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Ulrich Erlenmaier
&
Hans Gersbach

CESifo
Poschingerstr. 5, 81679 Munich, Germany
Phone: +49 (89) 9224-1410 - Fax: +49 (89) 9224-1409
E-mail: office@CESifo.de
Internet: http://www.cesifo.de

The Funds Concentration Effect and Discriminatory Bailout

Ulrich Erlenmaier and Hans Gersbach*†

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Abstract

In the presence of macroeconomic shocks severe enough to threaten the liquidity or solvency of the banking system, the regulator can rely on the funds concentration effect to save long-term investment projects. Some banks are forced into bankruptcy with the result that other banks obtain more new funds and remain solvent. We investigate two different implementations of the funds concentration effect and the corresponding discriminatory bailout scheme: "random bailout" and "bailout the big ones". While the latter can be problematic in terms of stability, it is superior to the former in terms of welfare and credibility.

Keywords: Financial intermediation, Macroeconomic risk, Banking regulation, Discriminatory bailout, Funds concentration, Aggregate liquidity, Consistent expectations.

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*Alfred-Weber-Institut, University of Heidelberg, Grabengasse 14, 69117 Heidelberg, Germany.

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1 Introduction

The frequency and virulence of financial crises has led to serious rethinking concerning the appropriate form of government intervention in financial markets. A major issue is whether such crises can or should be avoided or whether a workout approach is superior to prevention. In particular, it is unclear how financial intermediaries should be regulated when they are subject to large macroeconomic shocks, as has been the case in the recent crisis in Asia.[1]

While in the period before 1970 less intensive competition in banking in connection with interest rate ceilings created oligopoly profits which acted as a buffer against macroeconomic shocks, the present regulatory frameworks are focussed on the prevention of banking crises through cash-asset reserves and risk-sensitive capital requirements. If a banking crisis nevertheless occurs, a variety of approaches are applied. In the most common case of explicit or implicit deposit insurance, the taxpayers' money is used to bail out banks. In some cases, banking crises have been dealt with by closing some banks or by takeovers,[2] which smacks of a discriminatory approach to bailout.

Since the prevention of crises via restricted competition or ex-post bailout with taxpayers' money has costs of its own, and because equity will not always be sufficient to buffer severe macroeconomic shocks,[3] we will focus in this paper on the possibility of discriminatory bailout.

Under discriminatory bailout, the regulator forces only one or a small number of banks into bankruptcy while the remaining banks are allowed to continue with their operations although all banks may be identical with respect to their balance sheet. The rationale for discriminatory bailout can best be understood in an overlapping generation framework where banks invest short-term deposits in long-term productive investment. During the fruition time of the long-term investments, banks need to refinance themselves by taking the deposits from new generations of savers in order to pay back deposits from the old generation.

Suppose that, during the fruition time, new information reveals that the real return on long-term investment is low. This shock will not allow all banks to refinance long-term

[1]See e.g. Hellwig (1998) and Bhattacharya, Boot, and Thakor (1998)

[2]This has happened e.g. during the crisis in Asia and the Swiss regional bank crisis (see Radelet and Sachs (1998, 1999) and Staub (1998)).

[3]For example Hellwig (1995) notes (p. 723): "Given the difficulties of recapitalization after a spell of bad luck - and given the possibility of repeated bad spells - it is not clear what one means in asking a bank to follow a strategy of having more equity as a buffer. More equity at the beginning - certainly! But thereafter?" Moreover, Gersbach (2001) shows that requiring large equity buffers for banks reduces equity in firms, thereby increasing credit rationing which has negative macroeconomic consequences.

productive investment by taking new deposits since they cannot credibly promise sufficiently high interest rates and cannot attract enough deposits from the new generation. Since there is no coordination mechanism that allows depositors to concentrate savings on a fraction of banks in order to allow them to refinance, regulatory intervention is desirable because liquidation of all long-term investment projects is the worst case.

In order to save both banks and long-term investment projects, the regulator can rely on the following general equilibrium effect, which we call *funds concentration effect*: by forcing some illiquid banks into bankruptcy, the share of funds available for the remaining banks will increase since there are fewer banks competing for new deposits. Moreover, the surviving banks can buy investment projects from bankrupt banks at liquidation value, thus enabling them to credibly offer higher deposit rates to the second generation. The bailout policy of the regulator is discriminatory.

To concentrate on the funds concentration effect of bank closures, we start our analysis with a situation where the banks' insolvency is assumed to result solely from an exogenous macroeconomic shock. Since the realization of this shock is not under the control of the banks' managers, it would be most natural to decide randomly about which banks to close (RB). However, we find that closure policies feed back into the banks' competitive behavior and we therefore also consider two other bailout schemes, i.e. bailout of big banks (BB) and bailout of small banks (BS).

We compare the discriminatory bailout approach this regulation with scenarios where banking crises are prevented completely and with the no-regulation case. Moreover, we analyze the different implications of the discriminatory bailout schemes with respect to stability, welfare and credibility. We identify BB as the preferred bailout scheme if depositors can coordinate on maximum-return assessments. BS raises severe stability problems and may support low-return equilibria, which both can be avoided under BB. Moreover, BB dominates RB with respect to welfare and credibility of regulatory actions when negative macroeconomic shocks occur.

Finally, recognizing that the welfare implication of this paper can only be a first step towards a more complete assessment of the pros and cons of discriminatory bailout, we want to stress that an important aspect of this paper is the provision of a simple analytical framework and a clarification of the major conceptual issues involved. Given the possibility of a macroeconomic shock and discriminatory bailout, deposits are risky. If an individual bank raises deposit rates, it will affect its own bailout probability as well as that of all other banks, since the refinancing needs rise accordingly. Therefore, the expected returns for depositors of all banks are influenced by the decision of an individual bank. Moreover, the distribution of deposits among banks will affect expected returns on deposits as well, since some banks have higher refinancing needs under asymmetric distributions than others. These banks might have to offer higher second-period deposit rates than under symmetric distributions, forcing the other banks to offer higher rates as well in order to obtain any savings at all. Since expected returns for all banks are affected by individual bank decisions and by depositors' savings decisions, it is not a priori clear whether consistent assessments of depositors' expected returns actually exist. We establish a general existence result for consistent return assessments. We also identify the constellations in which such assessments may not exist.

2 Review of the Literature

The role governments should play in managing illiquid banks remains one of the main unresolved issues in banking regulation (see BHATTACHARYA, BOOT, AND THAKOR (1998)). The existing theoretical literature primarily draws on a partial equilibrium point-of-view where systemic consequences are accounted for only by *exogenous* factors. It has been stressed that closure policies have to weigh the costs of bailout (subsidies to uninsured debtholders) with the closure costs (direct bankruptcy costs, externalities). Excessive risk taking incentives can occur as both costs of bailout and costs of closure. On the one hand, bailout creates moral hazard, as the probability of surviving depends less on the bank's risk choice and more on the regulator's actions. On the other hand, it increases the bank's probability of survival, thus raising the value at stake and, in turn, the bank's incentive to protect it.[4]

Depending on how the different costs are weighed, authors come to different conclusions about the desirability of governmental intervention. While for example HUMPHREY (1986) and SCHWARTZ (1995) advocate a non-interventionist view on bank bailouts, the opposite view, namely that in some cases bailing out banks is socially desirable, has been put forward by MISHKIN (1995), SANTOMERO AND HOFFMAN (1998), FREIXAS, PARIGI, AND ROCHET (1998)) or CORDELLA AND YEYATI (1999).[5] Our paper gives a new slant to this debate. In our model, closing some banks is necessary so that others can survive without further government intervention. In this sense, putting the funds concentration effect to work is both interventionistic and non-interventionistic.

[4]See CORDELLA AND YEYATI (1999) for a formalization of the tradeoffs resulting from these two mutually offsetting effects.

[5]For a comprehensive discussion of this issue see GOODHART (1995).

A further question raised in the literature is how the decision to close a bank should depend on important bank-specific or macroeconomic variables such as the level of uninsured debt on a bank's balance sheet (Freixas 1999), the size of a bank (Goodhart and Huang 1999) or aggregate investment returns (Cordella and Yeyati 1999). Freixas (1999) finds that under optimal policies, banks will be closed either if they have a too low *or* a too high level of uninsured debt on their balance sheet. Whether the former or the latter of these policies should be applied depends on the respective dominance of two counteracting effects: the costs of the subsidies to uninsured debt holders on the one hand and the monitoring incentives for debtholders (which are increasing in the level of uninsured debt) on the other hand. Cordella and Yeyati (1999), investigating how closure policies can minimize the risk taking incentives of banks, find that banks should be bailed out if the macroeconomic variable describing aggregate investment returns falls below a certain threshold level. The intuition behind their conclusion is that if the aggregate variable is high (high aggregate returns) and a bank fails nevertheless, this will signal excessive risk taking which is discouraged by threatening closure. Bailing out banks in low states of the variable will increase a bank's charter value and therefore decrease risk taking incentives.

While the conditionality introduced in Cordella and Yeyati (1999) would have no sensible application in the crises scenarios we are mainly interested in,[6] distinguishing between the relative levels of insured deposits and uninsured debt on a bank's balance sheet would be a further useful step for the analysis of bank closure policies in general equilibrium frameworks.

Finally, Goodhart and Huang (1999) provide a framework that justifies a "bail out the big ones" policy as long as risk-taking incentives are not taken into account. If those incentives are taken into account the optimal rescuing policy may depend on the size of the bank in a non-monotonic way. While Goodhart and Huang (1999) derive their results by comparing the costs of bank failure (contagion) and of bailout (rescuing insolvent banks with the taxpayers' money), we stress the following advantages of BB: compared to RB it is more credible ex-post and helps to avoid low return equilibria; compared to BS it always guarantees the existence of consistent return assessments of depositors (which is not the case under BS). However, BB is subject to self-fulfilling prophecies and hence return assessments might not be unique. This is not the case under RB.

Besides the analysis of optimal bank closure policies, an important strand of the literature has investigated the regulator's incentives to apply such rules. Boot and Thakor (1993) examined the regulator's incentives to close banks in a manner that results in

[6] While the macroindicator would surely indicate that all banks should be rescued in such scenarios, it would still be too costly to do so.

socially optimal bank portfolio choices. They find that the regulator's optimal bank closure policy is less tight than is socially optimal. The analysis has been extended by ACHARYA AND DREYFUS (1989), FRIES, MELLA-BARRAL, AND PERRAUDIN (1997) and MAILATH AND MESTER (1994). Finally, REPULLO (1999) considers government agencies with different objective functions and investigating which of these agencies should make bailout decisions. He finds that central banks should be responsible for dealing with small liquidity shocks, while the deposit insurance agency should deal with large ones. In this paper, we do not address institutional design issues as in REPULLO (1999). but incentives for regulators are briefly discussed during the analysis of the bailout schemes' credibility.

On a conceptual level, this paper is related to the literature in three respects. First, discriminatory bailout can be interpreted as a version of the "creative ambiguity" principle, where regulators have full discretion to let one bank go bankrupt. Two concepts of creative ambiguity have been discussed in the literature. In FREIXAS (1999) the central bank deciding which banks are to be rescued follows a mixed strategy. In GOODFRIEND AND LACKER (1999) and REPULLO (1999) on the other hand, the bailout policy is not random from the perspective of the central bank but is perceived as such by outsiders that cannot observe the supervisory information that leads to the bailout decisions. Our closure policy RB introduces a creative ambiguity concept similar to FREIXAS (1999), since the regulator will choose to bail out a bank with a certain probability. The BB and the BS concept are subtle mixtures of predetermined bailout (if banks differ in size) and creative ambiguity (if banks are equal in size). In contrast to FREIXAS (1999), who considers a regulator that follows a mixed strategy when deciding about a *single* bank's bailout, we investigate the whole banking system and motivate creative ambiguity with aggregate solvency concerns. This makes the design of such a policy more demanding since bailout probabilities have to be chosen in a way ensuring that the banks which have not been closed will be able to survive.

Second, this paper is related to the literature discussing banking competition when deposits are risky and depositors' assessments about the performance of a bank might become self-fulfilling. In particular, MATUTES AND VIVES (1996) have highlighted the importance of the perceptions of depositors about the probability of success in banking competition. They show that return expectations can become self-fulfilling because of economies of scale or diversification effects. In this paper the probability of an individual bank failure is determined by the amount and distribution of savings obtained by banks in conjunction with the regulatory bailout approach. This creates existence problems for consistent return assessments by depositors. Moreover, we show that BB (but not RB) can lead to self-fulfilling prophecies. Under BB, a bank that is assessed as offering high returns on deposits attracts a larger share of depositors than competitors and hence increases the likelihood of bailout in the event of a negative macroeconomic shock. This may validate the higher expected returns that depositors have associated with this bank in the first place. As in MATUTES AND VIVES (1996), such self-fulfilling prophecies can create stability problems. These problems do not occur under RB.

Finally, our analysis bears on the recent tradition of integrating financial intermediation in overlapping generation models. An important strand in this literature (FULGHIERI AND ROVELLI (1998), BHATTACHARYA AND PADILLA (1996) and QI (1994)) has examined the relative merits of intermediaries and financial markets for the possibility of sharing liquidity risk across generations. In this paper we examine how macroeconomic shocks can be dealt with in OLG frameworks.

3 The Model

The model encompasses two overlapping generations; the first generation lives from $t = 0$ to $t = 1$ and the second from $t = 1$ to $t = 2$. Each generation consists of a continuum of households. There is one single physical good in the economy which can be used for production and consumption. Moreover, there is a number of banks owned and managed by bankers. Banks gather the households' savings and invest them in a production technology.[7] The key features of the model are:

1. returns on the production technology are subject to macroeconomic risk;

2. banks offer *uncontingent* deposit contracts to households, thereby exposing themselves to macroeconomic risk.

We first have to justify why some of the macroeconomic risk remains on the balance sheets of the banks. According to HELLWIG (1998), a bank could in principle reduce its exposure to macroeconomic risk traceable to easily observable indicators such as GDP or interest rates (either by offering state contingent deposit contracts or by transferring risk to third parties via hedging contracts). However, a full isolation of banks from macroeconomic risk does not occur and banks in reality bear substantial macroeconomic risk. HELLWIG (1998) offers a detailed account of why this is the case. First, available indicators are only an incomplete measure of exposure to aggregate risk. Second, in practice banks do not

[7]For simplicity of representation we do not model bank loans to entrepreneurs.

conclude contingent deposit contracts for the following reasons: the inflexibility of indexed deposit rates as a risk management tool, the existence of transaction costs, and the market-making role of banks. Moreover, the on-demand clause of deposit contracts may invite runs on banks if repayments are made contingent on the realization of macroeconomic variables such as GDP at a certain point in time. Third, hedging counterparties are often banks themselves and hence our analysis can be applied to the counterparty banks. Moreover, banks which shift their risk to third parties are still exposed to credit risk; this risk is likely to be correlated with the macroeconomic risk they want to insure themselves against.[8]

In order to keep the analysis as simple as possible, we do not focus on the moral hazard of banks or risk aversion of households as further possible explanations for aggregate risk exposure for banks. However, our analysis can be applied to the excessive risk-taking problem, which has been identified as one of the major problems of prudential banking regulation (see e.g. DEWATRIPONT AND TIROLE (1994)). If all banks in the industry undertake portfolio choices with a common macroeconomic risk component that cannot be diversified, regulatory intervention can follow a logic similar to the one outlined in this paper. The additional question emerging in this context is how regulatory bailout schemes affect bank's risk choices. Integrating this aspect will have to be left to future research.

Finally our model allows an alternative interpretation that draws on the uncertainty about the accuracy of banks' risk management systems rather than on uncontingent deposit contracts. Suppose that banks write contingent contracts which - according to their risk management tools - isolate them from macroeconomic risk. If banks use similar risk management tools, the aggregate uncertainty about future returns can be interpreted as aggregate uncertainty with respect to the accuracy of contingencies in the deposit contracts: risk management tools may overestimate production returns in the state $R_2 = r_{2l}$ while they underestimate them for $R_2 = r_{2h}$ which leaves the banking system exposed to macroeconomic risk.[9]

[8] GERSBACH (1998) describes two additional scenarios in which banks do not offer contingent deposit contracts. In the first scenario, the regulator can commit to the failure of insolvent banks. Macroeconomic shocks are then borne by risk-neutral entrepreneurs, as long as their inside funds are a sufficient buffer for these shocks. In the second scenario, banking crises are worked out. Banks offer uncontingent deposit rates that can only be paid back when the state of returns is good. Downturn macroeconomic risk is shifted to future generations.

[9] This view is for example substantiated by SHIN (1999) who suggests that the risk management tools of financial institutions tend to heavily underestimate risk during episodes of market turbulence since they do not take into account the endogeny of future market outcomes (i. e. the fact that outcomes depend on their own actions and that of other market participants).

3.1 Technology

We assume that there is a long-term technology that pays a random return of R_2 units of the good in $t = 2$ for each unit invested in $t = 0$. If liquidated in $t = 1$, returns are zero.[10] Production returns in $t = 2$ are subject to aggregate risk. Two different realizations of R_2 are possible. In the first state, occurring with probability p_l, we have low returns: $R_2 = r_{2l}$. In the second state, with probability $p_h := (1 - p_l)$, we have high returns $R_2 = r_{2h}$. The expected return $p_l r_{2l} + p_h r_{2h}$ is denoted by $\bar{R}_2$. We assume that the realization of the aggregate productivity shock is revealed in $t = 1$ and will be observed by all market participants. We assume (a) constant returns to scale and (b) that investment at arbitrary scale is possible.

3.2 Banks

The need for financial intermediation can arise for several reasons (see Bhattacharya and Thakor (1993) for a comprehensive overview). We take this need for granted and do not model it explicitly here. A special feature of our model is that banks finance long-term investments with short-term savings contracts. In contrast to the standard Diamond and Dybvig (1983) framework, there is no risk of consumption timing for the first generation in our model. The individuals of the first generation know that they will never see the fruits of their long-term investments. However, there is an aggregate production risk that makes consumption uncertain in the second period. The economic problem lies in enabling both generations to participate in the benefits of a risky long-term investment though only the second generation will see the returns of the investment.

In $t = 0$ there are n banks, denoted by $B_1, ..., B_n$. They are long-living institutions enabling both generations to participate in long-term investments. They offer deposit contracts at deposit rates d_1^i to the first generation and receive an amount D_1^i of deposits ($i = 1, .., n$); all deposits are invested in the production technology. In $t = 1$ banks have to pay back their debt $d_1^i D_1^i$ to first-generation depositors. To obtain new funds, they offer deposit contracts to the second generation at deposit rates d_2^i. After banks have received their second-period deposits, two cases can occur for each individual bank. First, it has raised enough funds from second-generation depositors to pay back its debt; in this case it receives investment returns in $t = 2$ and pays back its second-period depositors. If returns are not sufficient to service all depositors in $t = 2$, investment proceeds are uniformly distributed among depositors. Second, the bank cannot raise enough funds; in

[10] We will relax this assumption later on (see section 6.4.1 and appendix ??.

this case it has to declare bankruptcy, and the investments are liquidated. First-period depositors of such banks receive only the bank's cash, i.e. the deposits of second-period depositors if there are any. Second-period depositors receive nothing.

We complete the description of the banking sector by assuming (a) that banks are owned by risk-neutral bankers[11] who live for three periods and consume in $t = 2$ and (b) that bank managers maximize expected bank profits and hence internalize losses that accrue to depositors in case their claims cannot be fully served.[12]

3.3 Households

There are two overlapping generations of consumers (first and second generation), each consisting of a continuum of households living for two periods. They are risk-neutral but want to smooth consumption over time. The assumption of risk neutrality is made for convenience and tractability as in Bernanke and Gertler (1988) and Kijotaki and Moore (1997). Households deposit their savings in the banking system. We assume that the individual savings function s_{gh} that describes how much funds household h in generation g ($g = 1, 2$) is willing to deposit with banks is increasing in the *expected* return paid on bank deposits, which we denote by u.

Note that since in both periods some banks might not be able to fully pay back their debts to depositors, both generations of households have to assess the expected returns paid by each bank given first-period deposit rates (for the first generation) and given the first-period allocation and second-period deposit rates (for the second generation). Finally, we denote the resulting aggregate saving function for generation g as $S_g(\cdot)$ and assume that S_g is continuous and strictly increasing in u. Note that $S_g(u)$ can be represented as an integral of a savings density function $s_{gh}(u)$ over an interval on the real line (w.l.o.g. $[0, 1]$), each point on the interval representing one household: $S_g(u) = \int_0^1 s_{gh}(u)\, dh$. Later on we will use the expressions "*full measure of savings*" and "*zero measure of savings*". A certain bank has obtained the full measure of savings if it has not attracted all depositors

[11] Note that for the sake of tractability we have excluded the possibility of issuing equity. We could allow for equity as long as bank reserves cannot buffer losses completely in the event of negative macroeconomic shocks.

[12] Therefore, we deliberately abstract from the usual risk-taking incentive when bank managers do not take the losses accruing to depositors into account. For instance, we can assume that the regulatory body imposes some kind of penalty on failing banks or bank managers that induces them to internalize losses (see e.g. Dewatripont and Tirole (1994)). If bank managers only maximized returns for shareholders and sought excessive risks, banks would bid up deposit rates even higher than derived in this paper, thereby aggravating the refinancing problems of banks in $t = 1$. Consequently, the benefits of regulation, based on funds concentration, would be even more pronounced when risk-taking incentives are present

but if the integral of the savings density function over all the banks' depositors is equal to the integral over all households ("full-measure bank"). If a bank has attracted some depositors but the integral of the savings density function over the banks' depositors is zero, then we say that the bank has obtained a zero measure of savings ("zero-measure bank"). In the sequel we will use functions of the type $S(u) = au^{\alpha}$ with $a, \alpha \in (0, \infty)$ as an example for the saving functions of both generations.

Finally, note that the saving functions S_g for deposits can can be interpreted as a result of a portfolio decision. Deposits may only be one of several saving possibilities[13] that are imperfect substitutes. In this case, the expected-return elasticity of deposits can be quite high.

3.4 Example

Throughout the paper we will use the example presented in table 1 to illustrate our results. The example is given by the following parameters.

$S_1(u) = u$	$p_l = 0.2$	$r_{2l} = 1.03$	$\bar{R}_2 = 1.18$
$S_2(u) = 1.07 \cdot u$	$p_h = 0.8$	$r_{2h} = 1.22$	

Table 1: Example A.

3.5 Regulatory Policy

We will derive the necessity of regulation precisely in sections 4.1 and 5.1. For the time being, note that it will result from the fact that in the case of low production returns it might not be possible for all banks to refinance in $t = 1$, since they cannot credibly offer sufficiently high second-period deposit rates in order to attract enough new funds. Nevertheless, it might be possible for a fraction of the banks to refinance if depositors concentrate their savings on those banks. Without regulation though, depositors have no possibility of coordinating their savings on such a fraction of banks; equilibria in which no bank is able to refinance can therefore not be excluded. We will therefore consider three regulatory scenarios designed to avoid these problems. The first one (prudential banking) ensures that the whole banking system is able to refinance in both states of production

[13] The others are not modeled explicitly but enter the model via the specification of the saving functions.

returns by encouraging banks to offer low deposit rates in the first period. The other two (discriminatory bailout via random bailout or bail out the big ones) allow for situations where the banking system is not able to refinance itself and the regulator solves the coordination problem of depositors by closing a fraction of banks in order to make sure that the others can survive. Closing some banks will have two effects; first, it will reduce the amount of second-period deposits needed by the banking system; second, by taking over investment projects of closed banks, surviving banks can offer higher returns on deposits. In this section we describe the different regulatory approaches formally .

3.5.1 Bailout Schemes

Suppose that there are $m \leq n$ banks in $t = 1$ that have received deposits. The regulator observes the realization r_2 of the macroeconomic shock, i.e. the future prospects of aggregate production returns. The banking system is able to refinance if and only if

$$S_2(r_2/d_1^{\max}) \geq \sum_{i=1}^{n} d_1^i D_1^i. \tag{1}$$

$d_1^{\max}$ is the highest deposit rate that has been offered by a bank in the first period. Note that $r_2/d_1^{\max}$ is the highest return that *all* banks can credibly offer to the second generation in $t = 1$ (because $d_2^i d_1^i$ cannot exceed r_2). If refinancing condition (1) holds, then all banks can survive (for example, if a uniform deposit rate of $r_2/d_1^{\max}$ is offered to second-period depositors) and the regulator will not intervene. Consequently all banks will be allowed to compete for second-generation deposits in this case. In the following we will use the the matrix $\mathbf{\Delta} := (\Delta_i)_{i=1}^n$, where $\Delta_i = (\Delta_{iD}, \Delta_{iI})$ to summarize deposits and investments of the banks after the regulatory decision. Δ_{iD} denotes the obligations to first-period depositors and Δ_{iI} denotes the units of investment projects that a bank holds. Hence, if (1) holds, deposits and investments are given by $\Delta_i = (d_1^i D_1^i, D_1^i)$ for $i = 1, .., n$.

If condition (1) does not hold, then not all banks will be able to refinance themselves because new funds at the largest credible uniform deposit rate are less than the aggregate obligations of the banking system.[14] In this case the regulator will close a certain number $(m - k)$ of banks and additionally eliminate a fraction $(1 - b)$ of the surviving banks' deposits. Depositors whose deposits have been eliminated will loose their claims on the bank. The bailout schemes only differ with respect to the manner in which the subset of surviving banks, which we denote by $\mathcal{B}^+$, is determined. Under all schemes the regulator

[14] If higher deposit rates would be credibly offered by some banks, then at least the bank that has offered $d_1^{\max}$ in $t = 0$ would not be able to receive any deposits.

first determines the number k of surviving banks and the fraction b of deposits which will remain on the surviving banks' balance sheets. Under **random bailout (RB)**, $\mathcal{B}^+$ is then determined by *randomly* drawing k banks (out of the m banks which have received any deposits). Under **prudential banking (PB)**, the regulator applies RB and additionally imposes a penalty P on all banks that had to be closed. P is assumed to be so high that a strategy of a bank which has a positive probability of leading to P will always be eschewed in favor of any strategy that does not involve the possibility of illiquidity, including exiting from the market.[15] While the surviving banks are chosen randomly under RB and PB, banks are ordered with respect to the amount of first-period deposits they have gathered under **bail out the big ones (BB)**:

$$D_1^{\tau(1)} > \cdots > D_1^{\tau(\underline{k})} = \cdots = D_1^{\tau(k)} = \cdots = D_1^{\tau(\bar{k})} > \cdots > D_1^{\tau(m)}.$$

The set $\mathcal{B}^+$ will contain the banks $B_{\tau(1)}, ..., B_{\tau(\underline{k}-1)}$ and another $k - (\underline{k} - 1)$ banks which are chosen randomly from the set $\{B_{\tau(\underline{k})}, ..., B_{\tau(\bar{k})}\}$. The scheme **bail out the small ones (BS)** is defined in a symmetric way; the only difference is that the ordering scheme is reversed, i.e. big banks will be closed first.[16] Since both bailout schemes BB and BS work completely symmetrically, we will describe only BB in more detail; also all examples that will be discussed refer to BB.

The investment projects of closed banks are distributed among surviving banks in proportion to the amount of deposits they have gathered. Hence, after regulatory intervention, the balance sheet of a surviving bank i consists of obligations $bd_1^i D_1^i$ to first-period depositors and of $b_I D_1^i$ units of investment projects where[17]

$$b_I = b_I(\mathcal{B}^+) := \frac{\sum_{i=1}^n D_1^i}{\sum_{i \in \mathcal{B}^+} D_1^i}. \tag{2}$$

Hence

$$\Delta_i := \begin{cases} (bd_1^i D_1^i \, , \, b_I D_1^i) & \text{if i} \in \mathcal{B}^+ \\ (0 \, , \, 0) & \text{else.} \end{cases}$$

[15]Note that by offering very unfavorable deposit rates a bank can always ensure that it will never become illiquid, regardless of the behavior of the other banks.

[16]Regarding BB and BS we might also envisage ordering banks with respect to the value of their obligations instead of ordering them with respect to their deposits, as above. Our arguments would not be affected by a switch to outstanding debt ordering.

[17]Note that we use $\mathcal{B}^+$ not only to denote the set of surviving banks but also to denote the set of indices $i_1, ..., i_k$ that identify the surviving banks. Note also that if all banks in $\mathcal{B}^+$ have only a zero measure of deposits on their balance sheet, then the denominator of equation (2) is zero. In this case investment projects are uniformly distributed among all banks in $\mathcal{B}^+$.

Recall now that regulatory policy must ensure that all remaining banks are able to pay back their first-period depositors with the savings of the second generation. Note that if a bank i receives exactly the amount of second-period deposits that it needs to service its obligations (i.e. $D_2^i = bd_1^i D_1^i$), then it will be able to credibly offer a deposit rate $r_2 b_I/(bd_1^i)$ to its second-period depositors; hence the rate $r_2 b_I/(bd_1^{\max})$ can be offered by *all* surviving banks and the total amount of second-period savings that can be attracted is at least $S_2\Big(r_2 b_I/(bd_1^{\max})\Big)$. Since b/b_I is equal to the fraction of overall deposits which have not been eliminated (we denote this fraction by q), we conclude that all remaining banks will be able to refinance if

$$S_2\left(\frac{r_2}{qd_1^{\max}}\right) \geq qd_1^{\max} \sum_{i=1}^{n} D_1^i.$$

The highest possible fraction $\bar{q}$ of first-period deposits that can be bailed out under the constraint that the surviving banks shall be able to refinance is therefore given as solution of the equation[18]

$$S_2\left(\frac{r_2}{qd_1^{\max}}\right) = qd_1^{\max} \sum_{i=1}^{n} D_1^i. \tag{3}$$

Note that under BB we have

$$q = \frac{b\sum_{i=1}^{k} D_1^{\tau(i)}}{\sum_{i=1}^{n} D_1^i}. \tag{4}$$

Hence, k and b can be chosen to ensure that $q = \bar{q}$. Obviously there is more than one combination of k and b that leads to $q = \bar{q}$. It is therefore important to note that allowing the regulator to additionally eliminate a fraction $(1-b)$ of all the surviving banks' balance sheets only serves technical purposes.[19] In principle we do not allow for the balance sheets of all banks to be scaled down arbitrarily without disruptive consequences for the banks to continue with their operations and thus for the economy. If such an arbitrary scale-down were possible, an alternative implementation of the funds concentration effect would be to scale down the balance sheets of all banks without completely closing any of them. But under severe macroeconomic shocks (in which our major interest lies), the scale-down needed would most likely disrupt operations of banks and thus shrinking all banks

[18]Note that the left-hand side of the equation is decreasing while the right-hand side is strictly increasing in q which, together with the fact that inequality (1) does not hold, implies that there is a unique solution $q \in [0,1]$ of equation (3).

[19]This assumption allows us to avoid discontinuities (see page 26).

simultaneously is no viable alternative. Therefore, under all bailout schemes we will try to choose b as high as possible. Hence, using equation (4) we determine k and b under BB by

$$k = \min\Big\{l \in \mathbb{N} \mid \sum_{i=1}^{l} D_1^{\tau(i)} \geq \bar{q} \sum_{i=1}^{n} D_1^i \Big\}, \quad (5)$$

$$b = \Big(\bar{q} \sum_{i=1}^{n} D_1^i\Big) / \Big(\sum_{i=1}^{k} D_1^{\tau(i)}\Big). \quad (6)$$

Contrary to BB, the fraction q of bailed out depositors under RB is in general *not* determined by the choice of k and b since it is not clear which banks will be chosen to survive. To determine k under RB, we must therefore take into account that the fraction of remaining deposits should not exceed $\bar{q}$, regardless of which banks have been chosen. The worst case that can be thought of in terms of remaining deposits is that - as under BB - the k largest banks have been chosen to survive. Hence, to ensure that the fraction of bailed out deposits is equal to $\bar{q}$ in this case, k and b are determined as under BB, i.e. according to equations (5) and (6).

3.5.2 Bailout Schemes: The Symmetric Case

In this section we illustrate the working of the bailout schemes for an arbitrary symmetric first-period allocation (d_1, D_1) where all banks have offered the same deposit rate d_1 and received the same amount of first-period deposits D_1. In this case all bailout schemes will proceed in the same way. First, $\bar{q}$ is determined as the solution of a simplified version of equation (3):

$$qnd_1D_1 = S_2(\frac{r_2}{qd_1}). \quad (7)$$

Figure 1 illustrates the solution of equation (7) for example A, which will be used for all illustrations unless otherwise indicated. Second, to achieve a fraction $\bar{q}$ of bailed out deposits with b as a high as possible, we obtain $k = \lceil nq \rceil$ and $b = (nq)/\lceil nq \rceil$.[20] The k banks that will survive are chosen randomly under both schemes since all banks have raised the same amount of first-period deposits; hence the bailout probability of each single deposit is equal to the fraction $\bar{q}$ of bailed out deposits.

As an example consider the case $n = 4$. Figure 2 depicts the function $q \to \lceil nq \rceil$. If e.g.

[20] Note that $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

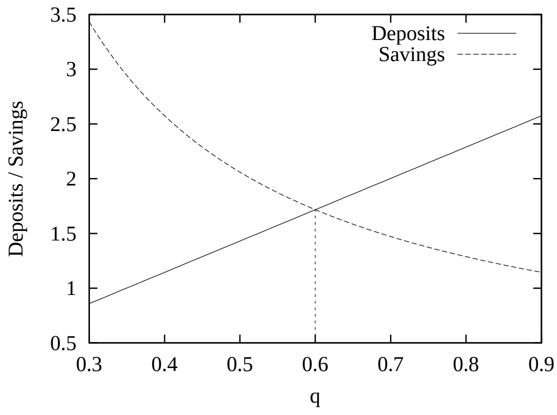

Figure 1: Overall first-period Deposits qnd_1D_1 and second-period savings $S_2(r_2/qd_1)$ as functions of q ($nD_1 = 1.6$ and $d_1 = 1.05$, example A).

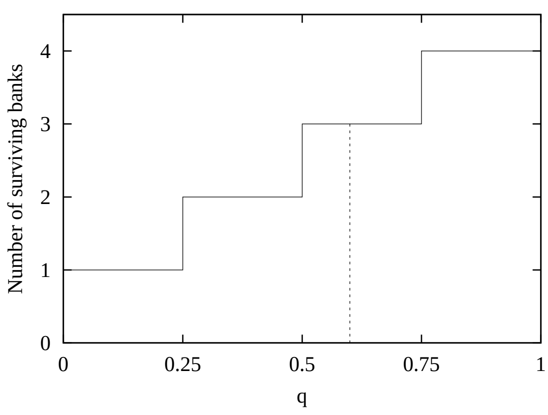

Figure 2: Number k of surviving banks as function of q.

$\bar{q} = 0.6$ then $\lceil n\bar{q} \rceil = 3$ and one bank will be closed. Hence $b = 1.8/3 = 0.6$ and 40% of the deposits of each surviving bank are eliminated.

3.5.3 Bailout Schemes: The Asymmetric Case

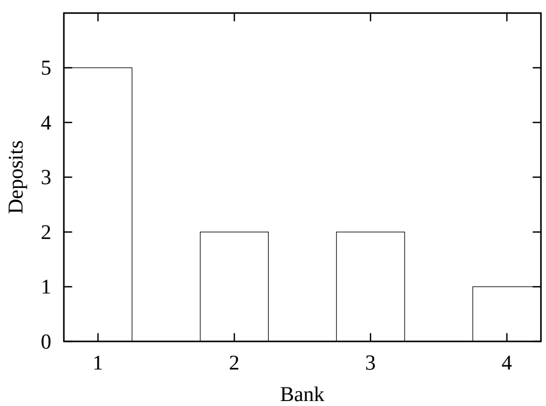

Figure 3: Example distribution of first-period bank deposits

If the first-period deposit distribution is asymmetric (i.e. if not all banks have received the same amount of deposits), the schemes RB and BB will generally produce different regulatory decisions; under BB, always a fraction $\bar{q}$ of depositors is bailed out and the bailout probabilities for deposits depend on the size of the bank at which the deposits are held. Under RB, in contrary, the bailout fraction can be lower than $\bar{q}$ and the bailout probability of each deposit is given by $(k - 1 + b)/m$.

We start, however, with an important case of asymmetric deposit distribution where RB and BB produce the same regulatory decision: the case where one bank has obtained all deposits and the other banks none. In this case we have $k = 1$ and $b = \bar{q}$ implying that the bailout probability of each deposit is equal to the fraction $\bar{q}$ of bailed out deposits.

As an example illustrating the differences between BB and RB, consider a deposit distribution as depicted in figure 3 and suppose that as above $\bar{q} = 0.6$. To determine k, note that the sum of all first-period deposits is 10. Since $5/10 < 0.6$ and $(5+2)/10 > 0.6$ we have $k = 2$ and $b = 10 \cdot 0.6/(5+2) \approx 0.85$. Under BB, the regulator will therefore close bank 4 and either bank 2 or bank 3; the choice between those two banks is performed randomly with each bank having a probability of 0.5 to survive. After that a fraction $(1-b) = 0.15$ of the surviving banks' deposits is eliminated. Hence, the bailout probability for deposits at bank 4 is zero, it is $0.5 \cdot 0.85 = 0.425$ for deposits with banks 2 and 3, and it is given by 0.85 for deposits at banks 1. Under RB, in contrast, the bailout probability is equal to $(2-1+0.85)/4 = 0.462$ for all deposits. Moreover, the fraction of bailed out deposits is 0.6 if bank 1 and bank 2 have been chosen to survive while it drops to $[0.85 \cdot (2+1)]/10 = 0.255$ if banks 3 and 4 have been chosen.

Finally, consider the case where one bank has obtained the full measure of savings and the other $(n-1)$ have obtained zero measures of savings. As for the case where only one bank has received any deposits, we obtain $k = 1$ and $b = \bar{q}$. But while under BB the $(n-1)$ small banks are closed and the bailout probability for the depositors of the big bank is $\bar{q}$, the bailout probability under RB drops to $\bar{q}/n$. Moreover, with probability $(n-1)/n$, the full measure of deposits is eliminated.

3.6 Summary: Sequence of Events

We now summarize the sequence of events.

1. **Banks offer first-period deposit rates.**
   In $t = 0$ banks simultaneously offer their first period-deposit rates d_1^i $(i = 1, .., n)$. $\mathbf{d_1} = (d_1^i)_{i=1}^n$ denotes the vector of all first-period deposit rates.

2. **Households (first generation) assess expected returns and make their savings decisions.**
   First-generation households make assessments $\mathbf{u_1} = (u_1^i)_{i=1}^n$ about the expected returns that will be paid on deposits by each bank. Based on these assessments, they decide on the amount of savings they want to deposit with each bank. Denote the deposits that each bank obtains by D_1^i $(i = 1, .., n)$ and the vector of all first-period deposits by $\mathbf{D_1} = (D_1^i)_{i=1}^n$. Finally, banks invest the deposits obtained in the production technology.

3. **Regulatory policy**
   The regulator observes the realization of the productivity variable r_2. If the refinanc-

ing condition (1) is fulfilled, the regulator will not intervene. In this case the deposits and investments of bank i is given by $\Delta_i = (d_1^i D_1^i, D_1^i)$. If condition (1) is not fulfilled, then one of the bailout schemes will be applied and some banks will be closed. The set of surviving banks is denoted by $\mathcal{B}^+$. Investment projects of closed banks are distributed among surviving banks in proportion to the amount of first-period deposits they have gathered. Deposits and investments of bank i after regulatory policy are given by $\Delta_i = (0,0)$ if it has been closed and by $\Delta_i = (bd_1^i D_1^i, b_I D_1^i)$ if it has survived.

4. **Surviving banks offer second-period deposit rates.**
   Surviving banks simultaneously offer their second-period deposit rates $d_2^i(\mathbf{\Delta})$ $(i \in \mathcal{B}^+)$. Denote the vector of all second-period deposit rates by $\mathbf{d_2} = (d_2^i)_{i \in \mathcal{B}^+}$.

5. **Households (second generation) assess expected returns and make their savings decisions.**
   Second-generation households make assessments $\mathbf{u_2} = (u_2^i)_{i \in \mathcal{B}^+}$ about the expected returns that will be paid on deposits by each bank. Based on this assessment, they decide on the amount of savings they want to deposit with each bank. Denote the deposits that each bank gathers by D_2^i $(i \in \mathcal{B}^+)$ and the vector of all second-period deposits by $\mathbf{D_2} = (D_2^i)_{i \in \mathcal{B}^+}$.

6. **Surviving banks pay their second-period depositors back.**
   In $t = 2$ surviving banks receive returns from investments and pay their second-period depositors back. Profits are consumed by managers.

We call steps 4 -6 the second-period subgame of the intermediation game.

3.7 Equilibrium Concept

In order to derive the subgame-perfect equilibrium of the game described in section 3.6, some subtle points have to be taken into account. In particular we need to discuss how the households' return assessments can be derived.

First, given an assessment $\mathbf{u_g}$ by households in generation g $(g = 1, 2)$ about expected returns paid by each bank, the deposit distribution $\mathbf{D_g} = \mathbf{D_g}(\mathbf{u_g})$ is derived from the households' utility maximization. We use $\mathcal{B}_g^{\max}$ to denote the subset of all banks that are assessed to pay the maximal expected return $u_g^{\max}$ among all banks for generation g. Hence the banks in $\mathcal{B}_g^{\max}$ will receive all the savings of the households: $\sum_{i \in \mathcal{B}_g^{\max}} D_g^i(\mathbf{u_g}) = S(u_g^{\max})$ and $D_g^i(\mathbf{u_g}) = 0$ for all $i \notin \mathcal{B}_g^{\max}$.

Since depositors are indifferent with regard to all banks in $\mathcal{B}_g^{\max}$, it is unclear how deposits are distributed among those banks. We will assume that if two banks are in $\mathcal{B}_g^{\max}$, they will receive the same amount of deposits if all of their characteristics are identical.[21] This means that indifferent depositors will randomize among their preferred banks with equal probability and independently of each other.

Second, the households' assessments have to be consistent. In order to give a precise definition of consistency, we use $\mathbf{U_1}(\mathbf{d_1}, \mathbf{D_1})$ to denote the vector of expected returns on first-period deposits resulting from the allocation $(\mathbf{d_1}, \mathbf{D_1})$, and from regulatory policy. Furthermore, given deposits and investments $\mathbf{\Delta}$ after regulatory policy and given second-period deposit rates $\mathbf{d_2}$ and deposit distribution $\mathbf{D_2}$, we can define the resulting vector of expected second-period returns as $\mathbf{U_2}\Big(\mathbf{\Delta}, \mathbf{d_2}, \mathbf{D_2}\Big)$. We will show in section 4.1 that the functions $\mathbf{U_1}(\cdot)$ and $\mathbf{U_2}(\cdot)$ are well defined for all entries i with $i \in \mathcal{B}_g^{\max}$, i.e for banks that are assessed to pay the maximum expected returns on deposits. However, if $i \notin \mathcal{B}_g^{\max}$ then bank i will receive no deposits and exit the market; it is therefore unclear whether the assessment was correct in the first place. To deal with this problem, we introduce a so called "*zero-measure test*": we calculate the expected returns for each bank resulting from a deposit distribution $\hat{\mathbf{D}}_{\mathbf{g}}(\mathbf{u_g})$ where all banks $i \notin \mathcal{B}_g^{\max}$ receive a zero measure of savings and the full measure of savings is equally distributed among the banks in $\mathcal{B}_g^{\max}$:

$$\hat{\mathbf{D}}_{\mathbf{g}}(\mathbf{u_g}) := \begin{cases} D_g^i(\mathbf{u_g}) & \text{if i} \in \mathcal{B}_g^{\max} \\ \text{zero measure} & \text{else.} \end{cases}$$

Definition 1 (Consistent assessments)
Given first-period deposit rates $\mathbf{d_1}$*, an assessment* $\mathbf{u_1}$ *is consistent if and only if*

$$\mathbf{U_1}\Big(\mathbf{d_1}, \hat{\mathbf{D}}_{\mathbf{1}}(\mathbf{u_1})\Big) = \mathbf{u_1}.$$

Given post-regulation deposits and investments $\mathbf{\Delta}$ *and second-period deposit rates* $\mathbf{d_2}$*, an assessments* $\mathbf{u_2}$ *is consistent if and only if*

$$\mathbf{U_2}\Big(\mathbf{\Delta}, \mathbf{d_2}, \hat{\mathbf{D}}_{\mathbf{2}}(\mathbf{u_2})\Big) = \mathbf{u_2}.$$

Note that we will only consider different assessments for two banks if they are different with regard to at least one of their characteristics.

[21] In $t = 0$, banks are identical if they have offered the same first-period deposit rates and in $t = 1$ they are identical if their balance sheets are identical and if they have offered the same second-period deposit rate.

Consistent assessments mean that depositors make optimal savings decisions[22] and that expected returns are equal to returns generated when depositors distribute themselves among the preferred banks. Whether or not consistent assessments exist will be discussed at length in the next section. If more than one consistent assessment exists, we apply the Pareto selection criterion and assume that the assessment which generates the highest returns will be realized. We therefore define:

Definition 2 (Optimal assessments)
An assessment $\mathbf{u_g}$ $(g = 1, 2)$ *is called optimal if it is consistent and if* u_g^{max} *is at least as high as the maximal expected return resulting from any other consistent assessment.*

We will see that under regulation the best assessment and the corresponding deposit distribution are always unique. We conclude this section by summarizing our equilibrium concept. Note that, since banks are identical ex-ante, we constrain ourselves to the analysis of symmetric equilibria.

Definition 3 (Equilibrium concept)
For any given regulatory policy, a symmetric subgame-perfect Bayesian equilibrium is a set of first-period deposit rates $\mathbf{d_1} = (d_1, .., d_1)$*, assessments* $\mathbf{u_1} = (u_1, ..., u_1)$*, a deposit distribution* $\mathbf{D_1} = (D_1, ..., D_1)$*, a set of reaction functions* $\mathbf{d_2} = \mathbf{d_2}(\mathbf{\Delta})$ *that assign a vector of second-period deposit rates* $\mathbf{d_2}$ *to each possible set* $\mathbf{\Delta}$ *of post-regulation deposits and investments, and a second-period deposit distribution* $\mathbf{D_2} = (D_2, .., D_2)$*. It has to fulfill the following conditions:*

1. *Given* $\mathbf{\Delta}$*, second-period deposit rates* $\mathbf{d_2}(\mathbf{\Delta})$ *constitute an equilibrium in the subgame.*
2. *The second-period subgame equilibrium is symmetric, i.e. banks which are identical in* $t = 1$ *offer the same second-period deposit rate.*
3. *The strategies* $\left(\mathbf{d_1}, \mathbf{d_2}(\cdot)\right)$ *constitute a subgame-perfect Bayesian Nash equilibrium in the entire game.*
4. *Assessments are optimal.*

The equilibrium concept is a subgame-perfect Bayesian Nash equilibrium involving two subtleties. First, individual deposit decisions have no influence on return assessments since the contribution of each single depositor to overall deposits has zero measure. However,

[22] I.e. savings decisions that lead to the highest expected returns, given the deposit rates offered by the banks.

the *distribution* of deposits matters. Second, different deposit distributions for the same vector of deposit rates can imply different probabilities for bank defaults which also feed back into the return assessment. Both subtleties raise considerable problems for the determination of return assessments. These problems will be addressed in the following section.

4 Equilibria in the Second Period and Consistent Assessments

In this section we first solve the second-period subgame and then analyze the existence of consistent assessments in the first period. All proofs in this and the next sections are deferred to the appendix.

4.1 Equilibria in the Second Period

Recall that a surviving bank i in $t = 1$ has $bd_1^i D_1^i$ first-period deposits and $b_I D_1^i$ units of investment projects. If the refinancing condition (1) holds or if no regulation is applied in $t = 1$, then $b = b_I = 1$ and all banks that have received any deposits in $t = 0$ compete for second-period deposits. If, on the other hand, condition (1) does not hold and regulation is applied, then $b \leq 1$, $b_I > 1$, and the regulator will have closed all banks outside of $\mathcal{B}^+$.[23] The surviving banks' profits in both cases are given by[24]

$$\Pi_2^i := \begin{cases} r_2 b_I D_1^i - (1-b) d_1^i D_1^i - d_2^i D_2^i & \text{if } D_2^i \geq bd_1^i D_1^i \\ -d_1^i D_1^i & \text{else.} \end{cases}$$

To analyze the second-period subgame equilibrium we define $\tilde{d}_1^{\max} := \max_{i \in \mathcal{B}^+}\{d_1^i\}$, $\bar{d}_2^* := S_2^{-1}\left(\sum_{i \in \mathcal{B}^+} bd_1^i D_1^i\right)$ and

$$\mathcal{E}_2^* := \left\{ d_2^i = \bar{d}_2^*, \;\; D_2^i = bd_1^i D_1^i \;\; (i \in \mathcal{B}^+) \right\}.$$

Note that $\bar{d}_2^*$ is the lowest deposit rate that generates enough second-period deposits for

[23]Note that if no bank has been closed by the regulator, then $\mathcal{B}^+$ simply denotes the set of all banks which have received any deposits in $t = 0$.

[24]Note again that we assume that bank managers internalize losses in order to abstract from excessive risk taking incentives.

all surviving banks to refinance and that $\mathcal{E}_2^*$ is the (potential) second-period equilibrium where all surviving banks offer $\bar{d}_2^*$.

We start with the analysis of the no-regulation case. This case is only presented to derive the necessity of regulation, and we will restrict ourselves in this case to first-period constellations where all banks have offered the same deposit rate d_1 and therefore have received the same amount D_1 of deposits. Note that this implies that $\tilde{d}_1^{\max} = d_1$ and $\bar{d}_2^* = S_2^{-1}(md_1D_1)$ where m is the number of banks which have received any deposits in $t = 0$.

Proposition 1 (No-regulation case)
Suppose that banks have offered the same deposit rates d_1 and therefore have received the same amount D_1 of deposits in $t = 1$. Then the following statements hold:

(i) If $r_2/d_1 \geq \bar{d}_2^*$, then $\mathcal{E}_2^*$ is an equilibrium. Moreover, from the point of view of the banks, $\mathcal{E}_2^*$ Pareto-dominates all other possible equilibria.

(ii) If $r_2/d_1 < \bar{d}_2^*$, then there is no equilibrium where all banks can refinance themselves. Moreover, in all symmetric equilibria, where all banks offer the same deposit rates, no bank can refinance itself and we have $D_2^i = 0$ for all $i \in \mathcal{B}^+$.

Intuitively statement (i) stems from the following reasoning. First, deviations from $\mathcal{E}_2^*$ are not profitable, since higher deposit rates increase repayment obligations; deviation to lower deposit rates leads either to the loss of all second-period deposits to the other banks or to the concentration of all savings on the deviating bank, which takes the deviating bank's profits down to zero. Second, $\mathcal{E}_2^*$ Pareto-dominates all other equilibria, since equilibria with higher deposit rates lead to an increase in repayment obligations and since in equilibria with lower deposit rates no bank will receive any deposits. The mechanism leading to the second observation is also responsible for the second part of statement (ii) and can be explained as follows.

Assume that the refinancing condition were fulfilled for $\tilde{m}$ banks ($\tilde{m} < m$), i.e. that

$$\frac{r_2}{d_1} \geq S_2^{-1}\Big(\tilde{m}d_1D_1\Big).$$

If depositors could manage to deposit their savings only with a subset of $\tilde{m}$ banks, these banks would be able to refinance. But since all banks are identical, depositors cannot coordinate to deposit with a particular subset of banks; rather they would randomize independently between banks and, by the law of the large numbers, every bank would receive the same amount of savings, which is not enough to refinance. This in turn implies that none of the banks will receive *any* savings. Discriminatory bailout solves this coordination problem by closing some of the banks so that the remaining ones can raise enough new funds to refinance.

Of course there can be asymmetric constellations where one bank is able to refinance. Imagine the case where there are only two banks and $r_2/d_1 \geq S^{-1}(d_1 D_1)$. If one bank offers u_*, defined as the positive solution of $u = r_2 D_1/S(u)$, and the other bank offers a lower deposit rate, the depositors' coordination problem is solved since they know that the bank which has offered u_* can pay strictly higher returns. Without regulation, however, there is a severe coordination problem, since both of them would like to be the bank that is able to pay depositors back. Therefore, in our analysis of the no-regulation case in section 5.1 we will assume that no bank will receive any second-period savings if $r_2/d_1 < \bar{d}_2^*$. On the other hand, if $r_2/d_1 \geq \bar{d}_2^*$, we assume that $\mathcal{E}_2^*$ is played which by proposition 1 can be justified by the Pareto selection criterion.

We now turn to the case were regulation (PB, RB, BB or BS) ensures that always $r_2/\tilde{d}_1^{\max} \geq \bar{d}_2^*$. The regulation case will be analyzed in the general setting where banks may have offered different deposit rates in $t = 0$. We have already derived that under symmetric first-period allocations, banks will offer second-period deposit rates that are just sufficient to attract enough second-period savings to pay back their obligations to first-period depositors. Deviations to lower deposit rates can be excluded since the bank which has offered lower deposit rates will receive no second-period savings. Under asymmetric first-period constellations, however, deviations to lower deposit rates could be profitable for big banks since the smaller non-deviating banks cannot cope with all second-period savings alone. This in turn could lead to non-existence of equilibria or to equilibria where not all banks can refinance despite the fact that all banks would be able to refinance if offered deposit rates were high enough. To avoid these problems, we assume that the regulator imposes a **lower bound on second-period deposit rates** (**LBD**), i.e. she guarantees that no banks offers a deposit rate lower than $\bar{d}_2^*$.[25] This ensures that refinancing of all banks indeed occurs in equilibrium under asymmetric first-period constellations as the next proposition indicates.

Proposition 2 (Regulation case)
Suppose that regulation ensures that $r_2/\tilde{d}_1^{\max} \geq \bar{d}_2^*$ and that LBD is applied in $t = 2$. Then $\mathcal{E}_2^*$ is a second-period equilibrium. Moreover, from the point of view of the banks, $\mathcal{E}_2^*$ Pareto-dominates all other possible equilibria.

[25] Again we could think that a high enough penalty is imposed in case that banks do not obey.

Regulation LBD ensures that banks do not undercut the rate $\bar{d}_2^*$. Moreover, banks which have offered higher deposit rates than $\bar{d}_2^*$ have higher repayment obligations than under $\mathcal{E}_2^*$. This implies that deviations from $\mathcal{E}_2^*$ are not profitable and that all other possible equilibria are Pareto-dominated by $\mathcal{E}_2^*$.

Throughout the paper we will assume that under regulation, banks will play $\mathcal{E}_2^*$ which can be justified by the Pareto criterion. In this case, second-period deposits just suffice to cover the refinancing needs of the banks and we can describe expected returns on first-period deposits of bank i $(i = 1, ..., n)$ as $u_1^i = (p_l q_l^i + p_h q_h^i) d_1^i$. q_l^i (q_h^i) denotes the bailout probability for bank i in the case of low (high) production returns.

4.2 Consistent Assessments in the First Period

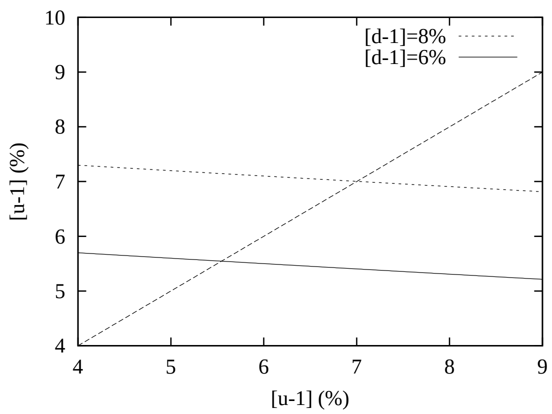

Figure 4: The left-hand and the right-hand side of equation (8) as functions of u for different values of d. Note that u is represented by the percentage points by which it exceeds 1.

In this section we analyze the existence and uniqueness of consistent assessments in the first period, assuming that one of the regulatory schemes is applied.[26] Consider a situation where there are two groups of banks, $\mathcal{B}_l$ and $\mathcal{B}_h$, that have offered first-period deposit rates d_{1l} and d_{1h} $(d_{1l} \leq d_{1h})$ respectively. Note that the assessments and deposits for all banks in $\mathcal{B}_l$ and for all banks in $\mathcal{B}_h$ must be identical. Note also that this scenario includes two important cases that need to be considered in order to analyze symmetric equilibria of the intermediation game: first, the symmetric *non-deviation case* where all banks offer the same first-period deposit rate $d_{1l} = d_{1h} = d$ and second, the *deviation case* in which one bank deviates to a lower or a higher deposit rate. Note that in the non-deviation case all banks are in one group (w.l.o.g. in $\mathcal{B}_h$) while in the deviation case either the deviating bank is in $\mathcal{B}_h$ while the the non-deviating banks are in $\mathcal{B}_l$ or vice versa.

We will now examine expected first-period returns on deposits in the non-deviation case and in the deviation case if one group of banks receives all deposits.[27] We denote the de-

[26]The no-regulation case will be summarized in section 5.1.

[27]We will see later (in propositions 3 and 4) that we do not have to consider the case where one bank deviates and both the deviating and the non-deviating bank receive deposits.

posit rate in the non-deviation case and the deposit rate offered in the group of banks that has received all savings in the deviation case by d. The corresponding return assessment is denoted by u and the bailout probability when productivity is low (high) is denoted by q_l (q_h). Using equation (7) we observe that in both cases u can only be consistent if it solves the following system $\mathcal{S}(d)$, which consists of the equations

$$\begin{aligned} u &= (p_l q_l + p_h q_h)d & (8)\\ q_l &= \min\Big\{ \frac{1}{dS_1(u)} S_2\Big(\frac{r_{2l}}{q_l d}\Big) \ , \ 1 \Big\} & (9)\\ q_h &= \min\Big\{ \frac{1}{dS_1(u)} S_2\Big(\frac{r_{2h}}{q_h d}\Big) \ , \ 1 \Big\} & (10) \end{aligned}$$

and of the constraints $q_l > 0$ and $q_h > 0$. Note that refinancing condition (1) will hold in both states of production returns if and only if $dS_1(d) \leq S_2(r_{2l}/d)$. We denote the highest first-period deposit rate at which this is the case by $d_{\mathbf{L}}$. Hence, $d_{\mathbf{L}}$ is the unique solution of the equation

$$S_2^{-1}\Big(dS_1(d)\Big) = \frac{r_{2l}}{d}.$$

Moreover, we use $\underline{u} := \min\{u | S(u) \geq 0\}$. The next lemma is crucial for the analysis of consistent first-period assessments.

Lemma 1

Suppose that $d > \underline{u}$. Then the system $\mathcal{S} = \mathcal{S}(d)$ has a unique solution which we denote by $(\bar{u}_d, \bar{q}_{l,d}, \bar{q}_{h,d})$. Moreover, this solution has the following properties:

(i) $\bar{u}_{(\cdot)}$, $\bar{q}_{l,(\cdot)}$ *and* $\bar{q}_{h,(\cdot)}$ *are continuous functions of* d.

(ii) $\bar{u}_d = d$ *for* $d \leq d_{\mathbf{L}}$ *and* $\bar{u}_d < d$ *for* $d > d_{\mathbf{L}}$.

(iii) $\bar{q}_{l,d} < 1$ *for* $d > d_{\mathbf{L}}$; $\bar{q}_{l,d}, \bar{q}_{h,d} > 0$ *for all* d.

(iv) $\bar{q}_{l,(\cdot)}$ *is strictly decreasing in* d *for all* $d \in \mathcal{D}_M$ *where*

$$\mathcal{D}_M := \Big\{ d \mid dS_1(\bar{u}_d) \leq S_2(r_{2h}/d) \text{ and } d > d_{\mathbf{L}} \Big\}.$$

The existence and uniqueness of a solution for $\mathcal{S}$ is derived from a fixed-point argument: the right-hand-side of equation (8) is a decreasing function of u, since bailout probabilities q_l and q_h are decreasing in u while the left-hand-side is strictly increasing. This implies existence and uniqueness of $\bar{u}_d$ because of the continuity of the bailout probabilities as functions of u. Figure 4 illustrates the solution of $\mathcal{S}$. Note that u and d are represented by the percentage points by which they exceed 1, i.e. $u = 1.06$ is represented by 6. This scale will be used for u and d in all following illustrations.

Note that at this point our technical device which allows for a fraction of $(1 - b)$ of the surviving banks' deposits to be eliminated guarantees the continuity of the bailout probabilities as functions of u and thus the existence . If only entire banks could be closed, the bailout probabilities would not be continuous in u. Discontinuities would appear for all u where a marginal higher value of u requires to close an additional bank: in such points bailout probability would fall by $1/n$ and consistent assessments might not exist for some values of d.

For the description of the deviation case we need to analyze the system $\tilde{\mathcal{S}} = \tilde{\mathcal{S}}(d_{1l}, d_{1h})$ consisting of equations (8) - (10) where d is replaced by d_{1l} in equation (8) and by d_{1h} in equations (9) and (10).[28]

Lemma 2
Suppose that $d_{1l}, d_{1h} > \underline{u}$. Then the system $\tilde{\mathcal{S}}$ has a unique solution which we denote by $(\tilde{u}_{d_{1l},d_{1h}}, \tilde{q}_{l,d_{1l},d_{1h}}, \tilde{q}_{h,d_{1l},d_{1h}})$.

In the next two propositions we characterize consistent and optimal assessments. Note that in the non-deviation case where all banks have offered the same first-period deposit rate d_1, assessments for expected returns of banks are denoted by u_1. In the deviation case there are two groups of banks $\mathcal{B}_l$ and $\mathcal{B}_h$ that have offered different first-period deposit rates d_{1l} and d_{1h} respectively ($d_{1l} < d_{1h}$). Here we denote the corresponding assessments by u_{1l} and u_{1h} respectively.

Proposition 3 (Consistent assessments: Non-deviation case)
If all banks have offered the same first-period deposit rate d_1 then $\mathbf{u_1} = (\bar{u}_{d_1}, ..., \bar{u}_{d_1})$ is the only consistent assessment under all bailout regimes.

Proposition 4 (Consistent assessments: Deviation case)
In the deviation case only the following types of assessments can be consistent:

(a) $u_{1l} < u_{1h} = \bar{u}_{d_{1h}}$, (b) $u_{1h} < u_{1l} = \bar{u}_{d_{1l}}$ and (c) assessments of the type $u_{1l} = u_{1h}$.

More specifically, we obtain:

(i) *Under the RB or PB bailout scheme, $u_{1l} < u_{1h} = \bar{u}_{d_{1h}}$ is the* only *consistent assessment.*

[28] This will become apparent when going through the proof of proposition 4.

(ii) *Under the BB bailout scheme,* $u_{1l} < u_{1h} = \bar{u}_{d_{1h}}$ *is a consistent assessment and* $u_{1h} < u_{1l} = \bar{u}_{d_{1l}}$ *is a consistent assessment if and only if* $p_h d_{1h} < \bar{u}_{d_{1l}}$. *Moreover, an assessment* $u_{1l} = u_{1h}$ *is never optimal.*

(iii) *Under the BS bailout scheme,* $u_{1l} < u_{1h} = \bar{u}_{d_{1h}}$ *is a consistent assessment if and only if* $\bar{u}_{d_{1h}} > d_{1l}$*; an assessment* $u_{1h} < u_{1l}$ *can never be consistent. Finally, if* $\bar{u}_{d_{1h}} < d_{1l}$ *and* $p_h d_{1h} < \tilde{u}_{d_{1l},d_{1h}}$*, then no consistent assessments exist.*

Corollary 1

If $\bar{q}_{h,d_{1h}} = 1$ *and* $d_{1l} > d_{\mathbf{L}}$ *then no consistent assessments exist under BS if* $d_{1h} - d_{1l}$ *is sufficiently small.*

Proposition 3 follows directly from lemma 1, since expected returns for depositors can be expressed by equations (8) - (10). Moreover, under RB, bailout probabilities for all banks are the same, implying that the banks that have offered the highest deposit rates will always pay the highest returns. Therefore assessments are also unique in the deviation case. Under BB, however, we cannot generally exclude assessments that assign higher expected returns to banks in $\mathcal{B}_l$ despite the fact that those banks have offered lower deposit rates than the banks in $\mathcal{B}_h$. This is due to a *self-fulfilling prophecy* effect caused by BB. Suppose that a bank is assessed to pay higher expected returns than the other banks. This bank will then obtain more deposits than the others and hence will be "bigger" in terms of the bailout regime. Therefore this bank will have a higher bailout probability in the case of illiquidity of the whole banking system. This effect can indeed compensate for lower deposit rates. To see why an assessment $u_{1l} = u_{1h}$ cannot be optimal under BB, note that in this case banks in $\mathcal{B}_h$ must be smaller with respect to first-period deposits than banks in $\mathcal{B}_l$; otherwise bailout probability *and* offered deposit rates would be higher for $\mathcal{B}_h$ banks. But this implies that $\mathcal{B}_h$ banks have a lower bailout probability than in the case where the $\mathcal{B}_h$-banks receive all deposits. The formalization of these arguments leads to statement (ii) in proposition 3.

Finally, the mechanism leading to consistency problems under BS is the following. If some banks are assessed to pay higher returns than the other ones, the former will attract all deposits. But this implies that that these banks are bigger than the banks associated with lower expected returns and will therefore have a lower bailout probability. This could proof the initial assessment to be incorrect even if higher assessed banks have offered higher deposit rates. In analogy to the self-fulfilling prophecy effect under BB, this mechanism could be termed *self-contradicting prophecy* since by assessing a bank to pay high returns, depositors lower its bailout probability and hence its expected returns. To give an impression of the size of effects, we note that if (for our example A) $d_{1l} = 1.05$

then no consistent assessments exist for $d_{1h} \leq 1.059$, a difference of nearly 1 percentage point. This can prevent banks from biding up deposit rates in low-return equilibria. Suppose that depositors will not switch to deviating banks in case the deviation deposit rate lies in the non-consistency region. Then, if e.g. all banks have offered 1.05, a deviating bank would have to offer at least 1.059 to attract the other banks' depositors. Such a rate may proof to be too large to be attractive, since a deviating banks would have to trade off the higher amount of deposits with a lower bailout probability (in our case it would decline from 0.97 to 0.958) and higher deposit rates. Because of these problems we will not further analyze the BS bailout scheme and rather add some additional comments when we summarize our results.

Propositions 3 allow us to characterize symmetric equilibria under regulation solely in terms of the first-period deposit rate d_1 offered by all banks:

1. Banks offer $\mathbf{d_1} = (d_1, ..., d_1)$ which leads to the assessments $\mathbf{u_1} = (\bar{u}_{d_1}, ..., \bar{u}_{d_1})$ and to the deposit distribution $\mathbf{D_1} = \Big(D_1, ..., D_1\Big)$ where $D_1 = S_1(\bar{u}_{d_1})/n$.

2. The regulator observes the realization r_2 of the aggregate productivity shock and determines $\bar{q}$ as the positive solution of

$$q d_1 S_1(\bar{u}_{d_1}) = S_2(\frac{r_2}{q d_1})$$

   if that solution is lower than 1; otherwise $\bar{q}$ is set equal to 1 . k and b are determined by $k = \lceil n\bar{q} \rceil$ and $b = (n\bar{q})/\lceil n\bar{q} \rceil$.

3. A set $\mathcal{B}^+$ of k banks is randomly chosen from all n banks; each bank has the same probability of being chosen. Investment projects of closed banks are uniformly distributed among surviving banks. Deposits and investments Δ of surviving banks are given by

$$\Delta = \Big(b d_1 D_1\,,\ (n/k) D_1^i\Big).$$

4. Banks offer $\mathbf{d_2} = (d_2, ..., d_2)$ where

$$d_2 := S_2^{-1}\Big(\bar{q} S_1(\bar{u}_{d_1})\Big).$$

We will therefore characterize symmetric equilibria by using the short form $\mathcal{E} = (d_1)$.

5 Allocations Under Different Regulatory Approaches

Note that from a $t = 1$ perspective expected profits for bank i are given by[29]

$$\Pi_1^i := -\operatorname{Prob}(A_i)(d_1^i D_1^i + P) + \Big(1 - \operatorname{Prob}(A_i)\Big)\,\mathbb{E}\Big[R_2 b_I D_1^i - (1-b) d_1^i D_1^i - d_2^i D_2^i | A_i^c\Big].$$

A_i denotes the eventuality of bank i being closed by the regulator or no being able to meet its obligation in $t = 1$ and A_i^c denotes the complement of A_i, i.e. the possibility of bank i living on until $t = 2$. Recall that P is the penalty imposed by the regulator under A_i. While $P > 0$ under PB, we have $P = 0$ under RB and BB. Note again that under all regulatory approaches, banks are assumed to internalize losses that accrue to depositors. The penalty P under PB will be imposed additionally to any other penalties which might be used to force banks to internalize losses.

5.1 No Regulation

In this section we analyze the no-regulation case to motivate the potential benefits of regulation. Consider a symmetric equilibrium where all banks have offered the same deposit rate $d_1 = d$ in $t = 0$. Note that according to proposition 1 only three first-period return assessments are possible, namely d_1, $p_h d_1$ and zero. Obviously, if $d_1 \leq d_{\mathbf{L}}$ then only $u_1 = d_1$ is consistent and banks can refinance in both states of production returns.[30] If $d_1 > d_{\mathbf{L}}$ then $u_1 = d_1$ is no longer consistent since under this assessment banks would go bankrupt for $r_2 = r_{2l}$ (since $\bar{d}_2^* > r_{2l}/d$) which would lead to $u < d_1$. Since equilibria where banks are correctly assessed to pay zero returns can also be excluded, the only other possible assessment is $u_1 = p_h d_1$. Such an assessment will be correct if and only if $r_{2h}/d_1 \geq \tilde{d}_2^*(d_1) > r_{2l}/d_1$ where $\tilde{d}_2^*(d) := S_2^{-1}\Big(dS_1(p_h d)\Big)$. Hence,

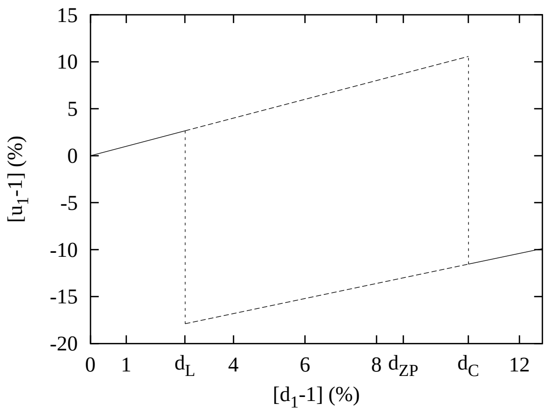

Figure 5: Expected returns u_1 for the first generation as function of the first-period deposit rate d_1 (no-regulation case, example A). d_{ZP} stands for $\tilde{d}_{\mathbf{ZP}}$.

[29]Note that given A_i bank i cannot pay anything to depositors since the liquidation value of the project is zero.

[30]Note that in this case first-period savings amount to $S_1(d)$ and hence $\bar{d}_2^* = S_2^{-1}\Big(dS_1(d)\Big)$ which by definition of $d_{\mathbf{L}}$ is not higher than r_{2l}/d.

by defining $d_{\mathbf{C}}$ as the unique solution of $\tilde{d}_2^*(d) = r_{2l}/d$, we have derived that without regulation no consistent assessments exist if $d_{\mathbf{L}} < d_1 \leq d_{\mathbf{C}}$.

Using the parameter values from example A, we illustrate the consistent-assessment problem in figure 5: if $d_{\mathbf{L}} < d_1 \leq d_{\mathbf{C}}$, and depositors assume that all banks will survive in both states of production returns, then return assessments are given by the upper (broken) line in figure 5. Actual returns paid are represented by the lower (broken) line; if depositors assume that banks can only refinance in the high state, then assessments are given by the lower line and actual returns paid by the higher line. Finally, if $d_1 \leq d_{\mathbf{L}}$ or $d_1 > d_{\mathbf{C}}$, the solid lines represent the respective consistent return assessments for first-generation depositors.

In order to assess the benefits of regulation we also want to compare expected returns for depositors resulting with and without regulation. For our purposes it is sufficient to observe that banks will not bid deposit rates higher than $d_1 = \tilde{d}_{\mathbf{ZP}}$ which is defined as the unique solution of the equation $\tilde{\pi}(d) = 0$. $\tilde{\pi}(d)$ are the banks' profits per deposit in a symmetric equilibrium if $d_1 > d_{\mathbf{C}}$. They can be described by

$$\tilde{\pi}(d) = -p_l d + p_h \Big(r_{2h} - d\tilde{d}_2^*(d) \Big).$$

Symmetric equilibria with higher deposit rates will not occur since such equilibria would imply negative bank profits (because $\pi(\cdot)$ is strictly increasing in d). Our results are summarized in the following proposition.

Proposition 5

Suppose that banks play a symmetric strategy $\mathbf{d_1} = (d_1, ..., d_1)$ in the first period and that there is no regulation. Then the following statements hold:

(i) If $d_{\mathbf{L}} < d_1 \leq d_{\mathbf{C}}$, then no consistent assessments exist.

(ii) For both generations, the highest possible symmetric equilibrium returns are either achieved if $d_1 = d_{\mathbf{L}}$ or $d_1 = \tilde{d}_{\mathbf{ZP}}$. The corresponding unique first-period assessments are $d_{\mathbf{L}}$ and $p_h \tilde{d}_{\mathbf{ZP}}$ respectively.

(iii) If $\tilde{d}_{\mathbf{ZP}} \leq d_{\mathbf{C}}$ then the highest possible symmetric equilibrium returns are achieved for $d_1 = d_{\mathbf{L}}$.

Note that in example A we have $\tilde{d}_{\mathbf{ZP}} = 1.079 < d_{\mathbf{C}} = 1.105$ and hence statement (iii) applies. Proposition 5 points to the potential benefits of regulation. Without regulation, the existence of consistent assessments is not guaranteed and it can occur that none of the banks is able to refinance in $t = 1$, implying that intermediation services break down completely for the second generation. In the following, we discuss how regulatory approaches can avoid the breakdown of intermediation. In section 5.2, we consider the enforcement of prudential equilibria with $d_1 \leq d_{\mathbf{L}}$, and in section 5.3 we analyze the case of discriminatory closure of some banks in order to allow the others to refinance. Both scenarios also help to avoid the problem of nonexistent assessments, as we have already observed in proposition 3. In section 6 we explicitly compare the no-regulation and the different regulatory approaches with respect to stability and expected returns paid on deposits.

5.2 Prudential Banking

In this section we assume that the regulatory regime forces banks to avoid the possibility of default.

Proposition 6

The unique symmetric equilibrium under prudential banking is $\mathcal{E}_L$.

Obviously, prudential banking can heavily depress deposit rates and investments if a serious productivity shock can occur. Moreover, in the case $r_{2l} < \underline{u}$, intermediation is impossible. In the next sections we therefore examine work-out type regulatory approaches to banking crises and their implications.

5.3 Discriminatory Bailout

In this section we investigate the equilibria which occur under discriminatory bailout. In order to describe the banks' profits under discriminatory regulation schemes, we recall the definition of the set $\mathcal{D}_M$ and additionally introduce the set $\mathcal{D}_H$:

$$\begin{aligned} \mathcal{D}_M &:= \Big\{ d \mid dS(\bar{u}_d) \leq S(r_{2h}/d) \text{ and } d > d_{\mathbf{L}} \Big\} \\ \mathcal{D}_H &:= \Big\{ d \mid dS(\bar{u}_d) > S(r_{2h}/d) \Big\}. \end{aligned}$$

The sets $\mathcal{D}_M$ and $\mathcal{D}_H$ refer to a situation where all banks have symmetrically offered a deposit rate $d_1 = d$ in $t = 0$. If $d \in \mathcal{D}_M$ then all banks can refinance in the good state but not in the bad state of production returns while banks cannot refinance in both states for $d \in \mathcal{D}_H$. Moreover, $d_2^*(d) := S^{-1}\Big(dS(\bar{u}_d)\Big)$ is the second-period deposit rate that - if symmetrically offered by all n banks in $t = 1$ - generates just enough savings for all banks to refinance.

Now consider a potential symmetric equilibrium $\mathcal{E} = (d_1)$, or a deviation d_1^{dev} from such a symmetric equilibrium, where the deviating bank receives all savings. Then the expected profits that a bank makes on each unit of deposits are given by $\pi(d_1)$ and $\pi(d_1^{\text{dev}})$ respectively, where π is defined by

$$\pi(d) := \begin{cases} \bar{R}_2 - dd_2^*(d) & \text{if } d \leq d_{\mathbf{L}} \\ -p_l(1-\bar{q}_{l,d})d + p_h\Big(r_{2h} - dd_2^*(d)\Big) & \text{if } d \in \mathcal{D}_M \\ -p_l(1-\bar{q}_{l,d})d - p_h(1-\bar{q}_{h,d})d & \text{if } d \in \mathcal{D}_H, \end{cases}$$

and $\bar{R}_2 := p_l r_{2l} + p_h r_{2h}$.

Lemma 3
$\pi(\cdot)$ is a continuous function of d.

We note that $\pi(d) > 0$ if $d \leq d_{\mathbf{L}}$ and that $\pi(d) < 0$ if $d \in \mathcal{D}_H$. Hence, by the continuity of $\pi(\cdot)$, there is a first-period deposit rate d with $\pi(d) = 0$. We will work with the following assumption:

Assumption 1 (UZP)
There is a unique first-period deposit rate d where profits are zero ($\pi(d) = 0$). We denote this deposit rate by $d_{\mathbf{ZP}}$ and further assume that $\pi(d) < 0$ for $d > d_{\mathbf{ZP}}$.

Note that assumption UZP is satisfied if $\bar{u}_{(\cdot)}$ is increasing in d for all $d \in \mathcal{D}_M$ which can be verified for our example savings functions.

Lemma 4
If $S_i(u) = a_i u^{\alpha_i}$ $(i = 1, 2)$ then $\bar{u}_{(\cdot)}$ is increasing in d for all $d \in \mathcal{D}_M$.

We can now turn to the analysis of the random bailout regime:

Proposition 7
Suppose that UZP holds. Then the unique symmetric equilibrium under random bailout is $\mathcal{E}_{ZP} := (d_{\mathbf{ZP}})$.

$\mathcal{E}_{ZP}$ is the zero profit equilibrium. Equilibria with higher deposit rates imply negative profits for banks and will thus not be played. Equilibria with lower deposit rates do not exist, since banks will then have an incentive to deviate to slightly higher deposit rates thereby collecting all savings. Figures 6 and 7 show expected returns for the first

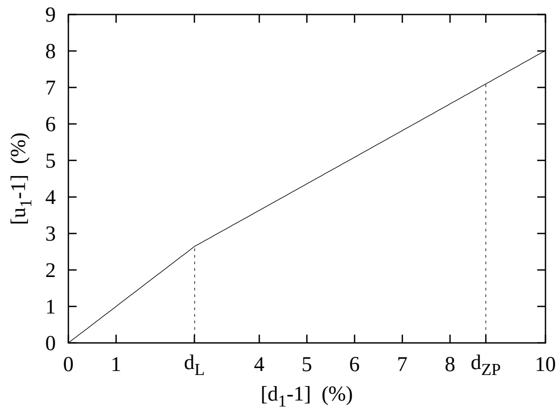

Figure 6: Expected returns u_1 for the first generation as function of the first-period deposit rate d_1 (example A).

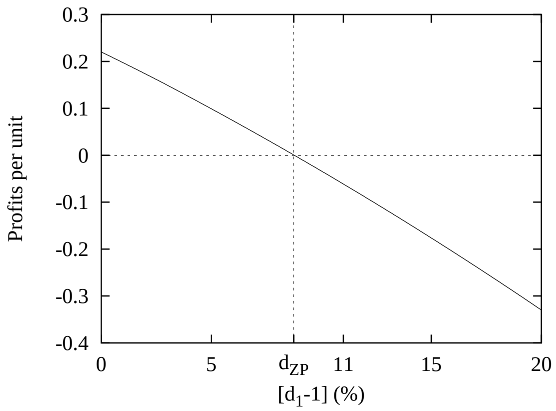

Figure 7: Profits per unit of deposits π as function of the first-period deposit rate d_1 (example A).

generation and banks' expected profits as functions of the first-period deposit rate d_1 that has been offered.

Having derived this result, we now set out to examine whether and how allocations are affected if the regulator follows BB instead of RB. We have already indicated that BB can lead to self-fulfilling prophecy effects when first-period deposit rates are set asymmetrically. Banks which have offered lower deposit rates can consistently be assessed to pay higher returns than banks which have offered higher deposit rates. To present our results we introduce the following tie-breaking rule:

(TR) *If depositors receive the same expected returns when depositing with non-deviating banks as when depositing with the deviating bank, they choose the non-deviating ones.*

Moreover, we introduce the function

$$\Pi^{\text{dev}}(d) := \max\Big\{\pi(\tilde{d})S(\bar{u}_{\tilde{d}}) \ : \ \bar{u}_{\tilde{d}} > \bar{u}_d\Big\},$$

which describes the maximum profits that can be obtained when deviating from a symmetric equilibrium where all banks have offered a first-period deposit rate d. Finally we distinguish the following cases for the relationship between expected equilibrium returns for the first generation and offered first-period deposit rates:

1. There is a $d_{\mathbf{UH}}$ ($d_{\mathbf{L}} \le d_{\mathbf{UH}}$) such that $\bar{u}_{(\cdot)}$ is strictly increasing in d for $d < d_{\mathbf{UH}}$ and strictly decreasing for $d_{\mathbf{UH}} < d \le d_{\mathbf{ZP}}$. (**UID**)

2. There is a $d_{\mathbf{UL}}$ ($d_{\mathbf{L}} \le d_{\mathbf{UL}}$) such that $\bar{u}_{(\cdot)}$ is strictly decreasing in d for $d < d_{\mathbf{UL}}$ and

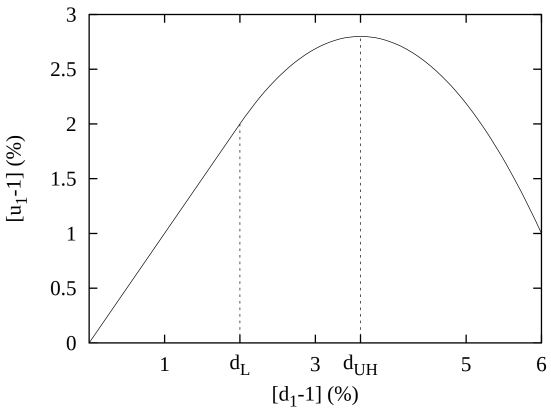

Figure 8: The case UID.

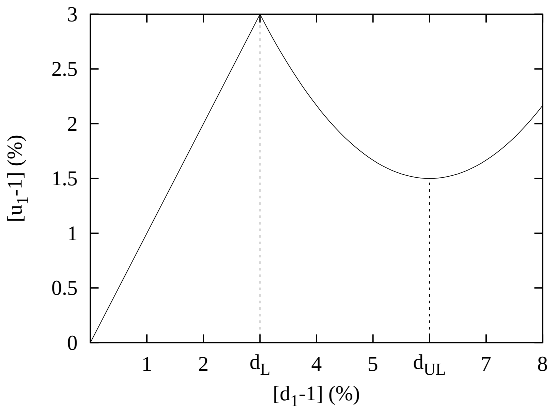

Figure 9: The case UDI.

strictly increasing for $d_{\mathbf{UL}} < d \leq d_{\mathbf{ZP}}$. $(\mathbf{UDI})$

These cases are illustrated in figures 8 and 9. Note that under UID (UDI), both constellations are possible: (a) $d_{\mathbf{UH}} < d_{\mathbf{ZP}}$ $(d_{\mathbf{UL}} < d_{\mathbf{ZP}})$ and (b) $d_{\mathbf{UH}} \geq d_{\mathbf{ZP}}$ $(d_{\mathbf{UL}} \geq d_{\mathbf{ZP}})$.

Proposition 8

Suppose that the assumption UZP holds and that TR is applied. Then the following holds under bail out the big ones:

(i) $\mathcal{E} = (d)$ *is an equilibrium for each deposit rate* $d \in \mathcal{U}_{\max} := \operatorname{argmax} \bar{u}_{(\cdot)} |_{\pi(d) \geq 0}$.

(ii) Under UID we obtain that $\mathcal{E}_{UH} := \Big(\min\{d_{\mathbf{UH}}, d_{\mathbf{ZP}}\}\Big)$ *is the unique symmetric equilibrium.*

(iii) Under UDI we obtain:

- *If* $d_{\mathbf{L}} > \bar{u}_{d_{\mathbf{ZP}}}$, *then* $\mathcal{E}_L$ *is the unique symmetric equilibrium.*
- *If* $d_{\mathbf{L}} < \bar{u}_{d_{\mathbf{ZP}}}$, *then* $\mathcal{E}_{ZP}$ *is a equilibrium and* $\mathcal{E}_L$ *is a equilibrium if and only if* $\Pi^{dev}(d_{\mathbf{L}}) \leq \pi(d_{\mathbf{L}})S(d_{\mathbf{L}})/n$. *No other equilibria exist.*

The next corollary is an immediate consequence of proposition 8:

Corollary 2

Suppose assumption UZP holds and that TR is applied. Then under bail out the big ones we obtain:

(i) Under UI, $\mathcal{E}_{ZP}$ *is the unique equilibrium.*

(ii) Under UD, $\mathcal{E}_L$ is the unique equilibrium.

What is the economic intuition behind the results in proposition 8? Let us first turn to statement (i). Under BB, maximal expected return equilibria are supported, since even if banks deviate to higher deposit rates, depositors can consistently assess non-deviating banks as paying higher returns, thereby securing maximal expected returns. This is not possible under RB. Let us now turn to the interesting case of statement (ii) where $d_{\mathbf{UH}} < d_{\mathbf{ZP}}$. Equilibria with higher deposit rates are not possible, since banks would deviate to lower rates and depositors would switch to the deviating banks as they can guarantee higher expected returns. Again this is made possible by the self-fulfilling prophecy effect of BB. Lower deposit rates are not possible because banks will deviate to higher rates. Statement (iii) can be explained by the same reasoning.

6 Comparison

In this section we compare the three regulatory scenarios (prudential banking, random bailout and bail out the big ones) with the no-regulation scenario. Our comparison is concerned with three issues: fragility issues, credibility issues and expected returns. For two points of the analysis we have relied on simulation results: first, for the determination of the shape of $\bar{u}_{(\cdot)}$ as function of d; second, for the comparison of expected returns in the $\mathcal{E}_L$ and the $\mathcal{E}_{ZP}$ equilibrium.

For the time being, we will focus on what we call the "normal case", namely the case where $\bar{u}_{(\cdot)}$ is strictly increasing in d. The label "normal" is justified by the fact that $\bar{u}_{(\cdot)}$ has this property for our family of example saving functions and because $\bar{u}_{(\cdot)}$ has always behaved in this way for a wide range of other numerical examples. In section 6.4 we will briefly consider scenarios where $\bar{u}_{(\cdot)}$ is not increasing in d.

6.1 Fragility Issues

Regulation improves the stability of intermediation: first, the existence of second-period equilibria is guaranteed under regulation. Moreover, these equilibria can be ranked by banks according to the Pareto criterion. Without regulation, neither are guaranteed. Second, with regulation the nonexistence of consistent assessments in the first period, which may occur without regulation (see proposition 5), can be avoided.

The crucial question in comparing the stability across regulatory schemes is whether the proposed coordination mechanism for depositors return assessments works if there is more than one consistent assessment. We have assumed that if there is more than one consistent assessment, then depositors choose the assessment that promises the highest expected returns (optimal assessment). If this is the case, all regulatory regimes yield the same stable result, namely unique assessments and a unique equilibrium: $\mathcal{E}_{ZP}$ (under RB and BB) and $\mathcal{E}_L$ (under PB). The result for PB and RB is independent of whether this selection criterion holds or not. The stability of the BB regime on the other hand depends upon it heavily: if it does not hold, uniqueness is not guaranteed (see proposition 3).

6.2 Return Issues

Recall that in the normal case we have to compare the equilibria $\mathcal{E}_L$ (implemented by prudential banking possibly implemented without regulation), $\tilde{\mathcal{E}}_{ZP}$ (possibly implemented without regulation) and $\mathcal{E}_{ZP}$ (implemented by RB and BB). The expected returns for the first generation (u_1) and for the second generation (u_2) in the different equilibria are presented in table 2.

	First Generation	Second Generation
$\mathcal{E}_L$	$d_{\mathbf{L}}$	$d_2^*(d_{\mathbf{L}})$
$\tilde{\mathcal{E}}_{ZP}$	$p_h\tilde{d}_{\mathbf{ZP}}$	$p_l\underline{u} + p_h\tilde{d}_2^*(\tilde{d}_{\mathbf{ZP}})$
$\mathcal{E}_{ZP}$	$p_l\bar{q}_{l,d}d_{\mathbf{ZP}} + p_h d_{\mathbf{ZP}}$	$p_l S_2^{-1}\left(\bar{q}_{l,d}d_{\mathbf{ZP}}S_1(\bar{u}_{d_{\mathbf{ZP}}})\right) + p_h d_2^*(d_{\mathbf{ZP}})$

Table 2: Expected returns under different equilibria

The most important question is whether regulation can improve expected returns for both generations. We observe that returns $u_i(\mathcal{E}_{ZP})$ in $\mathcal{E}_{ZP}$ are higher for both generations than returns $u_i(\tilde{\mathcal{E}}_{ZP})$ in $\tilde{\mathcal{E}}_{ZP}$ $(i = 1, 2)$. This is stated in the next proposition:

Proposition 9
$u_i(\mathcal{E}_{ZP}) > u_i(\tilde{\mathcal{E}}_{ZP})$ *for* $i = 1, 2$.

Proposition 9 implies that regulation can improve welfare. On the other hand, it is not clear whether $\mathcal{E}_{ZP}$ also delivers higher returns than $\mathcal{E}_L$. Obviously, $u_1(\mathcal{E}_{ZP}) > u_1(\mathcal{E}_L)$, but the effect for the second generation is ambiguous since $\bar{q}_{l,d}d_{\mathbf{ZP}}$ might be smaller than

$d_{\mathbf{L}}$ and hence might offset the effect that $d_2^*(d_{\mathbf{L}}) < d_2^*(d_{\mathbf{ZP}})$. However, in all simulation exercises $\mathcal{E}_{ZP}$ also improves returns for the second generation compared to $\mathcal{E}_L$. Hence, in these cases discriminatory bailout improves expected returns for both generations compared to PB and the non-regulation case. As illustration we show in figure 10 expected returns for the first and the second generation under discriminatory bailout as function of offered first-period deposit rates for example A. The returns resulting under $\mathcal{E}_L$ and $\mathcal{E}_{ZP}$ are presented in table 3.[31]

	d_1	u_1	u_2	q_l
$\mathcal{E}_L$	2.64	2.64	0.34	1
$\mathcal{E}_{ZP}$	8.75	7.00	9.24	0.92

Table 3: First-period deposit rates, expected returns, and fraction of bailed out depositors for example A.

6.3 Credibility Issues

The issue of credibility obviously only has a bearing on the three regulatory schemes. The most important difference with respect to the credibility of those schemes is the *out-of- equilibrium* strategies that are required. While the credibility of PB depends first of all on the credibility of the non-pecuniary punishment of managers (which will not be taken up here), the credibility of BB and RB depends on the impact of the respective out-of-equilibrium closure rules.

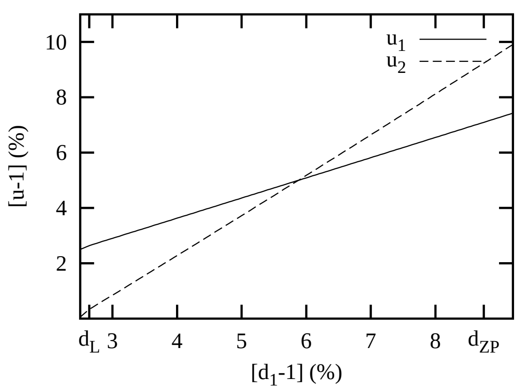

Figure 10: Expected returns for both generations as function of the first-period deposit rate d_1 (example A).

In section 3.5.3 we have already illustrated that while the maximum fraction of depositors is always bailed out under BB, under RB it might be necessary to bail out a significantly lower fraction of first-period deposits than would be possible. This occurs if the deposit distribution is very unequal. The necessity to commit to lower-than-possible bailout fractions might well reduce the credibility of the RB scheme, since agents might expect the

[31]Note that the equilibrium $d_1 = \tilde{d}_{\mathbf{ZP}}$ does not exist in the no-regulation case for example A. Hence highest possible equilibrium returns are achieved under $\mathcal{E}_L$.

regulator to abandon RB and bail out more depositors if an asymmetric deposit distribution occurs. As mentioned above, this kind of credibility problem does not occur under BB.

6.4 Extensions

In this section we discuss three important extensions of the current model: positive liquidation values and takeover costs, differences in banks sizes that might stem from other sources than considered in this paper, and risk-taking incentives.

6.4.1 Liquidation Value and Takeover Costs

In this section we will briefly explore cases where $\bar{u}_{(\cdot)}$ is not increasing in d and discuss the consequences for the comparison of the schemes RB and BB. In order to obtain such scenarios we had to relax two implicit assumptions: (a) that the takeover of investment projects from closed banks does not involve any deadweight costs; and (b) that the $t = 1$ liquidation value of investments is zero. In a more general setting we could assume that a fraction $(1 - \delta)$ of project returns is consumed by the takeover procedure. Hence returns of such projects in the second period would be given by δr_2.[32] Moreover, we could allow for a positive liquidation value R_1 of investments when liquidated in $t = 1$; the realization of the liquidation value can be either high ($R_1 = r_{1h}$ if $R_2 = r_{2h}$) or low ($R_1 = r_{1l}$ if $R_2 = r_{2l}$). We require that depositors whose claims have been eliminated by the regulator will receive a return equal to the liquidation value of investments.

In ERLENMAIER (2001) we show that if $r_{1h} \leq \delta d_{\mathbf{L}}$, our whole analysis also applies for this more general case. Moreover, concerning the expected returns achieved under the different regulatory schemes, our numerical exercises have generated the following pattern: First, profits per deposits are always decreasing in d, i.e. UZP is always fulfilled and hence propositions 7 and 8 apply. Second, in all cases investigated, either UID or UDI applies. If UDI holds then $d_{\mathbf{L}} > \bar{u}_{d_{\mathbf{ZP}}}$ and therefore BB implements $\mathcal{E}_L$. Hence, for all examples considered, BB implements either the same expected first-period returns as RB or higher returns. The same is true for second-period returns: they qualitatively behave in the same way as expected first-period returns (increase when u_1 increases and decrease when u_2 decreases). Moreover, we observe that scenarios where the normal case UI is not fulfilled only occur if δ is sufficiently small *and* p_l (the probability of low returns) is high enough.[33]

[32] Note that we have employed the most simple and plausible specification of takeover costs, namely that they are proportional to the *ex-post* size of the project.

[33] This result is intuitively convincing. Return losses due to lower deposit rates are decreasing in p and increasing in δ.

$S_1(u) = u$	$p_l = 0.8$	$r_{1l} = 0.7$	$r_{2l} = 1.08$	$\bar{R}_2 = 1.14$
$S_2(u) = 1.05 \cdot u$	$p_h = 0.2$	$r_{1h} = 0.7$	$r_{2h} = 1.40$	$\delta = 0.6$

Table 4: Example B.

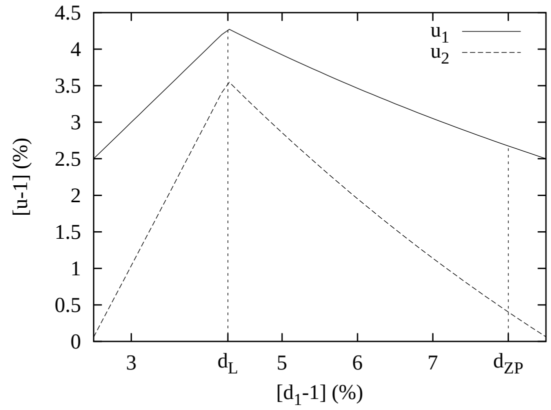

Figure 11: Expected returns for both generations as function of the first-period deposit rate d_1 (example B).

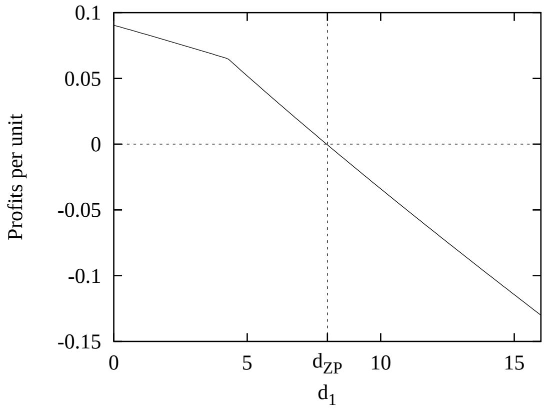

Figure 12: Profits per unit of deposits as function of the first-period deposit rate d_1 (example B).

For illustration we use the example that is presented in table 4. Figures 11 and 12 show expected returns for the first and the second generation and profits per unit of deposits as function of offered first-period deposit rates under discriminatory bailout. The case UD applies (i.e. $\bar{u}_d$ is decreasing in d if $d \in \mathcal{D}_M$). Hence BB implements the equilibrium $\mathcal{E}_L = (d_{\mathbf{L}})$ and RB implements $\mathcal{E}_{ZP} := (d_{\mathbf{ZP}})$. Expected returns for both generations are therefore higher under BB as can be inferred from table 5.

	d_1	u_1	u_2	q_l
$\mathcal{E}_L$	4.28	4.28	3.84	1
$\mathcal{E}_{ZP}$	8.00	2.67	0.4	0.82

Table 5: First-period deposit rates, expected returns, and fraction of bailed out depositors for example B.

6.4.2 Bank Size and Growth Rates of Deposits

In this section we discuss how the interpretation of the proposed bailout policies has to be adapted if the size of a bank depends not only on the attractiveness of its offered deposit rates - as in our model. In a more realistic setting, banks sizes will differ for many other reasons.[34] It is therefore important to recall the actual mechanism which characterizes the BB scheme, namely that banks which are assessed to pay higher returns (or, in a more general view, to provide better intermediation services) will also have a higher bailout probability. Conditioning bailout policies on the *growth rates* rather than on the actual size of a bank's deposits should therefore be an alternative implementation of the spirit of BB in realistic banking competition environments.

6.4.3 Risk-Taking Incentives

The risk taking incentives generated by bailout policies are an important focus of the existing literature on bank closures.[35] In this section we briefly discuss the consequences of such considerations for the policy suggestions derived by now.

If banks can decide about the riskiness of their projects after they have received deposits, BB will have the drawback of providing risk taking incentives for big banks since they can anticipate to be bailed out with high probability. RB - in contrast - will provide less incentives for risk taking since banks are uncertain about the regulator's bailout decision. This foundation of a constructive ambiguity approach to bailout has been highlighted by FREIXAS (1999). In FREIXAS (1999), creative ambiguity is achieved by assuming that the regulator follows a mixed strategy when deciding about a *single* banks bailout. In our general equilibrium framework, bailout probabilities have to be chosen in a way ensuring that the remaining banks will be able to survive which makes the design of such a policy more demanding. The simple version of RB we have proposed in this paper requires that - in out-of-equilibrium strategies - the regulator must commit to bail out significantly less deposits than would be possible. This in turn undermines the credibility of the RB policy (see sections 3.5.3 and 6.3). However, it might be possible to construct RB-type policies which do not share this drawback.

To explore this possibility, recall how bank closures where determined under our RB specification. We required the regulator to randomly draw a subset of k banks from the complete sample with each bank having the same probability to be be drawn. The

[34] See TIROLE (1994) and FREIXAS AND ROCHET (1999) for general industrial organization and bank specific reasons.

[35] See CORDELLA AND YEYATI (1999) and references therein.

integer k was chosen in way which ensured that the surviving banks would always be able to refinance, independently of which subsample had been drawn. Thus the choice of k had to ensure that even if the biggest banks had been chosen, the fraction q of bailed out deposits would never exceed $\bar{q}$. Hence, in cases where small banks had been drawn, the fraction of bailed out deposits could drop strongly leading to the credibility problems of RB.

To avoid this problem, the regulator could *sequentially* draw from the sample of all banks. Each bank which is drawn will survive and the regulator continues drawing until the critical fraction $\bar{q}$ is reached. All other banks, which have not been drawn, will be closed and eventually a fraction of the last drawn banks' balance sheet will be eliminated (see section 3.5.3). The most intuitive application of the RB principle to sequential drawing would be to give all banks remaining in the sample the same probability to be drawn in the next step. For example, if there are 10 banks and 2 have already been drawn to survive, the remaining 8 banks will all have a probability of 1/8 to be drawn next.

However, this sequential drawing procedure would lead to a higher bailout probability for big banks than for small banks as is illustrated in the following example. Suppose that there are only two banks which have obtained deposits of $D_1^1 = 0.8$ and $D_1^2 = 0.2$ and that $\bar{q} = 0.6$. If the regulator draws the big bank first, then the small will be closed and a fraction 0.6/0.8 of the big bank's deposits will be bailed out. In the small bank is drawn first, the maximum fraction $\bar{q}$ will not yet have been reached and a fraction 0.4/0.8 of the big bank's deposits will be bailed out. Hence, the a-priori bailout probability for deposits at the big bank is given by $q_1 = 0.5 \cdot 0.6/0.8 + 0.5 \cdot 0.4/0.8 = 0.625$ while the respective probability for the small bank is $q_2 = 0.5 \cdot 0 + 0.5 = 0.5$. In general, big banks will have a higher bailout probability since, given that small banks have already been drawn, their conditional bailout probability will still be high while the contrary is true for small banks.

Note that in our example it would still be possible to achieve an equal bailout probability of $\bar{q} = 0.6$ for depositors of both banks. This could be done by altering the probabilities with which the two banks are drawn. Denoting the respective probabilities by μ_i $(i = 1, 2)$ we have $\mu_2 = 1 - \mu_1$ and hence

$$q_1 = \mu_1 \cdot 0.6/0.8 + (1 - \mu_1) \cdot 0.4/0.8 \stackrel{!}{=} 0.6,$$

implying that $\mu_1 = 0.4$ and $\mu_2 = 0.6$. The most simple generalization to the case of n banks would be to assign a certain probability weight μ_i to each bank i $(i = 1, .., n)$ and to define the probability of bank i to be drawn, given that banks $B_{i_1}, .., B_{i_k}$ are still in the sample by

$$\frac{\mu_i}{\sum_{j=1}^{k} \mu_{i_j}}.$$

It is, however, unclear whether the probabilities $\mu_1, .., \mu_n$ can be chosen in way that a-priori bailout probabilities are equal for all banks.[36] An even more general probabilistic structure could be formulated by allowing for conditional default probabilities to depend on any piece of information available from draws which have already been conducted (e.g. on the size or number of banks which have already been drawn). Our conjecture is therefore that it should be possible to construct a drawing mechanism that produces uniform bailout probabilities while bailing out the maximum fraction $\bar{q}$ of deposits under all circumstances. *How* to find such a mechanism in general seems to be, however, far from straightforward and is left to future research.

Putting together the observations from this section and from those in section 6.4.1, a mixture of BB and RB might be sensible. Whether the actual policy will more closely resemble the former or the latter will depend on whether return or risk-taking consideration are the major focus of interest. However, to make such combinations of BB and RB possible, an algorithm has to be found which - for a given deposit distribution - can deliver drawing mechanisms generating *any* pattern $q_1, .., q_n$ of bailout probabilities.

7 Conclusion

We have attempted to provide a general equilibrium analysis of the funds concentration effect and the corresponding regulatory bailout schemes in an overlapping-generations framework. Banks invest in long-term projects that are subject to macroeconomic risk. The projects are financed by short-term deposit contracts, offered to both generations of households. The banks' obligations to the first generation have to be served with the savings of the second generation. Since deposit contracts are not contingent on the realizations of investment returns, banks are exposed to macroeconomic risk: if investment returns are expected to be low, banks might not be able to attract enough second-generation savings to pay back first-generation depositors.

Without regulation, banks might bid up first-period deposit rates so that they are able to refinance if investment returns are high but are not able to refinance if they are low.

[36]Note that it is quite complicated to actually calculate these probabilities. Moreover, even once this is done one would very likely end up with a system of n polynomial equations of degree n.

Hence, all banks may go bankrupt in the state of low production returns since they cannot offer sufficiently high returns to second-period depositors. This has the unfavorable effect that long-term projects are liquidated in the state of low returns. Moreover, consistent return assessments do not exist for first-period depositors for a range of first-period deposit rates.

In this paper we have considered two different regulatory approaches to deal with these problems. First, prudential banking, where the regulator ensures that banks bid up first-period deposit rates only to the point where they are still able to refinance themselves in both future states of production returns. Second, discriminatory bailout where in the low state of production returns the regulator closes some banks and distributes the investment projects of these banks among the surviving ones. This increases returns that can be paid by the surviving banks and decreases the amount of second-period savings needed by the banking system to refinance.

Our results are as follows. First, discriminatory bailout improves the stability of intermediation by guaranteeing the existence of consistent assessments for first-period depositors. Moreover, it dominates both prudential banking and the no-regulation case with respect to expected returns paid on deposits. In the case where expected returns $\bar{u}_d$ on first-period deposits are an increasing function of first-period deposit rates d, the two different variants of discriminatory bailout, random bailout (RB) and bail out the big ones (BB) implement the same equilibrium. Non-increasing functional forms of $\bar{u}_{(\cdot)}$ occur if takeover of investment projects involves deadweight costs and the probability of low returns is high. In this case BB dominates RB with respect to expected returns. The reason for this result is that under RB, banks will always bid up deposit rates until profits are zero, independently of the shape of $\bar{u}_{(\cdot)}$. Under BB, in contrast, banks stop the bidding when expected returns for first-generation depositors are at their maximum since - due to a self-fulfilling-prophecy effect - depositors will not switch to a bank that offers slightly higher deposit rates.

We have also considered a third variant of discriminatory bailout, bail out the small ones (BS), and have identified that under such a regime the existence of consistent assessments cannot be guaranteed when banks deviate from a symmetric equilibrium. This result stems from what we call a self-contradicting-prophecy effect. While under BB, a bank which is assessed to pay higher returns than an other bank will also have a higher bailout probability (which can validate the initial assessment), under BS the same bank will have a lower bailout probability than the bank which has been assessed to pay lower returns (an effect that undermines the initial assessment). Additional to the resulting instability, the non-existence problem may also prevent banks from biding up deposit rates which might support low-return equilibria.

In order to ultimately assess which of the discriminatory bailout schemes, BB or RB, should be preferred, two further issues have to be taken in account. First, to what degree does return maximization help to coordinate depositors' assessments if more than one consistent assessment exists? Under RB, only one consistent assessment exists for each vector of first-period deposit rates while under BB there might be more than one consistent assessment. We have suggested that if more than one consistent assessment exists, depositors will choose the assessment which promises the maximum expected return (optimal assessment). Under this assumption assessments are unique under BB.

Second, to what extent do bailout policies influence the risk-taking incentives for banks? We have argued that BB will will have the drawback of providing risk taking incentives for big banks since these banks can anticipate to be bailed out with high probability. RB - in contrast - will provide less incentives for risk taking since banks are uncertain about the regulator's bailout decision. This parallels the „*constructive ambiguity*" approach to bailout highlighted by FREIXAS (1999). In FREIXAS (1999), creative ambiguity is achieved by assuming that the regulator follows a mixed strategy when deciding about a *single* bank's bailout. In the general equilibrium framework, bailout probabilities have to be chosen in a way ensuring that the banks which have not been closed will be able to survive. This makes the design of such a policy more demanding. The simple version of RB we have proposed in this chapter requires that - in out-of-equilibrium strategies - the regulator must commit to bail out significantly less deposits than would be possible and optimal. This undermines the credibility of the regulatory policy. We have indicated, however, that a RB-type policy may be found which circumvents these drawbacks. The comprehensive construction of such a policy seems, however, to be far from obvious and is left to future research.

In summary, our findings suggest that closure rules in severe crises should be a mixture of BB and RB. Whether the actual policy will more closely resemble the former or the latter will depend on whether return consideration or stability consideration (risk-taking incentives, unique assessments) are more important.

The most important road for future research is the integration of risk taking incentives into the general equilibrium framework developed in this paper. In particular, the following two issues should be addressed. First - as has already been outlined above - the construction of bailout schemes that allow for arbitrary combinations of RB and BB and that do not share the credibility problem of the RB scheme introduced in this paper. Second, the analysis of closure rules where the bailout decision may depend on the level of uninsured debt on a bank's balance sheet or on other bank-specific variables.

Finally, future research should also allow for banks to differ with respect their positions in the matrix of interbank connections. ALLEN AND GALE (2000) and FREIXAS, PARIGI, AND ROCHET (2000) have argued that this position can be crucial for the stability of the financial system.

A Appendix

In this appendix, we present the proofs of all lemmata and propositions. We start with the proofs for section 4.

A.1 Proofs for Section 4

Proof of proposition 1.

(i) The proof will proceed in two steps.

Step 1: $\mathcal{E}_2^*$ is an equilibrium.

First, we note that if $d_2^i = \bar{d}_2^*$ is played ($i \in \mathcal{B}^+$), the assessment $u_2^i = \bar{d}_2^*$ ($i \in \mathcal{B}^+$) is optimal and the proposed deposit distribution is the only one which is consistent with this assessment. Second, we have to show that deviations from $d_2^i = \bar{d}_2^*$ are not profitable. Deviations to higher deposit rates can be excluded, since they raise repayment obligations. If a bank deviates to $d_2^{\text{dev}} < \bar{d}_2^*$, it is not possible for all banks to receive a positive measure of deposits.[37] But then either all or none of the non-deviating banks receive a positive measure of deposits.[38] Hence, there can only be one case where the deviating bank receives a positive measure of savings, namely if it can attract the full measure of second-period savings. But in this case the deviation cannot be profitable for the following reasons. Depositors only choose to give resources to the deviating bank if returns are at least as high as returns at the non-deviating banks. But if depositors chose to deposit with the non-deviating banks, returns are given by $\min\{u_*, \bar{d}_2^*\}$, where u_* is the positive solution of

$$u = \frac{(m-1)r_2 D_1}{q S_2(u)}. \tag{11}$$

If, on the other hand, depositors deposit with the deviating bank, returns cannot be higher than $\min\{u_*^{\text{dev}}, d_2^{\text{dev}}\}$, where u_*^{dev} is the positive solution of equation (11) when $m-1$ is replaced by 1. Hence, inequality $u_*^{\text{dev}} \geq u_*$ can only be fulfilled if $m = 2$ and $d_2^{\text{dev}} \geq u_*$. But in this case the deviating banks' profits cannot be higher than zero.

Step 2: Pareto-dominance.

[37] If that were the case, all banks would have to be assessed paying the *same* return $u_2 \leq d_2^{\text{dev}}$. But then $S_2(u_2) < m d_1 D_1$, and hence at least one bank cannot refinance, which implies that $u_2 = 0$ and $S_2(u_2) = 0$ in contradiction to the assumption that all banks receive a positive measure of deposits.

[38] Because those banks are identical with respect to all their characteristics and will therefore receive the same amount of deposits.

If $d_2 > \bar{d}_2^*$ then banks obtain lower profits than in $\mathcal{E}_2^*$ because repayment obligations are higher. If $d_2 < \bar{d}_2^*$ then the amount of overall savings is bounded by $S_2(d_2)$ and hence second-period deposits of a single bank are limited by $S_2(d_2)/m$ (since depositors cannot coordinate on a subset of banks). But this implies that no bank can refinance and the only consistent assessment is $u_2 = 0$ for all banks, implying that no bank receives any savings.

(ii)

The first observation is obvious and the second has already been derived under step 2 in (i).

□

Proof of proposition 2.

Step 1: $\mathcal{E}_2^*$ is an equilibrium. As in the last proposition, we observe that if $d_2^i = \bar{d}_2^*$ is played, the assessment $u_2^i = \bar{d}_2^*$ ($i \in \mathcal{B}^+$) is optimal and the proposed deposit distribution is the only one which is consistent with this assessment. Deviations to higher deposit rates would increase repayment obligations and can therefore not be profitable. Deviations to lower rates are excluded by LBD.

Step 2: Pareto-dominance. Equilibria where some banks have offered lower deposit rates than $\bar{d}_2^*$ are excluded by LBD and all other equilibria are Pareto-dominated by $\mathcal{E}_2^*$ since repayment obligations are higher than under $\mathcal{E}_2^*$.

□

Proof of lemma 1.

Step 1: The system $\mathcal{S}$ has a unique solution.

First, note that if $\bar{u}_d$ is a solution of the system $\mathcal{S}$, then $\bar{u}_d > \underline{u}$. But for all $u > \underline{u}$, equations (9) and (10) have unique, strictly positive solutions $q_l = q_l(d,u)$ and $q_h = q_h(d,u)$.[39] Figure 13 illustrates the argument by depicting the left-hand and the right-hand side of equation (9) for $d = 1.05$. Figure 13 also illustrates that the functions $q_l(d,\cdot)$ and $q_h(d,\cdot)$ are increasing in u for fixed d by depicting the solutions of the equation for $u = 1.05$ and $u = 1.07$. Moreover, $q_l(\cdot,\cdot)$ and $q_h(\cdot,\cdot)$ are continuous in d and u since the left and

[39] The left-hand sides of the equations are strictly increasing in q_l and q_h respectively and they take all values in $(0,\infty)$; the right-hand sides are strictly positive non-increasing in q_l and q_h respectively.

the right-hand side of the equations (9) and (10) are continuous functions of d and u. Inserting $q_l(d,u)$ and $q_h(d,u)$ in equation (8), we obtain an implicit equation for u. As we saw above, we have to restrict the range of this equation to $u > \underline{u}$. Hence the left-hand side of this equation is strictly increasing in u and takes all values in $\mathbb{R}$ that are strictly higher than $\underline{u}$; the right-hand side is decreasing in u and higher than $\underline{u}$ if u is close enough to $\underline{u}$.[40] Hence, by the mean value theorem, a unique solution $\bar{u}_d$ of this implicit equation exists. This solution is a continuous function of d, since the left and the right-hand side of the equation are continuous functions of d. Finally insert $\bar{u}_d$ in q_l and q_h to obtain $\bar{q}_{l,d} := q_l(d,\bar{u}_d)$ and $\bar{q}_{h,d} := q_h(d,\bar{u}_d)$.

Step 2: Proof of statements (i) - (iv)

The continuity of $\bar{u}_{(\cdot)}$ has already been shown in step 1 and the continuity of $\bar{q}_{l,(\cdot)}$ and $\bar{q}_{h,(\cdot)}$ follows directly from the continuity of $\bar{u}_{(\cdot)}$, $q_l(\cdot,\cdot)$ and $q_h(\cdot,\cdot)$. Statements (ii) and (iii) are straightforward.We now need to substantiate (iv), i.e. the monotony of $\bar{q}_{l,(\cdot)}$ for all $d \in \mathcal{D}_M$. Recall that

$$\mathcal{D}_M = \left\{ d \mid dS_1(\bar{u}_d) \leq S_2(r_{2h}/d) \text{ and } d > d_{\mathbf{L}} \right\},$$

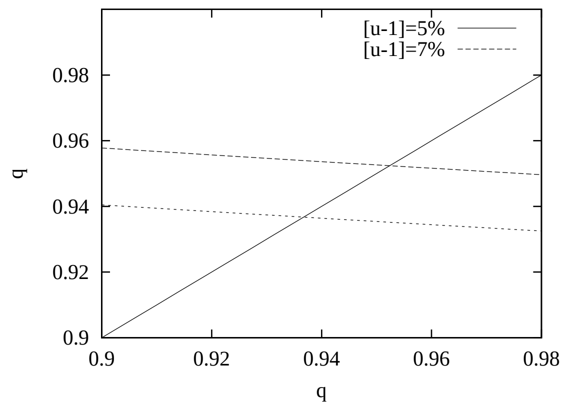

Figure 13: The left-hand and the right-hand side of equation (9) as functions of q ($d = 1.05$).

and note that for $d \in \mathcal{D}_M$ we have

$$\bar{q}_{l,d} = \frac{1}{dS_1(\bar{u}_d)} S_2\left(\frac{r_{2l}}{\bar{q}_{l,d}d}\right), \tag{12}$$

$\bar{q}_{h,d} = 1$ and

$$\bar{u}_d = (p_l\bar{q}_{l,d} + p_h)d. \tag{13}$$

Suppose now that $d < \tilde{d}$. If $\bar{u}_d \leq \bar{u}_{\tilde{d}}$, then we obtain $\bar{q}_{l,d} > \bar{q}_{l,\tilde{d}}$ from equation (12). If on the other hand $\bar{u}_d > \bar{u}_{\tilde{d}}$, then $\bar{q}_{l,d} > \bar{q}_{l,\tilde{d}}$ follows from equation (13).

□

Proof of lemma 2.

The proof is along the same lines as the proof for lemma 1.

[40]Note that if $u \to \underline{u}$, the right-hand side approaches d and $d > \underline{u}$.

□

Proof of proposition 3.

Since banks are identical, assessments and deposit distribution have to be symmetrical: $\mathbf{u_1} = (u, ..., u)$ and $\mathbf{D_1} = (D, .., D)$ where $D = S(u)/n$. Hence the expected return on first-period bank deposits is given by equations (8) - (10). But we know from lemma 1 that in this case $u = \bar{u}_{d_1}$ is the only consistent assessment.

□

Proof of proposition 4 .

We denote the share of deposits that the banks in $\mathcal{B}_l$ receive by λ_l . First, note that an assessment $u_{1l} < u_{1h}$ always leads to $\lambda_l = 0$ and the symmetric distribution of all savings among banks in $\mathcal{B}_h$. Hence, by lemma 1, this assessment is consistent if and only if $u_{1h} = \bar{u}_{d_{1h}}$ and if the assessment passes the zero measure test: if all banks in $\mathcal{B}_l$ receive a zero measure of deposits, then depositors at $\mathcal{B}_l$-banks must receive lower expected returns than $\bar{u}_{d_{1h}}$. Of course, the same is true for the converse assessment $u_{1h} < u_{1l}$. It leads to $\lambda_l = 1$ and $u_{1l} = \bar{u}_{d_{1l}}$ and is consistent if and only if the return paid by zero measure $\mathcal{B}_h$-banks is smaller than $\bar{u}_{d_{1l}}$.

The assessment $u_{1l} < u_{1h} = \bar{u}_{d_{1h}}$ is consistent under RB, PB and BB since under all of those bailout schemes the bailout probability for depositors at zero-measure banks is never higher than the bailout probability for deposits at positive-measure banks. Under BS, however, a zero-measure bank is bailed out with probability 1. Hence, an assessment $u_{1h} < u_{1l} = \bar{u}_{d_{1l}}$ will never pass the the zero-measure test, and an assessment $u_{1l} < u_{1h} = \bar{u}_{d_{1h}}$ passes this test if and only if $d_{1l} < \bar{u}_{d_{1h}}$. Furthermore, under PB and RB bailout probabilities are the same for all deposits. Hence expected returns on deposits of $\mathcal{B}_h$ banks are strictly higher than those on deposits of $\mathcal{B}_l$ banks for *any* consistent assessment. Finally, under BB, the assessment $u_{1h} < u_{1l} = \bar{u}_{d_{1l}}$ passes the zero-measure test if and only if $p_h d_{1h} < \bar{u}_{d_{1l}}$.

It remains to show that an assessment $u_{1l} = u_{1h}$ cannot be optimal under BB and that no consistent assessments exist under BS if $\bar{u}_{d_{1h}} < d_{1l}$ and $p_h d_{1h} < \bar{u}_{d_{1l}}$. Note that, concerning the last point, we have already shown that $u_{1l} < u_{1h}$ and $u_{1h} < u_{1l}$ are not consistent under these circumstances. Hence it remains the analyze assessments of the type $u_{1l} = u_{1h}$ under BB and BS. Note above that such assessments can only be consistent if $\lambda_l > n_l/n$ (under BB) and $\lambda_l < n_l/n$ (under BS) respectively, where n_l denotes the number of banks in $\mathcal{B}_l$. Hence, the statement that $u := u_{1l} = u_{1h}$ is a consistent assessments , is equivalent to the existence of a real number $\lambda_l > n_l/n$ ($\lambda_l < n_l/n$) with u solving the system $\bar{\mathcal{S}}(\lambda_l)$ described by equations (14) - (19):

$$
\begin{aligned}
u &= (p_l q_{l,l} + p_h q_{h,l}) d_{1l} && (14)\\
u &= (p_l q_{l,h} + p_h q_{h,h}) d_{1h} && (15)\\
q_{i,l} &= \begin{cases} 1 & \text{if } q_i \geq \lambda_l \\ q_i/\lambda_l & \text{else} \end{cases} \quad (i = l, h) && (16)\\
q_{i,h} &= \begin{cases} (q_i - \lambda_l)/(1-\lambda_l) & \text{if } q_i \geq \lambda_l \\ 0 & \text{else} \end{cases} \quad (i = l, h) && (17)\\
q_l &= \min\left\{ \frac{1}{d_{1h} S_1(u)} S_2\left(\frac{r_{2l}}{q_l d_{1h}}\right), 1 \right\} && (18)\\
q_h &= \min\left\{ \frac{1}{d_{1h} S_1(u)} S_2\left(\frac{r_{2h}}{q_h d_{1h}}\right), 1 \right\}. && (19)
\end{aligned}
$$

Note that $q_{i,l}$ and $q_{i,h}$ denote the bailout probabilities of banks in $\mathcal{B}_l$ and $\mathcal{B}_h$ respectively ($i = l, h$ denotes the state of production returns). The remaining statements therefore follow from lemma 5.

□

Lemma 5

Suppose that $\underline{u} < d_{1l} < d_{1h}$.

(i) If $\bar{\mathcal{S}}(\lambda_l)$ has a solution u for arbitrary $\lambda_l \in (0,1]$ then $u < \bar{u}_{d_{1h}}$.

(ii) If $\bar{u}_{d_{1h}} < d_{1l}$ and $p_h d_{1h} < \tilde{u}_{d_{1l}, d_{1h}}$ then $\bar{\mathcal{S}}(\lambda_l)$ has no solution for all $\lambda_l \in [0,1]$.

Proof.

First of all note that a solution of the complete system has to solve the subsystems $\bar{\mathcal{S}}_l$ (consisting of equations 14, 16, 18 and 19) and the subsystem $\bar{\mathcal{S}}_h$ (consisting of equations 15, 17, 18 and 19).

(i) The proof of part (i) rests on the observation that the sub-system $\bar{\mathcal{S}}_h$ consists of the same equations as the system $\mathcal{S}(d_{1h})$ which has the solution $\bar{u}_{d_{1h}}$. The only difference is that in the system $\mathcal{S}(d_{1h})$, $q_{i,h}$ is replaced by q_i ($i = l, h$) in equation (15). The statement $u < \bar{u}_{d_{1h}}$ therefore follows from the fact that $q_{i,h} \leq q_i$.

To present this argument in a more formal way, we use the index i to indicate both $i = l$ and $i = h$. Note that a solution of $\bar{\mathcal{S}}_h$ can be derived by solving equations (18) and (19) and inserting the solutions in equation (17) which yields $q_{i,h}(u, \lambda_l)$. Since the right-hand side of equation (17) is decreasing in λ_l for arbitrary $q_i \leq 1$, we obtain that $q_{i,l}(u, \cdot)$ is decreasing in λ_l. By inserting $q_{i,l}$ in equation (15) we can therefore conclude that the solution $u(d_{1h}, \lambda_l)$ of the resulting equation is decreasing in λ_l. Hence $u(d_{1h}, \lambda_l) \leq u(d_{1h}, 0) = \bar{u}_{d_{1h}}$. In order to show that the inequality holds strictly for $\lambda_l > 0$, we assume that $u(d_{1h}, \lambda_l) = \bar{u}_{d_{1h}}$ for $\lambda_l > 0$. By inserting $u = \bar{u}_{d_{1h}}$ in equations (18) and (19) we can then see that $q_i = \bar{q}_{i,d_{1h}}$ and therefore that $q_{i,h}(\lambda_l) < \bar{q}_{i,d_{1h}}$. But then insertion into equation (15) would imply that $u(d_{1h}, \lambda_l) < \bar{u}_{d_{1h}}$, in contradiction to our assumption.

(ii) We denote the solutions of $\bar{\mathcal{S}}_i(\lambda_l)$ by u_i $(i = l, h)$. Note that if $u_h > p_h d_{1h}$ then $q_{l,h} > 0$ implying that $q_{l,l} = q_{h,l} = 1$.[41] Therefore $u_l = d_{1l} > \bar{u}_{d_{1h}}$ which by statement (i) is not possible if $u = u_l$ solves $\bar{\mathcal{S}}$. But the case $u \leq p_h d_{1h}$ can be excluded by observing that $u_l \geq \tilde{u}_{d_{1l},d_{1h}}$ which by our assumption leads to $u = u_l > p_h d_{1h}$. To see that $u_l \geq \tilde{u}_{d_{1l},d_{1h}}$, assume to the contrary that $u = u_l < \bar{u}_{d_{1l}}$. Then, by equations (18) and (19), we have $q_i > \tilde{q}_{i,d_{1l},d_{1h}}$, implying that $q_{i,l} > \tilde{q}_{i,d_{1l},d_{1h}}$. This in turn leads to $u_l > \tilde{u}_{d_{1l},d_{1h}}$ in contradiction to our assumption.

□

Proof of corollary 1.

Since $\tilde{q}_{i,d_{1l},d_{1h}} \geq \bar{q}_{i,d_{1h}}$ $(i = l, h)$ we obtain that $\tilde{u}_{d_{1l},d_{1h}} \geq p_h d_{1l} + \bar{q}_{l,d_{1h}} d_{1l}$ which implies the statement of the corollary, since $\bar{q}_{l,d_{1h}} > 0$.

□

A.2 Proofs for Section 5

The propositions in section 5 are concerned with symmetric equilibria in $t = 1$. Hence we will either have to analyze the case where all banks offer the same first-period deposit rate d_1 or the deviation case where $(n-1)$ banks offer the same first-period deposit rate d_1 and one bank j offers a different rate d_1^{dev}. We will always denote the assessment for the

[41] If $q_{l,h} > 0$ then $q_l \geq \lambda_l$ by equation (17) and, since $q_l \leq q_h$, we also obtain that $q_h \geq \lambda_l$. Hence $q_{l,l} = q_{h,l} = 1$ by equation (16).

non-deviating banks by u_1 and that for the deviating bank by u_1^{dev}. Given the assessment, we denote the deposits finally received by those banks by D_1 and D_1^{dev} respectively.

Proof of proposition 5.

It only remains to substantiate (ii) and (iii). Statement (ii) is obvious for first-period returns and follows for second-period returns since they are given by $S_2^{-1}\Big(dS_1(d)\Big)$ if $d_1 \leq d_{\mathbf{L}}$ and by $p_l\underline{u}+p_h\tilde{d}_2^*(d_1)$ if $d_1 > d_{\mathbf{L}}$. Both expressions are increasing in d_1. Statement (iii) follows immediately from (ii) since if $d_{\mathbf{ZP}} \leq d_{\mathbf{C}}$, then only symmetric equilibria with $d_1 \leq d_{\mathbf{L}}$ are possible.

□

Proof of proposition 6 .

We use the results of proposition 3 about existence and uniqueness of assessments. The proof will proceed in two steps.

Step 1: $\mathcal{E}_L$ is an equilibrium.

Given $\mathbf{d_1} = (d_{\mathbf{L}}, ..., d_{\mathbf{L}})$, the only consistent assessment is $\mathbf{u_1} = (d_{\mathbf{L}}, ..., d_{\mathbf{L}})$. Hence equilibrium deposit distribution is $D_1^i = S_1(d_{\mathbf{L}})/n$ $(i = 1, ..., n)$ and each bank's expected profits are given by

$$\Pi_1 = \frac{S_1(d_{\mathbf{L}})}{n} p_h(r_{2h} - r_{2l}) > 0.$$

Consider now a deviation of one bank. Deviation to $d_1^{\text{dev}} < d_1$ leads to $u_1^{\text{dev}} < u_1$ and hence to $D_1^{\text{dev}} = 0$, which cannot be profitable. On the other hand $d_1^{\text{dev}} > d_1$ leads to $u_1 < u_1^{\text{dev}} = \bar{u}_{d_1^{\text{dev}}}$ and hence to $D_1^{\text{dev}} = S_1(\bar{u}_{d_1^{\text{dev}}})$. If $S_1(\bar{u}_{d_1^{\text{dev}}}) > S_1(d_{\mathbf{L}})$, then the refinancing condition is not fulfilled in the case of low production returns. If $S_1(\bar{u}_{d_1^{\text{dev}}}) \leq S_1(d_{\mathbf{L}})$, then at least $\bar{q}_{l,d_1} < 1$. Therefore there is a positive probability of being closed in both cases. Hence deviation to $d_1^{\text{dev}} > d_1$ cannot be profitable since extreme punishment in the event where the bank is closed would outweigh all possible gains.

Step 2: No other equilibria $\mathcal{E} = (d_1)$ with $d_1 \neq d_{\mathbf{L}}$ exist.

If $d_1 < d_{\mathbf{L}}$, then deviation to a slightly higher interest rate d_1^{dev} $(d_1 < d_1^{\text{dev}} < d_{\mathbf{L}})$ would lead to the only possible assessment $u_1 < u_1^{\text{dev}} = \bar{u}_{d_1^{\text{dev}}} = d_1^{\text{dev}}$. Therefore $D_1^{\text{dev}} = S_1(d_1^{\text{dev}}) < S_1(d_{\mathbf{L}})$ and the deviating bank would still be able to pay depositors back in $t = 1$. The collection of all savings outweighs the slightly higher interest payment. If $d_1 > d_{\mathbf{L}}$, the only possible assessment is $\mathbf{u_1} = (u_1, ..., u_1)$ where $u_1 < d_1$. But this implies a positive probability of being closed and suffering the punishment P, which leads to negative expected profits; of course this is not possible in equilibrium.

$\square$

Proof of lemma 3 .

The proof rests on the continuity of the functions $S_i(\cdot)$ $(i = 1, 2)$, $\bar{u}_{(\cdot)}$ and $\bar{q}_{i,\cdot}$ $(i = l, h)$. First we prove continuity for a point $d \in \mathcal{D}_M$. Consider a sequence $(d_n)_{n \in \mathbb{N}}$ with $d_n \to d$ $(n \to \infty)$. If $dS_1(\bar{u}_d) < S_2(r_{2h}/d)$, then $d_n S_1(\bar{u}_{d_n}) < S_2(r_{2h}/d)$ for sufficiently large n. If $dS_1(\bar{u}_d) = S_2(r_{2h}/d)$, we have $\bar{q}_{h,d_n} \to 1$ by equation (10) and $d_2^*(d_n) \to r_{2h}/d$ $(n \to \infty)$. Hence, in both cases we obtain $\pi(d_n) \to \pi(d)$.

Now suppose that $d \le d_{\mathbf{L}}$. Again, only the case $d = d_{\mathbf{L}}$ is interesting. If $d_n \to d$ then $q_{l,d_n} \to 1$ by equation (9) and $d_n d_2^*(d_n) \to d_{\mathbf{L}} d_2^*(d_{\mathbf{L}}) = r_{2l}$. Hence $\pi(d_n) \to p_h(r_{2h} - r_{2l})$.

$\square$

Proof of lemma 4.

If $d \in \mathcal{D}_M$ then by equation (9) we obtain

$$q_l = \frac{a_2}{d a_1 u^{\alpha_1}} (\frac{r_{2l}}{q_l d})^{\alpha_2} ,$$

implying that $q_l = c d^{-1} u^{-\tilde{\alpha}_1}$ where

$$c := \Big(\frac{a_2 r_{2l}^{\alpha_2}}{a_1}\Big)^{1/(1+\alpha_2)},$$

and $\tilde{\alpha}_i := \alpha_i/(1 + \alpha_i)$ $(i = 1, 2)$. Inserting q_l in equation (8) we find that $\bar{u}_d$ can be described as solution of the equation $F(u, d) = 0$ where

$$F(u, d) := u - p_l c u^{-\tilde{\alpha}_1} - p_h d.$$

But since $F(\bar{u}_d, d) = 0$ we find that $F_u \bar{u}'_d + F_d = 0$ implying that $\bar{u}'_d = -F_d/F_u$.[42] But

$$\begin{aligned} F_d(u, d) &= -p_h d \\ F_u(u, d) &= 1 + \tilde{\alpha}_1 p_l u^{-(\tilde{\alpha}_1 + 1)}. \end{aligned}$$

[42] F_u and F_d denote the partial derivatives of F with respect to u and d respectively.

□

Proof of proposition 7 .

The proof follows the same arguments as the proof of proposition 6:

(1) $\mathcal{E}_{ZP}$ is an equilibrium. Consider a deviation of one bank. Deviation to $d_1^{\text{dev}} < d_{\mathbf{ZP}}$ leads to $u_1^{\text{dev}} < u_1$ and hence to $D_1^{\text{dev}} = 0$, which cannot be profitable. On the other hand, $d_1^{\text{dev}} > d_1$ leads to $u_1 < u_1^{\text{dev}} = \bar{u}_{d_1^{\text{dev}}}$ and hence to $D_1^{\text{dev}} = S(\bar{u}_{d_1^{\text{dev}}})$. But since $\pi(d_1^{\text{dev}}) < 0$, deviation profits are negative.

(2) No other equilibria $\mathcal{E} = (d_1)$ where $d_1 \neq d_{\mathbf{ZP}}$ exist. If $d_1 < d_{\mathbf{ZP}}$, then deviation to slightly higher interest d_1^{dev} ($d_1 < d_1^{\text{dev}} < d_{\mathbf{ZP}}$) leads to $D_1^{\text{dev}} = S(\bar{u}_{d_1^{\text{dev}}})$. By continuity, losses in profits per unit of deposits can be offset by the collection of all savings if $(d_1^{\text{dev}} - d_1)$ is small enough. If $d_1 > d_{\mathbf{ZP}}$, then $\pi(d_1) < 0$, which cannot be the case in equilibrium.

□

Proof of proposition 8 .

Consider a deviation d_1^{dev} from a symmetric equilibrium $\mathbf{d_1} = (d_1, ..., d_1)$ and corresponding assessments (u_1^{dev}, u_1). We make the following preliminary remarks:

(1) From propositions 3 and 4 we know that only the following two assessments and deposit constellations are possible:

$$\begin{array}{llll} (A1) & u_1^{\text{dev}} < u_1 & D_1^{\text{dev}} = 0 & D_1 = S_1(\bar{u}_{d_1}) \\ \text{(A2)} & u_1^{\text{dev}} > u_1 & D_1^{\text{dev}} = S_1(\bar{u}_{d_1^{\text{dev}}}) & D_1 = 0. \end{array}$$

Furthermore, assessment A1 is always consistent if $d_1^{\text{dev}} < d_1$ or $\bar{u}_{d_1^{\text{dev}}} \leq \bar{u}_{d_1}$ and assessment A2 is always consistent if $d_1^{\text{dev}} > d_1$ or $\bar{u}_{d_1^{\text{dev}}} \geq \bar{u}_{d_1}$. This follows directly from proposition 3 and the fact that (say for A1) $p_h d_1^{\text{dev}} < \bar{u}_{d_1^{\text{dev}}} \leq \bar{u}_{d_1}$.

(2) We can exclude equilibria $\mathcal{E} = (d_1)$ where $d_1 > d_{\mathbf{ZP}}$ because they are negative expected profits equilibria.

(3) We do not have to consider deviations to $d_1^{\text{dev}} \geq d_{\mathbf{ZP}}$ since they would lead to zero profits in the case of A1 and to zero or negative profits in the case of A2 and hence cannot be profitable.

Now we turn to the proof of the proposition.

(i) Suppose that $d_1 \in \mathcal{U}_{\max}$. Deviation to $d_1^{\text{dev}} \neq d_1$ cannot be profitable, since by TR depositors would always choose to deposit with the non-deviating banks because A1 is consistent and $\bar{u}_{d_1} \geq \bar{u}_{d_1^{\text{dev}}}$.

(ii) We define $\tilde{d} =: \min\{d_{\mathbf{UH}}, d_{\mathbf{ZP}}\}$. From statement (i) we know that $\mathcal{E}_{UH}$ is a Nash equilibrium because $\mathcal{E}_{UH} = (\tilde{d})$ with $\tilde{d} \in \mathcal{U}_{\max}$. No other equilibrium with $d_1 \neq \tilde{d}$ exists, since for $d_1 < \tilde{d}$ deviation to a slightly higher deposit rate $d_1^{\text{dev}} > d_1$ leads to A2 (because $\bar{u}_{d_1^{\text{dev}}} > \bar{u}_{d_1}$) and hence such a deviation is always profitable if $(d_1^{\text{dev}} - d_1)$ is small enough. The same argument applies for $d_1 > \tilde{d}$ if $\tilde{d} = d_{\mathbf{UH}}$. In this case, deviation to slightly lower deposit rates is profitable. Finally, equilibria with $d_1 > \tilde{d}$ for $\tilde{d} = d_{\mathbf{ZP}}$ have already been excluded.

(iii) Case (1) follows from the same arguments as used under (ii). Consider now case (2). $\mathcal{E}_{ZP}$ is a Nash equilibrium according to statement (i). Now turn to the question whether $\mathcal{E}_L$ is an equilibrium. Obviously, only deviations to $d_1^{\text{dev}} > d_{\mathbf{L}}$ with $\bar{u}_{d_1^{\text{dev}}} > d_{\mathbf{L}}$ can be profitable. Hence deviation is profitable if and only if $\Pi^{\text{dev}}(d_{\mathbf{L}}) - \pi(d_{\mathbf{L}})S_1(d_{\mathbf{L}})/n > 0$. Moreover, no other equilibria $\mathcal{E} = (d_1)$ can exist because for $d_1 < d_{\mathbf{UL}}$, deviation to slightly lower, and for $d_1 \geq d_{\mathbf{UL}}$ deviation to slightly higher deposit rates is profitable by the same arguments as under (ii).

□

Proof of proposition 9.

Suppose that $u_1(\tilde{\mathcal{E}}_{ZP}) \geq u_1(\mathcal{E}_{ZP})$. Then $\tilde{d}_{\mathbf{ZP}} > d_{\mathbf{ZP}}$ (see table 2) and hence $\tilde{d}_2^*(\tilde{d}_{\mathbf{ZP}}) > d_2^*(d_{\mathbf{ZP}})$. But since under $\tilde{\mathcal{E}}_{ZP}$ banks' profits per deposit are given by

$$p_h\Big(r_{2h} - \tilde{d}_2^*(\tilde{d}_{\mathbf{ZP}})\Big),$$

this would imply that those profits are smaller than $\pi(d_{\mathbf{ZP}}) = 0$, which is impossible in equilibrium. To prove that $u_2(\tilde{\mathcal{E}}_{ZP}) < u_2(\mathcal{E}_{ZP})$, we draw on the fact that $\tilde{d}_2^*(\tilde{d}_{\mathbf{ZP}}) < d_2^*(d_{\mathbf{ZP}})$.[43] This implies that

$$\begin{aligned} u_2(\tilde{\mathcal{E}}_{ZP}) &= p_l\underline{u} + p_h\tilde{d}_2^*(\tilde{d}_{\mathbf{ZP}}) \\ &\leq p_l\underline{u} + p_h d_2^*(d_{\mathbf{ZP}}) \\ &< u_2(\mathcal{E}_{ZP}). \end{aligned}$$

□

[43]Suppose that $\tilde{d}_2^*(\tilde{d}_{\mathbf{ZP}}) \geq d_2^*(d_{\mathbf{ZP}})$ then $\tilde{d}_{\mathbf{ZP}} \geq d_{\mathbf{ZP}}$ implying that $\pi(\tilde{d}_{\mathbf{ZP}}) < 0$.

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