A joint Initiative of Ludwig-Maximilians-Universität and Ifo Institute for Economic Research

Venice Summer Institute 2006

Perspectives on the Performance of the Continent's Economies

21 - 22 July 2006

Venice International University, San Servolo

Lifetime Aggregate Labor Supply with Endogenous Workweek Length

Edward C. Prescott, Richard Rogerson, and Johanna Wallenius

Federal Reserve Bank of Minneapolis
Research Department

Lifetime Aggregate Labor Supply with Endogenous Workweek Length

Edward C. Prescott, Richard Rogerson,
and Johanna Wallenius*

Working Paper

July 2006

Preliminary and incomplete:
Not to be quoted without authors' permission

ABSTRACT

There is a non linearity in the mapping from hours supplied to the market to labor services. This is in large part due to the fixed costs of getting to and getting set up in a job. The non linearity gives rise to an endogenous standard workweek that can differ across occupations. This paper studies lifetime aggregate labor supply with endogenous workweek length. Such a theory is needed to evaluate retirement systems and to determine the consequences of constraints on workweek length. Empirically the principal margin of adjustment is the fraction of weeks a person works in their lifetime. The theory determines what fraction of the lifetime an individual works, not when. Our results suggest that constraints on the workweek length will depress aggregate labor supply. The extent to which this will occur depends on the nature of the mapping from hour supplied to the market to labor services. Also, we find that policies designed to increase the retirement age may result in people working a smaller fraction of their lifetime. Hence, aggregate lifetime labor supply may not increase significantly as a result of an increase in the retirement age.

---

*Prescott, Arizona State University and Federal Reserve Bank of Minneapolis; Rogerson and Wallenius, Arizona State University. We thank the National Science Foundation for supporting the research (Grant 0422539) and our colleagues with whom we discussed this research. We particularly want to thank Jay H. Hong for providing economic statistics and Simona Cociuba for providing exceptional research assistance services.

Perhaps the most important parameter for purposes of evaluating tax policies is the aggregate intertemporal leisure substitution elasticity. This elasticity is a function of parameters that specify not only individuals' preferences, but also household production technology as specified by the consumption sets.

Aggregate applied general equilibrium studies find this aggregate elasticity to be near one. One set of these studies is business cycles studies, which use variants of the neoclassical growth model to evaluate the contribution of various shocks to business cycle fluctuations. The finding of these studies is that if and only if this intertemporal elasticity parameter is near one do shocks, whatever they might be, give rise to fluctuations that display the key business cycle facts. These studies date back to Kydland and Prescott (1982).

More recently there have been a number of studies of large movements in output and market hours relative to the secular trend initiated by the study of Cole and Ohanian (1999).[1] Unlike business cycle studies these studies of depressions and prosperities are concerned with the path of the economy, and not with certain statistical properties of the aggregate time series. They also find the aggregate intertemporal leisure substitution elasticity near one.

Still another class of studies (Prescott, 2002, 2004) examines cross economy labor supplies and effective marginal tax rates on labor income. These studies find that with an intertemporal elasticity of leisure near one, differences in this marginal tax rate account for most of the current differences in aggregate labor supply across the major industrial economies. These studies find that changes in this marginal tax rate also accounts for the

[1] See the Kehoe and Prescott (2006) volume with 15 such studies.

large change in aggregate labor supply in the major continental European countries. Thus, macro theory accounts for both cross sectional and time series observations.

We emphasize that factors other than this tax rate can and have reduced labor supply by a large amount. As found by Cole and Ohanian (1999, 2004), U.S. labor supply was not depressed by 25 percent from 1931-1939 because of a high marginal tax rate on labor income. It was depressed for other reasons, such as New Deal cartelization policies.

This high aggregate intertemporal leisure substitution parameter implies a large elasticity of aggregate labor supply. A one percent change in the real wage, holding wealth constant, gives rise to a three percent change in hours worked for the United States and even higher for Western Europe, where the fraction of the time endowment allocated to the market is smaller.

Until recently these findings of the macro economists concerning the aggregate elasticity of labor supply were in apparent conflict with those of micro economists, who used observations on individual behavior and the prices individuals face to estimate an individual Frischian labor supply elasticity. This apparent conflict disappears once aggregation theory is used to derive the implication of individual's preferences and technology for the preferences of the aggregate stand-in household.

There is an analogy to this result on the production side. The fact that the sum of strictly convex technology sets asymptotically is a convex cone – that is displays constant returns to scale – has long been known, going back at least to Alfred Marshall and Knut Wicksell. Inference based on plant supply is invalid for aggregate supply if the number of plants operated can be varied.

Only recently has aggregation theory on the household side been developed. Rogerson (1984, 1988) discovered that the same aggregation result holds for the stand-in household if people are constrained to work a standard workweek or not at all in a given week. The analogue to the number of physical machines operated is the number of human machines being used to produce market output. Rogerson found that the aggregate substitution is much larger than the individual elasticity if the margin of adjustment is the number of people working in the market sector.

Empirically the principal margin of adjustment is the fraction of people employed and not hours worked per employed person in a given period. Once finite lifetimes are introduced, what gets chosen is the fraction of lifetime worked. This assumption that most people either work a standard workweek or not at all matches well with observations. Most people work a forty hour workweek when they work.

The restriction to a standard workweek is arbitrary and the question that naturally arises is why the principal margin of labor supply adjustment is the fraction employed and not the number of hours worked per person employed? Hornstein and Prescott (1993) figure out why the principal margin of adjustment is the number working per week.[2] Key, following Rosen (1978), is a non linearity in the mapping from hours supplied to labor income. The non convexity arises because, for many, labor compensation increases more than proportional to the length of the workweek in the relevant range. Technically workweeks of different lengths are different commodities.

If the nature of the technology is such that an appropriate commodity space has hours as the *labor* input then aggregate labor supply is a statement about preferences

[2] In this essay we say the workweek. With the number of holidays endogenous, the appropriate work period must be shorter, say a day. But the number of holidays, and any sick days that are in fact holidays, margin is secondary for most international comparisons across countries and time.

only. But there is not a single labor input. Compensation typically is not proportional to the hours an individual works. The line between preferences and technology is not clean. But in using general equilibrium theory in modeling, the way the data is reported and the empirical facts dictate the way things are handled. It is desirable to have prices for the things exchanged, as prices are what we use to value things.

In this paper, people's preferences are ordered by the expected value of time additively separable utility functions. We deal with a finite number of types with a measure of each type.

Rogerson uses a Prescott-Townsend (1984a, 1984b) lottery equilibrium as did Hornstein and Prescott (1993). Equivalently an Arrow-Debreu equilibrium could have been used to support that allocation (see Prescott and Shell, 2002), but then symmetry is lost and the notation becomes almost impossibly complex. Here we show that it holds without the need to index to some sunspot variable or to introduce lotteries, provided that there is borrowing and lending. If there is aggregate uncertainty, Arrow securities are needed as well.

To summarize, the result that the *aggregate* supply elasticity for the stand-in household has large Frischian labor supply elasticity is a robust one and does not require lottery allocations. The beauty of lottery competitive equilibria is that they are so easy to deal with, while having the same equilibrium allocations as sequence-of-market, recursive, and Arrow-Debreu competitive equilibria.

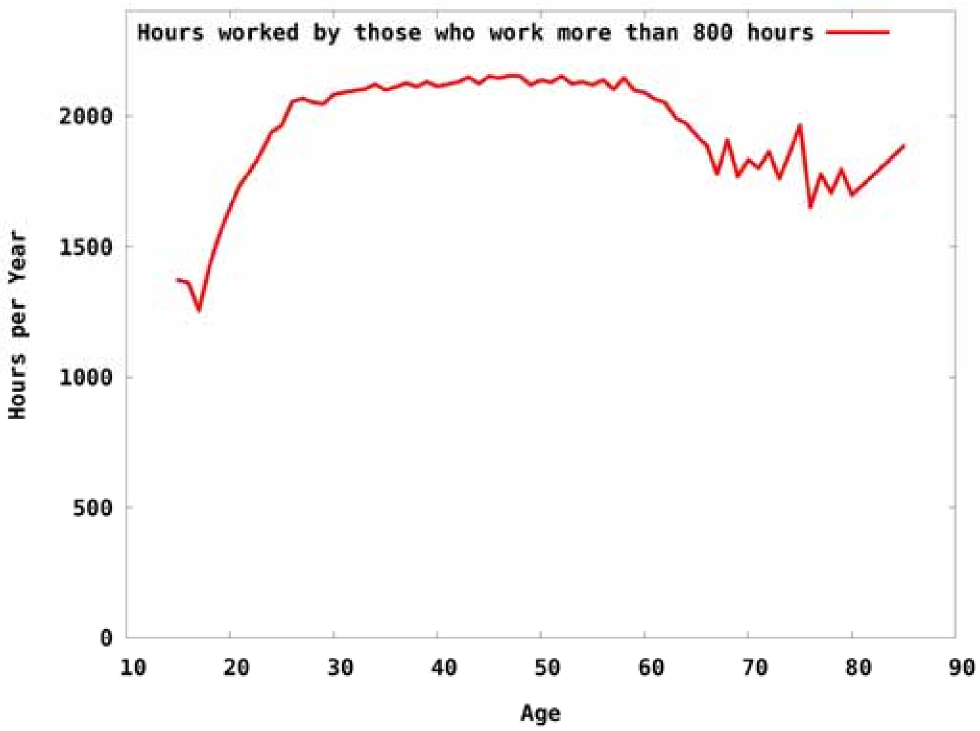
The great labor economist Jacob Mincer pointed out there was good theory of how much secondary wage earners work over their lifetime, but not when they work. A significant fraction of these secondary wage earners are deciding what fraction of their potential working life to work in the market. Two early studies that concluded that the aggregate labor supple elasticity is or is probably high are the following ones. Becker and Ghez (1975) conclude that it is quite likely large based upon micro theoretic reasoning and individual time allocation studies. Lucas and Rapping (1968) conclude it is large based upon the behavior of aggregate time series and real wages. The Lucas-Rapping measure of the real wage is aggregate compensation divided by aggregate hours.

In a recent study in which the Ben-Porath (1967) human capital investment part of compensation is taken into consideration, Imai and Keane (2004) estimate that the male labor supply elasticity for males is 3.7. Ignoring human capital accumulation on the job reduces the estimate to 0.3 (see McCurdy (1981) and Atonji (1986)). To summarize, the micro and macro evidence is that the aggregate intertemporal elasticity of substitution for leisure is large. This implies that the aggregate labor supply elasticity is large.

Section 1: The Principal Margin of Adjustment is the Fraction Employed

Fact 1: In the United States hours worked per person who work at least 800 hours declines little with age. This is shown in Figure 1.

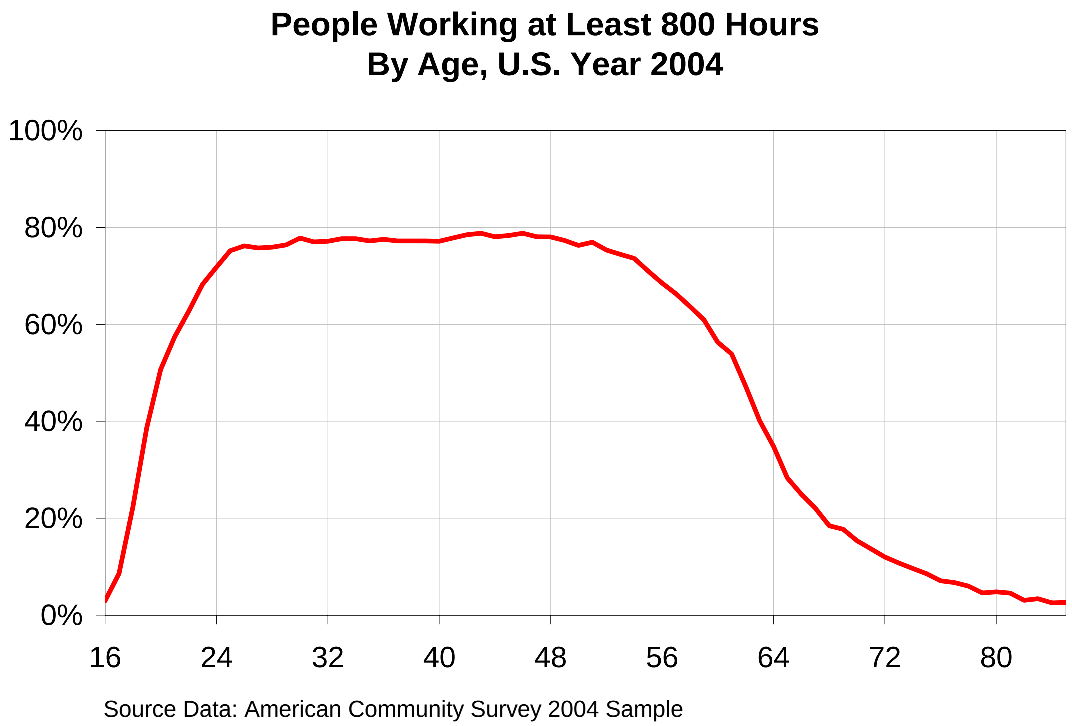
Figure 1



U.S. Source: CPS March Demographic Files 2004, Jay H. Hong, University of Pennsylvania

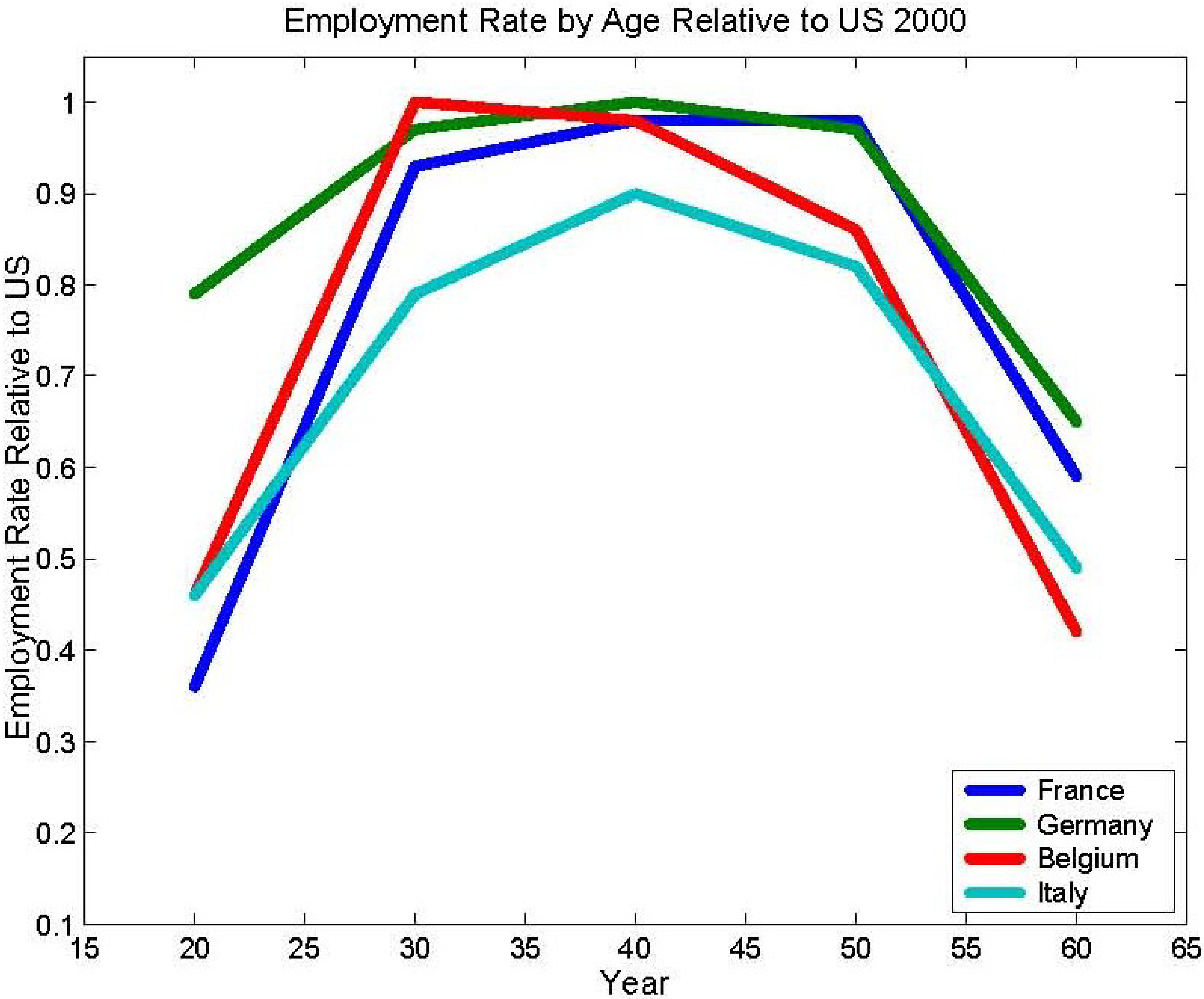
Fact 2: In the United States the fraction of people working at least 800 hours declines significantly with age. This shows that the decline in hours as people age is mostly due to the fraction working and not hours per employed person.

Figure 2



Fact 3: About half the difference in hours worked between the U.S. and Europe is accounted for by differences in the fraction of lifetime worked.

Figure 3



Fact 4: Cyclically in the United States the principal margin of adjustment is the fraction of the working age population employed. Hours worked per worker in a given week vary little.

The most important margin of adjustment over time is the employment rate. Cho and Cooley (1994) find that three quarters of the variation in total hours of employment takes the form of movements in and out of the labor force rather than adjustments in average hours of work.

Fact 5: International comparisons find big variations in market hours per working age person as shown in Table 1. This variation is split between the employment rate and hours per *year* per employed person. A year is not the workweek length. Much of the difference in hours worked per employed person is due to the number of weeks of vacation, the number of holidays, and the number of days that are categorized as sick days when in fact they are holidays.

Table 1

A Decomposition of Hours per Person into Hours per Employed Person and Employment Rate

Country	Hours per person	Hours per employed person	Employment rate
France	904	1441	.63
Germany	951	1443	.66
Italy	904	1585	.57
Spain	1102	1791	.61
United Kingdom	1195	1669	.72
United States	1305	1824	.72

Fact 6: There are sizable differences in workweek lengths across categories of workers based upon occupation and education.

Section 2: Aggregation Theory and Aggregate Stand-in Household's Preferences

In static situations with labor indivisibilities Prescott-Townsend lotteries or sunspot contracts are needed to achieve efficiency. In dynamic situations this is not the case. Here we show that borrowing and lending contracts or Arrow securities, if there is aggregate uncertainty, suffice to achieve efficiency. Individuals have finite lives. What is determined is the fraction of time the individual works, not when a person works. With the addition of some feature of reality that leads people to work the first part of their lifetime, the theory determines when individuals retire, that is switch for the rest of their life from working some fixed number of hours per unit of time to not working.

The non convex preferences arise because the disutility of labor supply flow, $v(h)$, is not convex. Labor supply and hours allocated to the market are not proportional. Thus, if the productive time endowment is normalized to 1, $1-h$ is *not* in general leisure. The reason it is not is that the mapping from hours allocated to the market to labor services typically is not linear. The empirical evidence that this mapping is non linear is in many cases overwhelming. Someone who chooses to work 40 hours per week typically will earn more than twice as much as a similar person who chooses to work 20 hours per week. One reason is fixed costs associated with getting set up for a job and associated with being supervised. Another reason is that supervisory time falls when one full time worker is doing a job rather than two half time workers.

We could have, following Honrstein and Prescott (1993) treated workweeks of different lengths as different commodities with a technology that resulted in the demand price of a workweek of length *h* being an increasing, initially convex function. In particular, Hornstein and Prescott assume a single worker produces $h\,k^{\theta}$ units of output, where *k* is the number of units of capital that the individual uses. Here we operate in the labor service space, because it is convenient to keep the budget constraint simple.

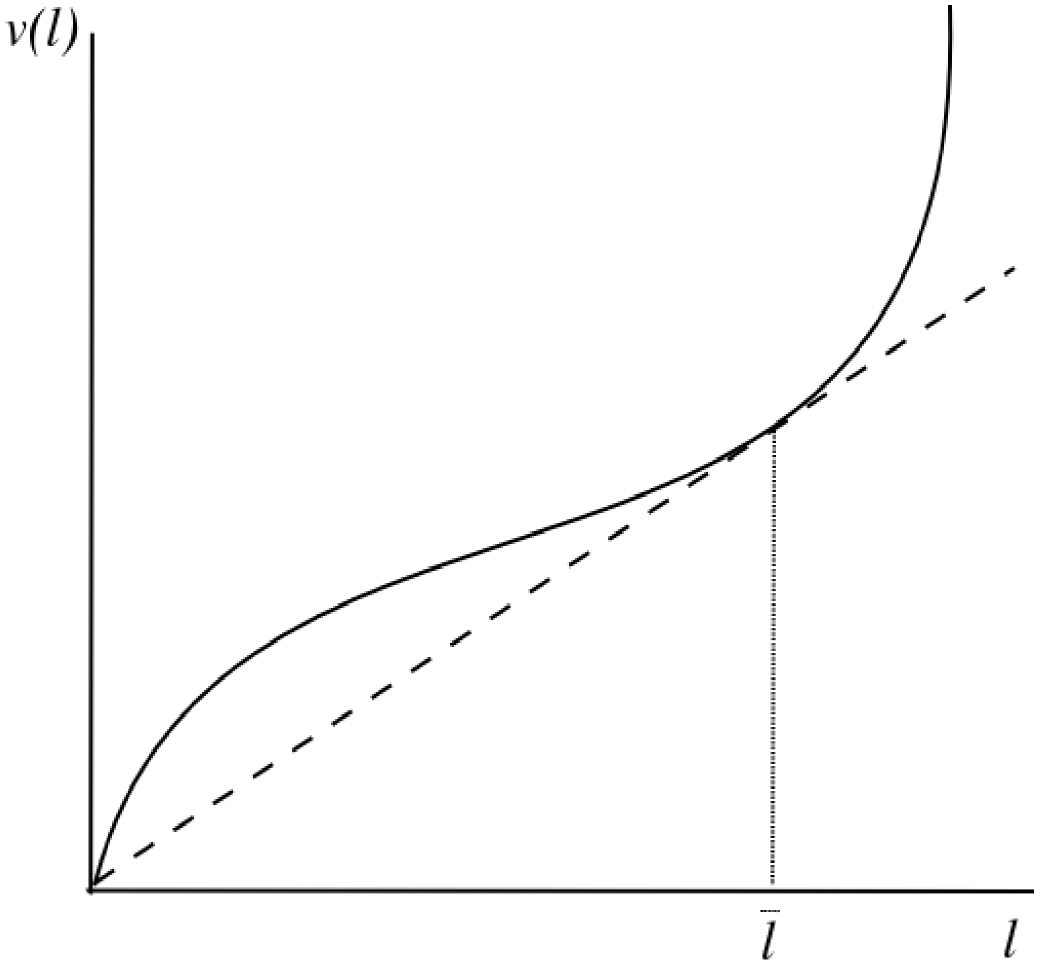
Time is denoted by $t \in [0,1]$, where the length of the lifetime has been normalized to 1. A person's preferences are ordered by

$$\int_0^1 (\log(c) - v(h))\, dt \tag{1}$$

where $c(t)$ is the consumption path and $h(t)$ is units of *labor* services supplied to the market. The $v(h)$ function reflects both preferences for non market time and the mapping from time allocated to the market to units of labor services produced. The mapping from hours allocated to the market to labor services typically has a convex region and becomes concave for long workweek lengths. The derivative of the function v *is* positive and strictly quasi convex. The function *v* satisfies $v(0) = 0$, is increasing and differentiable, is initially concave and then convex. Figure 1 depicts the shape of the function *v*.

Figure 4

Disutility of Labor Supply



The non linear mapping from hours supplied to the market to labor services results in an endogenous standard workweek. The length of the standard workweek is determined by the *v* function. The mapping can be different for different occupations, thus giving rise to different workweeks across occupations or education groups. As noted in section 1, we observe significant differences in workweek length across occupations. Our theory is consistent with this observation. If $g(h)$ is the time required to provide h units of labor services, then the standard workweek length is $g(\bar{h})$.

The budget constraint is

(2) $$\int_0^1 (c - w\,h)\,dt \le 0\,.$$

In the budget constraint the interest rate is 0 and the wage rate is constant over time. We deal with this simple case first in order to make clear what is going on. The generalization is straightforward and is outlined.

Solutions to the household's maximization problem take two possible forms. The first and empirically interesting one is $c(t) = w\bar{e}\,\bar{h}$ for all *t* and $\int_0^1 h(t)\,dt = \bar{e}\,\bar{h}$ for some $\bar{e} \le 1$ and $\bar{c}$. The empirically relevant case is when $\bar{e} < 1$. To find $\bar{e}$ and $\bar{c}$, two equations in these two unknowns can be solved. The first is the binding budget constraint. The second is obtained by equating the marginal rate of substitution between consumption and the fraction of time employed in some short interval of length dt ; that is

(3) $$v(\bar{h})\,we\bar{h} = w\,g(\bar{h})\,.$$

If the solution to equation (3) is an $\bar{e} \le 1$, then $h(t) = \bar{h}$ and $\bar{e}$ is the fraction of the lifetime the individual works the standard workweek. The aggregate employment is $E(t) = \bar{e}$ for all t.

If the solution to equation (3) exceeds one, then $\bar{e} = E(t) = 1$, $h(t) = \hat{h}$ where $\hat{h} \in \arg\max\{\log h - v(h)\}$, and $c(t) = w\hat{h}$ is the optimum. Everyone works all the time in this case.

If the interest rate is *r*, the discount rate ρ, and the growth rate of consumption γ, then consumption grows at rate γ rather than being constant. However, the labor supply correspondence does not change.

Proposition 1: The theory determines what fraction of the lifetime an individual works, not when.

Comment: The equilibria can be supported with a borrowing and lending arrangement.

Section 3: Generalizing the Result

We now deal with an overlapping generation structure and introduce capital accumulation and technology. We deal with the deterministic case first and then outline the extension to uncertainty in the TFP parameter A. The assumed technology is

(4) $$C(t) + X(t) \le A(t)e^{(1-\theta)\gamma t} K(t)^{\theta} L(t)^{1-\theta}$$

(5) $$\dot{K}(t) = (1-\delta)K(t) + X(t)$$

Capital letters denote aggregates. The initial capital stock $K(0)$ is given. We deal with the parametric case for which the equilibrium employment is always less than one, which means mans that $K(0)$ is not too small, the $A(t)$ path is not variable, and lifetimes are sufficiently long. In this case $L(t)$ equals the employment rate $E(t)$.

Preferences are modified to introduce discounting, so for those born at *t* their utility function is

(5) $$\int_t^{t+1} e^{-\rho t}[\log c - v(h)]\,dt\,,$$

where the length of a lifetime is 1. Provided that the parameters are such that not all are employed at every point in time, the interest rate must be $\rho+\gamma$. The wage rate and consumption grow at rate γ.

We deal with the deterministic case first. The rental rate of capital is

(6) $$r(t) = \rho+\gamma+\delta\,.$$

The equation equating it to the marginal product of capital can be solved for the capital to labor ratio. The equation equating the marginal product of labor to the wage can be solved for the wage. Remaining to be determined is the split of output between consumption and investment.

The initial investment and consumption rates either imply next instance outcomes or that there is no equilibrium consistent with the previous instance outcomes. If the differential equation, given the initial split between investment and consumption, is feasible and satisfies the transversality condition, it is the equilibrium path.

A Digression:

Only under special conditions is the aggregate production function – that is the production function of the stand-in firm – a Cobb-Douglas production function.[3] To rule out the use of the Cobb-Douglas production function just because these special conditions are not literally true, even though the Cobb Douglas production function works well in

[3] See Prescott (2004) for conditions. The proof is in Lecture 10 in Macro Theory on the web page http://minneapolisfed.org/research/prescott/

aggregate analyses, makes no sense. The same is true for the aggregate utility function of the stand-in household. There is a parallel.

Some interesting complications include dealing with married households, which at a point in time can have zero, one, or two wage earners. Still another problem is that many people work for organizations where all work the same periods. On this problem, Fitzgerald analyses the consequences of constraints on workweek length in a world where supervisors and workers must work the same workweek length. There is team or club production and externalities are internalized within organizations that are small relative to the economy and the invisible hand works.

An implication of this is that when estimating individual elasticity of substitution between consumption and leisure, only people with the choice of hours worked should be considered. It is well know that construction workers working for organizations get paid less per hour and work more hours on average per year than those not working for organizations. The seasonal variation in labor supply of construction workers not working for General Motor type organizations is high, yet the wage varies little. In many cases seasonal variation in the production of say housing could be cheaply reduced, thereby smoothing employment with little reduction in wage. But this is not done, indicating a high intertemporal elasticity of labor supply. When someone temporarily works at a higher paying job in an organization, that individual does not have the option to work as much as he/she wants at that higher wage. All these issues suggest that regression results at the individual level must be interpreted with caution when drawing inference as to the sensitivity of labor supply to incentives.

Section 4: Not Puzzling that People Work in Europe

We now establish that even if someone has the preferences of the stand-in household and is offered the option of receiving one-half the wage, the individual will choose to work. We emphasize that the wage is after labor tax and the transfer is not taxed. The following utility function is obtained when calibrating to the U.S. marginal effective tax rate on labor income, which includes both the labor income tax and the consumption tax, of 40 percent (See Prescott 2004), and to the fraction of productive time allocated to market of 0.25:

$$\log c + 1.8 \log(1-h) .$$

In Europe the consumption tax rate is approximately 25 percent and the marginal labor income tax rate 50 percent. The labor income tax rate includes social security taxes.

The technology is $c = Ah$. The consumption tax rate is 0.30 while the labor income tax rate including social security taxes is 0.48. This implies an effective tax rate of 0.60. Labor supply is $h = 0.182$ (after tax wage). Now we consider a system that offers people the option of being paid and not working. The amount paid for not working is 0.50 of the after income tax wage. As the after income tax wage is $0.52A$, this payment is $0.26A$. *This* payment is not subject to the income tax, but consumption purchases are subject to the consumption tax.

The optimal decision is to work with this system, though the loss in utility associated with being paid not to work is small. This matches with the fact that in Europe countries with this type of system are on margin between working and not working.

Section 5: Consequences of Constraints on Workweek Length

A question is what are the consequences of a binding constraint upon workweek length? Absent taxes, aggregate labor supply is

$$e\,l = \frac{l}{v(l)}\,.$$

Labor supply is maximized in the unconstrained case. Precisely how much aggregate labor supply is reduced as a consequence of the constraint on the workweek length requires knowledge of both preferences for leisure and the mapping from time allocated to market activities to units of labor services, as they determine the function *v*. It is important to recognize that a constraint that sets a minimum workweek length above $\bar{l}$ or a maximum workweek length below $\bar{l}$ both reduces aggregate labor supply.

Section 6: Increasing the Retirement Age may not Increase Labor Supply Much

is being written

Section 7: Concluding Comments

The infinitely lived generation abstraction, when calibrated to U.S. data accurately predicts the current large difference in labor supply between Europe and the other advanced industrial countries as well as the large decline in hours worked per working age person in Europe during the 30 year period beginning in the early 1970s. This paper finds that the overlapping generation abstraction does as well.

The assumptions underlying rigorous aggregation on the business side to obtain an aggregate production function are extreme. The aggregation theory does provide insight as to the nature of the aggregate production technology based upon the technologies being aggregated. The same is true on the household side. Aggregation theory along with micro observations say that the aggregate labor supply elasticity will be large, as the macro observations say it is.

Our results suggest that constraints on the workweek length will depress aggregate labor supply. The functional form of the mapping from hours supplied to the market to labor services will determine the extent to which this happens. Also, we find that policies designed to increase the retirement age may result in people working a smaller fraction of their lifetime. Thus we can conclude that policies designed to increase aggregate labor supply by raising the retirement age will be largely offsetting and aggregate lifetime labor supply will not increase significantly.

References

Altonji, Joel, S. 1986 "Intertemporal Substitution Substitution in Labor Supply: Evidence from Micro Data," *J. Polit. Economy* 94, Part 2 (June): S176-S215.

Becker, Gary S. and Gilbert Ghez. 1975. "The Allocation of Time and Goods over the Business Cycle." Reiter's.

Ben-Porath, Yoram. 1967. "The Production of Human Capital and Life Cycle of Earnings." *J. Polit. Economy* 75 (August): 352–65.

Cho, Jang-Ok and Thomas F. Cooley. 1994. "Employment and Hours over the Business Cycle." *J. Econ. Dynam. and Control* 18(2): 411-432.

Cole, Harold L., and Lee E. Ohanian. 1999. "The Great Depression in the United States from a Neoclassical Perspective." *Fed. Reserve Bank Minneapolis Q. Rev.* 23 (Winter): 2–24.

Cole, Harold L., and Lee E. Ohanian. 2004. "New Deal Policies and the Persistence of the Great Depression: A General Equilibrium Analysis." *J. Polit. Economy* 112 (August): 779–816.

Cooley, Thomas F., ed. 1995. *Frontiers of Business Cycle Research*. Princeton, NJ: Princeton Univ. Press.

Fisher, Jonas D. M., and Andreas Hornstein. 2000. "(S, s) Inventory Policies in General Equilibrium." *Rev. Econ. Studies* 67 (January): 117–45.

Fitzgerald, Terry J. 1998. "Work Schedules, Wages, and Employment in a General Equilibrium Model with Team Production." *Rev. Econ. Dynam.* 1 (October): 809–34.

Hansen, Gary D. 1985. “Indivisible Labor and the Business Cycle.” *J. Monetary Econ.* 16 (November): 309–27.

Hornstein, Andreas, and Edward C. Prescott. 1993. “The Firm and the Plant in General Equilibrium Theory.” In *General Equilibrium, Growth, and Trade II: The Legacy of Lionel McKenzie*, edited by Robert Becker, Michele Boldrin, Ronald Jones, and William Thomson. San Diego: Academic Press.

Imai, Susumu, and Michael P. Keene. 2004. “Intertemporal Labor Supply and Human Capital Accumulation.” *Int. Econ. Rev.* 45 (May): 601–41.

Kehoe, Timothy J., and Edward C. Prescott. 2002. Introduction to “Great Depressions of the 20th Century.” *Rev. Econ. Dynam.* 5 (January): 1–18.

Kydland, Finn E., and Edward C. Prescott. 1982. “Time to Build and Aggregate Fluctuations.” *Econometrica* 50 (November): 1345–70.

Kydland, Finn E., and Edward C. Prescott. 1991. “Hours and Employment Variation in Business Cycle Theory.” *Econ. Theory* 1 (January): 63–82.

Lucas, Robert E. and Leonard Rapping. 1969. "Real Wages, Employment, and Inflation." *Journal of Political Economy* 77(5): 721-754.

MaCurdy, Thomas E. 1981. “An Empirical Model of Labor Supply in a Life-Cycle Setting.” *J. Polit. Economy* 89 (December): 1059–85.

Prescott, Edward C. 2002. “Prosperity and Depressions.” *American Econ. Rev.* 92(2): 1-15.

Prescott, Edward C. 2004. “Why Do Americans Work So Much More than Europeans?” *Fed. Reserve Bank Minneapolis Q. Rev.* 28 (July): 2–13.

Prescott, Edward C., and Karl Shell. 2002. “Introduction to Sunspots and Lotteries.” *J. Econ. Theory* 107 (November): 1–10.

Prescott, Edward C., and Robert M. Townsend. 1984a. “General Competitive Analysis in an Economy with Private Information.” *Int. Econ. Rev.* 25 (February): 1–20.

Prescott, Edward C., and Robert M. Townsend. 1984b. “Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard.” *Econometrica* 52 (January): 21–45.

Rogerson, Richard. 1984. *Topics in the Theory of Labor Markets*. Ph.D. thesis, University of Minnesota, September.

Rogerson, Richard. 1988. “Indivisible Labor, Lotteries and Equilibrium.” *J. Monetary Econ.* 21 (January): 3–16.

Rosen, Sherwin. 1978. “The Supply of Work Schedules and Employment.” In *Work Time and Employment*. Washington, DC: National Commission for Manpower Policy.

Weiss, Yoram. 1972. “On the Optimal Lifetime Pattern of Labor Supply.” *Economic J.* 82 (December): 1293–1315.