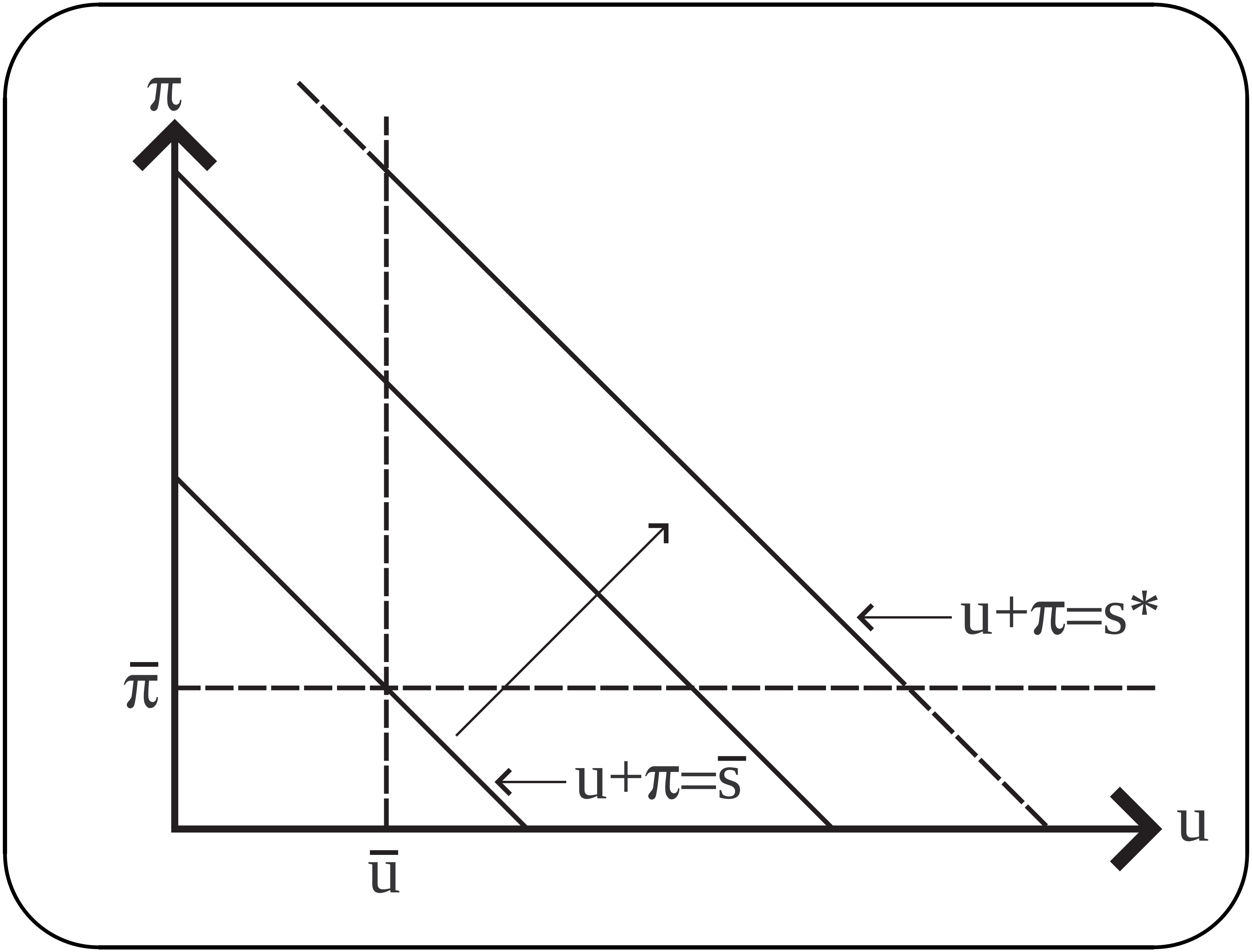
  (ii) the agent's acceptance decision

  (iii) the agent's action

- Focus: self-enforcing relational contracts

“Optimal” Contracts

- There may exist many self-enforcing relational contracts.
- *From a self-enforcing contract with $s > \bar{s}$, one can construct another self-enforcing contract that divides s in any way between them.*
- We can thus naturally focus on optimal self-enforcing contracts, which maximize the expected joint surplus s.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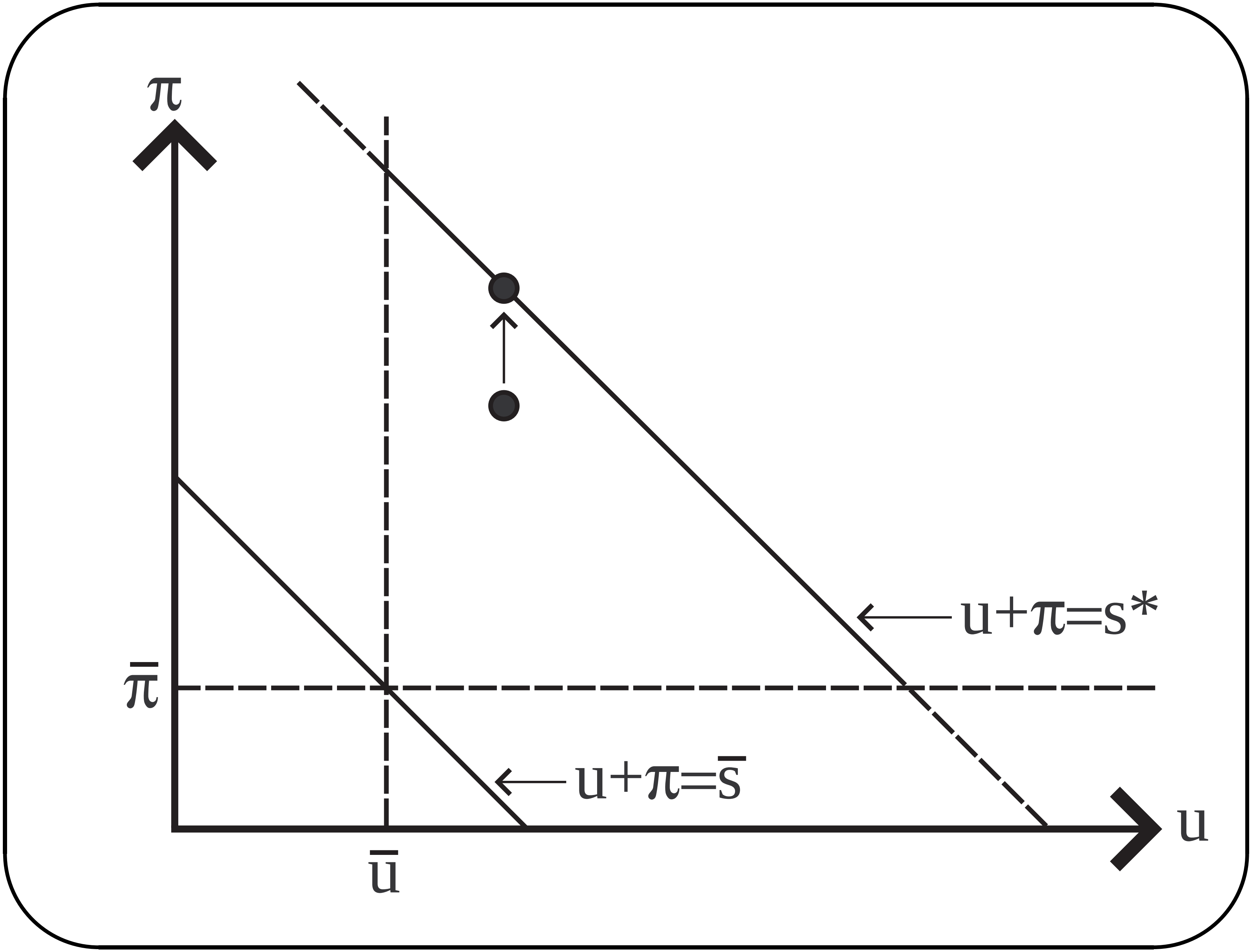
Stationary Contracts

- Stationary contract: On the equilibrium path,
  - the principal always offers the same plan: $W_t = w + b(\varphi_t)$
  - the agent always follows the same action: $a_t = a$
- *If an optimal contract exists, there are optimal stationary contracts.*

Why Stationary Contracts Are Optimal?

1. Following any history with positive probability in equilibrium, $u_t + \pi_t = s^*$ holds.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2. Given an optimal contract $\{w_t, b_t(\cdot)\}$ and action profile $\{a_t(\cdot)\}$, note $y(a_1) - d(a_1) = s^*$ and $s_2(\cdot) = s^*$

3. For a given $u^* \geq \overline{u}$, construct a period 1 contract $\{w^*, b^*(\cdot)\}$
   - that implements a_1;
   - that gives the agent the future per period payoff u^* on the equilibrium path

4. Now the stationary contract: $\{w^*, b^*(\cdot), a_1\}$ in every period
   $\rightarrow$ It is optimal by construction.

$t = 1$ | $t = 2, 3, \ldots$

$w_1, b_1(\cdot), a_1 \xrightarrow{\text{no deviation}} s(\cdot) = u(\cdot) + \pi(\cdot) \quad a_t(\cdot)$

$\downarrow$ deviation

$\overline{s} = \overline{u} + \overline{\pi}$

$w^*, b^*(\cdot), a_1 \xrightarrow{\text{no deviation}} s^* = u^* + \pi^* \quad a_1$

$\downarrow$ deviation in payment

$\overline{s} = \overline{u} + \overline{\pi}$

Example

- $a_t \in \{L, H\}$: observable but unverifiable
- No uncertainty: $y_H = x_H$, $y_L = x_L$
- $s^* = y_H - d_H > y_L - d_L$
- Suppose an optimal relational contract is such that
  - only a fixed payment w_1 in period 1;
  - the future per period payoff depends on a_1;
  - each continuation contract is self-enforcing.

- The contract must satisfy the following:

$$w_1 - d_H + \frac{\delta}{1-\delta} u(H) \geq w_1 - d_L + \frac{\delta}{1-\delta} u(L) \qquad \text{(E1)}$$

- Given $u^* \geq \overline{u}$, construct a stationary contract as follow.

$$b^*(H) = \frac{\delta}{1-\delta}[u(H) - u^*] \qquad \text{(E2)}$$

$$b^*(L) = \frac{\delta}{1-\delta}[u(L) - u^*] \qquad \text{(E3)}$$

$$w^* = u^* - [b^*(H) - d_H] \qquad \text{(E4)}$$

- And if someone cheats on $b^*(\cdot)$, then revert to no trade

- It implements $a_t = H$ because

$$
\begin{aligned}
&[b^*(H) - d_H] - [b^*(L) - d_L] \\
&= \frac{\delta}{1-\delta}[u(H) - u(L)] - (d_H - d_L) \\
&\geq 0
\end{aligned}
$$

- Note that under the stationary contract, deviation from $a_t = H$ has no effect on the future payoffs.

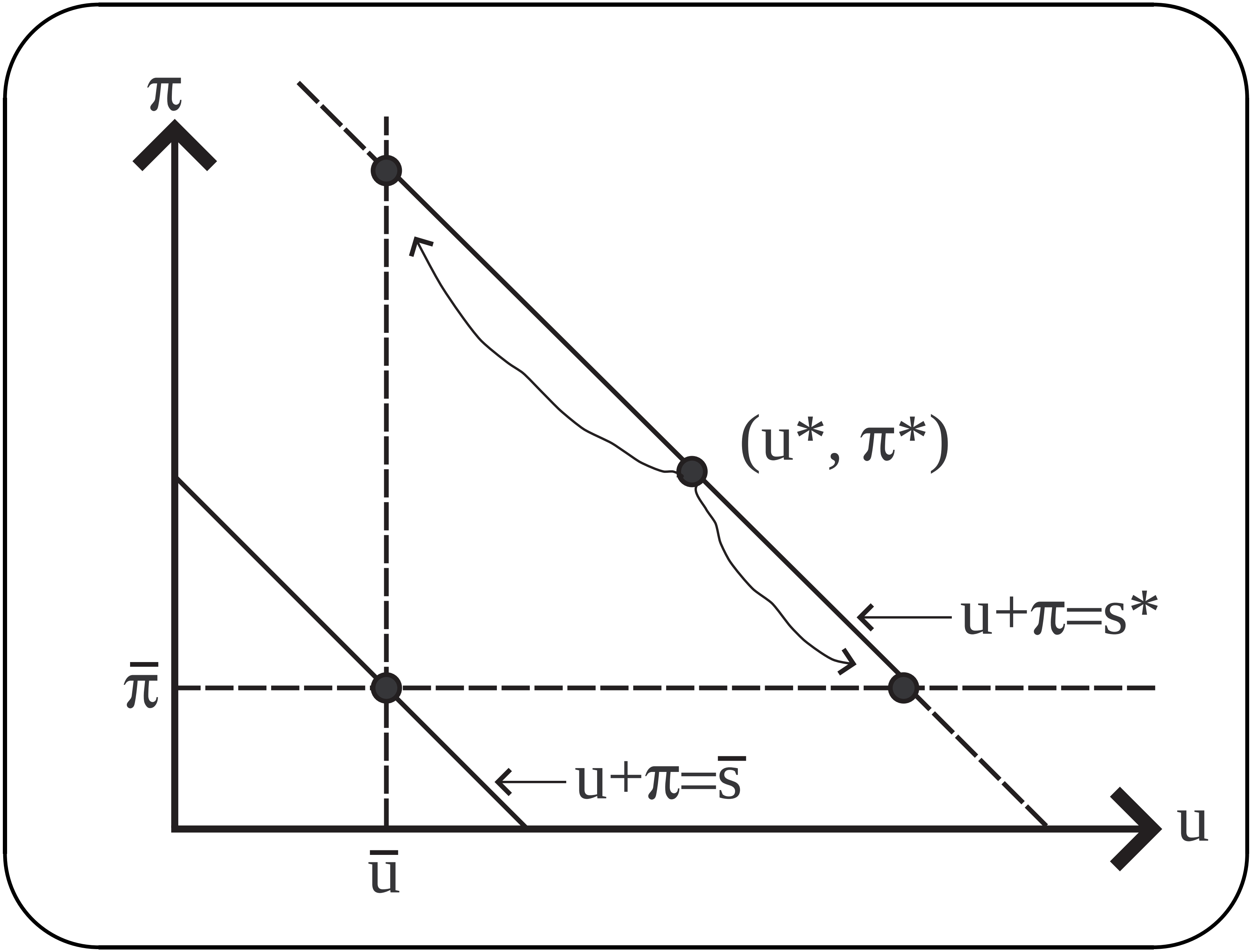
Key Features

1. Quasi-linear utility and monetary transfers

2. No expected surplus is destroyed on the equilibrium path

Off the Equilibrium Path

- Off the equilibrium path
  $\rightarrow$ no trade (the worst equilibrium outcome)
  $\rightarrow$ inefficient
  $\rightarrow$ possibility of renegotiation
- *The optimal stationary contract can be modified so that no gain from renegotiation*
  - Agent deviates $\Rightarrow$ the worst equilibrium for the agent ($u = \overline{u}, \pi = s^* - \overline{u}$)
  - Principal deviates $\Rightarrow$ the worst equilibrium for the principal ($\pi = \overline{\pi}, u = s^* - \overline{\pi}$)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Implementation

- Consider the implementation of $s^* = s(a^*)$
- The agent chooses a^* if and only if

$$a^* \in \arg\max_a E[b(\varphi) \mid a] - d(a) \qquad \text{(IC)}$$

- The principal's reneging temptation: $\sup_\varphi b(\varphi)$
- The principal's future loss:

$$\frac{\delta}{1-\delta}(\pi - \overline{\pi})$$

- The principal does not deviate if and only if

$$\sup_{\varphi} b(\varphi) \leq \frac{\delta}{1-\delta}(\pi - \overline{\pi}) \qquad \text{(DE-P)}$$

- The agent's reneging temptation: $-\inf_{\varphi} b(\varphi)$
- The agent's future loss:

$$\frac{\delta}{1-\delta}(u-\overline{u})$$

- The agent does not deviate if and only if

$$-\inf_{\varphi} b(\varphi) \leq \frac{\delta}{1-\delta}(u-\overline{u}) \qquad \text{(DE-A)}$$

$$\sup_{\varphi} b(\varphi) \leq \frac{\delta}{1-\delta}(\pi - \overline{\pi}) \qquad \text{(DE-P)}$$

$$-\inf_{\varphi} b(\varphi) \leq \frac{\delta}{1-\delta}(u - \overline{u}) \qquad \text{(DE-A)}$$

- Therefore, both do not deviate
  $\Rightarrow$ (IC) and the following dynamic enforcement constraint (DE) must hold

$$\sup_{\varphi} b(\varphi) - \inf_{\varphi} b(\varphi) \leq \frac{\delta}{1-\delta}(s^* - \overline{s}) \qquad \text{(DE)}$$

- Conversely, *the relational contract satisfies (IC) and (DE)* $\Rightarrow$ *both do not deviate*

Back To Example

- The principal and the agent want to implement $a = H$ if implementable.
- Suppose $\Delta d \equiv d_H - d_L > 0$
- WLOG: $b(x, H) \equiv b_H$, $b(x, L) \equiv b_L$.
- Then

$$b_H - b_L \geq \Delta d \tag{IC}$$

$$b_H - b_L \leq \frac{\delta}{1-\delta}(s_H - \overline{s}) \tag{DE}$$

- a_H can be implemented if and only if

$$\frac{\delta}{1-\delta}(s-\overline{s}) \geq \Delta d \quad \Leftrightarrow \quad s \geq \overline{s} + \frac{1-\delta}{\delta}\Delta d$$

- If the principal can make a take-it-or-leave-it offer, then $u = \overline{u}$ and hence $(b_L, b_H) = (0, \Delta d)$. (performance bonus)

- If the agent can make a take-it-or-leave-it offer, then $\pi = \overline{\pi}$ and hence $b_H = 0$ and $b_L < 0$ solve (DE) with equality ("efficiency wage").

Observable Actions: General Case

- (IC) is satisfied by

$$b(a) = d(\underline{a}) \quad \text{for all } a \neq a^*$$
$$b(a^*) = d(a^*)$$

- The dynamic enforcement constraint:

$$d(a^*) - d(\underline{a}) \leq \frac{\delta}{1-\delta}(s^* - \overline{s}) \qquad \text{(DE1)}$$

Extension: Partially Enforceable Actions

- $a = (a_1, a_2)$.
- a_1 is verifiable while a_2 is not.
- What will happen in the one-period case?
  - Define $\alpha_2(a_1) = \arg\min_{a_2} d(a_1, a_2)$
  - Define $\hat{a}_1 = \arg\max_{a_1} s(a_1, \alpha_2(a_1))$.
  - Define $\hat{s} = s(\hat{a}_1, \alpha_2(\hat{a}_1))$

Case 1: No Formal Contract

- Suppose that no formal contract is written at the beginning (except w).
- Suppose $\hat{s} > \overline{s}$
  $\rightarrow$ The principal and the agent then agree to implement $\hat{a}_1$ after either reneges.
  $\rightarrow$ The agent chooses $\hat{a}_2 = \alpha_2(\hat{a}_1)$.
- The dynamic enforcement constraint:

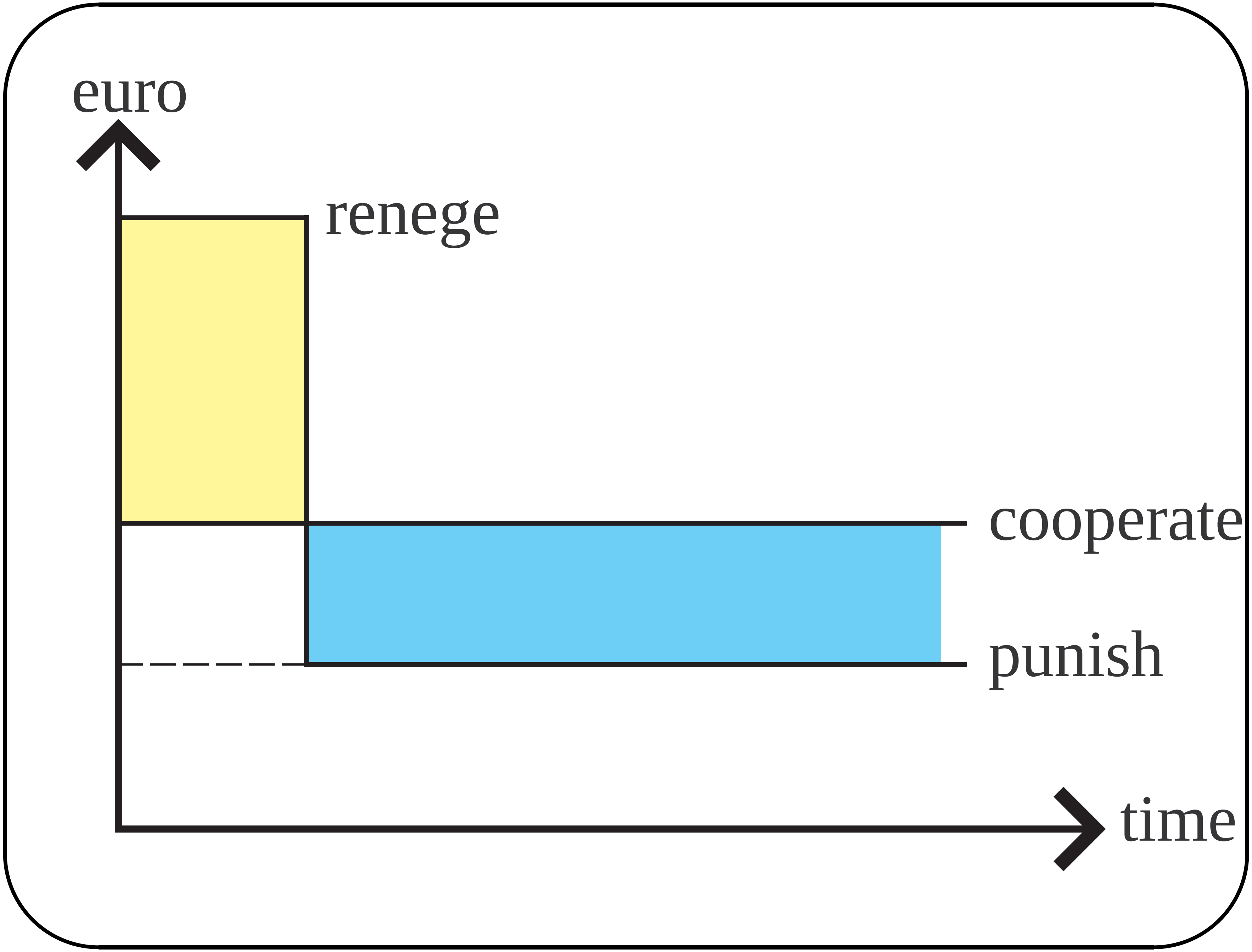
$$d(a^*) - d(\underline{a}) \leq \frac{\delta}{1-\delta}(s^* - \hat{s}) \qquad \text{(DE2)}$$

- Comparison with the fully unenforceable case.

$$d(a^*) - d(\underline{a}) \leq \frac{\delta}{1-\delta}(s^* - \overline{s}) \qquad \text{(DE1)}$$

$$d(a^*) - d(\underline{a}) \leq \frac{\delta}{1-\delta}(s^* - \hat{s}) \qquad \text{(DE2)}$$

- Result: It is more difficult to implement a^*
- If $\hat{s} \leq \overline{s}$, no change from the case of unenforceable actions.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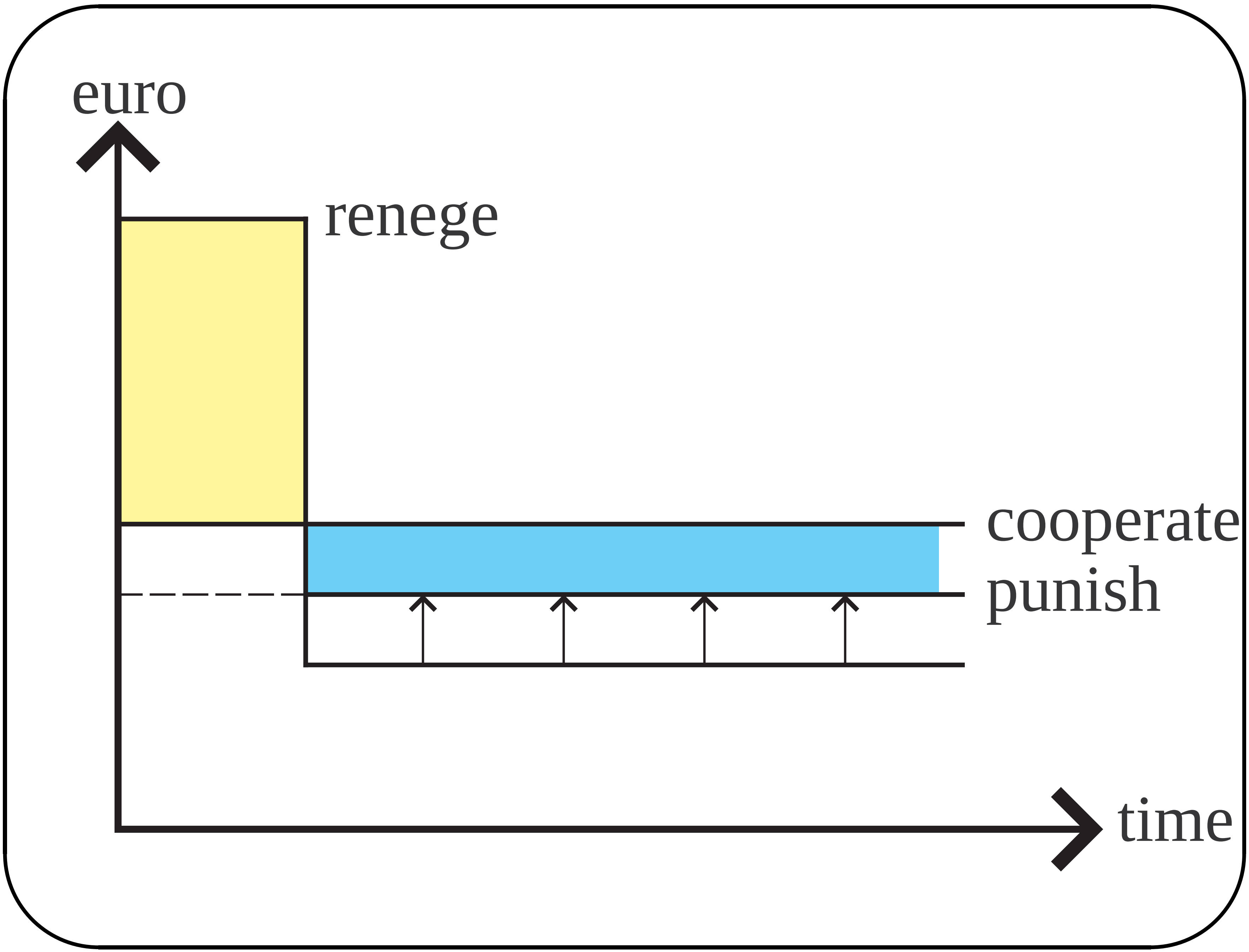
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Case 2: Writing A Formal Contract

- Suppose that the principal and the agent write a formal contract to implement a_1^*
- The dynamic enforcement constraint:

$$d(a^*) - \min_{a_2} d(a_1^*, a_2) \leq \frac{\delta}{1-\delta}(s^* - \max\{\hat{s}, \overline{s}\}) \quad \text{(DE3)}$$

- Comparison:

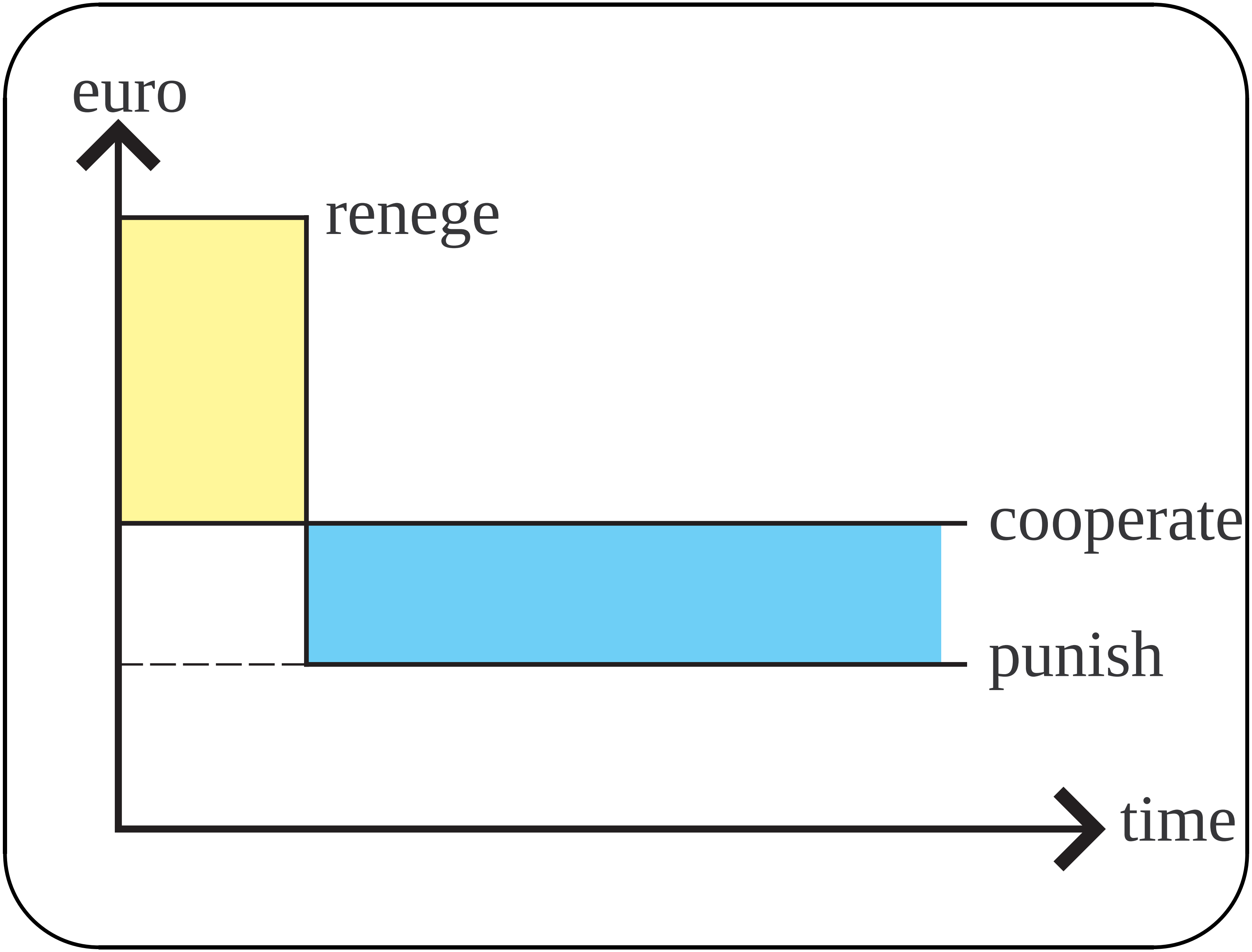
$$d(a^*) - d(\underline{a}) \le \frac{\delta}{1-\delta}(s^* - \overline{s}) \qquad \text{(DE1)}$$

$$d(a^*) - \min_{a_2} d(a_1^*, a_2) \le \frac{\delta}{1-\delta}(s^* - \max\{\hat{s}, \overline{s}\}) \qquad \text{(DE3)}$$

- If a_1 becomes costlessly enforceable, then

  (i) the reneging temptation (of the agent) decreases;

  (ii) the future loss from deviation may decrease.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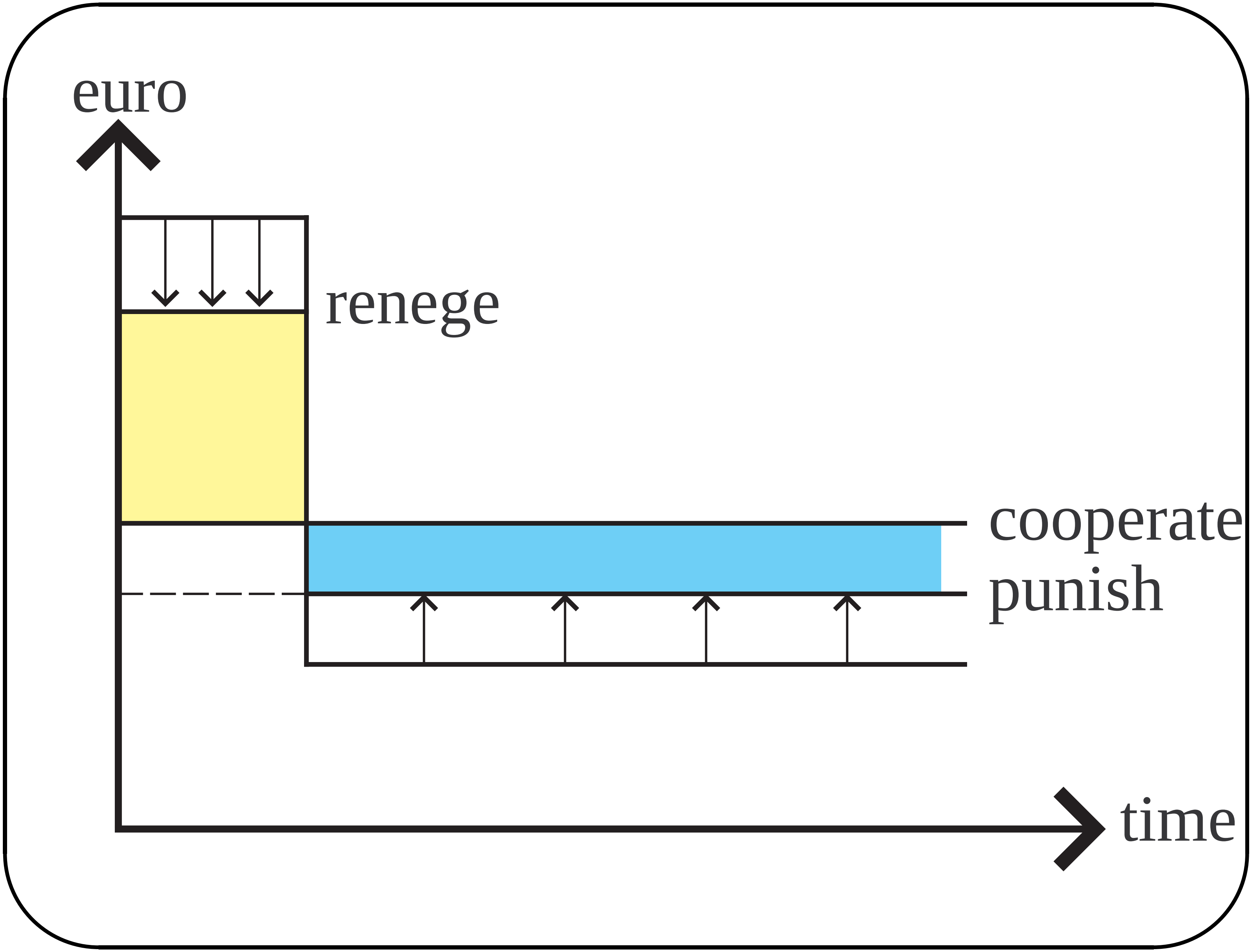
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