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Spectrum Auctions: How They Should and How They Should Not Be Shaped

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Abstract

Spectrum Auctions are multiple–unit auctions where the objects auctioned are not necessarily identical. It is shown that the requirement of independent bids in such auctions can prevent the existence of pure strategy equilibria (which implies inefficiency). Thus, we suggest that spectrum auctions should allow combinatorial bids. Additionally, all the auctions mentioned were multiple–round auctions—those are shown to offer an invitation to collude. A Folk Theorem, as in repeated, games can be proven. Preferable to a multiple–rounds scheme is a Vickrey Auction. The paper concludes with two alternative suggestions on the improvement of auction procedures.

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1 Introduction

In 1994, the Federal Communications Commission (FCC) started spectrum auctions, thereby abolishing its former lotteries and comparative hearings (also known as beauty contests). Mainly, the FCC intended to allocate their spectra more efficiently. In addition, the federal income has been increased, which is always desirable (and especially was in 1994). Similar mechanisms to allocate spectra have been adopted in lots of other countries since then (e.g. in Germany and Great Britain).

Basically, spectrum auctions are multiple–unit auctions where the objects are not necessarily identical. Similarly complex as the allocation of a spectrum (which is subdivided into several frequency domains or, as we will abbreviate it, frequencies) are the allocations of landing rights in air traffic, of experiments in spacecraft, and of transport capacity in networks. Each of these resembles the knapsack problem: the question of how to put the most valuable load into the restricted space of the "knapsack". Usually, in these cases the value of the pieces put in (e.g. the value of the frequencies for the bidders) is unknown to the one who puts them in (e.g. the FCC). Auctions, however, may lead the applicants to reveal their true valuations, thus enabling the auctioneer to solve the knapsack problem correctly (and, for that matter, to allocate the objects efficiently).

Thanks to the revenues the FCC auctions raised and some other reasons (cf. Cramton, 1998) the adopted auction format is currently considered the *state-of-the-art* in auctioning a spectrum. The stylized rules of the FCC auctions are the following (Cramton et. al. 1998, and FCC, 2000):

(1) Multiple individual licenses are sold simultaneously.

(2) The auction progresses through multiple, discrete rounds, and it closes when no (serious) bids are received on any license.

(3) After each round all new bids and the respective bidder's identities are published.

(4) The objects are allocated to the highest bids, and the pricing rule is "pay–your–bid".

(5) There can only be bids on individual licenses (i.e. not on packages of them).

Additionally, the bidding eligibility depends on an upfront payment and on the activity shown throughout the course of the auction. As the activity rules are not to be discussed here, their explicit description is skipped.

Six of these FCC–style simultaneous multiple–round auctions took place in Europe in the year 2000 (in order to allocate the European UMTS–spectra); some results are given in Table 1. These results were quite surprising. The revenue was either far above (in Great Britain and Germany) or far below (elsewhere) the initial forecasts. Auction theory cannot easily explain the joint results if we assume that the licenses' values per capita were about equal. Yet, what do we know about theory in such complex cases at all? The simple truth is: there is no theory for spectrum auctions. All inferences have to rely on often inadequate simplifications, which are necessary to cope with the complex auction form chosen. There is a large body of theory concerning auctions for single objects and a small amount of theory for multiple but identical objects. Only two papers have been written (Bernheim and Whinston, 1986, and Bolle, 2000) on auctions for multiple non-identical objects, both assuming complete information. And there is some discussion of examples.

First we will review the recent spectrum auctions shortly. The British auction happened to be the first one and generated extraordinarily high revenues. It did not go as nicely (for the treasury) in the Netherlands, where the next auction took place. Initially, 13 companies explained their interest to participate, but five of them withdrew even before the qualified bidders were announced. Further two pulled back before the auction eventually started. In the auction, six companies competed for the 5 licenses. One of them quit bidding comparatively (to Great Britain) early, saying that it had not expected to be successful, as it (a would–be newcomer) had been unable to compete with the five incumbents. This auction was deemed a failure. A week later, the German auction started. Eleven bidders qualified, but only seven took part in the auction. The number of licenses was subject to the auction's result, between four and six. Many believed in a *deja vu*—five incumbents leaving no room for a newcomer, perhaps even less competitive than the Dutch auction. The German auction has yielded a huge revenue, however, as the newcomers were rather determined to get a license—the first one left the auction at around DM 63bn (the sum of bids), the second one actually got a license. Already in the final stages of the auction, though, it was questioned whether the winners would ever be capable of earning the tremendous amounts of money they paid.

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Insert Table 1 about here

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The auction train was then to leave for Italy. Its optimistic days, though, were over. Several prospective bidders left before the event, and the government had to be content with six competitors for its five licenses (even that was in doubt for a while). The sixth competitor withdrew already in the 11th round of the auction. It was quite similar in Austria and in Switzerland. The Austrians used the same 4–6–licenses scheme as the Germans. There were, however, just six qualified bidders and those came to an (implicit) agreement quite early. In Switzerland ten bidders qualified for the auction, but six of them have been "merged away" or pulled back in the last week before the start. It took merely four rounds of the auction to allocate the four licenses to the four remaining bidders.

In this paper we are going to criticize several issues. In all auctions (as has been told already) the simultaneous multiple–rounds format was used, which, as we will argue, may be an invitation to tacit collusion. Certainly, in Great Britain and Germany it worked well, and the Swiss and Italian auctions had a lack of competitive bidders in the first place. Yet, the mess in the Netherlands may be partly due to the multiple–rounds component of its auction design and in Austria it certainly was the main cause.

In a single–round auction, six bidders for six licenses may be enough if every bidder can compete for more than one licence. In a multiple–round auction, low–price tacit collusion may be an equilibrium. In Austria, a main point of the auction design was that the number of licenses was subject to the result of the auction. Each of the bidders could acquire two or three of the 12 frequency domains—allowing between four and six licensed companies. In the auction, this point had not been disputed at all. In the multiple–round format it was easy (and safe) to agree on low prices, rather than revealing the true valuations and paying accordingly. In Germany, where this 4–6 licenses design had been used before, it worked quite well. At the time when six bidders were left in the auction the sum of bids had already reached DM 63bn. In particular the dominant incumbent Deutsche Telekom seemed to expect that at least one further competitor would draw back soon—bidding for three of the domains was promising.

It is difficult to say whether the inefficiencies of the auction format (which we will derive) can really be found in these recent auctions. We do not know the valuations of the bidders, and we do not know their informational status. This is a systematic problem which can hardly be solved for 'real' auctions. In laboratory experiments, however, the experimenters can influence these variables to a large extent. They can try different bidding formats, too. Thus, the authors have started to conduct such experiments.

Before the remainder of this paper is outlined, some pairs of alternatives in auction design are introduced. An auctioneer who is to sell several objects may auction them *sequentially*, that is one auction has to be completed before the next may start. He may, alternatively, auction the objects *simultaneously*, that is all auctions start in the same moment. Additionally, as is common practice (e.g. in the FCC auctions), all auctions may be closed simultaneously as well (that is, all auctions are opened for new bids till nobody bids on any object).

In *multiple–round* auctions (as the FCC ones) the bids are unsealed at the end of each round and provisional high bidders are published. The auction progresses until no new bids are put in throughout a whole round (according to the FCC design). In a *single–round* (also: sealed–bid) auction all the bids are collected till a given date, but they are not published before that date. Whoever has put in the highest bid then wins the object(s).

If (as in the FCC auctions) a bidder can bid only on single licenses, independent of the outcome of the auctioning of other licences, he places *independent bids.* If he can bid on a package, then he is required to pay the amount only if he gets all of the licenses in this package (otherwise he gets none and pays nothing). This is a *combinatorial bid.*

The last but certainly not the least important pair of alternatives concerns the pricing rule. A winner (as those in the FCC auctions) might be obliged to pay what he had bid (*first–price* auction), or, in the case of independent bids, what the runner–up had bid (second–price or *Vickrey* auction). The Vickrey auction for combinatorial bids requires a payment scheme which is a bit more complicated (see Section 4).

In the following, these alternatives will be discussed. Finally, some conclusions are drawn.

2 Simultaneous vs. Sequential Auctions

Since we agree with the FCC with respect to the question of auctioning the frequencies simultaneously (instead of sequentially), we will shorten the discussion of this topic. Sequential auctions in their early stages can reveal information about later sold objects' prices. This information effect reduces the winner's curse and, as the bidders can bid more aggressively, it tends to increase the sellers' expected revenues. Yet, the bidders do not want their valuations to be revealed too early, therefore they have an incentive to underbid (Hausch, 1986).

Furthermore, substitutes' prices tend to decline when the auction progresses, as risk premia decrease (McAfee, 1993). Additionally, a predatory bidder might try to drive the early licenses' prices above reasonable limits to harm budget–constrained rivals (Pitchik and Schotter, 1988).

If the participants in a sequential auction do not agree on the complementary character of the frequencies, it becomes even harder (or impossible) to acquire a desired package. This actually is the main obstacle when conducting spectrum auctions sequentially, as the values might well be super–additive.

All in all, there are lots of reasons that prevent bidders from revealing their true valuations, which, on the other hand, prevents the auctioneer from solving the knapsack–problem. Therefore, the less additive the values of the individual frequencies are and the more frequencies are sold, the better it is to auction them simultaneously.

3 Single– vs. Multiple–Round Auctions

In multiple–round auctions, the bids of others provide bidders with common value information. Furthermore the bidders are able to switch between substitutes if their prices differ, and they can look for the most profitable aggregations of licenses. Especially, if one aggregation gets unaffordable they can look for another.

Unfortunately, the revelation of information (as a seemingly positive consequence of the multiple–round structure of the auction) facilitates finding a way which bidders are always eagerly looking for—the way around competition: collusion. Cramton and Schwartz (2000) show that competitors indeed colluded in the FCC spectrum auctions. In the following, we want to explain the outstanding cause of this phenomenon. In a multiple–round auction collusion is an *equilibrium.*

Multiple–round auctions seem to be very close to infinitely repeated games with discount factors arbitrarily close to 0. Hence, it is not surprising that, if a pure strategy equilibrium exists, a Folk Theorem applies—independently of whether an auction with either a combinatorial or independent bid scheme is used. At first we will assume that bidders are *completely* informed.

Before we proceed with spectrum auctions let us consider a rather general example, the multiple–round Prisoners' Dilemma game (Table 2).

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Insert Table 2 about here

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In the single–round game, the unique equilibrium is (D, D). In a game with several rounds of announcing one's choice (and without restrictions on changing it, but with closing the game when no changes are observed), things are different. Again, both always choosing D is an equilibrium, but a very unlikely one. Both bidders can signal that they are willing to cooperate. Cheating is impossible if, for example, the following strategies are used.

(i) Start with bidding C.

(ii) If the other one had bid C in the last round, then bid C this round.

(iii) If the other had bid D in the $k < m$ last rounds, then change your bid (so that the stopping rule does not apply); if he had bid D in $k \geq m$ rounds, then also bid D this round.

m is a measure of a bidder's patience to "lure" the other bidder into cooperation. Both players using such a strategy (with an arbitrary m) results in a cooperative equilibrium, i.e. (C, C).

From the view of competitors in a market, competition usually *is* a kind of Prisoners' Dilemma game. Thus, this simple example is characteristic for the effect of introducing several rounds of bidding together with the above termination rule.

Let us now assume that $N = \{1, \ldots, n\}$ is the set of frequencies to be sold. $b_i = \big(b_i(F)\big)_{F \subset N}$ is a combinatorial bid. The auctioneer selects that combination of bids from the m bidders which maximizes her revenue. In order to facilitate the notation of the following proposition, the minimum bid increment is assumed to be 0.

Proposition 1 (Folk Theorem) *Let $(b_i^*)_{i=1,\ldots,m}$ be a pure strategy equilibrium of a single–round spectrum auction with expected profits r_i for bidder i. If there are Pareto-superior pure strategies $\{\tilde{b}_i\}$,i.e. strategies which imply profits $\tilde{r}_i > r_i$ for all i, then the following trigger strategies are equilibrium strategies:*

(i) i starts with bids $\{\tilde{b}_i(F)\}$for all $F \subset N$

(ii) i repeats his bid if all $j \neq i$ bid $\{\tilde{b}_j(F)\}$

(iii) otherwise i switches to $\{b_i^*(F)\}$ and sticks to it.

(Without proof.)

Note that exactly the same theorem can be formulated for independent bids.

According to this Theorem, if there is an equilibrium in pure strategies, *anything* better than that (from the view of the bidders) can be the outcome of a multiple–round auction as well. The situation is rather complicated if the single–round auction has no equilibrium in pure strategies. A trigger strategy relying on a mixed strategy equilibrium is impossible if bids can be revised. It seems very plausible, however, that in many cases the bidders would coordinate their bids as well, instead of risking a bidding war[1], which might leave all of them with just a small profit or even with a loss.

An example of the working of such collusion strategies is the German 1999 auction of 10 mobile phone frequencies (Jehiel and Moldovanu, 2000).

[1] Such cases may remind us of the Dollar auction (Shubik, 1971, and Leininger, 1989). The Dollar auction is an oral auction for one dollar with the special rule that the winner *and* the second highest bidder have to pay their bid to the auctioneer, but only the highest bidder wins the dollar. In a generalization, all bidders pay their bids. Dollar auctions are used to describe arms races or races for a patent.

4 Independent vs. Combinatorial Bids

Clearly, the more interdependent the values of the frequencies are, the less appropriate an independent bid auction is. Furthermore, there may be no equilibrium in pure strategies. If one bids independently for the licenses of a desired package, one risks being allocated just a fraction of this package and is, thus, unable to benefit from crucial complementarities (and may be facing a loss).

Let us now present some introductory remarks on single–round auctions with combinatorial bids (for a deeper analysis see Bolle, 2000) and independent bids. This will lead to a general method to compute equilibria in these auctions. Discussing two examples, we are able to reveal fundamental properties of such auctions.

4.1 Combinatorial Bids

Assumption 1 *There are m bidders $i = 1, \dots, m$ and n frequencies $1, \dots, n$. The valuation associated with the frequency set $F \subset N = \{1, \dots, n\}$ is described by strictly positive and monotonous functions $\beta_i(F)$. Strictly monotonous means*

$$G \subset F \text{ and } G \neq F \quad \Rightarrow \quad \beta_i(G) < \beta_i(F). \tag{1}$$

The functions $\beta_i(.)$ are normalized by

$$\beta_i(\emptyset) = 0.$$

Now, we want to analyze combinatorial bids in single–round, first–price auctions, which are close to the menu auctions analyzed by Bernheim and Whinston (1986).

Definition 1 *A* combinatorial bid *by bidder i is a set–valued function $b_i(F)$, for all $F \subset N$ with $F \neq \emptyset$. For notational convenience we set all $b_i(\emptyset) = 0$. Each value $b_i(F)$ is called a* bid *for F.*

Definition 2 *$\{F_i\}_{i=0,1,\dots,m}$ with $F_i \cap F_j = \emptyset$ for all $i \neq j$ and $\cup_i F_i = N$ is called an* allocation of the frequencies. *F_i, $i \geq 1$, is the set of frequencies which bidder i gets. F_0 is the set of frequencies that is not allocated.*

Assumption 2 *The auction is a single–round auction. First, the m bidders independently submit combinatorial bids; then the auctioneer allocates the frequencies. The bidders have to pay their successful bids. The auctioneer chooses an allocation* $\left\{F_i^A\right\}$ *which maximizes her revenue* $\sum b_i\left(F_i^A\right)$ *from the auction, while the bidders want to maximize their profit* $\beta_i\left(F_i^A\right)-b_i\left(F_i^A\right)$.

Thus, the set of allocations with its m^n elements is the strategy set of the auctioneer.

Assumption 3 *The valuations* $\beta_i(F)$, $i = 1, \ldots, m$ *are* common knowledge.

In the following, we will look for subgame–perfect equilibria of the two–stage auction game. We denote these equilibria by $\{b_i^*(.), F_i^*\}$. F_i^* is the set of frequencies allocated to bidder i, i.e. in the case of the auctioneer only her best reply to the equilibrium strategies is given and not her best reply function to all combinations of combinatorial bids.

Definition 3 *In an equilibrium* $\left\{b_i^*(.), F_i^*\right\}$ *the allocation* $\left\{F_i^*\right\}$ *is called the* winning allocation. *If i gets* $F_i^* \neq \emptyset$ *he is an* important member *of the winning allocation.*

Definition 4 $\{G_i\}$ *is called a* competing allocation *if*

$$\sum_{i=1}^{m} b_i^*\left(F_i^*\right) = \sum_{i=1}^{m} b_i^*\left(G_i\right).$$

Definition 5 *An allocation* $\{F_i\}$ *is called* efficient *if*

$$\sum_{i=1}^{m} \beta_i\left(F_i\right) \geq \max_{\{G_i\}}\left\{\sum_{i=1}^{m} \beta_i\left(G_i\right)\right\}.$$

Winning allocations are supported by *winning bids* and competing allocations by *competing bids.*

One may ask what makes the auctioneer choose the winning bids with certainty when there is a competing bid with the same revenue. The problem is the same as in Bertrand competition with two producers A and B who have constant marginal costs $c_A < c_B$: the unique equilibrium price is $p_A = p_B = c_B$ with producer A getting the whole market. If the customers should allocate a positive share to producer B, this was not an equilibrium because A would lower his price, while B cannot lower his. Winning and competing allocations are distinguished in the same way. If there were a positive probability that a competing allocation would be chosen, then at least one of the bidders would increase his "winning bid". The (non–generic) cases of equilibria with more than one winning allocation correspond to Bertrand competition with $c_A = c_B$; in Bertrand competition as well as in spectrum auctions this is connected with zero profits of the bidders.

The following Lemmata 1 and 2, and Proposition 2 correspond to Bernheim and Whinston's (1986) Lemma 2[2]. As the proofs are rather short they are repeated here for the convenience of the reader.

Lemma 1 *Let $\{b_i^*(.), F_i^*\}$ be an equilibrium and let $\{F_i\}$ be an alternative allocation. Then there must be a $\Delta\big(\{F_i\}\big) \geq 0$ so that*

$$\sum_{i=1}^{n} b_i^* \left(F_i^*\right) = \sum_{i=1}^{n} b_i^* \left(F_i\right) + \Delta\big(\{F_i\}\big), \tag{2}$$

but there must not be a bidder who would be willing to pay this $\Delta\big(\{F_i\}\big)$. That is,

$$\beta_i \left(F_i^*\right) - b_i^* \left(F_i^*\right) \geq \beta_i \left(F_i\right) - b_i^* \left(F_i\right) - \Delta\big(\{F_i\}\big) \qquad , \forall\, i. \tag{3}$$

Proof: Equation (2) results from the auctioneer's objective. If the left hand side of (3) were smaller than the right hand side, bidder i could profitably increase his bid $b_i^* \left(F_i\right)$ by a little more than Δ, thus making the selection of $\{F_i\}$ more profitable for the auctioneer and for bidder i himself.□

Corollary 1

$$F_0^* = \emptyset \tag{4}$$

[2]Note that the model of Bernheim and Whinston is slightly different, as they assume that bidders can bid on complete allocations (i.e. on who gets which frequency). Nonetheless, the arguments are similar.

$$F_i^* \neq \emptyset \quad \Rightarrow \quad b_i^*\left(F_i^*\right) > 0 \tag{5}$$

Proof: From (1) and Lemma 1 follows that a bidder j could profitably bid for $F_j^* \cup F_0^*$ or for $F_j^* \cup F_i^*$ if (4) or (5) were not fulfilled.□

Lemma 2 *In equilibrium, for every k there must exist a competing allocation $\{F_i^{-k}\}$ with $F_k^{-k} = \emptyset$ or with $b_k^*\big(F_k^{-k}\big) = 0$.*

Proof: If there was no competing allocation or if all competing allocations $\{G_i\}$ showed $G_k \neq \emptyset$ and $b_k^*\big(G_k\big) > 0$, then bidder k could jointly decrease all his positive bids by the same $\varepsilon > 0$ so that he would be allocated F_k^* at lower cost. ε must be smaller than the smallest bid $b_i^*(G_k)$ with G_k from competing allocations $\{G_i\}$ and smaller than all the positive $\Delta\big(\{F_i\}\big)$ from non–competing allocations. Because of the finite number of alternative allocations, such an $\varepsilon > 0$ would exist.□

Lemmata 1 and 2 provide the necessary conditions for equilibria of spectrum auctions. On the other hand, these conditions are sufficient.

Proposition 2 *$(b_i^*(.), F_i^*)$ is an equilibrium iff for every allocation $\{F_i\}$ equation (3) is fulfilled with $\Delta\big(\{F_i\}\big) \geq 0$ and competing allocations $\{F_i^{-k}\}$ with $F_k^{-k} = \emptyset$ or $b_k^*\big(F_k^{-k}\big) = 0$ exist for every k.*

Proof: The existence of a competing allocation $\big\{F_i^{-k}\big\}$ with $F_k^{-k} = \emptyset$ or $b_k^*\big(F_k^{-k}\big) = 0$ makes it impossible for bidder k to prevent the auctioneer from earning a profit,

$$\sum_i b_i^*\left(F_i^*\right) = \sum_i b_i^*\left(F_i^{-k}\right). \tag{6}$$

Therefore, he has to increase the bid $b_i\big(\tilde{F}_i\big)$ if he wants to make her select an allocation with $\tilde{F}_i$ for him. Under (3), however, this cannot be profitable. Thus, the above conditions are not only necessary but also sufficient for an equilibrium.□

Remark: Basically, Proposition 2 provides us with a method for deriving all possible equilibria. We can simplify this task if we restrict our interest to equilibria with monotonous bids, i.e. $b_i(F) \geq b_i(G)$ if $G \subset F$. In this case we can require, in Proposition 2, the existence of competing allocations f^{-j} with $F_j^{-j} = \emptyset$ because the reallocation of $F_j^{-j} = \emptyset$ to any other bidder would not diminish the auctioneer's revenue.

In the case of combinatorial bids it is possible to show that, with completely informed bidders, efficient equilibria always exist. With additional assumptions on equilibrium selection (Trembling Hand Perfectness) and on the set of value functions (which is fulfilled in the above example) a unique equilibrium with an efficient allocation is selected (Bolle, 2000).

Without the uniqueness of the equilibrium there may be a coordination problem. Palfrey (1983) and Bykowsky et.al. (2000) note that in a first–price auction with combinatorial bids, small bidders may face serious difficulties in displacing a large bidder, as nobody wants to pay more than his "just" share of the required amount. This resembles a free–rider problem, but one where cooperation is strictly dominant (as it is possible to earn at least some profit). The problem is, however, more severe. It is possible that *every* allocation of frequencies is supported by subgame–perfect equilibrium bids.

Let us investigate a simple example of a spectrum auction with $n = m = 2$ (number of frequencies and of bidders, resp.) and the valuations

$$\beta_1(\{1\}) = 4, \qquad \beta_1(\{2\}) = 3, \qquad \beta_1(\{1,2\}) = 5, \tag{7}$$
$$\beta_2(\{1\}) = 3, \qquad \beta_2(\{2\}) = 4, \qquad \beta_1(\{1,2\}) = 5. \tag{8}$$

The only efficient allocation is

$$F_1^* = \{1\} \qquad \text{and} \qquad F_2^* = \{2\}. \tag{9}$$

Is this an equilibrium? If we concentrate on equilibria with monotonous bids, then, according to the remark concerning Proposition 2, the "crucial" competing allocations $\{F_i^{-k}\}$ are described by

$$F_2^{-1} = \{1,2\} \qquad \text{and} \qquad F_1^{-2} = \{1,2\}.$$

Thus, Proposition 2 requires

$$b_1^*(\{1\}) + b_2^*(\{2\}) = b_1^*(\{1,2\}) = b_2^*(\{1,2\}), \tag{10}$$
$$4 - b_1^*(\{1\}) \geq \max\{3 - b_1^*(\{2\}) - \Delta, 5 - b_1^*(\{1,2\}), 0\}, \tag{11}$$
$$4 - b_2^*(\{2\}) \geq \max\{3 - b_2^*(\{1\}) - \Delta, 5 - b_2^*(\{1,2\}), 0\}, \tag{12}$$
$$\Delta = b_1^*(\{1\}) + b_2^*(\{2\}) - b_1^*(\{2\}) - b_2^*(\{1\}) \geq 0. \tag{13}$$

The symmetric solutions of this system of equations and inequalities are contained in a two–dimensional set. Let us abbreviate

$$b_1^*\big(\{1\}\big) \equiv b_2^*\big(\{2\}\big) \quad =: x^*, \tag{14}$$
$$b_1^*\big(\{2\}\big) \equiv b_2^*\big(\{1\}\big) \quad =: y^*, \tag{15}$$
$$b_1^*\big(\{1,2\}\big) \equiv b_2^*\big(\{1,2\}\big) =: z^*. \tag{16}$$

Applying this on (10) – (13) results in

$$x^* + x^* = z^* = z^*, \tag{17}$$
$$4 - x^* \geq \max\{3 - y^* - \Delta, 5 - z^*, 0\}, \tag{18}$$
$$4 - x^* \geq \max\{3 - y^* - \Delta, 5 - z^*, 0\}, \tag{19}$$
$$\Delta = x^* + x^* - y^* - y^* \geq 0. \tag{20}$$

Thus, the space of equilibrium bids is described by

$$1 \leq x^* \leq 4 \qquad\qquad y^* \leq x^* \qquad\qquad z^* = 2x^*. \tag{21}$$

Are there also inefficient equilibria? If we investigate whether

$$F_1^* = \{2\} \qquad \text{and} \qquad F_2^* = \{1\} \tag{22}$$

can be an equilibrium allocation, again with the competing allocations

$$F_2^{-1} = \{1,2\} \qquad \text{and} \qquad F_1^{-2} = \{1,2\}.$$

We have to interchange x and y and also 4 and 3 in (17) – (20) and we obtain

$$y^* + y^* = z^* = z^*, \tag{23}$$
$$3 - y^* \geq \max\{4 - x^* - \Delta, 5 - z^*, 0\}, \tag{24}$$
$$\Delta = y^* + y^* - x^* - x^* \geq 0. \tag{25}$$

and the allocation (22) is supported by symmetric bids

$$x^* \leq y^* - 1, \qquad\qquad 2 \leq y^* \leq 3, \qquad\qquad z^* = 2 * y^*. \tag{26}$$

The equilibrium allocation

$$F_1^* = \{1,2\} \qquad \text{and} \qquad F_2^* = \emptyset, \tag{27}$$

with the competing allocation

$$F_2^{-1} = \{1,2\} \qquad \text{and} \qquad F_1^{-1} = \emptyset,$$

translates into the following system

$$z^* = z^*, \tag{28}$$
$$5 - z^* \geq \max\{4 - x^* - \Delta_1, 3 - y^* - \Delta_2, 0\}, \tag{29}$$
$$0 \geq \max\{4 - x^* - \Delta_1, 3 - y^* - \Delta_2, 5 - z^*\}, \tag{30}$$
$$\Delta_1 = z^* - 2 * x^* \geq 0, \tag{31}$$
$$\Delta_2 = z^* - 2 * y^* \geq 0, \tag{32}$$

and is supported by

$$x^* \leq 1, \qquad y^* \leq 2, \qquad z^* = 5. \tag{33}$$

What do we learn from this example? First, we learn that there is a tremendous number of equilibria; in this example, every possible allocation is an equilibrium allocation. Secondly, many of these equilibria do not seem to be plausible. Take, for example, a special case of (33), namely

$$(x^*, y^*, z^*) = (1, 1, 5).$$

Both bidders earn nothing because they pose too small bids on the efficient allocation—they "block" each other. If bidder 1 had bid 3 on frequency 1 and if bidder 2 had bid 3 on frequency 2, the auctioneer would have preferred this allocation and both would have earned 1. The same is true for a number of other equilibrium bids. There are further peculiarities. In the case of the efficient allocation, for example, the following bids form an equilibrium:

$$(x^*, y^*, z^*) = (3, 3, 6). \tag{34}$$

Why should the bidders pose such large out–of–equilibrium bids as in (34) for z? If, mistakenly, one is allocated the frequency set $\{1, 2\}$ one would have a loss.

Some of these equilibria, for instance (34), can be ruled out by excluding weakly dominated strategies. But this principle is not even sufficient to restrict the space of equilibria to efficient ones. Let us take the set of inefficient equilibria described by (26), and select one of the equilibrium strategies of bidder 1, $s_1^* = (x^*, y^*, z^*)$. We ask whether any strategy

$$s_1' = (x_1', y_1', z_1')$$

can weakly dominate s_1^*. When we compare the performance of both strategies when confronted with the equilibrium strategy s_2^* from the equilibrium $\left(s_1^*, s_2^*\right)$ it is clear that $y_1' = y^*$ is necessary because y^* determines the equilibrium payoff and every departure from y^* means loss of payoff relative to s_1^*.

With $y_1' = y^*$ we can ask whether s_1' can outperform s_1^* by a bid resulting in $\{1,2\}$ for 1. This is impossible if $z_1' > 2y^*$, because the profit in $\left(s_1', s_2^*\right)$ would be smaller than $5 - 2y^*$ which again has to be smaller than or equal to $3 - y^*$, the profit under $\left(s_1^*, s_2^*\right)$, see (23)–(25).

If $z_1' < 2y^*$, $\{1,2\}$ may be allocated to bidder 1 when s_1' is confronted with

$$s_2' = (y_2', x_2', z_2'),$$

if

$$z_1' > \max\left\{x_1' + x_2',\ y_1' + y_2',\ z_2'\right\}.$$

In these conditions, however, s_1 would outperform s_1' against a bid s_2'' with

$$\left(y_2'', x_2'', z_2''\right) = \left(0,\ 0,\ y^* + \frac{z_1'}{2}\right),$$

where the profit of s_1' would be zero while s_1^* would earn a positive profit after the allocation of $\{1,2\}$. Hence, $z_1' = 2y^*$ is necessary.

On the same lines we can exclude $x_1' \neq x^*$ and thus conclude that s_1^* is not weakly dominated.

Thus, it is clear that we need another selection criterium in order to rule out implausible equilibria. In the face of the tremendous space of equilibria, this seems to be difficult. A generalization of Trembling Hand Perfectness (Selten, 1975), however, allows us to boil down the number of equilibria considerably. The resulting *unique* "Sensible Equilibrium" in this example is (see Bolle, 2000)

$$F_1^* = \{1\} \qquad \text{and} \qquad F_2^* = \{2\}$$

and

$$(x^*, y^*, z^*) = (1, 0, 2).$$

In general, it can be shown that a Sensible Equilibrium always exists, that all Sensible Equilibria are efficient, and that often a unique equilibrium results.

With independent bids, which is our next point, the problems are even more serious. Again, there may be a plethora of equilibria; but the contrary is possible as well: the non-existence of pure–strategie equilibria.

4.2 Independent Bids

Formally, no more than the restriction

$$a_i(F) = \sum_{j \in F} a_i\big(\{j\}\big) \qquad \forall\, F \subset N,\ i = 1, \ldots, m \tag{35}$$

makes up the difference between combinatorial and independent bids. This, however, is quite decisive. Obviously, any equilibrium in independent bids is an equilibrium in combinatorial bids. The other direction, though, does not work generally—as (35) claims a certain bid structure. The rest of this section is mainly devoted to the discussion of an example of an auction with independent bids and its relevance for the general problem.

Let us regard a second example with $n = m = 2$.

The valuations are

$$\beta_1\big(\{1\}\big) = 1, \qquad \beta_1\big(\{2\}\big) = 1, \qquad \beta_1\big(\{1,2\}\big) = r, \tag{36}$$

$$\beta_2\big(\{1\}\big) = s, \qquad \beta_2\big(\{2\}\big) = s, \qquad \beta_2\big(\{1,2\}\big) = t. \tag{37}$$

As abbreviations for the bids are defined

$$b_1\big(\{1\}\big) =: x_1, \qquad b_1\big(\{2\}\big) =: x_2, \tag{38}$$

$$b_2\big(\{1\}\big) =: y_1, \qquad b_2\big(\{2\}\big) =: y_2. \tag{39}$$

Monotonicity requires $s < t$. Let us, in addition, assume $t < r$. If

$$F_1^* = \{1\} \qquad \text{and} \qquad F_2^* = \{2\} \tag{40}$$

should be the equilibrium allocation, then

$$\begin{aligned} x_1 + y_2 &= x_1 + x_2 = y_1 + y_2 \\ 1 - x_1 &\geq \max\{1 - x_2 - \Delta_1, r - x_1 - x_2 - \Delta_2, 0\} \\ s - y_2 &\geq \max\{s - y_1 - \Delta_1, t - y_1 - y_2 - \Delta_3, 0\} \\ \Delta_1 &= x_1 + y_2 - x_2 - y_1 \geq 0 \\ \Delta_2 &= x_1 + Y_2 - x_1 - x_2 \geq 0 \\ \Delta_3 &= x_1 + y_2 - y_1 - y_1 \geq 0 \end{aligned}$$

is necessary because of Proposition 2 and (35). Therefore

$$r \leq 2 * (1 + s) - t \tag{41}$$

has to be fulfilled in order to obtain (40) as an equilibrium allocation. The same condition is necessary for the equilibrium allocation $F_1^* = \{2\}$ and $F_2^* = \{1\}$. For the equilibrium allocation

$$F_1^* = \{1, 2\} \qquad \text{and} \qquad F_2^* = \emptyset \tag{42}$$

we need (besides $r \geq t$, which we assume to be true)

$$r \geq 2s. \tag{43}$$

Because we have assumed $t < r$, there are no equilibrium bids leading to the allocation $F_1^* = \emptyset, F_2^* = \{1, 2\}$.

Consequently, for

$$2 * (1 + s) - t < r < 2s \tag{44}$$

there is no independent–bid equilibrium in pure strategies (see Figure 1). Such a case might be interpreted in the following way. There is a small firm (bidder 2) which can use only one frequency efficiently, that is

$$r < 2s.$$

Otherwise, $F_1^* = \{1, 2\}, F_2^* = \emptyset$ would be an equilibrium allocation. There is a large firm (bidder 1) which needs both frequencies to operate efficiently:

$$t > 2$$

Otherwise, (44) would not hold as $2 * (1 + s) - t \geq 2s$. Independently of r, at least one of the inequalities (41) and (43) would be fulfilled. Finally, the economies of scale of bidder 1 should offset the diseconomies of bidder 2:

$$t - 2 > 2s - r$$

Otherwise, $F_1^* = \{1\}, F_2^* = \{2\}$ would be an equilibrium allocation. It is obvious that, on this line of argument, plausible examples without pure–strategy equilibria can be constructed for arbitrary numbers of frequencies and bidders. Equilibria in mixed strategies, however, bear a positive probability that an inefficient allocation results.

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Insert Figure 1 about here

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4.3 Comparison

It can be shown (Bernheim and Whinston, 1986, and Bolle, 2000) that under complete information and additional selection criteria[3], auctions with combinatorial bids always have efficient equilibria. This is not the case for auctions with independent bids, where pure strategy equilibria need not exist. As in such a case inefficient allocations are possible, combinatorial bids are preferable.

A special class of spectrum auctions are multiple–bid auctions, in which many identical objects are allocated (such as bonds, treasury bills, or electricity). The absurdity of using independent bids is very clear in these circumstances: such a bid structure means that every bidder had to submit only one price for an arbitrary number of objects. *Thus, all extant multiple–bid auctions use combinatorial bids which allow the bidders to restrict the price offered to a certain number of objects.*

With incompletely informed bidders, however, combinatorial bids may result in inefficient allocations, too, when combined with the above use first–price payment scheme (Bolle, 1997). In the general case, only a Vickrey auction shows efficient equilibria.

[3]Coalition Proofness in Bernheim and Whinston (1986), Trembling Hand Perfectness in Bolle (2000).

5 First–Price vs. Vickrey Auctions

As has been stated, even completely informed bidders may have difficulties displacing a large package bidder in a first–price auction. Incompletely informed bidders, additionally, have to guess the distribution of other bidders' valuations in order to place an ex ante optimal bid.

In Vickrey Auctions, the amount a successful bidder has to pay is independent of what he has bid. Truthfully bidding one's valuation is shown to be a dominant strategy and would enable the auctioneer to allocate the licenses efficiently. Most importantly, these strategies do not incorporate guesses about other bidders' valuations and no coordination of bids.

Ironically, this truthful bidding is the main obstacle of second–price auctions (Vickrey Auctions). Nobody is eager to reveal his actual valuation, for example in order to hide the expected profits and have a better standing in following negotiations (cf. Rothkopf et.al., 1990).

Despite this counter–argument, we want to introduce and evaluate the Vickrey Auction rule for combinatorial bids[4]:

Allocation Rule The auctioneer chooses the allocation that maximizes the sum of bids, i.e.

$$\{F_i^*\} \in \arg\max_{\{F_i\}} \sum_{i=1}^{m} b_i(F_i). \tag{45}$$

Payment Rule Let $\{F_i^*\}$ denote the allocation the auctioneer chooses. Let $\{F_i^{-k}\}$ denote the revenue maximizing allocation if bidder k's bids were removed from the auction. Then bidder k has to pay

$$c_k(F_k^*) = \sum_{i \neq k} \left[b_i(F_i^{-k}) - b_i(F_i^*) \right]. \tag{46}$$

Because $F_i^{-k} = F_i^*$ can always be chosen for $i \neq k$ and because $\{F_i^*\}$ maximizes the sum of bids we have

$$b_i(F_i^*) \geq c_i \geq 0 \qquad \forall\, i.$$

[4]In the case of incomplete information, private values are assumed.

For identical objects and for bids referring only to the number of objects, this definition coincides with the definition of the Vickrey Auction in multiple–bid auctions (cf. Vickrey, 1961, and Bolle, 1997).

Proposition 3 *In the above defined, generalized Vickrey Auction it is a dominant strategy for every bidder to bid his true values, i.e.*

$$b_i(F) = \beta_i(F) \qquad \forall\, i,\ F \subset N. \tag{47}$$

Proof: Let $\{F_i^*\}$ be the allocation selected by the auctioneer after receiving bids according to equation (47) and let $\{G_i\}$ be the allocation if i is removed from the set of the bidders.

(i) Should bidder k increase an unsuccessful bid $b_k(F)$ so that he is allocated F instead of F_k^*? The allocation rule implies that

$$\beta_k(F_k^*)+\sum_{i\neq k}\beta_i(F_i^*) \geq \beta_k(F)+\sum_{i\neq k}\beta_i\big(F_i^*(F)\big) \qquad \forall\, F \subset N \tag{48}$$

where $\{F_i^*(F)\}_{i\neq k}$ denotes the revenue–maximizing allocation of $N\backslash F$ to the bidders $i \neq k$.

From (48) follows

$$\beta_k(F_k^*)-\sum_{i\neq k}\Big[\beta_i(F_i^{-k}) - \beta_i(F_i^*)\Big] \geq \beta_k(F)-\sum_{i\neq k}\Big[\beta_i(F_i^{-k}) - \beta_i\big(F_i^*(F)\big)\Big]$$

and, because of equation (46),

$$\beta_k(F_k^*)-c_k(F_k) \geq \beta_k(F)-c_k(F).$$

Thus, bidder i cannot increase his profit by increasing any of his bids.

(ii) Can bidder k increase his profit by reducing his successful bid $a_k(F_k)$? No, of course not: he would either get another set of frequencies which is not related to a higher profit as (i) shows, or the reduction of his bid has no effect at all because he gets the same set of frequencies F_k at the same price $c_k(F_k)$. □

If dominant strategy equilibria exist they are rather convincing candidates for the outcomes of games. But they need not be the only equilibria. In the case of Vickrey Auctions it has been remarked that there are also "collusive" equilibria, for independent bid auctions see Robinson (1985), Ungern-Sternberg (1988), and Güth and van Damme (1986), and for multiple–bid auctions see Bolle (1997).

6 Conclusion

We suggest to auction spectra *simultaneously*, allowing for *combinatorial bids*, either

- in a (single–round) *Vickrey Auction*, or otherwise
- in a multiple–round, first–price auction with a *publicly announced last round* (after the activity has ceased).

The first suggestion is a simple consequence of the previous analysis. The second one is new.

Announcing the last round in a multiple–round auction is necessary in order to destroy collusive equilibria. But who should be eligible to bid in this finale? We suggest allowing any winner of the first part to increase any of his bids (not just the winning ones) in the second part. Thus, in the first part an upper limit for the number of competitors would be worked out and the common value information could be exploited. After that, in the second part, the exact allocation would be derived. This suggestion, of course, is subject to further theoretical and perhaps experimental examination.

References

Bolle, Friedel. "Necessary Conditions for Efficient Multiple-Bid Auctions". In: *Economic and Environmental Risk and Uncertainty: New Models and Methods* (edited by R. Nau, E. Gronn, M. Machina, and O. Bergland), Kluwer Academic Publishers, 123–143. 1997.

Bolle, Friedel. "On Unique Equilibria of Team Selection Problems, Frequency Auctions, and Menu Auctions". Working Paper. 2000.

Bykowsky, Mark M., Robert J. Cull, and John O. Ledyard. "Mutually Destructive Bidding: The FCC Auction Design Problem". *Journal of Regulatory Economics*, 17(3), 205–228. 2000.

Cramton, Peter. "The Efficiency of the FCC Spectrum Auctions". *Journal of Law and Economics*, 41, 727–736. Oct. 1998.

Cramton, Peter, and Jesse Schwartz. "Collusive Bidding: Lessons from the FCC Spectrum Auctions". *Journal of Regulatory Economics*, 17(3), 229–252. 2000.

Cramton, Peter, John McMillan, Paul R. Milgrom, Bradley Miller, Bridger M. Mitchell, Daniel Vincent, and Robert Wilson. *Simultaneous Ascending Auctions with Package Bidding.* Report 2, Charles River Associates and Market Design Inc. 1998.

FCC. "Auction of Licenses for 800 MHz Specialized Mobile Radio (SMR) Service Frequencies in the Lower 80 Channels". Public Notice DA 00-1388. June 2000. Güth, Werner, and Eric van Damme. "A Comparison of Pricing Rules for Auctions and Fair Division Games". *Social Choice and Welfare*, 3, 177–198. 1986.

Hausch, Donald B. "Multi–Object Auctions: Sequential vs. Simultaneous Sales". *Management Science*, 32(12), 1599–1610. Dec. 1986.

Jehiel, Philippe, and Benny Moldovanu. "A Critique of the Planned Rules for the German UMTS/IMT–2000 License Auction". mimeo, University of Mannheim, 2000.

Leininger, Wolfgang. "Escalation and Cooperation in Conflict Situations". *Journal of Conflict Resolution*, 33, 231–254. 1989.

McAfee, R. Preston. "The Declining Price Anomaly". *Journal of Economic Theory*, 60, 191–212. 1993.

Mueller, Milton. "New Zealand's Revolution in Spectrum Management". *Information Economics and Policy*, 5(2), 159–177. July 1993.

Palfrey, Thomas R. "Bundling Decisions by a Multiproduct Monopolist with Incomplete Information". *Econometrica*, 51(2), 463–483. March 1983.

Pitchik, Carolyn, and Andrew Schotter. "Perfect Equilibria in Budget–Constrained Sequential Auctions: An Experimental Study." Rand Journal of Economics, 19, 363–388. Autumn 1988.

Robinson, Marc S. "Collusion and the Choice of Auction" *Rand Journal of Economics*, 16, 141–145. 1985.

Rothkopf, Michael H., Thomas J. Teisberg, and Edward P. Kahn. "Why

Are Vickrey Auctions Rare?". *Journal of Political Economy*, 98(11), 94–109. 1990.

Shubik, Martin. "The Dollar Auction Game: A Paradox in Non-cooperative Behavior and Escalatio". *Journal of Conflict Resolution*, 15, 545–547. 1971.

Ungern–Sternberg, Thomas von. "Cartel Stability in Sealed Bid Second Price Auctions". *The Journal of Industrial Economics*, 36, 351–358. 1988.

Vickrey, William. "Counterspeculation, Auctions, and Competitive Sealed Tenders". *Journal of Finance*, 16, 8–37. 1961.

7 Tables and Figures

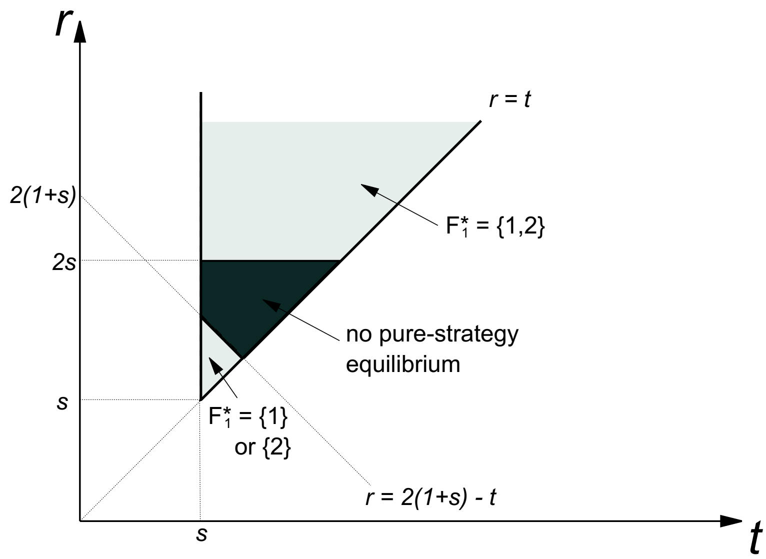

Figure 1: Equilibrium allocations in Example 2. Note that $s < t < r$ is assumed.

Table 1: The European UMTS–Auctions in 2000

		Date			Revenue		
		Number of Bidders			Start–End–Ratio*		
Country	Licences	Qualification	Start Auction	End Auction	Overall	per Capita	Rounds
Great Britain	5	Feb. 14th	Mar. 6th	Apr. 27th	$35.3bn	$607	150
		13	13	5	41.15		
The Netherlands	5	June 26th	July 6th	July 24th	$2.7bn	$173	305
		8	6	5	8.54**		
Germany	4–6	Apr. 28th	July 31st	Aug. 17th	$45.8bn	$559	173
		11	7	6	45 914		
Italy	5	Sep. 2nd	Oct. 20th	Oct. 23rd	$11.6bn	$203	11
		7	6	5	1.15		
Austria	4–6	Sep. 13th	Nov. 2nd	Nov. 3rd	$0.728bn	$90	14
		6	6	6	1.14		
Switzerland	4	Aug. 30th	Dec. 6th	Dec. 6th	$0.12bn	$17	4
		10	4	4	1.025		

*This is the ratio of the overall revenue and the sum of the first bids for each of the objects.
**Instead of using the sum of the first bids, the overall revenue is divided by the sum of the bids in the ninth round.
Most of the data has been acquired via the official websites of the respective regulatory agencies. Some websites proved not to be informative enough, information from newspapers has been taken for supplement.

Table 2: The Prisoners' Dilemma game. The vectors describe the pay-offs of Player 1 and Player 2, respectively.

		Player 2 C	Player 2 D
Player 1	C	(2,2)	(0,3)
	D	(3,0)	(1,1)