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Abstract

This paper presents a model of two competing local telecommunications networks which are mandated to interconnect. After negotiating the access charges, the companies engage in price competition. Given the prices, each consumer selects a network and determines the consumption of phone calls. Using a discrete/continous consumer choice model, it is shown that a pure strategy equilibrium exists quite generally and satisfies desirable properties. This equilibrium can be implemented by a simple rule that sets the access charges at a common discount from the retail prices. It requires no information and the discount factor is chosen by the companies through negotiations. Finally, if the networks are highly substitute, the retail prices obtained by imposing this rule will approximate the efficient prices.

Keywords: Access charges; Interconnection; Network competition; Telecommunications.

JEL Classification: D43, L51, L41, L43, L96, K21.

Introduction

In the early eighties, the telecommunications industry experienced the first major structural change with the emergence of competitive long distance providers. Interconnection of these competitive providers to the bottleneck local networks proved to be a challenging question for both academics and practitioners. This is a one-way interconnection since only one of the interacting parties need to connect to the network of the other party. The one-way interconnection problem has been analyzed in Laffont and Tirole (1994, 1996a) and the references therein.

The introduction of fiber optic lines, developments in the mobile telephony, convergence of the cable and telephone networking technologies, and finally the tremendous growth of the Internet made the definition of natural boundaries between various services obsolete. As a result, there have been policy changes in several countries to open the telecommunications market to full blown competition and gradually phase out regulations. The operators which provide similar services need to connect with each other, therefore the type of interconnection required is two-way. It is this type of interaction that will be analyzed in this paper.

New Zealand, serving as an experimental ground for the rest of the world, deregulated the telecommunications industry in April 1989. As Armstrong, Doyle and Vickers (1996) reports, interconnection negotiations proved to be a rather lengthy and complicated process only to be resolved by the intervention of the Privy Council in London, which suggested the use of the Efficient Component Pricing Rule of Baumol and Sidak (1994). During the same period, the UK and the US experienced a rapid growth in the number of competitive access providers, which led the governments to prepare the legislation for a competitive telecommunications industry. Most of the European Union countries removed the legal entry barriers and liberalized their telecommunications markets in January 1^{st}, 1998.

This paper studies a model of competition and interconnection in the telcommunications industry as suggested by the US legislation. The Telecommunications Act of 1996, sets the ground rules for competition in the US. The most discussed and controversial part of this law deals with the interconnection issues. The law states that interconnection is mandatory and the access prices should be cost-based, just, reasonable and nondiscriminatory. Reciprocal arrangements for interconnection prices are suggested. The interacting parties shall negotiate an access charge mechanism and upon the approval of regulatory bodies, they shall sign binding agreements. The law delegates any further policy suggestions to the FCC and the State Public Utility Commissions.(See Telecommunications Act of 1996 for details.) This type of procedures are being employed in several of the countries mentioned above. In August 1996, the FCC issued a document (composed of almost one thousand pages), that established the rules and guidelines for interconnection between competing telecommunications providers. As the size of this document suggests, interconnection policy will be the centerpiece of (de)regulation in this competitive era.

There are a number of papers that study the competition between telecommunications networks and the role of government agencies which facilitate their interaction. **?**) (hereafter LRT) have analyzed a model of two local network companies that possess different attributes for consumers.[1] In their model, the two companies, given access charges, set the local prices

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[1] **?**) build on LRT and examine the effects of brand loyalty.

competitively. The customers of a network are charged the same price independently of which network completes their calls. Each network pays access charges for terminating calls on the competitor network. The competition is only in prices and the other attributes are assumed to be fixed. Given the retail prices, consumers whose tastes are uniformly distributed along the unit interval select their preferred networks à la Hotelling.

It is shown in LRT that if the substitutability of the networks is not too small then the equilibrium with reciprocal access charges exists only when access charges are sufficiently close to the marginal cost of providing access. This result suggests that they should be set close to marginal cost prices, leaving very little room for access charge negotiations. In practice, the regulatory authorities require reciprocal access charges and allow for such negotiation with certain guidelines.

Our paper offers a different model of consumer choice within an industry model similar to that of LRT which quite generally results in equilibrium. Subscribers of each network have identical linear demand functions for calls. This provides a satiation point, which is natural for this type of services. This is different from the LRT model where demands are isoelastic and have no satiation point.

Instead of the Hotelling framework where consumers' tastes for networks are uniformly distributed, we employ a product differentiation model which has found wide applications in econometric studies. The model assumes that the surplus of a consumer is affected by a random taste variable which is double exponential distributed. Therefore, the choice between the two alternatives depends on the differences between these taste parameters as well as the consumer surplus. The literature offers several axiomatic derivations of the consumer choice model with double exponential distribution. For a survey see **?**).

We establish the existence of equilibrium for a large set of parameter values, even if, the access charges are well above marginal cost and/or the networks are close substitutes, provided that the fixed costs are covered. The price competition results in a market equilibrium as long as the access charges do not exceed the retail price which is sufficient to induce zero demand for calls. The equilibrium prices have intuitive properties. They decrease as the differentiation between the networks decreases, and they increase with the level of access charge.

Next it is shown that even if the regulator is not sufficiently informed, the implementation problem can be resolved by imposing a very simple rule which determines the access charges as a function of market prices. The rule determines the access charge of a network at a discount relative to its own retail price, i.e. the access charges are proportional to the retail prices. The networks negotiate a common discount that will be used to compute access charges. We call this rule the Reciprocal Proportional Access Charge Rule (the RPACR hereafter). The RPACR does not require any information about the parameters of the industry and therefore can easily be implemented. It is shown that when the proportion coefficient does not exceed one—namely, when access charges cannot exceed the retail prices—the RPACR coupled with our consumer choice model guarantees the existence of a symmetric equilibrium in retail prices.

Since the existence of equilibrium is obtained for all discount factors between zero and one, the outcome of the first stage negotiation between the networks becomes relevant. First we examine the case where the networks choose an access charge that maximizes their joint profits subject to RPACR and derive the subgame perfect equilibrium prices. We find that, if the substitutability between the networks is not too small,[2] they will select a discount factor equal to one, therefore the retail prices and access charges will coincide. The equilibrium prices are decreasing with the magnitude of substitutability, leading to low prices as the services of networks become similar. This result suggests that with a very mild regulatory rule low retail prices can be achieved and the implementation requires no information.

On the other hand, for low substitutability, the networks are able to sustain monopoly prices by an appropriate choice of the access discount. Similar results are also obtained in LRT and Armstrong (1998). In LRT, it is shown that access charge rules such as ECPR may allow the companies to sustain the monopoly prices in equilibrium. Armstrong (1998) developed a similar model to show how access charge agreements may lead to collusion between the firms.

Finally we calculate the access discounts that maximize social surplus. The regulator chooses the welfare maximizing access discount that will induce equilibrium and will allow the firms to cover their costs. In the presence of fixed costs, the financial viability requirement for the networks may suggest a markup above marginal costs especially when the substitutability is high. The determination of this markup requires information about both cost and demand parameters, therefore it is quite difficult to implement without introducing regulatory distortions. Under RPACR, the retail prices will approximate the efficient price levels for low differentiation between the services of the networks. Therefore by allowing the firms to negotiate access

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[2] This seems to be the case in telecommunications industry.

discounts subject to RPACR, socially desirable prices are obtained and the regulatory intervention is minimized.

The paper is organized as follows. Section 1 introduces the general model of the industry. Section 2 develops the model for consumer choice and derives their demands. Section 3 analyzes the price competition between the two network companies, derives the existence result and elaborate on the negotiation stage. Concluding remarks appear in Section 4.

1. The Industry Model

We deal with a simple case of two network companies which provide only local telecommunications services. We refer to them as Network 1 and Network 2. The networks incur zero marginal cost for each call. There is a fixed cost associated with building and operating a network. Operating and maintenance costs are assumed to be independent of the amount of service provided. Usually, operating and maintenance costs do depend on the number of customers of a network, but to simplify the analysis, we assume that these costs are included in the fixed cost component.[3] The fixed cost of each network is denoted by F. As we deal with two symmetric firms, our analysis better fits situations in the mature phase of competition in the industry.

Each company faces two demand functions. The demand function X_{11}, for calls initiated and completed at Network 1, and the demand function X_{12}, for the calls initiated at Network 1 and completed at Network 2. The demand functions for Network 2, X_{22} and X_{21}, are defined similarly. The

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[3] Computer simulations suggest that our results will remain true without this assumption, at least when these costs are not too large relative to the fixed cost. However the methods we use to prove our result does not apply to this case.

demand functions X_{ij} will be derived in the next section.

The Telecommunications Act of 1996 requires companies to negotiate and then to sign binding agreements concerning their access charges, before they engage in competition for local prices, services, etc. Accordingly, our model consists of two stages. The first is a cooperative stage where the two network companies negotiate access charges and sign a binding agreement. The second one is a competitive stage where they are engaged in price competition which determines the retail prices, p_k, for $k = 1, 2$. The retail prices p_k are the per unit charge of company k to each of its customers whether their calls are completed locally or in the other network. To be more general, we allow the companies to negotiate in the first stage access charge rules $a_k(p_1, p_2)$ $(k = 1, 2)$ which determine the access charges as a function of the second stage retail prices. Therefore, any change in local prices will have an immediate impact on the access prices. We focus on two rules in the sequel. The trivial rule $a_k(p_1, p_2) = a_k$ which generates fixed access charges and the proportional access charge rule $a_k(p_1, p_2) = a_k p_k$ leading to access charges which are proportional to the future determined retail prices.

The profit functions of the two networks are given by

$$\Pi_1 = p_1 X_{11} + (p_1 - a_2(p_1, p_2))X_{12} + a_1(p_1, p_2)X_{21} - F, \tag{1}$$

$$\Pi_2 = p_2 X_{22} + (p_2 - a_1(p_1, p_2))X_{21} + a_2(p_1, p_2)X_{12} - F. \tag{2}$$

The following is the sequence of events:

Stage 1. The network companies select an access charge rule by mutual consent.

Stage 2. The two companies choose their prices p_1 and p_2 simultaneously and independently, and announce them publicly.

Stage 3. After observing the prices p_1 and p_2, every consumer selects a network to subscribe to and chooses the amount of calls.

2. Consumer Demand For Local Telecommunications Services

In this section, we will specify the consumers' demands and model the process by which they select their networks. Suppose that there are N potential consumers. Each consumer is assumed to have idiosyncratic tastes which depend on the various attributes of the networks (like specific services offered by the companies, the intensity of advertising, accounting methods, etc.). This allows us to explain the existence of several networks with similar products but different prices. Consumer i who subscribes to network k has the following demand function

$$x_{ik} = \frac{1}{2w}(r - p_k), \tag{3}$$

where $r > 0$ and $w > 0$. This implies a bounded amount of calls demanded at prices close to zero and therefore provides a satiation point, which is natural for such services (there is a limit to the time one can spend on the phone).

The valuation of consumer i conditional on his subscription choice to network k is his consumer surplus which is given by

$$V_{ik} = \frac{1}{4w}(r - p_k)^2. \tag{4}$$

The subscription choice is affected by a random taste parameter known to the consumer but unobserved by the firms. The unconditional valuation of consumer i of network k is given by

$$\bar{V}_{ik} = V_{ik} e^{\sigma \epsilon_{ik}},$$

where the term ϵ_{ik} is random and measures the idiosyncratic tastes of consumer i for network k.[4]

Assumption 1. *The random variables, ϵ_{ik}, are drawn from the double exponential distribution, statistically independent for all i and k, and their realizations are private information of the consumers.*

Axiomatic justifications of the double exponential distribution for the discrete consumer choice model appear in **?**). From the companies point of view, the independence assumption implies that the individuals are symmetric in the way they value their services. Therefore, the companies can view the consumers' subscription decisions as a random rule, symmetric across individuals. Random choice models of this kind have been extensively employed in the literature starting with **?**); see also **?**) for a variety of examples.

Consumer i prefers Network k to Network $\bar{k}$ if and only if

$$V_{ik}e^{\sigma\epsilon_{ik}} \geq V_{i\bar{k}}e^{\sigma\epsilon_{i\bar{k}}}.$$

The probability that the last inequality holds is denoted by P_{ik}. Therefore, the companies assign probability P_{ik} that i will choose Network k over $\bar{k}$, where

$$P_{ik} = Probability(V_{ik}e^{\sigma\epsilon_{ik}} \geq V_{i\bar{k}}e^{\sigma\epsilon_{i\bar{k}}}). \tag{5}$$

Since the ϵ_{ik}'s are independent and double exponential distributed, this

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[4] The term σ is a measure of the dispersion of tastes,that is, σ measures the substitutability between the services of the two networks. As $\sigma \rightarrow 0$ the networks become perfect substitutes, while as σ increases indefinitely, the effect of prices on the consumers' decision diminishes.

probability is given by

$$P_{ik} = \frac{1}{1 + (\frac{V_{i\bar{k}}}{V_{ik}})^{\frac{1}{\sigma}}}. \tag{6}$$

For the derivation of (**??**) see **?**). By Assumption 1, we can drop the index i from P_{ik} and write P_k.

Let $m(p_1, p_2)$ be the ex-ante market share of Network 1. Clearly $P_1 = m(p_1, p_2)$ and by (**??**) and (**??**),

$$m(p_1, p_2) = \frac{(r - p_1)^\tau}{(r - p_1)^\tau + (r - p_2)^\tau}, \tag{7}$$

where $\tau = \frac{2}{\sigma}$. The market share of Network 2 is $P_2 = 1 - m(p_1, p_2)$. Observe by (**??**) and (**??**) that the aggregate subscriber demand faced by Network k is given by,

$$X_k = X_k(p_1, p_2) = \frac{N}{2w}(r - p_k)P_k, \qquad k = 1, 2, \tag{8}$$

where $P_1 = m(p_1, p_2)$ and $P_2 = 1 - m(p_1, p_2)$.

Assumption 2. *The fraction of calls from one network to the other is proportional to market shares.*

This assumption is reasonable especially for residential telephone services (it is also made in LRT). There may be other instances in which this assumption may not be very plausible. For example, telemarketing companies will have mostly outgoing calls, while credit card companies will mostly receive calls.

Assumption 2 allows us to compute the expected number of calls initiated in Network k, and completed in Network j where $k, j \in \{1, 2\}$. The fraction of calls made by customers of Network k to customers of Network j

is $m(p_1,p_2)$ if $j=1$ and $1-m(p_1,p_2)$ if $j=2$. Therefore by (**??**) and (**??**), we conclude that,

$$\begin{aligned} X_{11} &= \frac{N}{2w}(r-p_1)(m(p_1,p_2))^2, && (9)\\ X_{12} &= \frac{N}{2w}(r-p_1)m(p_1,p_2)(1-m(p_1,p_2)), && (10)\\ X_{22} &= \frac{N}{2w}(r-p_2)(1-m(p_1,p_2))^2, && (11)\\ X_{21} &= \frac{N}{2w}(r-p_2)m(p_1,p_2)(1-m(p_1,p_2)). && (12) \end{aligned}$$

We use these demand functions in the next section to analyze the competition between the two companies.

3. Network Competition

3.1. Second Stage Price Game with Reciprocal Constant Access Charges

In this section, we analyze the price competition when reciprocal constant access charges, $a_k(p_1,p_2)=\tilde{a}$, are used. That is, the competition following the networks' negotiation and agreement about the value of the access charge $\tilde{a}$ is considered. By (**??**), (**??**) and (**??**)-(**??**), the profit functions are

$$\begin{aligned} \Pi_1(p_1,p_2) &= \frac{N}{2w}\Big\{p_1(r-p_1)m(p_1,p_2)^2 \\ &\quad +(p_1-\tilde{a})(r-p_1)m(p_1,p_2)(1-m(p_1,p_2)) && (13)\\ &\quad +\tilde{a}(r-p_2)m(p_1,p_2)(1-m(p_1,p_2))\Big\}-F, \\ \Pi_2(p_1,p_2) &= \frac{N}{2w}\Big\{p_2(r-p_2)(1-m(p_1,p_2))^2 \\ &\quad +(p_2-\tilde{a})(r-p_2)m(p_1,p_2)(1-m(p_1,p_2)) && (14)\\ &\quad +\tilde{a}(r-p_1)m(p_1,p_2)(1-m(p_1,p_2))\Big\}-F. \end{aligned}$$

By simple algebraic manipulations (**??**) and (**??**) can be reduced to

$$\begin{aligned}\Pi_1(p_1,p_2) &= \frac{N}{2w}\Big\{p_1(r-p_1)m(p_1,p_2) \\ &\quad +\tilde{a}(p_1-p_2)m(p_1,p_2)(1-m(p_1,p_2))\Big\} - F, \qquad (15)\\ \Pi_2(p_1,p_2) &= \frac{N}{2w}\Big\{p_2(r-p_2)(1-m(p_1,p_2)) \\ &\quad +\tilde{a}(p_2-p_1)m(p_1,p_2)(1-m(p_1,p_2))\Big\} - F. \qquad (16)\end{aligned}$$

Let $\mathcal{A} = \{(\tau, r, w, F, N) | (\tau, r, w, F) \in \mathbf{R}_{++} \text{ and } N \in Z_+\}$.

Proposition 1. *For every $(\tau, r, w, F, N) \in \mathcal{A}$ and $0 \le \tilde{a} \le r$, there exists a unique symmetric pure strategy equilibrium, $p^* = p_1^* = p_2^*$, where*

$$p^* = \frac{2r+\tilde{a}}{\tau+4},$$

provided that the net profit of each company,

$$\Pi_i^* = \frac{N}{4w}\frac{(2r+\tilde{a})(r(\tau+2)-\tilde{a})}{(\tau+4)^2} - F,$$

is nonnegative.

Proof. The proof appears in the Appendix. It is not standard, since the profit functions are not quasi-concave.

The requirement that fixed costs are covered in equilibrium imposes some restrictions on the substitutability of the networks. If the substitutability is too large the fixed costs will not be covered. Given that the profits are nonnegative, a pure strategy equilibrium in retail prices exists whenever $0 \le \tilde{a} \le r$. The condition $\tilde{a} \le r$ is not too restrictive, since for market prices higher than r the demand for calls drops to zero.

The structure of the games in the this paper and in LRT are similar. Yet, the results are different, especially in regard to the existence of equilibrium.

The arguments for the existence in LRT apply mainly for low substitutability between the networks and for low access charge fees, without specifying how low it should be. Note that if we set $F = 0$, we will obtain an equilibrium as long as $0 \leq \tilde{a} \leq r$. However, this is not the case in LRT, since for high substitutability, the equilibrium does not exist even in the absence of fixed costs.

Computer simulations suggest that, if in the Hotelling framework of LRT one replaces the deterministic part which generates constant price elasticity of demand by our linear demands, equilibrium will exist under the same conditions it exists in our model. Unfortunately we did not succeed to prove this claim. The profit functions are not quasi-concave. Moreover, it is not possible to solve the first order necessary conditions analytically for a symmetric equilibrium, and thus one cannot even tell whether they are quasi-concave at the solution.

For the double exponential distribution, employed in this paper, it is possible to find the solution of the first order conditions explicitly and we have shown in the appendix that the profit functions are quasi-concave at this solution (they are not in general). It seems to us that the main source for the difference between the two models is their deterministic specification of the preferences of the consumers.

The equilibrium prices of our model have intuitive properties. First, they are decreasing in the substitutability, τ. The higher the substitutability τ, the stronger is the competition between the networks, and, therefore, the lower are the prices. Second, the prices are increasing in the demand intensity, r. Finally, they are increasing in the access charges, $\tilde{a}$. Viewing the access charges as the marginal cost of a call directed to the other network, this is an expected outcome. On the other hand for every value of $\tilde{a}$ such that $\tilde{a} \leq r$ the retail price approaches zero (which is the marginal cost in our model), as the substitutability parameter τ increases indefinitely.

3.2. Second Stage Price Game with the RPACR

A regulator ideally could force the outcome of the first stage negotiation to be in $[0, r]$ to induce market equilibrium. However, imposing such a restriction requires information about the parameter r and raises an implementation problem. We resolve this problem by means of a simple rule. We treat each network as a regular customer who only buys partial service (just completion of a call), and therefore, the access charges will be only a proportion of the full service price. Formally, consider the rule $a_k(p_1, p_2) = a_k p_k$, where $0 \leq a_k \leq 1$, for $k = 1, 2$. If $a_1 = a_2 = a$, then the access charge rules will be reciprocal. We call these rules the Reciprocal Proportional Access Charge Rules (RPACR). They essentially restrict the access charges not to exceed the retail prices and as the retail prices change, the access charges change proportionally. The RPACR should be viewed as a rule which is imposed by the regulatory agencies. The networks choose the parameter a, the proportion coefficient, in the first stage by mutual consent.

By (**??**), (**??**), (**??**)-(**??**), under the RPACR the profit functions are

$$\begin{aligned}
\Pi_1^R(p_1, p_2) &= \frac{N}{2w}\Big\{p_1(r - p_1)m(p_1, p_2)^2 \\
&\quad + (p_1 - ap_2)(r - p_1)m(p_1, p_2)(1 - m(p_1, p_2)) \qquad (17) \\
&\quad + ap_1(r - p_2)m(p_1, p_2)(1 - m(p_1, p_2))\Big\} - F, \\
\Pi_2^R(p_1, p_2) &= \frac{N}{2w}\Big\{p_2(r - p_2)(1 - m(p_1, p_2))^2 \\
&\quad + (p_2 - ap_1)(r - p_2)m(p_1, p_2)(1 - m(p_1, p_2)) \qquad (18)
\end{aligned}$$

$$+ap_2(r-p_1)m(p_1,p_2)(1-m(p_1,p_2))\Big\} - F.$$

By rearranging the terms in (**??**) and (**??**), these profit functions reduce to

$$\begin{aligned}
\Pi_1^R(p_1,p_2) &= \frac{N}{2w}\Big\{p_1(r-p_1)m(p_1,p_2) \\
&\quad +ar(p_1-p_2)m(p_1,p_2)(1-m(p_1,p_2))\Big\} - F, \qquad (19)\\
\Pi_2^R(p_1,p_2) &= \frac{N}{2w}\Big\{p_2(r-p_2)(1-m(p_1,p_2)) \\
&\quad +ar(p_2-p_1)m(p_1,p_2)(1-m(p_1,p_2))\Big\} - F. \qquad (20)
\end{aligned}$$

Observe the similarity between these profit functions and the profit functions Π_1 and Π_2 given in (**??**)-(**??**). For $\tilde{a} = ar$, both coincide. Therefore, the use of RPACR for $a \leq 1$ yields the same symmetric pure strategy equilibrium as with the constant reciprocal access charges $\tilde{a}$ provided that $\tilde{a} \leq r$. But now the information about the parameters of the demand functions is not used.

Proposition 2. *Suppose that the companies are restricted to RPACR. Then, for every $(\tau, r, w, F, N) \in \mathcal{A}$ and for every a, $0 \leq a \leq 1$, there exists a unique symmetric pure strategy equilibrium $p^* = p_1^* = p_2^*$, where*

$$p^* = \frac{(2+a)r}{\tau+4},$$

provided that the net profit of each company,

$$\Pi_i^* = \frac{Nr^2}{4w}\frac{(2+a)(\tau+2-a)}{(\tau+4)^2} - F,$$

is nonnegative.

The strategic equivalence of the games encountered with these two different access charge rules is a mere mathematical coincidence. The main reason behind this phenomenon is the linear demand functions. Although the RPACR maybe a useful regulatory device for some other models of consumer choice, in general, it might be necessary to employ different access charge rules for different demand systems to resolve the implementation problem. The RPACR demonstrates this point for the specific consumer choice model considered in this paper.

3.3. First Stage Negotiation of Access Charges

In this section, we focus on the first stage negotiation when RPACR is used as the access charge rule. The value of a is determined in the first stage by mutual consent. The bargaining between the two parties may result in essentially any number in $[0, 1]$ with the restriction that the revenue of a company covers the fixed cost. Let us first examine the case where the companies set a to maximize total profits. By Proposition 2, the equilibrium total profit level is

$$\Pi_T^R = \Pi_1^R + \Pi_2^R = \frac{Nr^2}{2w}\frac{(\tau+2-a)(2+a)}{(\tau+4)^2} - 2F, \qquad 0 \le a \le 1. \tag{21}$$

This function is concave in a and it attains its unconstrained maximum at $a = \frac{\tau}{2}$. To guarantee that $a \le 1$, the companies should select $a = min(\frac{\tau}{2}, 1)$ provided that they cover fixed costs. Hence

Proposition 3. *If a maximizes joint profits then*

$$a = \begin{cases} 1 & \tau \ge 2, \\ \frac{\tau}{2} & \tau \le 2, \end{cases} \tag{22}$$

and

$$p^* = \begin{cases} \frac{3r}{\tau+4} & \tau \ge 2, \\ \frac{r}{2} & \tau \le 2. \end{cases} \tag{23}$$

Consequently, when $\tau \geq 2$ then $a = 1$ and the access fee coincides with the retail price. That is, when the substitutability is relatively large, consumers will be charged a small price (reflecting strong competition) and each company will be treated as any other customer by the other company. If $\tau \leq 2$ (reflecting low substitutability), the equilibrium retail price is $\frac{r}{2}$, which is the monopoly price. The access charge in this case is $\frac{\tau p^*}{2}$ and this is smaller than the retail price.

Armstrong (1998) shows that the use of ECPR type of rules may lead to collusion between the companies if the services are sufficiently differentiated. Our results support his findings, i.e. for relatively low levels of substitutability, the access charges facilitate collusion between the firms and lead to high retail prices. Therefore, it is in the interest of a regulator to tighten the upper bound of the allowable access charges thereby forcing the retail prices to lower levels. This certainly still induces an equilibrium, however requires information about the companies costs as well as the consumer preferences, which may be quite costly.

Let us now consider the case where the regulator sets a to maximize total social surplus, $SS(a)$, subject to the constraints that the companies cover fixed costs and equilibrium is guaranteed to exist. The social surplus is the sum of the total industry profits and the total consumer surplus, $CS(a)$, where

$$CS(a) = \frac{N}{4w}(r - p^*)^2,$$

and p^* which is a function of a is given in Proposition 2. It is easy to verify that in equilibrium

$$SS(a) = \frac{Nr^2}{4w(\tau+4)^2}(\tau + 2 - a)(\tau + 6 + a) - 2F.$$

This is a quadratic function in a and it decreases for $-2 \leq a \leq 1$. Hence it is maximized for $a = -2$. But when $a = -2$ by (**??**), the total operating industry revenue is zero and the fixed costs are not covered at all. Thus the optimal a that induces a second stage equilibrium is the lowest number a, $0 \leq a \leq 1$, for which the total profit(net of fixed costs) is nonnegative. By (**??**), this is the lowest a such that $0 \leq a \leq 1$ and

$$(\tau + 2 - a)(2 + a)r^2 \geq 4\frac{F}{N}\frac{w}{r^2}(\tau + 4)^2.$$

The smallest solution of this inequality is

$$a^* = \max\left[\frac{(1-\kappa)\tau - 4\kappa}{2}, 0\right] \tag{24}$$

where

$$\kappa = \sqrt{1 - 16\frac{F}{N}\frac{w}{r^2}}.$$

If $\frac{F}{N} > \frac{r^2}{16w}$, the fixed costs will not be covered irrespectively of the value of a. In this case there is room for at most one network. If $\frac{(1-\kappa)\tau-4\kappa}{2} > 1$ then the fixed costs are covered for no $a \in [0, 1]$. Since our existence result apply only for $a \in [0, 1]$, we cannot add much to the analysis of this case. Assume therefore that $\frac{(1-\kappa)\tau-4\kappa}{2} \leq 1$ (which is likely to be the case if $\frac{Fw}{Nr^2}$ is relatively small).

Note that the difference between the total surplus for $a = 0$ and for $a = 1$ is

$$SS(0) - SS(1) = \frac{5}{4w}\frac{Nr^2}{(t + 4)^2}.$$

This declines sharply with τ and hence the difference in social surplus, for a^*, the constrained maximizer of SS(a), and $a = 1$, the maximizer of joint profits is small for large τ. Considering the informational requirements of setting the social surplus maximizing access discount, RPACR appears to be a rather simple rule that approximately achieves the regulatory goals with minimal intervention.

In this paper the attributes of the networks (other than the prices) are exogenously given. These attributes determine the value of σ and therefore τ. One could add another stage to the game, for instance before a is negotiated. In this stage the companies compete in attributes and their selections determine the value of τ. Then Proposition 3 determines the equilibrium prices in terms of a and these attributes. The equilibrium choice of attributes will be a function of their costs. If the costs of different attributes are quite similar and a is selected to maximize joint profits, then the companies will choose their attributes so that τ will be sufficiently small to guarantee monopoly local prices (See Proposition 3). This will eliminate the effectiveness of the price competition and will justify government intervention.

4. Conclusion

We have analyzed a model of competition between two network companies. The main result is that an equilibrium in retail prices always exist as long as the access charges do not exceed the prices which induce zero demand for calls. Since imposing such a restriction requires information about the consumers' demands, we offer a simple rule, RPACR, which induces an equilibrium irrespectively of the values of the parameters of our model.

The equilibrium prices exhibit desirable properties when the services of the networks are close substitutes—a reasonable assumption. In fact, for sufficiently high substitutability between the networks RPACR leads to retail prices which approximately maximizes social surplus subject to non-negativity constraints on the networks profits. The imposition of RPACR can be viewed as a mild regulatory policy. Since RPACR is responsive to future changes of the determinants of the market, it provides flexibility for the companies to react to changes in the environment and in strategies.

For low differentiation between the networks, the equilibrium retail prices are low, provided that both companies cover their fixed costs. If the industry is not too competitive—especially, if the companies are able to differentiate themselves considerably—then RPACR may not be desirable since it allows the companies to sustain monopoly prices.

Appendix

Proof of Proposition 1. Since the profit functions (**??**) and (**??**) are not globally concave or quasiconcave, the standard approach for the existence proof cannot be used. We show that the symmetric solution of the first order conditions are best responses to each other, and therefore constitute a Nash equilibrium of the game. This amounts to proving that, given that Network 2 chooses the equilibrium price level, it is the unique best response of Network 1 to choose the equilibrium price.

Let $\alpha = r - p_1$ and $\beta = r - p_2$. Also assume $\tilde{a} = ar$, therefore the restriction $\tilde{a} \in [0, r]$ is equivalent to $a \in [0, 1]$. Then the profit functions of the two companies are

$$\Pi_1(\alpha, \beta) = \frac{N}{2w} \frac{\alpha^\tau}{(\alpha^\tau + \beta^\tau)} \left[\alpha(r - \alpha) + ar(\beta - \alpha) \frac{\beta^\tau}{(\alpha^\tau + \beta^\tau)} \right] - F,$$

and

$$\Pi_2(\alpha, \beta) = \frac{N}{2w} \frac{\beta^\tau}{(\alpha^\tau + \beta^\tau)} \left[\beta(r - \beta) + ar(\alpha - \beta) \frac{\alpha^\tau}{(\alpha^\tau + \beta^\tau)} \right] - F.$$

Let

$$p_1^* = p_2^* = \frac{(2 + a)r}{\tau + 4}.$$

Then

$$\alpha^* = \frac{(\tau + 2 - a)r}{\tau + 4} = \beta^*.$$

We have to prove that $\Pi_1(\alpha, \beta^*)$ is maximized for $\alpha = \alpha^*$ and that $\Pi_2(\alpha^*, \beta)$ is maximized for $\beta = \beta^*$. By symmetry, it is enough to show this for $\Pi_1(\alpha, \beta^*)$. Since $\Pi_1(\alpha, \beta)$ is continuous in the parameter τ, it is sufficient to prove our claim for positive rational values of τ. Let $\tau = \frac{m}{n}$, where $m > 0$, $n > 0$ are integers. Let

$$t \equiv \alpha^{\frac{1}{n}} \quad \textit{and} \quad s \equiv \beta^{\frac{1}{n}}.$$

Then $t^* = (\alpha^*)^{\frac{1}{n}} = s^*$, where

$$s^* = (\alpha^*)^{\frac{1}{n}} = \left(\frac{m + 2n - an}{m + 4n} r\right)^{\frac{1}{n}}, \tag{25}$$

and

$$\Pi_1(t, s) \equiv \frac{N}{2w} \frac{t^m}{(t^m + s^m)} \left[t^n (r - t^n) + ar(s^n - t^n) \frac{s^m}{(t^m + s^m)} \right] - F.$$

Since t^n is an increasing function of t, it is sufficient to prove that $\Pi_1(t, s^*)$ is maximized for $t^* = (\alpha^*)^{\frac{1}{n}}$ when $s^* = (\beta^*)^{\frac{1}{n}}$.

It can be easily verified that

$$\frac{\partial \Pi_1}{\partial t} = \frac{N}{2w} \frac{t^m}{t(t^m + s^m)^3} P(t, s),$$

where the polynomial $P(t, s)$ is given by

$$\begin{aligned} P(t,s) &= ms^m t^n (r - t^n)(t^m + s^m) + arms^m (s^n - t^n)(s^m - t^m) \\ &\quad + (rnt^n - 2nt^{2n})(t^m + s^m)^2 - arnt^n s^m (t^m + s^m). \end{aligned} \tag{26}$$

After rearranging the terms we have

$$\begin{aligned} P(t,s) &= s^{2m} \left[-(m + 2n)t^{2n} + (rm - arm + rn - arn)t^n + arms^n \right] \\ &\quad s^m \left[-(m + 4n)t^{m+2n} + (rm + arm + 2rn - arn)t^{m+n} - armt^n s^n \right] \\ &\quad -2nt^{2m+2n} + rnt^{2m+n}. \end{aligned} \tag{27}$$

It is easy to verify that $t = t^* = s^*$ is the root of the polynomial $P(t, s^*)$. Therefore there exists a polynomial $g(t, s^*)$ such that

$$P(t, s^*) = (t - s^*)g(t, s^*), \tag{28}$$

for a certain polynomial $g(t, s^*)$. It can be verified that

$$\begin{aligned} g(t,s) \quad = \quad & -2n\sum_{j=0}^{n-1} s^j t^{2m+2n-j-1} + n(r-2s^n)\sum_{j=0}^{2m-1} s^j t^{2m+n-j-1} \\ & -ws^m\sum_{j=0}^{n-1} s^j t^{2n-j-1} + [y - ws^n + n(r-2s^n)]s^{2m}\sum_{j=0}^{n-1} s^j t^{n-j-1} \\ & -vs^m\sum_{j=0}^{n-1} s^j t^{m+2n-j-1} + (z - vs^n)\sum_{j=0}^{n-1} s^j t^{m+n-j-1} \\ & +(z - vs^n - arm)s^{m+n}\sum_{j=0}^{n-1} s^j t^{m-j-1}, \end{aligned} \tag{29}$$

where $s = s^* = \left(\frac{m+2n-an}{m+4n}r\right)^{\frac{1}{n}}$ and where

$$\begin{aligned} w &= m + 2n, \\ v &= m + 4n, \\ y &= rm - arm + rn - arn, \\ z &= rm + arm + 2rn - arn. \end{aligned} \tag{30}$$

It can be easily checked that

$$z - vs^n - arm = 0, \tag{31}$$

and

$$y - ws^n + n(r - 2s^n) = -arm. \tag{32}$$

Lemma 1. *Let* $0 \le t \le r^{\frac{1}{n}}$*. Then,* $g(t, s^*) < 0$.

Proof. Let us sum up the geometric series appearing in (**??**) and apply (**??**), (**??**), (**??**). We obtain

$$\begin{aligned} g(t,s) \quad = \quad & -2nt^{2m+2n-1}\frac{1-(\frac{s}{t})^{n}}{1-\frac{s}{t}} + n(r-2s^{n})t^{2m+n-1}\frac{1-(\frac{s}{t})^{2m}}{1-\frac{s}{t}} \\ & -(m+2n)s^{m}t^{2n-1}\frac{1-(\frac{s}{t})^{n}}{1-\frac{s}{t}} - arms^{2m}t^{n-1}\frac{1-(\frac{s}{t})^{n}}{1-\frac{s}{t}} \\ & -(m+4n)s^{m}t^{m+2n-1}\frac{1-(\frac{s}{t})^{n}}{1-\frac{s}{t}} + arms^{m}t^{m+n-1}\frac{1-(\frac{s}{t})^{n}}{1-\frac{s}{t}}, \end{aligned} \tag{33}$$

where $s = s^{*}$. First, we show that the sum of the last three terms of the right-hand side of (**??**) is negative. Consider the case where $t < s^{*}$. Observe that

$$arms^{2m}t^{n-1} > arms^{m}t^{m+n-1},$$

for all $s > t$. Therefore the sum of the last three terms of (**??**) is negative. Consider next the case where $t > s^{*}$. It is sufficient to show that

$$(m+4n)s^{m}t^{m+2n-1}\frac{1-(\frac{s}{t})^{n}}{1-\frac{s}{t}} > arms^{m}t^{m+n-1}\frac{1-(\frac{s}{t})^{n}}{1-\frac{s}{t}}.$$

Therefore, it is sufficient to show that

$$(m+4n)s^{n} - arm < 0. \tag{34}$$

It is easy to check that (**??**) holds for $s = s^{*}$. Consequently, the sum of the last three terms of (**??**) is negative for every $0 \le t \le r^{1/n}$, and $s = s^{*}$. Returning to (**??**), we are left to show that when $s = s^{*}$

$$\begin{aligned} g(t,s^{*}) \quad < \quad & -2nt^{2m+2n-1}\frac{1-(\frac{s}{t})^{n}}{1-\frac{s}{t}} + n(r-2s^{n})t^{2m+n-1}\frac{(1-\frac{s}{t})^{2m}}{1-\frac{s}{t}} \\ & -(m+2n)s^{m}t^{2n-1}\frac{1-(\frac{s}{t})^{n}}{1-\frac{s}{t}} < 0. \end{aligned}$$

Let

$$h(t,s) = -2t^{2m} - 2s^{2m} + (r-2s^{n})\frac{t^{2m}-s^{2m}}{t^{n}-s^{n}}. \tag{35}$$

It is easy to check that

$$g(t,s) \leq \frac{n(t^n - s^n)}{(t-s)} h(t,s).$$

Therefore, it is sufficient to show that $h(t,s) < 0$ for all $0 \leq t \leq r^{1/n}$ and for $s = s^*$. To this end, we need the following lemma.

Lemma 2. *Let*

$$k(t,s) = \frac{t^{2m} - s^{2m}}{t^n - s^n}.$$

Then, $\frac{\partial k}{\partial t}(t,s) > 0$ *if and only if* $(2m-n)(t-s) > 0$.

Proof. It can be easily verified that

$$\frac{\partial k}{\partial t}(t,s) = -\frac{2mt^{2m-1}}{(t^n - s^n)} \left[\frac{n}{2mt^{2m-n}} \frac{t^{2m} - s^{2m}}{t^n - s^n} - 1 \right].$$

By the mean value theorem

$$\frac{t^{2m} - s^{2m}}{t^n - s^n} = \frac{2m}{n} c^{2m-n}, \tag{36}$$

where $min(t,s) \leq c \leq max(t,s)$. Therefore

$$\frac{\partial k}{\partial t}(t,s) = -\frac{2mt^{2m-1}}{(t^n - s^n)} \left[(\frac{c}{t})^{2m-n} - 1 \right],$$

and it is now easy to verify that the condition of Lemma 2 holds.

We will use Lemma 2 to prove that $h(t,s) < 0$ for every $0 \leq t \leq r^{1/n}$ and $s = s^*$. First observe that by (**??**)

$$h(t,s) \leq \left[(r - 2s^n) \frac{t^{2m} - s^{2m}}{t^n - s^n} - 2s^{2m} \right]. \tag{37}$$

Consider the following four cases.

Case 1. $2m - n < 0$ and $t < s^*$.

By Lemma 2, $\frac{\partial k}{\partial t}(t, s) > 0$. Hence, it is sufficient to show that the right-hand side of (**??**) is negative at $t = s^*$. Indeed for $s = s^*$

$$h(s, s) \leq \frac{2m}{n}(r - 2s^n)s^{2m-n} - 2s^{2m} = s^{2m}\left[\frac{2m}{n}\frac{(r - 2s^n)}{s^n} - 2\right].$$

Using (**??**),we just need to show that

$$\frac{2m}{n}\frac{(2an - m)}{(m + 2n - an)} - 2 < 0, \tag{38}$$

holds for all m and n such that $2m - n < 0$. It is sufficient to show that this is true for $a = 1$ since the left-hand side of (**??**) is increasing in a. The last inequality holds if and only if

$$m^2 + n^2 > mn.$$

Clearly this is true for all $m, n > 0$; hence $h(t, s^*)$ is negative for $t < s^*$.

Case 2. $2m - n > 0$ and $t < s^*$.

Again by Lemma 2, $\frac{\partial k}{\partial t}(t, s) < 0$. Thus, we need to show that the right-hand side of (**??**) is negative at $t = 0$. Indeed for $s = s^*$

$$h(0, s) \leq (r - 2s^n)s^{2m-n} - 2s^{2m} < s^{2m}\left[\frac{(r - 2s^n)}{s^n} - 2\right]. \tag{39}$$

The right hand side of (**??**) is negative at $s = s^*$ if and only if

$$\frac{2an - m}{m + 2n - an} - 2 < 0,$$

for all m and n such that $2m - n > 0$. This is certainly true for $a = 1$ and for all $m, n > 0$.

Case 3. $2m - n < 0$ and $t > s^*$.

Since $\frac{\partial k}{\partial t}(t, s) < 0$, we need to show that the right-hand side (**??**) is negative at $t = s^*$. For $s = s^*$

$$h(s, s) \leq \frac{2m}{n}(r - 2s^n)s^{2m-n} - 2s^{2m} = s^{2m}\left[\frac{2m}{n}\frac{(r - 2s^n)}{s^n} - 2\right],$$

and this was shown to be negative in Case 1.

Case 4. $2m - n > 0$ and $t > s^*$. In this case, by (**??**)

$$h(t,s) \leq \left[(r - 2s^n) \frac{t^{2m} - s^{2m}}{t^n - s^n} 2t^{2m} \right]. \tag{40}$$

By (**??**), for all $s < t$ there exists c, $s \leq c \leq t$, such that

$$\frac{t^{2m} - s^{2m}}{t^n - s^n} = \frac{2m}{n} c^{2m-n}. \tag{41}$$

If $2m - n > 0$, c^{2m-n} increases in c. Therefore, whenever $s < t$,

$$c^{2m-n} < t^{2m-n} < \frac{t^{2m}}{s^n}.$$

Then by (**??**) and (**??**) we have

$$h(t,s) \leq (r - 2s^n) \frac{2m}{n} t^{2m} s^{-n} - 2s^{2m} = t^{2m} \left[\frac{2m}{n} \frac{(r - 2s^n)}{s^n} - 2 \right]. \tag{42}$$

The right-hand side of (**??**) is negative if and only if

$$\frac{2m}{n} \frac{(r - 2s^n)}{s^n} - 2 < 0,$$

holds for $s = s^*$. But in Case 1, it was shown that this condition is satisfied. This completes the proof of Lemma 1.

By Lemma 1 and (**??**), $P(t, s^*) > 0$ whenever $0 \leq t \leq s^*$ and $P(t, s^*) < 0$ whenever $s^* \leq t \leq r^{1/n}$. Therefore, $t = s^*$ is the unique maximizer of $\Pi_1(t, s^*)$ and hence α^* is the unique maximizer of $\Pi_1(\alpha, \beta^*)$. This completes the proof of Proposition 1.

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