The Equilibrium Real Exchange Rate of the Euro: An Evaluation of Research

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By *Jerome L. Stein*[*]

Contents

I. Introduction

1. Fundamental Determinants of the Equilibrium Real Rate of the Euro

We evaluate studies of the "fundamental determinants" of the real euro, and the underlying transmission mechanism. The authors combine theory with empirical evidence, test hypotheses and show the relevance of their analysis for policy. This article poses several interrelated questions.

- What are the contributions of the studies to the economics of exchange rates? Are they significant in answering important questions of policy? Do they provide useful frameworks to guide empirical and theoretical research?
- How can one explain the medium to longer run movements in the synthetic euro? The many theories of international finance published in the journals and graduate textbooks[1], wax and wane over time. To what extent have these theories been used, or have been found useful, by

[*] I thank the following for valuable suggestions for revision, which have led to a considerable improvement in the paper. My anonymous referee, *Carsten Detken*, *Alistair Dieppe*, *Jérôme Henry*, *Carmen Marin*, *Stylios Makrydakis*, *Bernd Schnatz* and *Frank Smets* at the European Central Bank, *Henri Bourguinat*, *Romain Duval*, *Thorvaldur Gylfason*, *Pascal Kauffmann*, *Giovanna Paladino* and *Karlhans Sauernheimer*.

[1] See the excellent book by *Gandolfo* (2001) that evaluates both the state of the art and the historical developments in international finance. *Stein* and *Paladino* (1997) evaluate the explanatory power of the "state of the art" theories in international finance.

empirical researchers who seek to answer the questions above concerning the exchange rate of the euro?

- What can we expect to happen to the euro with the enlargement of the euro area, under different policies/scenarios?

Many definitions of an "equilibrium" real exchange rate have been used in the literature. For purposes of responding to the questions marked by the bullets above, I define the *equilibrium* value of the real exchange rate as a *sustainable* rate that satisfies several criteria. Conditions (C1) – (C2) are referred to a *medium run equilibrium*. Conditions (C1) – (C4) are referred to as *longer run equilibrium.*

(C1) Internal balance prevails where the rate of capacity utilization is equal to its long run stationary mean.

(C2) External balance exists where there are no speculative capital movements or changes in reserves, and domestic and foreign long-term real rates of interest are equal.

(C3) The ratio of net[2] foreign liabilities/GDP is constant.

(C4) As a result of market forces, the actual exchange rate converges to a distribution whose conditional mean is the "equilibrium" rate.

The focus upon the equilibrium trajectory in the studies evaluated here has been motivated by several factors. (a) The short-term approaches rely very heavily upon anticipations. The explanatory power of the short-term models has been proven to be unsatisfactory. (b) Current macroeconomics, that aims to explain movements in the rate of capacity utilization or inflation, lacks a consensus; and the explanatory power of the dominant theories is weak. Therefore, our focus is upon the equilibrium/sustainable real exchange rate rather than short run exchange rate prediction.[3]

Define "misalignment" as the deviation of the actual real exchange rate from its equilibrium value. The underlying hypothesis is that actual real exchange rate converges to a distribution whose expectation is the equilibrium rate. Misalignment is ephemeral and can be eliminated by a linear combination of changes in the nominal exchange rate and by changes in relative prices.

2. Organization

In all of the studies evaluated here, the researchers constructed a synthetic Euro exchange rate. The hypothesis is that a valid theory concerning the actual real value euro, whose birth was only a few years ago, should be able to explain trends in the *real* value of the synthetic euro based upon many years of data.

[2] Net foreign assets are negative foreign liabilities. We include debt and equity in "net foreign liabilities", and call it "foreign debt".

[3] Our approach should be useful to institutional investors/fund managers who have longer term horizons.

The advent of the European Central Bank can be expected to change monetary policy and relative prices in the euro area. However, the change in monetary policy should not affect the longer-run equilibrium real value of the synthetic euro.

The nominal exchange rate is $N(t)$ = dollars/euro, where a rise is an appreciation of the euro. The real exchange rate $R(t)$ of the euro can be defined in several ways. Generally, the researchers use equation (1), where the ratio $p(t)/p^*(t)$ is the euro/foreign GDP price deflators.[4] The period covered is usually 1973:1 to 2000:1.

$$R(t) = N(t)p(t) / p^*(t) \tag{1}$$

The researchers divided the world into two blocs. The Euro bloc consists of a weighted average of the countries that currently comprise the Euro area, and use as the second bloc[5] a weighted average of the US, UK, Japan and Switzerland. Liliane Crouhy-Veyrac considered the $US vis-a-vis a weighted average of the euro-11. Since we have Crouhy-Veyrac's data, we shall use them as a basis for our empirical estimates.[6]

Figure 1 graphs the two exchange rates: the nominal $N(t)$ = $US/euro denoted EUUSNERMA, and the real $R(t)$ denoted EUUSREDPMA value of the euro, measured as four quarter moving average (MA). Several points should be noted immediately. (a) The real and nominal exchange rates have similar trends. (b) Neither variable reverts to a constant mean. Both are integrated of order I(1), where we use a 4 quarter lag, since the variables are 4Q MA. (c) Both nominal and real exchange rates display significant variation over the sample period. (d) The half-life of the speed at which the real exchange rate would revert to a constant mean, which is the PPP hypothesis, is 23 quarters. This is much too long a period of time[7] to attribute to wage-price inflexibility. Points (a)-(d) induced the researchers to think beyond the PPP hypothesis.

The researchers carefully examined the theoretical literature concerning the determination of exchange rates, in order to find models that are potentially useful to explain the observed movements in the euro. They discarded those models that were: (a) non-operational, in the sense that the crucial variables were not objectively measurable, or (b) whose structural equations have been shown to be inconsistent with the evidence. They ended up by going in two directions. In one direction, they took an empirical/econometric approach that is not model specific. In the other direction they used a theoretical stock-flow-growth model that takes into account conditions (C1) – (C4) above, and which implies explicit econometric equations. The former set of studies may be grouped under the heading BEER, an acronym for the behavioral equilibrium

[4] Some researchers use labor costs instead of broad based indexes. There are advantages and disadvantages to each measure. See, for example, *Clostermann* and *Friedmann* (1998) and Deutsche Bundesbank (1995).

[5] See table 1 for the different reference currencies used in the empirical studies.

[6] They are similar to those used by the ECB researchers.

[7] The unit root equation, $\Delta R(t) = -0.03[R(t-1) - C]$ implies that $R(t) = C + (.97)^t [R(0) - C]$. Therefore the half-life is 22.8 quarters and C is the PPP value.

real exchange rate[8], and the latter set of studies takes as a point of departure the Natural Real Exchange Rate NATREX model. Each one has a very different concept of "equilibrium".

Table 1

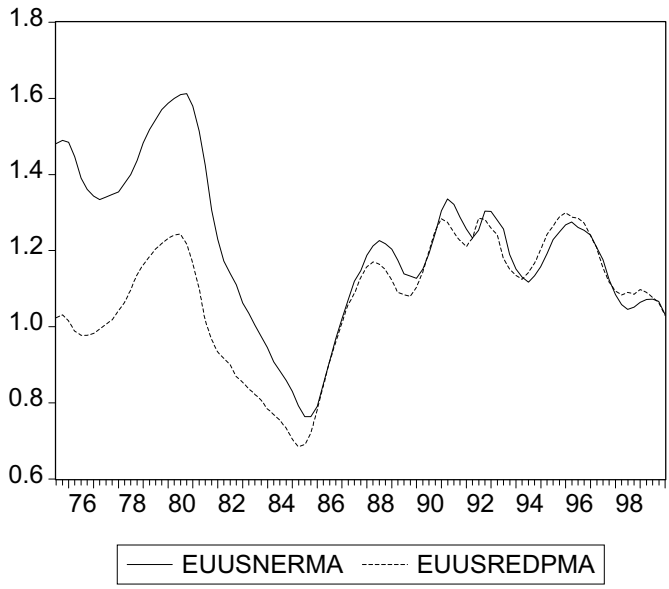
Survey of Beer Studies of Equilibrium Real Value of Euro

Study	Methodology	Fundamentals Z(t)	Reference Currency
Gern et al (2000)	BEER, UIP	Interest rate differential	$US
Clostermann, Schnatz (2000)	BEER	Internal price ratio, interest rate differential, government spending, oil prices	$US
Makrydakis et al (2000)	BEER	Interest rate differential, relative labor productivity, net foreign assets	Effective partners
Lorenzen, Thygesen (2000)	BEER	Net foreign assets, dependency ratio, Internal price ratio, R&D expenditures; interest rate differential; output gap, interest rate differential	$US
Koen et al (2001)	BEER	GDP per capita, dependency ratio, interest rate differential, oil prices	Effective partners
Maseo-Fernandez, Osbat and Schnatz (2002)	BEER/PEER	Productivity, internal price ratio, interest rate differential, government spending, oil prices	Effective partners
OECD (2001)	BEER	internal price ratio, net foreign assets, government consumption/GDP, real oil price, demographic balance	Effective partners; $US
Alberola et al (1999)	PEER	internal price ratio, net foreign assets	$US
Teiletche (2000)	BEER	Productivity, government spending, interest rate differential	$US
Hansen, Roeger (2000)	PEER	internal price ratio, net foreign assets	Effective partners
Wren-Lewis and Driver (1998)	FEER	Internal/external balance	$US
Borowski and Couharde (2000)	FEER	Internal/external balance	$US
IMF (2000)	Saving-investment		$US

Sources: See European Central Bank (2002: Box 2) for the references and a critique of many of these studies.

[8] The BEER approach is based upon *Clark* and *MacDonald* (1999).

Figure 1



Note: The real value of the euro relative to the $US: $R(t) = N(t)p(t)/p^*(t)$ denoted EUUSREDPMA, and nominal value of the euro relative to the $US: $N(t)$ = $US/euro denoted EUUSNERMA. A rise is an appreciation of the euro. MA = 4Q moving average.

Most of the empirical studies take the BEER approach, where one does not specify the underlying model. A partial list of these studies is presented in table 1. These studies take an eclectic approach that simply searches for cointegrating equations in "sensible" variables[9], which pass the usual econometric tests. The VECM vector error correction method used by the authors restricts the long run behavior of the endogenous variables to converge to their cointegrating equations, while allowing a wide range of short run dynamics. As MacDonald (2002) noted, the authors obtain a *statistical/atheortetic* concept of the "equilibrium". The statistical/atheoretical approach does not correspond to any particular analytical framework. The concept of equilibrium in the BEER studies is just a cointegrating equation.

Both approaches are positive, and not normative, economics. There is no welfare significance, or value judgments, that the derived equilibrium real ex-

[9] Outstanding papers using this approach are by *Clostermann* and *Schnatz* for the D-Mark, and by *Maseo-Fernandez*, *Osbat* and *Schnatz* for the synthetic euro. See *MacDonald* and *Stein* (1999, ch.1) and *Stein* and *Lim* (2002) for a discussion of these approaches.

change rate is "optimal" in some sense. There is no implication that exchange rates should be managed. The principal difference between the BEER and the NATREX, is that the NATREX takes as its point of departure a specific theoretical dynamic stock-flow growth model where the equilibrium satisfies conditions (C1) – (C4), and the underlying structure, linking endogenous, exogenous and control variables is explicit. The NATREX model is an analytical framework: it comes with a story. On the other hand, the BEER is a statistical/atheoretical approach that does not correspond to any particular analytical framework.

In part II the BEER results are evaluated, and are compared in summary table 2. Part III is a brief exposition of the NATREX model that is used in the studies listed in table 3. In part IV we evaluate the research that examine the structural equations as well as the implied reduced form equations. The results are summarized in table 4. A terse summary of our message is the conclusion, part V.

II. The Statistical/Atheoretical Approach

The "B" for "behavioral" in BEER means that there is no explicit underlying structural model. It is exclusively a quest for a cointegrating equation for the real exchange rate. There are differences in the approaches and results in the various papers, but I shall try to present them in terms of their common characteristics.

The first thing to observe from table 1 is that there are many variables that the authors consider as "fundamentals". The methodology is clearly specified by the authors of the OECD study.

"Different sub-sets of determinants were tested in turn…until a significant and economically meaningful cointegrating vector was identified…The long-run relationship was then tested for cointegration directly with the Johansen test…the residuals of the cointegrating vector were also tested for stationarity using the augmented Dickey-Fuller and Phillips-Perron tests" (OECD 2001, p. 29).

Second, there are therefore $n > 2$ candidate variables for a cointegrating equation $R = R[Z]$, where Z is a vector of candidate variables listed in table 1. Since the number of sensible elements in Z is n, the number of possible regressions is 2^n. When $n = 8$, there are 256 possible regressions, using a given technique and lag structure. If there are $m > 2$ techniques, lag structures and choice of smoothing variables into "permanent" and "transitory" components, then there are $2^{(m+n)}$ possible regressions. If $m = 4$, then there are $4096 = 2^{12}$ possible regressions. That is why the number BEER studies grows as a power function. We restrict our evaluation to a small number of representative BEER studies, which are listed in table 2.

A strength of the statistical/atheoretic approach is that the underlying model is not specified, so the results do not depend upon any one model that may be misspecified. This has many useful advantages, since one can never be sure that he has the correct model. But it comes at a high price. We shall show two things. (1) The qualitative/sign results concerning the significance and sign of variable z_i depend significantly which other variables are or are not included in

the specific regression. (2) It is often very difficult to provide an economic interpretation of the results. That is why it is difficult to use the statistical/atheroetical method to answer the questions in bullets at the beginning of this article. This is the subject of the next sections.

Table 2

BEER: Longer Run Determinants of Real Value of Euro and DM

Fundamental	Makrydakis, de Lima, Claessens and Kramer (2000)	Maseo-Fernandex, Osbat and Schnatz (2002): model I	Maseo-Fernandex, Osbat and Schnatz (2002): model IV	Lorenzen and Thygesen (2000)	Clostermann and Friedmann DM (1998)	OECD (2001: pp.29-31)
Relative labor productivity	Appreciate	Appreciate	-		Appreciate	
Real price of oil		Depreciate	Depreciate			Depreciate
Relative government expenditures		ns	Depreciate			Depreciate
Net foreign assets NFA	Not significant	Depreciate (wrong sign); weakly exogenous	-	Appreciate	Not significant	Not significant, depreciate (wrong sign)
Real long term interest differential	Appreciate	Appreciate	-		Appreciate	
Internal price ratio CPI/WPI	Not significant		Appreciate	Appreciate	Feeble effect	Not significant
Dependency ratio				Appreciate		

1. An Evaluation of the Statistical/Atheoretic Studies

Table 2 compares six carefully executed studies that use the BEER approach in terms of their common characteristics. All the studies agree that there are real variables that can produce a cointegration equation with the real exchange rate. Each cointegrating equation passes the usual econometric tests and does track the real value of the synthetic euro and the real value of the D-Mark. The authors often show that their equation for R[Z(t)] outperforms a random walk and the superiority improves as the horizon increases. The real value of the euro/$US is not a stationary, *constant mean reverting*, variable, as claimed by the PPP hypothesis.

Six main variables, vector $Z(t)$ in the rows in table 2, are considered as possible fundamentals. Maseo-Fernandez, Osbat and Schnatz considered five mod-

els, using the same data set. We have selected their models I and IV to show the sensitivity of the results, the effect of variable z_i upon the real exchange rate, depending upon what are the other regressors. Although each study succeeds in finding a cointegrating equation, the studies arrive at contradictory and often puzzling results. Consider several variables across the four studies.

Two measures of productivity are used. One is measured directly as GDP per worker. The other concerns the Balassa/Samuelson effect discussed in section II.2 below. The direct measure of productivity – GDP per worker – appreciates, and the real price of oil, depreciates the real exchange rate. These are "sensible" results within almost any model.

Net foreign assets NFA is the sum of the current accounts. NFA is included as a variable in five of the six studies in table 2. In three studies it is not significant, in one study it depreciates the real exchange rate and in only one study does it appreciate the real exchange rate. Moreover, Maseo-Fernandez, Osbat and Schnatz found that the net foreign assets variable was weakly exogenous. These results are not consistent with basic economic theory. In almost any transmission process, the current account is an endogenous variable that reacts to, and subsequently affects, the real exchange rate. Net foreign assets generate interest and dividend income and should appreciate the exchange rate. How can net foreign assets be exogenous in an equation/economic model that that determines the long run equilibrium real exchange rate?

Another variable is relative government expenditures, or fiscal policy. The role of fiscal policy in exchange rate determination is important for theory and policy. For example, will high employment budget deficits appreciate or depreciate the real exchange rate? In the Maseo-Fernandez, Osbat and Schnatz article, the significance of government expenditures depends upon what measure of productivity is included in the regression. When relative labor productivity is used (row 1), then the fiscal variable is not significant. If the ratio of the CPI/WPI (row 6) replaces labor productivity, then the fiscal variable is significant. What shall one conclude from these regressions about the effects of an expansionary fiscal policy upon the real exchange rate? The authors of the statistical/atheoretic studies focus upon econometric technique rather than upon the economic meaning of the results.

2. The Balassa/Samuelson B/S Effect

The B/S effect claims that the law of one price (PPP) applies to the traded goods sector, but not to the non-traded goods sector. Therefore the real exchange rate, measured in terms of broad based indices such as the CPI deflators, will vary in proportion to the relative prices of non-traded/traded goods in the two countries. Explanations of trends in the real exchange rate would then have to be based upon the determinants of relative prices in the two sectors. The latter varies insofar as productivity of inputs differs between the two sectors in the two countries. Then a hypothesis is added that labor productivity grows more rapidly in the traded goods sector. Hence the more productive countries tend to have appreciating real exchange rates. The questions are: how to

measure the B/S effect? Is this effect economically significant? Recent work casts serious doubt concerning the importance of the B/S effect for the European economies.

We formalize the B/S effect in equations (2), (2a), (2b) and (2c). Then we explain how different authors measure the variables. Finally, we describe the empirical results. The real exchange rate $R(t) = R(CPI)$ based upon broad based price indexes such as the *CPI* is the product of the constant "external" price ratio $R(T)$ of traded goods in the two countries and an "internal" price ratio $R(NT)$, the price ratio of nontraded/traded goods in the two countries The "law of one price" for traded goods is that $R(T) = C$, a constant. The weight of non-traded goods in the *CPI* is fraction *w*. The B/S hypothesis is that variations in the real exchange rate $R(CPI)$ derive from variations in $R(NT)$. The real exchange rate $R(CPI)$ changes in proportion to $R(NT)$, equation (2c). Most authors use the ratio of *CPI/WPI* as their measure of the internal price ratio $R(NT)$. This implies equation (2d).

(2) $$R(CPI) = N(t)p(t) / p^*(t) = R(T)R(NT)$$

(2a) $$R(T) = [N(t)p(T;t) / p^*(T;t)]\, p(T;t) = \text{ price of traded } (T) \text{ goods at time } t.$$

(2b) $$R(NT) = [p(N;t) / p(T;t)]^w [p^*(N;t) / p^*(T;t)]^w$$

$$p(N;t) = \text{ price of non-traded } (N) \text{ goods at time } t.$$

(2c) $$\log R(CPI(t)) = C + w \log [p(N;t) / p(T;t)] / [p^*(N;t) / p^*(T;t)]$$

(2d) $$\log R[CPI(t)] = C + a \log [CPI(t) / WPI(t)] / [CPI^*(t) / WPI^*(;t)]$$

It is clear that the CPI is on both sides of the equation (2d). Therefore, regressions of this type tend to find statistical significance. Even on that level, consider row 6 in table 2. In two studies the ratio of the CPI/WPI is not significant, in two the variable appreciates the real exchange rate, and in one the effect is feeble.

A recent study by Duval (2002) measures the relative price R(NT) directly, using sectoral value added data from the OECD data base over 1970-96 for 19 sectors and 14 countries. He derives sectoral price deflators and then he classifies the sectors as tradable/nontradable based upon the degree of openness. Based upon direct measures of $p(N, t)/p(T; t)$ he arrives at the following conclusions.

- The long run fluctuations of the real exchange rates do not seem to be explained by those of relative prices of nontradables.
- Long run PPP does not seem to be verified for tradable goods, particularly for the main currencies (dollar, yen, mark).

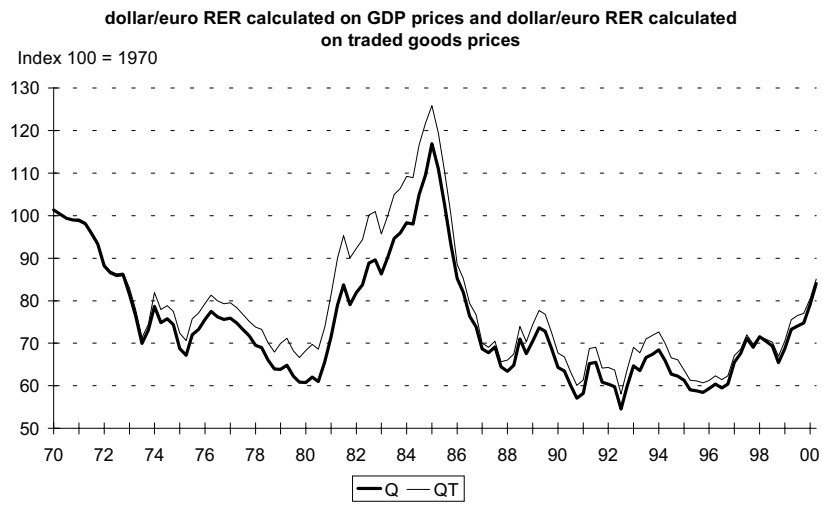
Duval's results are seen in figure 2 for the euro/$US real exchange rate, where a rise is an appreciation of the $US or a depreciation of the synthetic euro. The dark curve ($Q = R(t)$) is the real exchange rate calculated using GDP prices, and the light curve ($QT = R(T)$) is the real exchange rate calculated on traded goods prices. The B/S effect is the distance between the two curves. It is seen that the real exchange rate $R(t)$ = Q is primarily explained by the relative price of traded goods $QT = R(T)$. On the basis of figure 2, the PPP and B/S effects are seen to be incapable of explaining variations in the real synthetic

euro.[10] Duval's results are consistent with those of Clostermann and Friedmann (1998) for the D-Mark. They treated the prices of services and rents as prices of nontradables, and prices of consumer durables and other consumer goods as tradables. They wrote that:

> "[Figure 3 page 215] shows Germany's relative internal price ratio compared with a trade-weighted average of this group of 10 countries…[$R(NT)$]..is remarkably constant, and – accordingly – the real effective exchange rate on the basis of the overall *CP*I…[$R(CPI)$]..seems to be nearly identical with the real exchange rate based upon prices for tradables…[$R(T)$]. On balance so far, not much evidence in favour of a "Balassa-Samuelson effect" in broadly defined real effective D-Mark exchange rates seems to exist in the data under consideration".

Figure 2

The Real Value of the $US relative to the Synthetic Euro Q – Rise is a Depreciation of the Euro or Appreciation of $US – and Real Value in Terms of Tradables QT.



What can we conclude from these six statistical/atheoretic studies concerning the questions in bullets at the beginning of this paper? The studies in table 2 yielded different and often contradictory results, even though each obtained a cointegrating equation with significant values for different vectors of "fundamentals" $Z(t)$ and each one tracked the euro. The variables in the cointegrating

[10] Figure 2 is copied from his paper. A rise in his measure of the exchange rate is an appreciation of the dollar or a depreciation of the synthetic euro. *Duval* found the same results for Germany and France.

equations are mixtures of endogenous, control and exogenous variables. Without an explicit theoretical structure telling us what variables should be included in the regressions, it is difficult to know how to interpret the econometric results.

III. Structural Equations Determining the Equilibrium Real Exchange Rate: NATREX

In view of the problems of interpretation of the statistical/atheoretic method, several authors listed in table 3 proceeded to examine structural equations of a coherent theoretical model to explain the fundamental determinants of the real euro, and the underlying currencies. Detken, Dieppe, Henry, Marin and Smets (2002) wrote the following:

> "A further step towards increasing the structure underlying the estimated model is to estimate a number of behavioural relations as commonly found in standard structural macroeconometric models. To begin with, we consider a small-scale model based upon the NATREX approach (NATural Real Exchange rate)...This approach tries to link the real exchange rate to a set of fundamental variables explaining savings, investment and the current account. Natrex is based upon a rigorous stock-flow interaction in a macroeconomic growth [model]. A distinction is made between a medium run equilibrium where external and internal equilibrium prevails (equivalent to the macroeconomic balance approach) and the long-run equilibrium where the budget constraint on net foreign debt is met and the capital stock has reached its steady state level".

Detken, Dieppe, Henry, Marin and Smets, and Detken and Marin Martinez, described the NATREX model and estimated several key structural equations. From these equations, they inferred the equilibrium real exchange rate and compared the inferred equilibrium rate with the actual synthetic real Euro. Other authors, such as Duval (2002), Maurin (2000), estimated the implied reduced form equations of the NATREX model, to explain the evolution of the synthetic real euro. Stein (1999) estimated the implied reduced form equation for the $US/G7. Several authors used the model to explain the trends in the underlying currencies of the euro area. Federici and Gandolfo (2002) estimated structural equations for the Italian Lira, and Verrue and Colpaert did something similar for the Belgian Franc. Fischer and Sauernheimer (2002) estimated reduced form equations for the D-Mark. Stein and Paladino(2001) did the same for the Franc/DM and Italian Lira/DM and explained the currency crises of 1992-93. Crouhy-Veyrac and Saint Marc used the model to explain the real French Franc/DM exchange rate. The studies in table 3 obtain consistent results.[11]

[11] The papers concerning the euro dated in the references (2002) are published together in a special issue of the Australian Economic Papers December 2002: Exchange rates in Europe and Australasia. The Introduction by *Stein* and *Lim* puts the papers into perspective. The authors: *Detken*, *Dieppe*, *Henry*, *Marin* and *Smets*, as well as *Maseo-Fernandez*, *Osbat* and *Schnatz*, are researchers at the European Central Bank. Their articles, in the journal issue cited, do not represent an official points of view. Their papers reflect what the staff thinks about the usefulness of existing theoretical literature for an ex-

Table 3

NATREX Studies of the Synthetic Euro and Component Currencies

Author	Currency	Estimation
Detken, Dieppe, Henry, Marin and Smets (2002)	synthetic euro	structural equations
Detken and Marin Martinez (2001)	synthetic euro	structural equations
Duval (2002)	synthetic euro	reduced form dynamics
Maurin (2000/01)	synthetic euro	reduced form dynamics
Fischer and Sauernheimer (2002)	D-Mark	reduced form dynamics
Stein and Sauernheimer (1997)	D-Mark	reduced form dynamics
Federici and Gandolfo (2002)	Italian Lira	structural equations
Crouhy-Veyrac and Saint Marc (1997)	French franc/DM	reduced form dynamics
Stein (1999)	$US/G7	reduced form dynamics
Verrue and Colpaert (1998)	Belgian franc	structural equations
Stein and Paladino (2001)	French franc/DM, Italian Lira/DM	reduced form dynamics

Part III.1 very tersely describes the crucial structural equations of the NATREX model. Part III.2 explains the transmission mechanism linking the endogenous real equilibrium exchange rate to the exogenous and control variables, and the implications for econometric testing. This is the structure that is ignored in the statistical/atheoretic studies above. Part IV evaluates the econometric results of these studies.

1. The NATREX Model[12]

Basic Structure

The specific characteristics of the NATREX model vary according to the economy, as explained by Allen (1997). The NATREX model has a rational expectations foundation for the *private sector*, based upon stochastic optimal control models developed by Fleming and Stein. For the economy as a whole, which includes the public sector where decisions are political, decisions may be far from optimal.

planation of the value of the euro. The issue also contains an article by *Otmar Issing* and *Vitor Gaspar*, of the ECB, which is policy oriented.

[12] The reader is directed to the following references for a full exposition of the NATREX model: *Stein*, *Allen* et al (1997 ed.), *Allen* (1997), *Stein* (1999), *Stein* and *Paladino* (2001). This analysis is positive, not normative, economics. An analysis of optimal growth and debt under uncertainty, using dynamic programming/stochastic optimal control is in *Fleming* and *Stein* and in a related context by *Infante* and *Stein*.

Equations (3) (4) and (5) are the core of the model. Macroeconomic balance equation (3) is evaluated at a rate of capacity utilization equal to its stationary mean – *internal balance* – and where real long term rates of interest are equal at home and abroad – *portfolio balance*. The medium run equilibrium real exchange rate equates the sum of the current account CA(t) plus the non-speculative capital inflow, equal to investment less social saving [$I(t)$ - $S(t)$], to zero. Social saving is the sum of the saving of firms, households and the government. Government saving is the high employment budget surplus (+)/deficit (-). The current account is the trade balance $B(t)$ less the net flow of income transfers $r(t)F(t)$ from the Euro area to the US (rest of the world). Variable $F(t)$ is the stock of net debt plus equity claims of the US on Europe. The left hand side of (3) is the excess demand for euros relative to the dollar. If it is positive, then the euro appreciates (rises) relative to the dollar, reduces the trade balance and restores the equality to zero.[13] The exogenous/control vectors Z are discussed below.

The NATREX model generalizes the macroeconomic balance models by adding dynamic equations (4) for the change in the debt plus equity, and (5) for the growth rate. At macroeconomic balance, the current account can be positive or negative. A current account deficit equals the capital inflow, which is the endogenous change in net foreign claims in the form of either debt or equity. Refer to debt plus equity as "debt" or net foreign liabilities. The change in the debt affects the current account and feeds back upon the macroeconomic balance equation (3).

Equation (4) implies that the percentage change in the ratio $F(t)$ of net debt(+)/GDP is equal to: (i) the capital inflow/debt, equal to investment less social saving, divided by $F(t)$ less (ii) the growth rate $g(t) = (\mathrm{d}Y(t)/\mathrm{dt})/Y(t)$ of GDP.

Equation (5) is the growth rate of GDP. The endogenous part is proportional to the investment/GDP ratio and the exogenous part is contained in vector z_4. It is a generalization of the AK production function.[14]

Whereas the macroeconomic balance models are exclusively concerned with equation (3), the NATREX adds dynamic equations (4) and (5), which *feed back upon the macroeconomic balance equation (3).* The implications of the additions are that many effects of changes in control and exogenous variables are more than reversed in the transition from the medium to the longer run.

[13] Equation (3) evaluates the actual real exchange rate at the medium run equilibrium. In reality, convergence occurs at a finite rate, because it takes time for the real long term interest rate differential and the deviation of the output gap to converge to zero. See *Federici* and *Gandolfo* (2002).

[14] The AK production function $Y(t) = AK(t)$, so the growth rate is proportional to the investment rate.

Box 1

Reduced Form Equations of the NATREX Model

$$[B(R(t); Z_3(t)) - r(t)F(t)] + [I(R(t); Z_2(t)) - S(F(t); Z_1(t))] = 0 \quad (3)$$

$$dF(t)/dt = [I(R(t); Z_2(t)) - S(F(t); Z_1(t))] - g(t)F(t \quad (4)$$

$$dY(t)/Y(t) = g(t) = b\, I(R(t); Z_2(t)) + Z_4 \quad (5)$$

$F(t)$ = net liabilities to foreigners in the form of debt plus equity/GDP; $R(t)$ = real exchange rate = $N(t)p(t)/p^*(t)$, $N(t)$ = \$US/euro, $p(t)/p^*(t)$ = GDP price deflators (euro/US); $Y(t)$ = real GDP, I(t) = investment/GDP; $S(t)$ = saving/GDP; B = trade balance/GDP. $Z(t)$ = vector of exogenous and control variables. Growth rate of GDP = $g(t)$ = [(d$Y(t)$/dt)/$Y(t)$]. The functions are evaluated at a rate of capacity utilization equal to its stationary mean and the equalization of domestic $r(t)$ and foreign $r^*(t)$ real long term interest rates.

Micro-Foundations

The micro-foundations of the saving, investment and trade balance equations in box 1 are briefly discussed. All variables except the exchange rate, growth rate and interest rate are measured as fractions of GDP.

Social saving is the sum of private saving by firms and households and government. Government saving is the high employment surplus. Private saving depends upon net worth, equal to "capital" less foreign debt. Capital $K(t) = Y(t)/b$ is measured as capitalized current GDP, where the discount factor is b in equation (5), the mean productivity of investment. A stability condition in this growth model is that social saving is negatively related to net worth. A rise in the foreign debt F lowers net worth, lowers social consumption and raises social saving. Write social saving as equation (6), where social consumption $c = C(F; Z_1)$. Parameter Z_1 reflects the fiscal policy and social policies that raise social consumption. We refer to Z_1 as "time preference", where a rise raises social consumption and lowers social saving. Stein and Sauernheimer, and Stein and Paladino (2001) showed how changes in "time preference" have been associated with political regime changes in France, Germany and Italy.

$$S = 1 - C(F; Z_1) = S(F;Z_1) \quad (6)$$

Investment depends upon the Keynes-Tobin q-ratio. It is the capitalized value of the increase in profits resulting from an investment. The capitalization factor is the real rate of interest r. Assume that the output resulting from investment is sold in the world market at price P*/N, where P* is the dollar price and N is the exchange rate \$US/euro. The investment good is produced at home at price P. The expected productivity of investment is b = ΔY/I, the nominal wage is W, the increment of labor is ΔL, the dollar price of imported materials is V* and the increment of materials is ΔM. The q-ratio is equation (7.1).

$$q = (1/r)\,[(P^*/N)\,\Delta Y/PI - W\,\Delta L/PI - (V^*/N)\,\Delta M/PI]. \quad (7.1)$$

Let $a_1 = \Delta L/I$ and $a_2 = \Delta M/I$ be inverse measures of the productivities of labor[15] and materials. The real exchange rate R = NP/P*, the real wage is w = W/P and the terms of trade T =NP/V* the domestic price level/price of imported materials. Then the q-ratio can be written as equation (7.2), and the investment function as equation (7). Investment I is: *positively* related to the expected productivity of investment b, the terms of trade T, and productivities of labor ($1/a_1$) and of materials ($1/a_2$), and is *negatively* related to the real exchange rate R, the real wage w and the real rate of interest r. It is convenient to write vector Z_2 = (b, a_2/T, wa_1, r) ~ (productivity, terms of trade).

(7.2) $$q = (1/r)\,[b/R - wa_1 - a_2/T] = Q(R, Z_2).$$

(7) $$I = I(q).$$

The current account function CA is the trade balance B less net income transfers to foreigners rF. The logic of the trade balance is similar to that of the investment function. The greater the profit from investment in tradables, the greater will be the output. It follows that the trade balance is positively related to the q-ratio. We also add foreign social consumption c* as an element determining the trade balance. Thereby, the current account function is equation (8). Write vector Z_3 = (Z_2, c*) as the relevant one for the tradable sector.

(8) $$CA = B(R,Z_2, c^*) - rF = B(R,Z_3) - rF.$$

Portfolio balance at the longer run equilibrium real exchange rate implies that domestic and foreign real long-term interest rates are equal, or differ by a constant. This is one condition for external equilibrium.

(9) $$r = r^*.$$

In order to evaluate and interpret the econometric results of all of the papers summarized in table 3, it is necessary to explain the economic and econometric implications of equations in box 1. This is the subject of the following sections.

2. Transmission Mechanism: How Changes in Exogenous and Control Variables Affect the NATREX in the Medium Run and in the Longer Run[16]

The quotation from the OECD above described the statistical/atheroetical approach as a search procedure to find a cointegrating equation. The NATREX model takes a very different approach. (i) It specifies exactly what are the endogenous, exogenous and control variables. (ii) The structural equations describe the transmission mechanism. (iii) The solution of the model implies a trajectory of the exchange rate, so that there is a distinction between medium run and long run effects.

Figure 3 is a simple graphic exposition to demonstrate these points. The crucial exogenous/control variables are the n-tuple Z of vectors Z_1 - Z_4. Elements in Z_1 describe the shift in the social saving function, equation (6). Elements in Z_3

[15] The productivity of labor $\Delta Y/\Delta L = (\Delta Y/I)/(\Delta L/I) = b/a_1$.

[16] The presentation here is intuitive. The proofs are in the NATREX papers, for example *Stein* (2002), appendix.

describe shifts in the trade balance function, equation (8). Elements in Z_4 describe changes in the growth function, equation (5). Elements Z_3 and Z_4 are related to element Z_2 that shifts the investment function, equations (7.2), (7). We describe two scenarios involving changes in the fundamental determinants Z.

Figure 3

Scenario I. Decline in Social Saving less Investment. Rise in Z_1.

R(0) initial real exchange rate; R(1) medium run; R(2) long run

R(1) > R(0) > R(2)

Scenario II. Rises in Productivity/Competitiveness. Rise in Z_3.

R(0) initial real exchange rate, R(1) medium run, R(3) long run.

R(3) > R(1) > R(0)

Medium run equilibrium is based solely upon equations (3) and (9). It is very similar to the standard macroeconomic balance equations underlying the Mundell-Fleming model. The difference between the Mundell-Fleming (M-F) approach and the NATREX model is seen clearly by considering the effects of an expansionary fiscal policy upon the real exchange rate. Whereas the M-F model claims that the real exchange rate will appreciate, the NATREX model claims that the medium run appreciation will be more than offset in the longer run. An expansionary fiscal policy will depreciate the longer run exchange rate.

The curve SI relates saving less investment to the real exchange rate, and curve CA relates the current account to the real exchange rate. They are evaluated when real interest rates have converged. Their intersection gives us the medium run equilibrium exchange rate. The negatively sloped CA curve describes how an appreciation of the real exchange rate decreases the trade balance and current account. The positively sloped SI curve describes how an appreciation of the real exchange rate raises domestic costs and prices relative to world demand and reduces the present value of expected profits. The Keynes-Tobin q-ratio declines (equation 7.2), and investment then declines relative to saving. That is why S-I rises with the real exchange rate[17]. Initially, saving less investment is described by curve SI(0), and the current account by curve CA(0). Real exchange rate R(0) produces internal balance, when there is portfolio balance r(t) = r*(t).

[17] If the real exchange rate were defined using unit labor costs as deflators, as in *Stein* (1999), the negative effect of exchange rate appreciation upon investment would be simpler and clearer.

Populist Scenario

In *Scenario I* social saving declines, as a result of social policies that favor consumption of goods and services for the present generation. There is a rise in Z_1. These are populist policies[18]. Either the high employment government budget deficit rises, there are loan guarantees for industries producing consumer goods, or the government intervenes in the wage bargaining process to favor rising wages and lower hours.

Initially, saving less investment is described by curve SI(0), and the current account by curve CA(0). They are equal at real exchange rate R(0). The rise in Z_1 shifts the SI curve from SI(0) to SI(1). If all of the increased demand were directed to home goods, then the CA curve is unaffected. At exchange rate R(0), the ex-ante current account CA(0) is zero but ex-ante saving is less than investment. Domestic firms/government borrow abroad what they cannot borrow at home. The desired capital inflow appreciates the exchange rate to R(1) > R(0), to restore internal and external balance. The appreciation can occur via a rise in prices or an appreciation of the nominal exchange rate.

Whereas the macroeconomic balance (Mundell-Fleming) models stop at the movement R(0)-R(1), the NATREX model continues by taking into account the *endogenous* changes in net foreign claims, which will feed back upon the macroeconomic balance equation. At appreciated real exchange rate R(1), the capital inflow equals the current account deficit 0A, equation (3). The foreign debt/equity is rising at rate 0A *per unit of time,* equation (4). *The first thing to note, concerning the results in figure 3a, is that the real exchange rate appreciates and net foreign debt rises or net foreign claims decline.* These are just medium run effects. The movement to the medium run equilibrium point [R(1), A] is evaluated at the initial level of net foreign assets F(0) and productivity.

The implications for the trajectory of the real exchange rate to the longer run are derived by combining equations (3) and (4). The mathematics of the dynamic process are presented elsewhere, and the discussion here is intuitive. There are both stabilizing and destabilizing effects of the debt. (i) The rise in the debt raises the interest payments rF(t) which shift the CA curve to the left of CA(0). The leftward shift along the SI(1) function depreciates the exchange rate below R(1) and widens the deficit on current account. The steady decline in the CA function – due to a steady rise in the debt – *along a given SI function produces instability.* The debt will explode and the euro exchange rate will depreciate steadily.

(ii) The stabilizing effect arises from the *endogenous* shift of the SI curve to the right. The rise in the debt/GDP lowers net worth and consumption, and increases saving/GDP, according to the stability condition that a rise in the debt raises saving. The SI curve in figure 3a shifts to the right of SI(1). Thereby, the capital inflow and the rise in the debt are reduced.

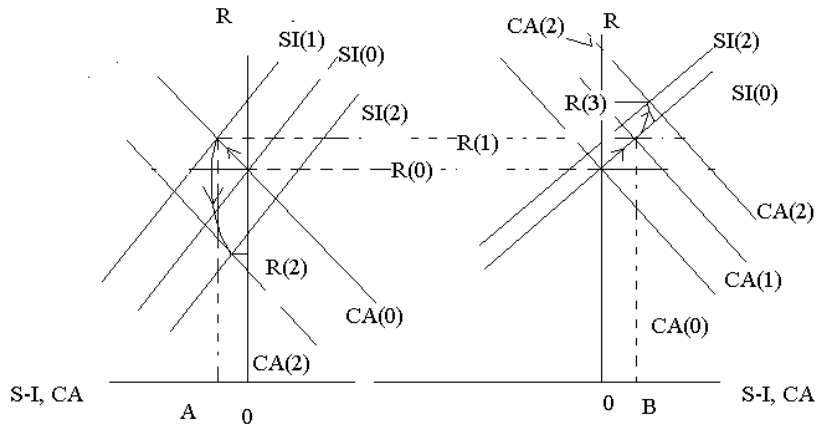
The long run debt/GDP rises, as a result of the accumulation of current account deficits. The current account curve declines below CA(0), due to the need

[18] For example, these were the policies followed by the first Mitterand government.

to make income transfers on the higher debt. The statistical/atheroetical studies in table 2 viewed net foreign assets NFA as an exogenous variable, whereas it is endogenous in the NATREX model.

Figure 3a, 3b

Effects of Changes in Fundamental Determinants



The longer run sustainable equilibrium real exchange rate depreciates below the initial level R(0). The explanation is as follows. From equations (3) and (4), the long run current account deficit/debt must equal the growth rate g. Equation (*10*a) describes this condition, where the current account deficit is the interest transfers rF* less the trade balance B(R;Z). Write (*10*a) as (*10*), where the asterisk denotes longer run values.

(10a) $$(rF^* - B(R^*;Z))/F^* = g$$

(10) $$(r - g)F^* = B(R^*;Z)$$

The long run debt F* is above its initial level, say F* > 0. If the present value of expected future income is positive, the interest rate must exceed the growth rate (r-g) > 0. The net income transfer (r-g)F* rises. Therefore the trade balance B(R* ; Z) must increase to generate the foreign exchange for the transfer. Consequently, the real exchange rate R* must be below its initial level to generate the greater trade balance.

In this manner, the NATREX model shows that scenario I – for example an expansionary fiscal policy – leads to longer run depreciation, not appreciation as claimed by the traditional Mundell-Fleming models.

Scenario of Improved Resource Allocation and Growth

Vectors Z_3 and Z_4 involve shifts in the trade balance and/or the growth function. I give an example based upon possible scenarios of enlargement of, and liberalization within, the euro area. Suppose that there is a greater liberalization of the economies, with a free and flexible system of prices and wages. The satisfaction of the marginal conditions for output and factor inputs leads to an improved allocation of resources. There is a rise in both the return on investment and total factor productivity, since the same total quantities of inputs produce a higher real GDP.

The SI curve remains at SI(0) for the enlarged euro area, but the rise in total factor productivity increases the trade balance and shifts the current account curve (equation 8) in figure 3b from CA(0) to CA(1). At the initial exchange rate for the euro R(0), the current account CA(1) exceeds the capital outflow SI(0), and the real exchange rate appreciates to R(1). The appreciation of the exchange rate to R(1) restores medium run equilibrium by equilibrating S(0)- I(0) = CA(1), equation (3).

In the *medium run* there is a difference between scenarios I and II. In scenario I, there is a capital *inflow* equal to the current account deficit of 0A per unit of time, which implies a rising foreign debt-equity. In scenario II, there is a capital *outflow* equal to a current account surplus of 0B per unit of time, which implies a declining foreign debt-equity.

The story is the reverse of scenario I. The debt declines. The endogenous decline in the net interest/dividend payments on the debt/equity shifts the CA curve further to the right to CA(2), and thereby strengthens the effect of the medium run impact upon the real exchange rate. The decline in the debt increases net worth and decreases the saving ratio. The SI curve shifts to the left to SI(2). The long run real exchange rate R(3) equilibrates CA(2) = SI(2) and the real exchange rate appreciates to R(3) > R(1) > R(0).

At the new long run equilibrium, the growth rate is higher and the endogenous debt is lower. The debt F* may become negative: net foreign assets may become positive. The right hand side of equation B(R*, Z) = (r-g)F* is algebraically lower. Hence the appreciation of the real exchange rate to R* = R(3) generates the required trade balance. Insofar as F* < 0, there are *sustainable* current account surpluses[19] and net foreign asset position grows at the rate g(t) of real GDP.

[19] If F* < 0, then CA/(-F*) = g > 0, and CA > 0. There are sustainable current account surpluses that keep F* constant. However B, the trade balance/GDP, is B = (r-g)F* < 0. The negative trade balance offsets the receipts of interest income from abroad, adjusted for growth.

Box 2 summarizes the effects of the fundamental determinants of the real exchange rate in the medium and longer terms in both scenarios.

Box 2

Implications of Changes in Fundamental Determinants

Scenario	Real exch. Rate, medium run R(1)	Real exch. Rate, long run R*	Debt-equity/GDP long run F*
I. Decline in parameter of Social saving/ rise in social consumption $dZ_1 > 0$	Appreciate	Depreciate below initial level	Increase debt ratio
II. Rise in Competitiveness/increase trade balance function $dZ_3 > 0$. Rise in growth, $dZ_4 > 0$	Appreciate	Appreciate above medium run level	Decrease debt ratio

IV. Empirical Estimates Based upon the NATREX Model

1. Structural Equations [20]

The studies in table 3 by Detken, Dieppe, Henry, Marin and Smets (2002) and by Detken and Marin Martinez (2001) estimate Vector Error Correction (VEC) models for the variables entering the investment, consumption and trade balance equations. The object was to examine structural equations (6) - (8) underlying those in Box 1, and from them estimate the equilibrium real exchange rate.

The consumption equation is (11), and the implied saving function is S(t)/Q(t) = 1 - C(t)/Q(t). The ratio of consumption to output C(t)/Q(t) depends positively upon net worth (K - F)/Q; capital less debt. Hence C(t)/Q(t) depends negatively upon the foreign debt/output F(t)/Q(t) measured as cumulative current account deficits. The stability of the system depends crucially upon the sign of the debt variable: social saving (consumption) must rise (fall) with the debt. The authors also assume that C(t)/Q(t) depends negatively upon the real rate of interest r(t), and positively upon the nominal interest rate i(t), which represents the business cycle.

[20] I am using the authors' notation, except for the growth in total factor productivity. I am not specifying when they use the long-term or the short-term interest rates.

Investment function equation (12) reflects a declining marginal product of capital and an estimate of the q-ratio. Investment/output I(t)/Q(t) is negatively related to the capital stock/output K(t)/Q(t) and to the real rate of interest r(t), and is positively related to a*, the growth of total factor productivity. Investment is negatively related to the real exchange rate. This is the investment crowding out effect, which produces a positively sloped SI curve in figure 3 above.

The trade balance equation (13) states that the trade balance TB(t)/Q(t) is negatively related to the real exchange rate R(t), the domestic social consumption ratio C(t)/Q(t) given in (11), and positively related to foreign social consumption ratio C*(t)/Q*(t) and to the terms of trade TOT.

Three equations are used, but not estimated directly. One is the uncovered real interest parity. In addition, there are two dynamic equations. One is the growth of the debt/GDP ratio F(t)/Q(t), corresponding to (4), and the second is the growth of capital/GDP ratio K(t)/Q(t), corresponding to (5).

Box 3

Estimated Structural Equations

(11) $C/Q = 1 - S/Q = a_6 + a_7\, K/Q - a_8\, F/Q - a_9\, r + a_{10}\, i$

(12) $I/Q = a_1 + a_2\, a^* - a_3 K/Q(-1) - a_4 r(-3) - a_3\, R(-4)$

(13) $TB/Q = -a_{11} - a_{12} R - a_{13}\, C/Q(-4) + a_{14} C^*/Q^*(-4) + a_{15}\, TOT$

Hypothesized values of regression coefficients are positive. C/Q = social consumption/GDP, I/Q = gross social investment/GDP, TB/Q = trade balance/GDP; quarterly lag is in parentheses; interest rates, r = real and i = nominal.

The authors estimate separate VEC models for the variables in box 3 over the period 1972:1 - 1997:4. The empirical results are as follows. The variables are of order I(1) except for the productivity growth rate. There is one cointegrating equation for each behavioral equation. *All of the crucial coefficients have the hypothesized sign and are significant.* There are certain crucial requirements for the validity of the structural aspects of the NATREX model, and others are not crucial. Results (a) - (d) below show that the crucial structural equations of the NATREX model are consistent with the evidence.

(a) The rate of investment is negatively related to the real exchange rate. Exchange rate appreciation crowds out domestic capital formation in the estimate of equation (12). This is consistent with the positively sloped SI curve in figure 3.

(b) The trade balance is negatively related to the real exchange rate in the estimate of trade balance equation (13). Exchange rate appreciation crowds out

the trade balance and tends to raise the debt. This is consistent with the negatively sloped CA function in figure 3.

(c) The stability of the system requires that the foreign debt reduce wealth, which reduces social consumption by the sum of households, firms and government. The debt significantly reduces social consumption in the estimate of social consumption equation (11). This is consistent with the dynamics in figure 3.

(d) A rise in the capital/output ratio reduces the rate of capital formation, in the estimate of investment equation (12).

The medium run equilibrium real exchange rate is derived from a solution of S(t) - I(t) = CA(t), using the estimates from the equations in BOX 3, evaluated with the current debt F(t)/Q(t) and capital K(t)/Q(t). The authors derive the longer run equilibrium real exchange rate by adding the conditions that: the current account deficit/debt is a constant, and that the ratio of investment to capital is a constant. The model is then simulated to compare the actual with the simulated estimates outside the sample period. The simulation results and conclusions are as follows.

> "Overall, the variability of the estimated … equilibrium is very high due to the volatility in the productivity growth rate, the terms of trade and domestic interest rates…On the positive side, this could be used to refute the claim that exchange rate models based on fundamentals are not volatile enough. Furthermore, the basic pattern of the synthetic euro has been traced by our version of the Natrex model. On the other hand one caveat is appropriate. The exact level of the resulting equilibrium rate is quite fragile to small changes in the behavioural equations…The basic pattern would remain similar, but the degree of over- or undervaluation can vary significantly."

Studies of the structural equations of the NATREX model for the Italian lira by Federici and Gandolfo (2002) and the Belgian franc by Verrue and Colpaert, cited in table 3, yield results that are consistent with (a) – (d) above. My conclusion is that the authors have shown that: (i) the crucial transmission mechanisms of the NATREX model are consistent with the evidence, but (ii) one should be hesitant in accepting the quantitative results from the simulation as precise estimates.

2. Reduced Form Dynamic Equation for NATREX

Table 3 cites the studies that use the NATREX model to obtain estimates of the reduced form dynamic equation for the equilibrium real exchange rate of the euro and for the underlying currencies. The NATREX model is a stock-flow dynamic model, where a distinction is made between the medium term and the longer- term trajectory of the exchange rate. In the medium term, the stock of debt and level of GDP are given, but they are endogenous in the longer run.

The system in box 1 implies a differential equation for the equilibrium exchange rate R(t) having the form of equation (14). This equation corresponds di-

rectly to the VEC method of finding a cointegrating equation. The NATREX theory and econometrics are now linked up directly[21].

$$\Delta R(t) = \alpha[R(t-1) - BZ(t-1)] + \Sigma b(i)\Delta Z(t-i) + \varepsilon(t) \quad (14)$$

Term BZ(t) is the longer run equilibrium associated with the "fundamentals" Z(t). Box 2 above states the signs of the elements of vector B linking the fundamentals Z to the longer run equilibrium real exchange rate. In the NATREX model graphed in figure 3, the cointegrating[22] equation R*(t) = BZ(t) reflects the long run movement from R(0) to R(2) resulting from a rise in time preference, and from R(0) to R(3) for a change in productivity/competitiveness. The asterisk denotes the longer run equilibrium.

The EC term α[R(t-1) – BZ(t-1)] represents the movement along the trajectories, resulting from endogenous variations in stocks. Term $\Sigma b(i)\Delta Z(t-i)$ represents short-term shocks from variables that are stationary/transitory I(0) and have zero expectations.

Vector B has been estimated[23] in several ways. (a)The most commonly used procedure is the Johansen/VEC method. The tests involve the following questions. (i) Are R(t) and vector Z(t) integrated I(1)? (ii) Is there just 1 cointegrating equation? (iii) Are the Z's weakly exogenous? (b) Some authors use a NLS method due to Phillips-Loretan. (c) A third method is the Engle-Granger 2-stage least squares. After establishing that [R(t),Z(t)] are I(1) and cointegrated, an OLS estimate of B is done directly. Then the residual [R(t-1) - BZ(t-1)] is used as the EC term. (d) Some authors use dynamic OLS estimators, the Stock-Watson approach, which involves leads-lags of the variables. This approach allows a standard interpretation of the t-statistics.

3. Measurement of the Variables[24]

A difficult problem, handled differently by the various authors, is how to measure the variables: the real exchange rate R(t), relative prices p(t)/p*(t) and the vector Z of disturbances to productivity and thrift that produce the change in the longer run equilibrium real exchange rate. Variables R and Z are I(1) – they do not revert to constant means. We generally measure the basic I(1) variables as four quarter moving averages (MA) to reduce the noise components.

(1) The real exchange rate R(t) can be measured in terms of alternative deflators. The Deutsche Bundesbank (1995) compared the differences in the trends of the real value of the DMark resulting from the use of the alternative

[21] Contrast this with the OECD quotation above concerning the statistical/atheoretic methods.

[22] Insofar as R(t) and vector Z(t) are integrated I(1), and satisfy the usual condition, we obtain R*(t) = BZ(t) as the cointegrating equation.

[23] See *MacDonald* (2002) and (1999) for an analysis and use of various estimation methods.

[24] I am using the data provided by Liliane Crouhy-Veyrac.

deflators: for example, GDP, CPI, unit labor costs in manufacturing, price indices of total expenditures.

(2) There are alternative measures of social "time preference". Theoretically, the measure should reflect shifts of the social consumption/saving function. We prefer to use the ratio c(t)/c*(t) of private plus public consumption to GDP in the two blocs. A question is raised whether this variable is truly exogenous. One can write c(t) as private consumption/GDP plus $c_g(t)$ government consumption/GDP. Some authors use $c_g(t)$ as their measure of time preference, since it is more exogenous than c(t). However, the ratio $c_g(t)/c_g^*(t)$ misses the effects of changes in tax policy and social policy upon total consumption, whereas c(t)/c*(t) corresponds directly to the variable in the theory.

(3) The measurement of the productivity disturbance is done in various ways. Theoretically, we want to find a measure for factors Z_2 in the investment function and Z_3 in the current account function that: initially raise the productivity of investment in tradables, induce investment, eventually raise output and growth Z_4 and ultimately shift the CA function to the right by more than it shifts the SI curve in figure 3.

Some authors use the productivity of labor y(t) = GDP/employment. Others use total factor productivity $y_s(t)$, the Solow residual, or its rate of change. The latter is appealing, but there are some problems. The magnitude of the residual depends upon assumptions concerning the aggregate production function and the ambiguous measurement of "capital". The measurement of "capital" is dubious, in a world of technical progress, obsolescence and depreciation. Insofar as most of growth cannot be explained by capital formation, the two measures y(t) and $y_s(t)$ can be expected to yield similar results.

It is theoretically appealing to use the differential rate of return on investment [b(t) – b*(t)] as the generator of growth. The problem arises from the econometrics. Variable b(t) is stationary/I(0) both in Europe and in the US. Therefore, we were constrained not to use it in the cointegrating equation based upon the VECM.

The NATREX model concerns the equilibrium exchange rate satisfying conditions (C1)-(C4) in part 1 above. The model assumes that real long term interest rate convergence has occurred and ignores the shorter-term transitory I(0)/disequilibrium effects that converge to zero. The most important disequilibrium effect involves uncovered real long term interest rate parity. Equation (15) combines the longer term fundamentals and the transitory effects. The actual real exchange rate is R(t) and the longer run equilibrium BZ(t) is the rational expectations value, derived from the cointegrating equation of the NATREX model. The real long term interest rate differential is [r(t) – r*(t)], and the speed of convergence to the long run rational expectations equilibrium is $1/a_2$.

(15) $$\log R(t) = \log BZ(t) + a_2 [r(t) - r^*(t)].$$

Equation (15) may be called the rational expectations/uncovered real long term interest rate parity estimate of the real exchange rate[25].

4. Summary of the Reduced Form Dynamics Studies for the Real Exchange Rate of Synthetic Euro

Two types of uncertainty must be considered: statistical uncertainty and model uncertainty. The former concerns confidence intervals of parameter estimates. The latter concerns differences in parameter estimates resulting from variations in the model specifications.

In all of the econometric studies listed in table 3, the relations among variables R(t) and Z(t) pass the econometric tests mentioned above. A unique cointegration equation is obtained. These are the longer run effects. The qualitative significant sign results for the euro are similar.[26] Each estimation finds that: (1) the ratio c(t)/c*(t) of Euro/US social consumption/GDP depreciates the longer run value of the Euro, (2) relative productivity y(t)/y*(t) or relative Solow residuals y_s*(t)/y_s*(t) appreciate the longer run real value of the euro. (3) When the terms of trade are included, they appreciate real euro.

An example of the model uncertainty is that the quantitative estimates differ according to the variables included, the way they are measured, the lag structures, and method of estimation – even though each one obtains a unique cointegrating equation and the same data set is used. Table 4 illustrates some of these points for the real synthetic euro. The first study is by Duval (2002). We refer the reader to his study for the econometric details relevant to most of the euro studies in table 3. The next two are by the author. Both contain relative social consumption and relative labor productivity in vector Z(t). Study [JLS-1] also contains the terms of trade, the reciprocal of the real price of oil, since it is included in vector Z_2 in equation (7.2) and carries over to Z_3 in equation (8). The conclusions from table 4 are summarized.

- The fundamentals Z, measured in different ways, have the same significant *qualitative/sign* effects. Each study is consistent with the conclusions of the NATREX model, Box 2.
- The longer run effect of the ratio c of social consumption/GDP depreciates the real exchange rate. This is movement R(0)-R(2) in figure 3a. However, the change in social consumption Δc appreciates the medium run real exchange rate. This is movement R(0)-R(1) in figure 3a. These results are consistent with the medium run and long run effects in Box 2,

[25] Some studies use the real long-term interest rate differential [r(t) - r*(t)] as the I(0) factor, as in equation (15) above. Others use the differential rate of return on investment [b(t) – b*(t)] as an important I(0) variable. The difference in approach concerns the assets that underlie the portfolio effect. Do investors look at long term interest rate differentials, or do they look at differential rates of return on real long term investment? These variables have a correlation coefficient of 0. 65, as one would expect from economic theory. Therefore, one cannot use both expected asset yields in equation (15).

[26] The studies of the Italian Lira, DMark and Belgian France as well as to the Franc/DM and Italian Lira/DM in table 3 obtain similar results.

and show how the Mundell-Fleming effect R(0) – R(1) is more than reversed in the longer run.

- The *quantitative* estimates of the parameters vary considerably among the studies. Compare the coefficients of relative social consumption and relative productivity in [JLS-1] when the terms of trade are included with [JLS-2] when the latter is excluded. The signs are the same, but magnitudes differ.

Similarly, the speed of response α in the error correction term α[R(t-1) - BZ(t-1)] differs in the two [JLS] columns. The half-life of convergence along the trajectories to the longer run equilibrium is 13.5 quarters in the [JLS-1] column and 22.8 quarters in the [JLS-2] column when the terms of trade variable is excluded. Varying the lag structure also changes the magnitudes of the coefficient estimates. The variables that are classified by a given econometric technique as being weakly exogenous also change with the lag structure.

The detailed studies of the reduced form dynamics for the real synthetic euro are listed in table 3. We provide the reader with a simple example, based upon equation (15), of how this approach explains the movements in the real synthetic euro.

The actual real exchange rate log R(t) converges in a statistical sense to the longer run equilibrium NATREX, log BZ(t), which is a rational expectations equilibrium based upon the real fundamentals Z(t). Figure 4 graphs the log of the actual real synthetic euro log R(t) denoted EUUSREDPMAL[27] and the longer run NATREX, R*(t) = log BZ(t) denoted NATJLR, which is the cointegrating equation in [JLS-1] table 4. Define “misalignment” Φ(t) as the deviation between the actual real exchange rate R(t) and the longer run/rational expectations equilibrium log BZ(t). The difference between the curves is "misalignment". Based upon the error correction estimates in table 4, the convergence to the longer run equilibrium BZ(t) may be about 3-4 years.

The NATREX model of "equilibrium" assumes that the real long term interest rates have converged, condition (C2) at the beginning of this paper. However, the convergence of real long term interest rates takes time. Therefore, the actual real exchange rate is also affected by uncovered real long term interest rate parity as well as by the rational expectations long run equilibrium.

Equation (15) can be written as (15a). A systematic part of "Misalignment" is explained by the real long term interest rate differential.

(15a) $$\Phi(t) = a_2[r(t) - r^*(t)], \quad \Phi(t) = \log R(t) - \log BZ(t).$$

[27] It is the EU relative to the US Real Exchange rate based upon GDP price Deflators, in Logarithms and 4Q MA. Hence the EUUSREDPMAL. The NATJLR is the Natrex Real exchange rate based upon the Johansen method in Logarithms.

Table 4

Log of the Longer Run Equilibrium Real Value of the Synthetic Euro Explained by the Log of Real Fundamentals. The $US is the Other Currency

Fundamental	Duval	JLS-1	JLS-2
Relative EU/US Social consumption Z_1= c/c* R(0)- R(2) fig. 3a	Depreciates	Depreciates dR/d(c/c*) = -4.13	Depreciates dR/d(c/c*) = -5.5
Relative productivity EU/US in Z_2, in eqn. (7.2), Z_3 in eqn. (8) R(0)-R(3) fig. 3b	Relative Solow residuals, ys(t)/ys*(t) Appreciates	Relative Labor productivity y(t)/y*(t) Appreciates dR/d(y/y*) = 2.9	Relative Labor productivity y(t)/y*(t) Appreciates dR/d(y/y*) = 3.6
Terms of Trade in Z_3, eq. (8); inverse of real price of oil		Appreciates	
Comments	Euro is a weighted average of French, German and Italian currencies.	Euro is a weighted average of EU-11	Euro is a weighted average of EU-11
Shorter run effects: ΔZ R(0)-R(1) fig. 3a	Change in Relative social consumption Δ(c/c*) appreciates	Change in Relative social consumption Δ(ë/c*) appreciates	Change in Relative social consumption Δ (c/c*) appreciates
Speed of response coefficient α3 equation (14)		α3 = -0.05 half-life 13.5 quarters	α = -.03 half-life 22.8 quarters

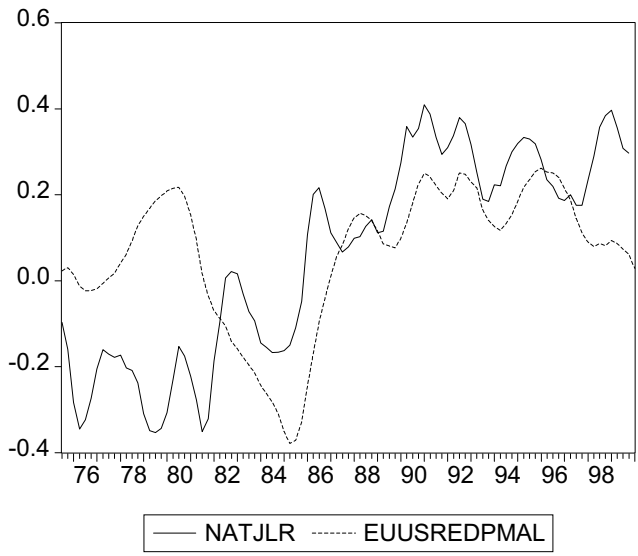
All of the NATREX studies in table 3 stress that one obtains consistent qualitative results, in terms of sign and significance. The quantitative estimates differ, which we refer to as model uncertainty. This type of uncertainty is an important motivation for us to demonstrate graphically the results in terms of normalized variables.[28] It is a convenient way to stress orders of magnitude.

Figure 5 graphs, in normalized form, the misalignment and the real long term interest rate differentials. When the euro is undervalued, misalignment $\Phi(t)$ = log R(t) – log BZ(t) is negative, then a systematic factor responsible for the undervaluation is that the real long term interest rate differential is negative. The hypothesis in (15a) is that both curves in figure 5 lie on the same side of the zero line. Overall, this condition is met. The two variables are significantly positively correlated.

[28] Variable X1(t) normalizes variable X(t), where X1(t) = (X(t) - mean)/standard deviation.

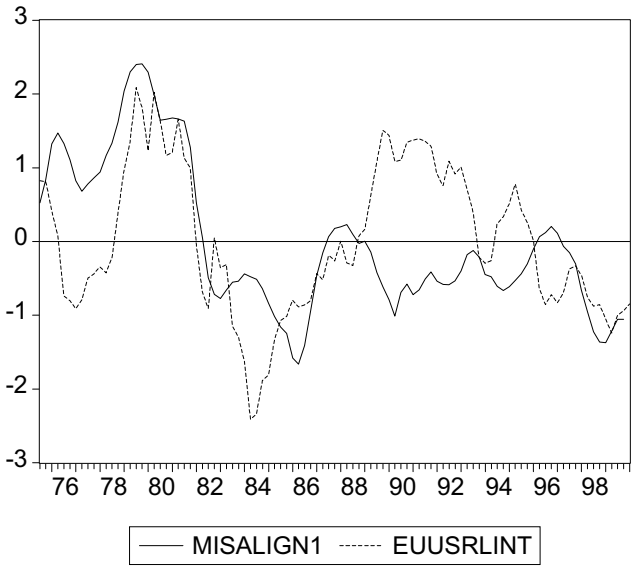
We have explained the movements in the real synthetic euro as resulting from explicit theoretically based systematic factors Z(t) that change the long run equilibrium rate and systematic factors such as the real long term interest rate differential. There are periods when the misalignment cannot be explained by real long term interest rate differentials. Clearly, there are non-systematic factors producing the misalignment. The non-systematic factors are seen as the difference between the curves in figure 5. At times, they are large; but they are transitory and hence ignored by our economic model.

Figure 4



Note: Logarithm of the Real value of the euro log R(t) = EUUSREDPMAL and estimate of longer run equilibrium NATREX [table 4, JLS-1] BZ(t) = NATJLR. The latter is the rational expectations equilibrium. Variables are 4Q MA.

Figure 5



Note: Misalignment [log R(t) - BZ(t)] = MISALIGN1 from longer run NATREX Real long term interest rate differential [r(t) - r*(t)] = EUUSRLINT.

V. Conclusions

To what extent have we answered the questions marked by bullets at the beginning of this article? First: the researchers agree that there are real fundamental determinants, that vary over time, of the longer run value of the synthetic euro. The trends in the nominal and the real euro are similar. The implication is that purchasing power parity and the monetary approaches are inadequate to explain the medium to longer term trends in the synthetic euro.

Second: the researchers studied the theoretical literature carefully to find useful frameworks for empirical analysis. They ended up taking two approaches. One was a statistical/atheoretical approach, where the equilibrium real exchange rate simply means the value of the real exchange rate implied by a cointegrating equation. One difficulty with a statistical/atheoretic approach is that researchers find different sets of fundamentals to be significant, and the sign/significance change as other regressors are added or dropped. A second serious difficulty is that it is often difficult to provide an economic explanation for the econometric results.

The theoretical framework that another set of researchers used was the NATREX model. This is a generalization of the more traditional macroeconomic balance models to take into account endogenous variations in the stock of debt and in the growth rate. The NATREX theoretical approach clearly specifies what are the exogenous and control variables, and which variables are endogenous. The model contains an explicit transmission mechanism linking the endogenous variables to the control variables. A very important result for theory and policy concerns the effects of fiscal policy. In the traditional Mundell-Fleming model, an expansionary fiscal policy appreciates the real exchange rate. In the NATREX model, this is just a medium run effect. As the stock of debt and growth rate vary endogenously, the real exchange rate depreciates below its initial level.

Another advantage of the NATREX model is that it provides an analytical framework to explain the effects of the enlargement of the euro area upon the equilibrium value of the euro. In our exposition of the model, we considered two possible scenarios of enlargement, which correspond to social policies. We then showed the transmission mechanism from the policies to the trajectories of the real exchange rate. No one can predict which social policies will be followed, so our predictions concerning the trajectory of the exchange rate are conditional upon which policies are followed.

Some studies of the NATREX model estimate the structural equations, and others estimate the reduced form dynamics for the synthetic euro as well as for the key underlying currencies. Our message can be tersely summarized.

- There are real fundamental factors - productivity, thrift and the terms of trade - that explain the longer term movements in the equilibrium real exchange rate of the synthetic euro. The qualitative/sign/significance results are similar across studies.
- Trends in real and nominal rates of the euro are highly correlated.
- Uncovered real long term interest rate parity helps to explain deviations from the longer term equilibrium.
- There is considerable model uncertainty. The quantitative estimates vary depending upon the method of estimation, and its details such as the lag structure. A direct implication of model uncertainty is that exchange rate management, the use of target zones, and other policies that require confidence in quantitative estimates, do not rest upon firm foundations.

Summary

We evaluate studies of the “equilibrium” value of the real euro. We ask: How can one explain the medium to longer run movements in the synthetic Euro? We compare the results derived from a statistical/atheoretic approach with those based upon a theoretical stock/flow/ growth model with an explicit transmission mechanism. Conclusions: There are real fundamental factors – productivity, thrift and the terms of trade – that explain the longer term movements in the equilibrium real exchange rate of the synthetic euro within a consistent theoretical framework. Uncovered real long term interest rate parity helps to

explain deviations from the longer term equilibrium. There is considerable model uncertainty. The quantitative estimates vary depending upon the method of estimation.

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