Notes on Excludable Public Goods

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The Oakland Model of Competitive Supply

In the small country of Couch Potato, people love to watch soap operas on cable television and eat hot dogs. They consume nothing else. Their currency is pegged to the hot dog, so that the price of a hot dog is 1. Television programming is financed entirely by paid subscriptions from viewers. The cost of producing soap operas is c per minute of program. There is free entry into the soap opera production industry and because of this, soap opera producers make zero profits in equilibrium.

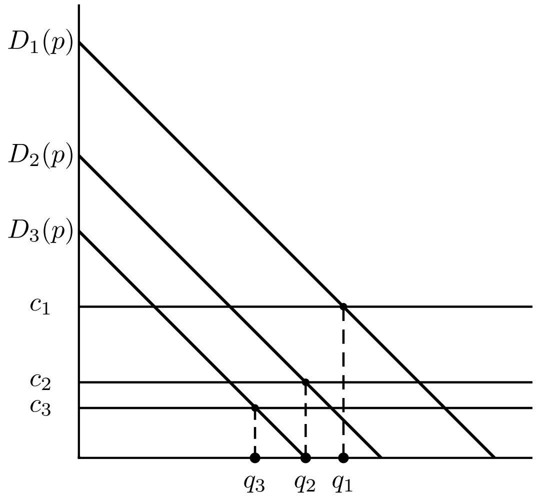
Couch Potato has three kinds of people, which we will call Types 1, 2, and 3. There are 1000 people of each type. People of type i have quasi-linear utility functions of the form

$$U_i(x_i, y_i) = x_i + A_i y_i - \frac{y_i^2}{2} \tag{1}$$

where x_i is the number of units of other $A_1 > A_2 > A_3$. With this utility function, we see that if a type i could buy access to soap operas at the price p per minute, she would purchase $q = A_i - p$ minutes of programming. The total amount demanded by type is at price p is therefore $D_i(p) = 1000(A_i - p)$ Figure 1 shows demand curves for each of the three types.

If all 3000 consumers purchase access to a soap opera, then the supplier could recover its costs by charging each subscriber $c/3000$ per minute. In Figure 1, we have drawn a horizontal line at $c_3 = c/3000$. At this price,

Figure 1: Soap Opera in Couch Potato



the type 3s, who have the lowest demand curve would want to purchase q_3 units, while the type 1s and type 2s would want to purchase more than that. But in order to be able to sell minutes of soap opera at a price of $c/3000$, suppliers must be able to sell to access for these minutes to all three types. Thus in equilibrium, suppliers will supply only q_3 units at price $c/3000$.

Suppose that some suppliers offer to sell additional units of soap opera at a price of $c_2 = c/2000$ in hopes that these units will attract demand from the type 1s and 2s. We see from Figure 1 that if they could buy only at price c_2 then the type 2s would want to buy q_2 units. In fact, type 2s can buy q_1 units at the lower price c_3. Since we have assumed quasi-linear utility, there are no income effects on demand and so the type 2s would buy the q_3 units that are available at price c_3 and would buy the remaining $q_2 - q_3$ units at the higher price c_2. Thus in equilibrium, suppliers would supply $q_2 - q_1$ units at price $c_2 = c/2000$.

There remains a possibility for producers to produce additional minutes of soap opera which would be consumed only by the highest demanders.

These would have to be sold at a price of $c_1 = c/1000$. From Figure 1, we see that at this price, type 1s would demand q_3 units in total. Since they can purchase a total of q_2 units at prices lower than c_1, the number of units they will buy at price c_1 is given by $q_1 - q_2$ in Figure 1

This example is a special case of the model of "competitive provision of public goods" proposed by William Oakland in the 1974 *Journal of Political Economy.*[1]

Monopoly models

A public good is produced by a monopolist who is able to exclude potential users from consuming any unit unless they pay the "price" that the monopolist charges for that unit. The monopolist is unable to tell his consumers apart and is not able to price discriminate. The monopolist can choose both a price to charge and a quantity to produce. Consumers of type z have demand functions for the public good that take the form $q(z,p)$ where p is the price that demanders must pay for the public good. We assume that demand functions are continuous in q and z and that curves of different types of people never "cross each other." That is, for all p, if $z' > z$, then $q(z',p) > q(z,p)$. The cost to the monopolist of producing Q units of public good is $C(Q)$, where $C'(Q) > 0$. Since the public good is "non-rival", each unit of the public good could be sold to every consumer in the economy. If the monopolist produces Q units and sells at a price p, then consumers for whom $q(z,p) > Q$ will be rationed to just Q units of the public good. Consumers for whom $q(z,p) \le Q$ will consume just $q(z,p)$ units.

With little loss of generality[2] , we assume that z is uniformly distributed

[1] Oakland describes equilibrium for a model in which there are many types whose demand curves may possibly cross each other, and where there may be income effects on demand.

[2] We can simply choose units of measurement for the public good so that the distribution of willingness to pay is uniform.

on the interval $[0, 1]$. Clearly the monopolist will never choose to produce a price-quantity pair (p, Q) such that for all z, $q(z, p) < Q$ nor would it choose (p, Q) such that $q(z, p) > Q$ for all z. (In the former case, he could increase his profits by reducing Q, in the latter case by increasing p.) Therefore for any profit-maximizing choice, $q(s, p) = Q$ for some type $s \in [0, 1]$. It follows that the monopolist's decision can described as a choice of a price p, and a marginal consumer type s, such that the quantity supplied is $Q = q(s, p)$.

Define $R(z, p) = pq(z, p)$. The monopolist's profits can be written as a function

$$\Pi(s, p) = \int_0^s R(z, p)dz + (1 - s)pq(s, p) - C(q(s, p)). \tag{1}$$

Let $q_1(z, p)$ and $q_2(z, p)$ denote the partial derivatives of q(z,p) with respect to its first and second arguments and let $R_2(z, p)$ be the partial derivative of $R(z, p)$ with respect to price. The first order conditions for profit maximization are found by differentiating with respect to p and s respectively. These are

$$\int_0^s R_2(z, p)dz + (1 - s)R_2(s, p) - C'(q(s, p))q_2(s, p) = 0 \tag{2}$$

and

$$[(1 - s)p - C'(q(s, p))]q_1(s, p) = 0. \tag{3}$$

From Equation (3) it follows that $C'(q(s, p)) = (1 - s)p$. Substituting into Equation (2), we have

$$\int_0^s R_2(z, p)dz + (1 - s)[R_2(s, p) - pq_2(s, p)] = 0 \tag{4}$$

But $R_2(s, p) = q(s, p) + pq_2(s, p)$. Therefore, Equation (4) simplifies to

$$-\int_0^s R_2(z, p)dz = (1 - s)q(s, p). \tag{5}$$

This condition looks like it might be useful, but what can we make of it. Let us monkey with some special functional forms and try to get explicit solutions and interpretations of this result.

One idea is to express things in terms of elasticities and then take a look at the special case of constant elasticity. Let $\xi(z,p)$ denote the price elasticity of demand for the public good by Type z consumers. Then a little bit of manipulation will show that $R_2(z,p) = (\xi(z,p)-1)q(z,p)$

Therefore Equation (5) could be written as

$$\int_0^s (\xi(z,p)-1)q(z,p)dz = (1-s)q(s,p). \tag{6}$$

Consider the special case where $q(z,p) = f(z)p^{-\xi}$. Then Equation (6) simplifies to

$$(\xi - 1)\int_0^s f(z)dz = f(s)(1-s). \tag{7}$$

It is not hard to show that Equation 7 has at least one solution. Let $H(s) = (\xi-1)\int_0^s f(z)dz - f(s)(1-s)$. Equation (7) will be satisfied if $H(s) = 0$. Then $H(0) = -f(0) < 0$ and $H(1) = \int_0^1 f(z)dz > 0$. From the continuity of H, it follows that there is at least one $\bar{s}$ such that $H(\bar{s}) = 0$. When is there a unique solution for $\bar{s}$? I don't know a good general answer. (By my calculations, $H'(s)$ is not necessarily positive everywhere.)

Having solved for a solution $\bar{s}$ to Equation 7, we can return to Equation 3 to find a solution for the profit maximizing price, $\bar{p}$. For simplicity, suppose that there is a constant marginal cost c of producing the public good. Then from Equation (3), we have $\bar{p} = c/(1-\bar{s})$. The monopolist will produce $\bar{Q} = f(\bar{s})\bar{p}^{-\xi}$.

We can go further if we assume that the function $f(z)$ takes the form $f(z) = \alpha z^\beta$ for some constants, α and β. Then Equation (7) is $\alpha(\xi - 1)s^{\beta+1}/(\beta+1) = \alpha s^\beta(1-s)$. This simplifies to $(\xi-1)/(\beta+1) = (1-s)/s$, which has a unique solution, $s = (\beta+1)/(\xi+\beta)$.

What comparative statics results can we find for this model? It would be good to have some applications in mind so that we can decide what are the interesting questions to investigate. We could compare this solution with various alternative solutions, like oligopoly, competition, regulated monopoly and so on.

Perhaps one should try some other simple special functional forms for demand.

A Problem to Solve

A city has 2 types of people, and 1000 people of each type. There is one private good and one public good. Let X_i denote the amount of private consumption consumed by citizen i and let Y denote the amount of public good available in the city. All type 1's have the utility function $U(X_i, Y) = X_i Y$, type 2's have the utility function $U(X_i, Y) = X_i Y^2$. The price of private goods is \$1 per unit. Type 1's have an income of \$10,000 and Type 2's have an income of $\$15,000$. Public goods can be made from private goods with constant returns to scale. It takes 30 units of the private good to make one unit of the public good. The following questions all relate to alternative arrangements for provision of public goods in this city.

A) Calculate the Lindahl equilibrium prices and quantities for this city. B) Suppose that the public good is excludable and marketed competitively as in the Oakland (1974) model. In the Oakland competitive equilibrium with free entry for firms, how many units will be consumed by the type 1's? the type 2's? What will be the total number of units produced? What will the competitive prices be? How many units of the public good will the low price seller sell? How much will the high price seller sell.

C) Suppose that the public good is excludable and marketed by a monopolist who must charge a uniform price per unit sold and must sell at the same price to all buyers, but who may choose to produce less than the total quantity that some buyers would like at the going price. (i) Suppose that the community has 1,000 type 1 people but no people of the other types, how much of the public good would the monopolist supply and what price will it charge per unit? (ii) Suppose that the community has 1,000 type 1's and 1,000 type 2's, how much of the public good will the monopolist supply, how much will each type consume, and what price will the monopolist charge?