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Abstract

This paper presents a sequential search model where consumers look for several products and multiproduct firms compete in prices. In a multiproduct search market, both consumer behavior and firm behavior exhibit different features from the single-product case: a consumer often returns to previously visited firms before running out of options; and prices can decrease with search costs and increase with the number of firms. The framework is then extended in two directions. First, by introducing both single-product and multiproduct searchers, the model can explain the phenomenon of countercyclical pricing, i.e., prices of many retail products decline during peak-demand periods. Second, by allowing firms to use bundling strategies, the model sheds new light on how bundling affects market performance. In a search environment, bundling tends to reduce consumer search intensity, which can soften competition and reverse the usual welfare assessment of competitive bundling in a perfect information setting.

Keywords: consumer search, oligopoly, multiproduct pricing, countercyclical pricing, bundling

JEL classification: D11, D43, D83, L13

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1 Introduction

Consumers often look for several products during a given shopping process. For example, they buy food, drinks and household products together during ordinary grocery shopping, or purchase clothes, shoes and other goods in high street shopping, or buy several presents when they go Christmas shopping. Sometimes a consumer seeks electronic combinations such as computer, printer and scanner, or several travel products such as flights and accommodation. On the other side of the market, there are many multiproduct firms such as supermarkets, department stores, electronic retailers, and travel agencies which often supply all the products a consumer is searching for in a particular shopping trip.¹ Usually the shopping process also involves non-negligible search costs. Consumers need to reach the store, find out each product's price and how suitable they are, and then may decide to visit another store in pursuit of better deals. In effect, in many cases a consumer chooses to shop for several goods together to save on search costs.

Despite the ubiquity of multiproduct search and multiproduct firms,² the search literature has been largely concerned with single-product search markets. There are probably two reasons why multiproduct search is under-researched. First, as we will discuss in more detail later, a multiproduct search model is less tractable than a single-product one. Second, people may also concern how useful a multiproduct search model will be. This paper develops a tractable model to study multiproduct search markets. We find that multiproduct consumer search actually has rich market implications, and the developed framework is useful in addressing several interesting economic issues. First, a multiproduct search market exhibits some qualitatively different properties compared to the single-product case. For example, in a multiproduct search market, prices can decline with search costs and rise with the number of firms. Second, the multiproduct search model can explain the phenomenon of countercyclical pricing, i.e., prices of many retail products fall during high-demand periods such as weekends and holidays. Third, the multiproduct search model provides an appropriate setting for studying bundling in search markets, and sheds new light on how bundling affects market performance.

Our framework is a sequential search model in which consumers look for several products and care about both price and product suitability. Each firm supplies all relevant products, but each product is horizontally differentiated across firms. By incurring a search cost, a consumer can visit a firm and learn all product and price information. In particular, the cost of search is incurred jointly for all products, and the consumer does not need to buy all products from the same firm, i.e., they can mix and match after

¹Marketing research also suggests that retailers regard consumer search behavior (such as how many stores they usually visit and how often they read ads and flyers) as an important determinant in their pricing and marketing decisions (e.g., Urbany, Dickson and Sawyer, 2000).

²Multiproduct search is also relevant in the labor market when a couple, as a collective decision maker, is looking for jobs in the same industry.

sampling at least two firms (if firms allow them to do so).

In the basic model, we assume linear prices are used, i.e., firms set separate prices for each product. A distinctive feature of consumer behavior in multiproduct search is that a consumer may return to previously visited firms to buy some products before running out of options. While in a standard single-product sequential search model, a consumer never returns to earlier firms before having sampled all firms. As far as pricing is concerned, with multiproduct consumer search, if a firm lowers one product's price, this will induce more consumers who are visiting it to terminate search and buy some other products as well. That is, a reduction of one product's price also boosts the demand for the firm's other products. We term this the *joint search effect*. As a result, even independent products are priced like complements.

Due to the joint search effect, prices can decline with search costs in a multiproduct search market. When search costs increase, the standard effect is that consumers will become more reluctant to shop around, which will induce firms to raise their prices. However, in a multiproduct search market, higher search costs can also strengthen the joint search effect and make the products in each firm more like complements, which will induce firms to lower their prices. When the latter effect dominates prices will fall with search costs. A related observation is that prices can rise with the number of firms. This is because when there are more firms, it becomes more likely that a consumer will return to previous firms to buy some products when she stops searching. This weakens the joint search effect and so the complementary pricing problem.

Another prediction of our model is that firms set lower prices in the multiproduct search environment than in the single-product case. This is for two reasons: first, due to economies of scale in search, consumers on average sample more firms in the multiproduct search case than in the single-product search case, which tends to increase each product's own-price elasticity; second, multiproduct search causes the joint search effect, which gives rise to the complementary pricing problem and so increases products' cross-price elasticities. There is a substantial body of evidence that prices of many retail products drop during high-demand periods such as weekends and holidays.³ Our model can offer a simple explanation for this phenomenon of countercyclical pricing. Suppose there are both single-product searchers and multiproduct searchers in the market, and suppose a higher proportion of consumers become multiproduct searchers during high-demand periods (e.g., many households conduct their weekly grocery shopping during weekends). Then the above result implies that prices will decline when demand surges.

The second part of this paper allows firms to engage in bundling (i.e., selling a package of goods in a particular price). Bundling is a widely observed multiproduct pricing strategy in the market. For example, many retailers offer a customer a discount or reward (e.g., free delivery) if she buys several products together from the same store.

³See section 3.5 for related literature and other possible explanations for countercyclical pricing.

Bundling is usually explained as a price discrimination or entry deterrence device,^{4,5} but in a search environment it has a new function: it can discourage consumers from exploring rivals' deals. This is because bundling reduces the anticipated benefit from mixing-and-matching after visiting another firm. As such, firms may have a greater incentive to adopt the bundling strategy in a search market. Moreover, this search-discouraging effect works against the typical pro-competitive effect of competitive bundling in a perfect information scenario (see the related literature below). When search costs are relatively high the new effect can be such that bundling benefits firms and harms consumers.⁶

Since the seminal work by Stigler (1961), there has been a vast literature on search, but most papers focus on single object search. There is a small branch of literature that investigates the optimal stopping rule in multiproduct search. In Burdett and Malueg (1981) and Carlson and McAfee (1984), consumers search for the lowest price of a basket of goods among a large number of stores. The former mainly deals with the case of free recall and the latter deals with the case of no recall. In both cases the optimal stopping rule possesses the reservation property.⁷ Gatti (1999) considers a more general setting with free recall in which consumers search for prices to maximize an indirect utility function. He shows that the reservation property holds in multiproduct search if the indirect utility function is submodular in prices, i.e., if a better offer in one dimension (weakly) reduces the search incentive in the other dimension. (The often adopted additive setting is a special case of that.) This branch of literature has emphasized the similarity between single-product and multiproduct search in the sense that in both cases the stopping rule often features the static reservation property. However, we argue that despite this similarity, consumer search behavior still exhibits substantial differences between the two cases.

More importantly, in the above works there is no an active supply side, and the price (or surplus) distribution among firms is exogenously given. According to our knowledge, the only genuine equilibrium multiproduct search model is McAfee (1995).⁸ It studies

⁴See, for instance, Adams and Yellen (1976), and McAfee, McMillan, and Whinston (1989) for the view of price discrimination, and Whinston (1990) and Nalebuff (2004) for the view of entry deterrence.

⁵In different settings, Carbajo, de Meza and Seidmann (1990) and Chen (1997) also argue that (asymmetric) bundling can create “vertical” product differentiation between firms, thereby softening price competition.

⁶The European Commission has recently branded all bundled financial products as anti-competitive and unfair. One of the main reasons is that the practice reduces consumer mobility. See the consultation document “On the Study of Tying and Other Potentially Unfair Commercial Practices in the Retail Financial Service Section”, 2009.

⁷However, with free recall consumers purchase nothing until search is terminated, while with no recall consumers may buy some cheap goods first and then continue to search for the other goods.

⁸Lal and Matutes (1994) also present a multiproduct search model where each product is homogenous across firms and each consumer needs to pay a location-specific cost to reach firms and discover the price information. Their setting is subject to the Diamond paradox. That is, no consumers will participate in the market given that they expect each firm is charging the monopoly prices. Lal and Matutes

multiproduct price dispersion by extending Burdett and Judd (1983) to the multiproduct case. Each product is homogenous across stores, and by incurring a search cost a consumer can learn price information from a random number of stores. In particular, some consumers only learn information from one store while others learn more. As a result, similar to the single-product case, firms adopt mixed pricing strategies, reflecting the trade-off between exploiting less informed consumers and competing for more informed consumers. However, multiproduct search generates multiple types of (symmetric) equilibria. In particular, there is a continuum of equilibria in which firms randomize prices on the reservation frontier such that one product's price decline must be associated with the rise of some other prices.⁹ Although the model offers interesting insights, both the multiplicity of equilibria and the complication of equilibrium characterization restrict its applicability. Our paper develops a more applicable multiproduct search framework with differentiated products and the standard sequential search technology, where the symmetric equilibrium is unique. We do not aim to address price dispersion. Instead, we use the developed framework to address other economic issues such as countercyclical pricing and bundling in search markets.

In terms of the modelling approach, our paper is built on the single-product search model with differentiated products. That framework was initiated by Weitzman (1979), and later developed and applied to a market context by Wolinsky (1986) and Anderson and Renault (1999). Compared to the homogeneous product search model, models with product differentiation often better reflect consumer behavior in markets that are typically characterized by nonstandardized products. Moreover, they avoid the well-known modelling difficulty suggested by Diamond (1971), who shows that with homogeneous products and positive search costs (no matter how small) all firms will charge a monopoly price and all consumers will stop searching at the first sampled firm. So rivalry between firms has no impact on price. In search models with product differentiation, there are some consumers who are ill-matched with their initial choice of supplier and then search

argue that firms can avoid the market collapse by employing loss-leading strategy, i.e., by advertising (and committing to) low prices of some products to persuade consumers to visit the store. However, in equilibrium each consumer still only samples one firm. Shelegia (2009) studies a multiproduct version of Varian (1980) in which for some exogenous reasons one group of consumers visits only one store while the other visits two. The presence of heterogeneously informed consumers can be a consequence of rational search, but without an explicit search model the main insights from our paper are absent there. Rhodes (2009) proposes a multiproduct monopoly model in which each consumer knows her private valuations for all products but needs to incur a cost to reach the firm and learn prices. He shows that selling multiple products can solve the Diamond hold-up problem which would unravel the market in a single-product case with unit consumer demand.

⁹In the other type of equilibria, firms randomize prices over the acceptance set (*not* just on its border). They are, however, qualitatively similar to the single-product equilibrium in the sense that the marginal price distribution for each product is the same as in the single-product search case, and so is the profit from each product.

further, so that the pro-competitive benefit of actual search is present.¹⁰ Recently this framework has been adopted to study various economic issues such as prominence and non-random consumer search (Armstrong, Vickers, and Zhou, 2009), firms' incentive to use selling tactics such as exploding offers and buy-now discounts (Armstrong and Zhou, 2010), how the decline of search costs affects product design (Bar-Isaac, Caruana, and Cuñat, 2009), and attention-grabbing advertising (Haan and Moraga-González, 2009). Our paper extends this literature to the multiproduct case.

Last but not least, our paper contributes to the literature on competitive bundling. Matutes and Regibeau (1988), Economides (1989), and Nalebuff (2000) have studied competitive pure bundling, and Matutes and Regibeau (1992), Anderson and Leruth (1993), Thanassoulis (2007), and Armstrong and Vickers (2010) have studied duopoly mixed bundling.¹¹ One important insight emerging from all these works is that bundling (whether pure or mixed) has a tendency to intensify price competition, and under the assumptions of unit demand and full market coverage (which are also retained in this paper) it typically reduces firm profits and boosts consumer welfare.¹² This paper is the first to study bundling in a search environment. Our findings indicate that assuming away information frictions (which usually do exist in consumer markets) may significantly distort the welfare assessment of bundling. In particular, when search costs are relatively high, bundling may actually benefit firms and harm consumers. As such, our work complements the existing literature.

The rest of the paper is organized as follows. Section 2 presents the basic model with linear pricing and analyzes consumer search behavior. Section 3 characterizes equilibrium linear prices in a duopoly and conducts comparative statics analysis, and an application to countercyclical pricing is then discussed. Section 4 studies bundling in a search market and examines its welfare impacts relative to linear pricing. Section 5 discusses the case with more firms and other extensions, and section 6 concludes. Omitted proofs and calculations are presented in the Appendix.

2 A Model of Multiproduct Search

There are $n \geq 2$ multiproduct firms in the market, each selling two products 1 and 2. Supplying each product involves a constant marginal cost which is normalized to zero. There are a large number of consumers with measure of one. Each of them has unit

¹⁰In the homogeneous product scenario, the main approach to avoid the Diamond paradox is to introduce heterogeneously informed consumers. See, for example, Burdett and Judd (1983) and Stahl (1989), where price dispersion arises endogenously and so consumers have incentive to search.

¹¹Most of these studies adopt the two-dimensional Hotelling setting and assume that consumers are distributed uniformly on the square and have unit demand for each product. Armstrong and Vickers (2010) consider a fairly general setting with arbitrary distributions and elastic demand.

¹²The welfare impact of bundling in the monopoly case is ambiguous (e.g., Schmalensee, 1984).

demand for both products, and each product is horizontally differentiated across firms. We model this scenario by extending the random utility model in Perloff and Salop (1985) to the two-product case. Specifically, a consumer's valuations for the two products in each firm are randomly drawn from a common joint cumulative distribution function $F(u_1, u_2)$ defined on $[\underline{u}_1, \bar{u}_1] \times [\underline{u}_2, \bar{u}_2]$ which has a continuous density $f(u_1, u_2)$. The valuations are realized independently across firms and consumers (but a consumer may have correlated valuations for the two products in a firm). For simplicity, we assume that the two products are neither complements nor substitutes, in the sense that a consumer obtains an additive utility $u_1 + u_2$ if product i has a match utility u_i , $i = 1, 2$. Let $F_i(u_i)$ and $H_i(u_i|u_j)$ denote the marginal and conditional distribution functions; $f_i(u_i)$ and $h_i(u_i|u_j)$ denote the marginal and conditional densities. Following Perloff and Salop (1985), we assume that in equilibrium all consumers buy both products, i.e., the market is fully covered.¹³ (This is the case, for example, when consumers have no outside options or when they have large basic valuations for each product on top of the above match utilities.) In the basic model, firms must charge a separate price for each product. We refer to this case as “linear pricing” henceforth. (We will consider bundling in section 4.)

We introduce imperfect information and consumer search as Wolinsky (1986) and Anderson and Renault (1999) did in a single-product framework. Initially consumers are assumed to have imperfect information about the price and match utilities of all products. But they can gather information through a sequential search process: by incurring a search cost $s \geq 0$, a consumer can visit a firm and find out both prices (p_1, p_2) and both match utilities (u_1, u_2) . The cost of search is assumed to be the same no matter how many products a consumer is looking for, which reflects economies of scale in search. At each firm (except the last one), the consumer faces the following options: stop searching and buy both products (maybe from firms visited earlier), or buy one product and keep searching for the other, or keep searching for both. In particular, consumers do not need to purchase both products from the same firm. (Otherwise, our multiproduct search model would degenerate to a single-product model with a match utility $u_1 + u_2$ and a price $p_1 + p_2$.) For simplicity, we also assume away other possible costs involved in sourcing supplies from more than one firm. Finally, following most of the literature on consumer search, we assume that consumers have free recall, i.e., there is no cost in going back to a store already visited.

Both consumers and firms are assumed to be risk neutral. We focus on symmetric equilibria in which firms set the same (linear) prices and consumers sample firms in a random order (and without replacement).¹⁴ We use the perfect Bayesian equilibrium

¹³Although the assumption of full market coverage is often adopted in oligopoly models, it is not innocuous for welfare analysis. In our model it highlights the match efficiency but ignores the output efficiency.

¹⁴As usual in search models, there exists an uninteresting equilibrium where consumers expect all

concept. Firms set prices simultaneously, given their expectation of consumers' search behavior. Consumers search optimally, to maximize their expected surplus, given the match utility distribution and their rational beliefs about firms' pricing strategy. At each firm, even after observing off-equilibrium offers, consumers hold the equilibrium belief about the unsampled firms' prices.

We have made several simplifying assumptions to make the model tractable.

Economies of scale in search. Our assumption that the cost of search is independent of the number of products a consumer is seeking is an approximation when the search cost is mainly for learning the existence of a seller or for reaching the store. In the other polar case where the cost of search is totally divisible among products (so no economies of scale in search at all), a multiproduct search problem degenerates to two separate single-product search problems. In reality, most situations are in between: the shopping process involves a fixed cost for reaching the store and also variable in-store search costs for finding and inspecting each product. Our simplification is both for analytical convenience and for highlighting the differences between multiproduct and single-product search.

Free recall. Free recall could be appropriate when a consumer can phone the visited firms (e.g., furniture retailers) to order the products she decides to buy, or when shopping online a consumer can leave the browsed websites open. In the search literature, sometimes we also assume no recall at all (especially in the job search case). In most consumer markets, however, there are usually positive returning costs but they are not so high that returning is totally banned. We choose to assume free recall both for tractability,¹⁵ and for facilitating the comparison between our model and the corresponding single-product search model in Wolinsky (1986) and Anderson and Renault (1999) (both of which assume free recall). We will discuss how costly recall or no recall could affect our results in section 5.2.

Two-stop shopping costs. Even if there are no search costs and returning costs, transacting with two firms may involve some other costs (e.g., the cost of paying two bills). But at least in some retail markets, these two-stop shopping costs seem less important than search costs. We will discuss the difference between this market friction and search frictions in section 5.3. (Two-stop shopping costs are also similar to the joint-purchase discount we will examine in the bundling part.)

firms to set very high prices which leave them with no surplus, consumers do not participate in the market at all, and so firms have no incentive to reduce their prices. We do not consider this equilibrium further.

¹⁵According to our knowledge, Janssen and Parakhonyak (2010) is the only paper in the economics literature which studies the optimal stopping rule in single-product search with costly recall. They find that when there are more than two (but a finite number of) firms, the stopping rule is non-stationary and depends on the historical offers in an intricate way. The optimal stopping rule in multiproduct search with costly recall and an arbitrary number of firms is still an open question.

2.1 The optimal stopping rule

We first derive the optimal stopping rule (which has been formally proved in Burdett and Malueg, 1981, or Gatti, 1999 in a price search scenario). The first observation is that given the indivisible search cost and free recall a consumer will never buy one product first and keep searching for the other. Hence, at any store (except the last one) the consumer faces only two options: stop searching and buy both products (one of which may be from a firm visited earlier), or keep searching for both.

Denote by

$$\zeta_i(x) \equiv \int_x^{\bar{u}_i} (u_i - x) dF_i(u_i) = \int_x^{\bar{u}_i} [1 - F_i(u_i)] du_i \quad (1)$$

the expected incremental benefit from sampling one more product i when the maximum utility of product i so far is x . (The second equality is from integration by parts.) Note that $\zeta_i(x)$ is decreasing and convex. Then the optimal stopping rule in a symmetric equilibrium is as follows.

Lemma 1 *Suppose prices are linear and symmetric across firms. Suppose the maximum match utility of product i observed so far is z_i and there are firms left unsampled. Then a consumer will stop searching if and only if*

$$\zeta_1(z_1) + \zeta_2(z_2) \leq s . \quad (2)$$

The left-hand side of (2) is the expected benefit from sampling one more firm given the pair of maximum utilities so far is (z_1, z_2) , and the right-hand side is the search cost. This stopping rule seems “myopic” at the first glance, but it is indeed sequentially rational. It can be understood by backward induction. When in the penultimate firm, it is clear that (2) gives the optimal stopping rule because given (z_1, z_2) the expected benefit from sampling the last firm is $\mathbb{E}[\max(0, u_1 - z_1) + \max(0, u_2 - z_2)]$, which equals the left-hand side of (2). (Note that we did not assume u_1 and u_2 are independent of each other. The separability of the incremental benefit in (2) is because of the additivity of match utilities and the linearity of the expectation operator.) Now step back and consider the situation when the consumer is at the firm before that. If (2) is violated, then sampling one more firm is always desirable. By contrast, if (2) holds, then even if the consumer continues searching, she will stop at the next firm no matter what she will find there. So the benefit from keeping searching is the same as sampling one more firm. Expecting that, the consumer should actually cease searching now. (This stopping rule also carries over to the case with an infinite number of firms.)

Figure 1 below illustrates the optimal stopping rule.

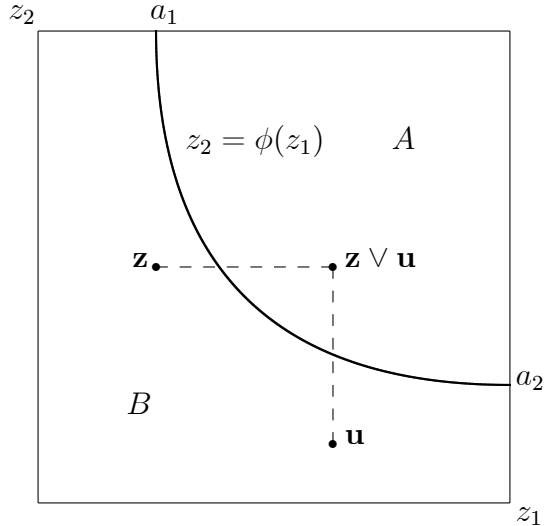


Figure 1: The optimal stopping rule in multiproduct search with perfect recall

A is the set of (z_1, z_2) which satisfies (2) and let us refer to it as the *acceptance set*. Then a consumer will stop searching if and only if the maximum utility pair so far lies within A . We define the border of A as $z_2 = \phi(z_1)$, i.e., $(z_1, \phi(z_1))$ satisfies (2) with equality, and call it the *reservation frontier*. One can show that A is a convex set, and the reservation frontier is decreasing and convex.¹⁶ Let B be the complement of A . Note that a_i on the graph is just the reservation utility level when the consumer is only searching for product i . It solves

$$\zeta_i(a_i) = s, \quad (3)$$

and satisfies $\phi(a_1) = \bar{u}_2$ and $\phi(\bar{u}_1) = a_2$. This is because when the maximum possible utility of one product has been achieved, the consumer will behave as if she is only searching for the other product.

It is worth mentioning that from (1) and (2), one can see that only the marginal distributions matter for the expected benefit of sampling one more firm. This implies that if the marginal distributions are fixed, the correlation of the two products' match utilities does not affect the reservation frontier.

Search behavior comparison. It is useful to compare consumer search behavior between single-product search and multiproduct search. The early literature has emphasized that in both cases (given additive utilities in the multiproduct case) the optimal stopping rule possesses the static reservation property. Despite this similarity, consumers'

¹⁶From the equality of (2), we have

$$\phi'(z_1) = -\frac{1 - F_1(z_1)}{1 - F_2(\phi(z_1))} < 0,$$

and this derivative increases with z_1 .

search behavior exhibits some differences between the two cases, which have not been noticed before.

In single-product search with perfect recall, the stopping rule is characterized by a reservation utility a . When a consumer is already at some firm (except the last one), she will stop searching if and only if the current product has a utility greater than a . Previous offers are irrelevant because they must be worse than a (otherwise the consumer would not have come to this firm). As a result, a consumer never returns to previously visited firms until she finishes sampling all firms. In particular, if there are an infinite number of firms, the consumer actually never exercises the recall option.

However, in multiproduct search, a consumer’s search decision may depend on *both* the current firm’s offer \mathbf{u} and the best offer so far \mathbf{z} . This can be seen from the example indicated in Figure 1, where the current offer \mathbf{u} lies outside the acceptance set A but the consumer will stop searching because $\mathbf{z} \vee \mathbf{u} \in A$ (where \vee denotes the “join” of two vectors). As a result, in multiproduct search (even with an infinite number of firms), although a consumer will buy at least one product at the firm where she stops searching, she may return to a previous firm and buy the other product (even if there are unsampled firms left). In the above example, the consumer will go back to some previous firm to buy product 2.

We summarize the discussion in the following table.

	Single-product search	Multiproduct search
Will previous offers affect the current search decision?	no	yes
Will a consumer retrieve previous offers before searching through all firms?	no	yes

Table 1: Search behavior comparison

These differences will complicate the demand analysis in multiproduct search. In particular, unlike the single-product search case, considering an infinite number of firms does not simplify the analysis (mainly because various types of returning consumers still exist). However, the complication can be avoided if there are only two firms. Moreover, as we will discuss in section 5.1, such a simplification does not lose the most important insights concerning firm pricing in a multiproduct search setting. Hence, in the following analysis, we restrict our attention to the duopoly case. (A detailed analysis of the general case is provided in the online supplementary document at <https://sites.google.com/site/jidongzhou77/research>.)

3 Equilibrium Prices

3.1 The single-product benchmark

To facilitate comparison, we first report some results from the single-product search model (see Wolinsky, 1986 and Anderson and Renault, 1999 for an analysis with n firms). Suppose the product in question is product i , and the unit search cost is still s . Then the reservation utility level is a_i defined in (3), and it decreases with s . That is, in a symmetric equilibrium, a consumer will keep searching if and only if the maximum match utility so far is lower than a_i , and a higher search cost will make the consumer less willing to search on. In the following analysis, we will mainly focus on the case with a relatively small search cost:

$$s < \zeta_i(\underline{u}_i) \Leftrightarrow a_i > \underline{u}_i \text{ for both } i = 1, 2 . \quad (4)$$

This condition ensures an active search market even in the single-product case.

The symmetric equilibrium price p_i^0 is then determined by the first-order condition¹⁷

$$\frac{1}{p_i^0} = \underbrace{f_i(a_i)[1 - F_i(a_i)] + 2 \int_{\underline{u}_i}^{a_i} f(u)^2 du}_{\alpha_0} . \quad (5)$$

(Its intuition will be clear soon.) It follows that p_i^0 increases with the search cost s (or decreases with a_i) if

$$f_i(a_i)^2 + f_i'(a_i)[1 - F_i(a_i)] \geq 0 .$$

This condition is equivalent to an increasing hazard rate $f_i/(1 - F_i)$. Then we have the following result (Anderson and Renault, 1999 have shown this result for an arbitrary number of firms).

Proposition 1 *Suppose the consumer is only searching for product i and the search cost condition (4) holds. Then the equilibrium price defined in (5) increases with the search cost if the match utility has an increasing hazard rate $f_i/(1 - F_i)$.*

3.2 Equilibrium prices in multiproduct search

We now turn to the multiproduct search case. Let (p_1, p_2) be the symmetric equilibrium prices. For notational convenience, let (u_1, u_2) be the match utilities of firm 1, the firm in question, and (v_1, v_2) be the match utilities of firm 2, the rival firm. In equilibrium, for

¹⁷The second-order condition is a delicate issue in the search model with differentiated products. One sufficient condition is that the density f_i is weakly increasing (see Appendix B in Anderson and Renault, 1999, for further discussion). But that condition is too stringent. For example, one can check that even in the exponential case the first-order condition is sufficient for the equilibrium price.

a consumer who samples firm 1 first, her reservation frontier $u_2 = \phi(u_1)$ is determined by

$$\zeta_1(u_1) + \zeta_2(\phi(u_1)) = s , \quad (6)$$

which simply says that the expected benefit of sampling firm 2 is equal to the search cost. Note that $\phi(u_1)$ is only defined for $u_1 \in [a_1, \bar{u}_1]$ (see Figure 2 below). For convenience, we extend its domain to all possible values of u_1 , but stipulate $\phi(u_1) > \bar{u}_2$ for $u_1 < a_1$.

We derive equilibrium prices by applying the following logic. Starting from an equilibrium, suppose firm 1 decreases p_2 by a small ε . How will this adjustment affect its profit? Since the equilibrium demand for its product 2 is $1/2$ (remember the assumption of full market coverage), this deviation first leads to a (first-order) loss $\varepsilon/2$. On the other hand, firm 1 gains from boosted demand. This includes two parts: (i) more consumers who visit firm 1 first will stop searching and buy *both* products immediately; (ii) the consumers who have already sampled both firms will buy product 2 from firm 1 more likely. In equilibrium, the loss and gain should be such that firm 1 has no incentive to deviate, which generates the first-order condition for p_2 .

Let us analyze in detail the (first-order) gain from the proposed small price reduction. We first need to understand how such a price adjustment affects the stopping decisions of those consumers who sample firm 1 first. (Note that the consumers who sample firm 2 first hold equilibrium beliefs and so their stopping decisions remain unchanged.) Denote by $\phi(u_1|\varepsilon)$ the new reservation frontier. Since reducing p_2 by ε is equivalent to increasing u_2 by ε , $\phi(u_1|\varepsilon)$ solves

$$\zeta_1(u_1) + \zeta_2(\phi(u_1|\varepsilon) + \varepsilon) = s ,$$

so $\phi(u_1|\varepsilon) = \phi(u_1) - \varepsilon$. That is, the reservation frontier moves downward everywhere by ε , and the stopping region A expands (i.e., more consumers buy immediately at firm 1) as illustrated in the figure below.

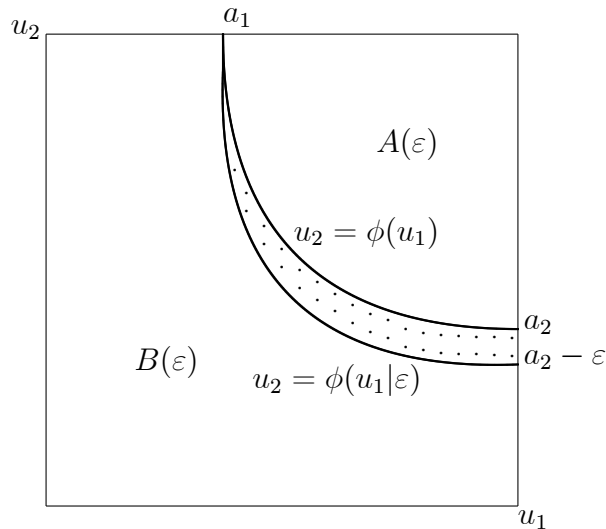


Figure 2: Price deviation and the stopping rule

For a small ε , the number of consumers who switch from keeping searching to buying immediately at firm 1 (i.e., the probability measure of the shaded area between $\phi(u_1)$ and $\phi(u_1|\varepsilon)$) is

$$\frac{\varepsilon}{2} \int_{a_1}^{\bar{u}_1} f(u, \phi(u)) du . \quad (7)$$

(Remember that half of the consumers sample firm 1 first. The integral term is the line integral along the reservation frontier in the u_1 dimension.) These marginal consumers now buy *both* products from firm 1 for sure, while before the price deviation they only bought one or two products from firm 1 with some probability (i.e., when they search on and find worse products at firm 2). This is the first source of gain from the price reduction.

To be specific, for a marginal consumer with $(u_1, \phi(u_1))$, if she searched on, she would come back and buy product 1 if $v_1 < u_1$ (of which the probability is $F_1(u_1)$) and buy product 2 if $v_2 < \phi(u_1)$ (of which the probability is $F_2(\phi(u_1))$). Hence, for a small ε the *net* benefit of the price reduction from the boosted demand for product 2 is

$$p_2 \frac{\varepsilon}{2} \int_{a_1}^{\bar{u}_1} [1 - F_2(\phi(u))] f(u, \phi(u)) du ; \quad (8)$$

and the *net* benefit from the boosted demand for product 1 is

$$p_1 \frac{\varepsilon}{2} \int_{a_1}^{\bar{u}_1} [1 - F_1(u)] f(u, \phi(u)) du . \quad (9)$$

The first own-price effect is a standard search effect, similar to the first term in (5) in the single-product case (where the mass of marginal consumers around the reservation point a_i is $f(a_i)$). The second cross-price effect is a new feature of our multiproduct search model, and it makes the two independent products like complements. This effect occurs because each consumer is searching for two products and the cost of search is incurred jointly for them. So we refer to it as the *joint search effect* henceforth. Also note that the mass of marginal consumers in (7) (and so both search effects) depends not only on the density function f but also on the “length” of the reservation frontier as indicated in Figure 2, which is another new feature of the multiproduct search model. As we shall see below, both new features play an important role in firms’ pricing decisions.

Now consider the second source of gain from the price reduction: the consumers who have sampled both firms. When firm 1 lowers p_2 , more of them will buy its product 2. This effect includes two parts: (a) The half of consumers who sample firm 2 first will come to firm 1 if $(v_1, v_2) \in B$, and then buy firm 1’s product 2 if $u_2 + \varepsilon > v_2$. For a given (v_1, v_2) and a small ε , the consumers who will be affected by the price adjustment are those who value firm 1’s product 2 at $u_2 \approx v_2$, and the density of them is $f_2(v_2)$. Hence, the demand increment from these consumers is $\frac{\varepsilon}{2} \int_B f_2(v_2) dF(\mathbf{v})$. (b) The other half of consumers who sample firm 1 first will continue to visit firm 2 if $(u_1, u_2) \in B(\varepsilon)$ (which converges to B as $\varepsilon \rightarrow 0$), and then come back to buy firm 1’s product 2 if $v_2 < u_2 + \varepsilon$.

The demand increment from these consumers is $\frac{\varepsilon}{2} \int_{B(\varepsilon)} f_2(u_2) dF(\mathbf{u}) \approx \frac{\varepsilon}{2} \int_B f_2(u_2) dF(\mathbf{u})$, which is the same as in (a). Therefore, the small price reduction generates a benefit

$$p_2 \varepsilon \int_B f_2(u_2) dF(\mathbf{u}) \quad (10)$$

from those consumers who sample both firms. (The counterpart in the single-product case is captured in the second term in (5).)

In equilibrium, the (first-order) loss $\varepsilon/2$ from the small price adjustment should be equal to the sum of the (first-order) gains (8), (9) and (10). This yields the first-order condition for p_2 :¹⁸

$$1 = \underbrace{2p_2 \int_B f_2(u_2) dF(\mathbf{u}) + p_2 \int_{a_1}^{\bar{u}_1} [1 - F_2(\phi(u))] f(u, \phi(u)) du}_{\text{standard effect}} + \underbrace{p_1 \int_{a_1}^{\bar{u}_1} [1 - F_1(u)] f(u, \phi(u)) du}_{\text{joint search effect}}. \quad (11)$$

The first two terms on the right-hand side capture the effect of a product's price adjustment on its own demand (which is similar to the single-product search case), and the last term captures the new joint search effect. Similarly, one can derive the first-order condition for p_1 as:

$$1 = 2p_1 \int_B f_1(u_1) dF(\mathbf{u}) + p_1 \int_{a_2}^{\bar{u}_2} [1 - F_1(\phi^{-1}(u))] f(\phi^{-1}(u), u) du + p_2 \int_{a_2}^{\bar{u}_2} [1 - F_2(u)] f(\phi^{-1}(u), u) du, \quad (12)$$

where ϕ^{-1} is the inverse function of ϕ .

Both (11) and (12) are linear equations in prices, and the system of them has a unique solution.¹⁹ Thus, the symmetric equilibrium, if it exists, will be unique.²⁰ Notice that if

¹⁸One can also derive the first-order conditions by calculating the demand functions directly. For example, when firm 1 unilaterally deviates to $(p_1 - \varepsilon_1, p_2 - \varepsilon_2)$, the demand for its product 1 is

$$\frac{1}{2} \int_{u_1}^{\bar{u}_1} [1 - H_2(\phi(u_1|\varepsilon)|u_1)(1 - F_1(u_1 + \varepsilon_1))] dF_1(u_1) + \frac{1}{2} \int_{u_1}^{\bar{u}_1} H_2(\phi(v_1)|v_1)(1 - F_1(v_1 - \varepsilon_1)) dF_1(v_1),$$

where $\varepsilon = (\varepsilon_1, \varepsilon_2)$ and $\phi(u_1|\varepsilon)$ is the reservation frontier associated with the deviation. (Consumers who sample firm 1 first will buy its product 1 if they stop searching immediately or if they search on but find firm 2's product 1 is worse. Consumers who sample firm 2 first will purchase firm 1's product 1 if they come to firm 1 and find firm 1's product 1 is better.) The deviation demand for product 2 is similar. However, this direct method will become less applicable in the case with more products, more firms, or mixed bundling.

¹⁹One can show that the standard-effect coefficient in each first-order condition is greater than the joint-search-effect coefficient. Then the claim follows.

²⁰The issue of equilibrium existence is not our central concern. In our multiproduct search model,

firms ignored the joint search effect, then the pricing problem would be actually separable between the two products. Let \tilde{p}_i denote the price of product i in this hypothetical case.

A special case is when $s = 0$ (so $a_i = \bar{u}_i$ and B equals the whole utility domain). Then both search effects (8) and (9) disappear, and we obtain

$$\frac{1}{p_i} = 2 \int_{u_i}^{\bar{u}_i} f_i(u)^2 du .$$

In this case, the multiproduct model yields the same equilibrium prices as the single-product model.

In the following analysis, we will often rely on the case of symmetric products. Slightly abusing the notation, let the one-variable functions $F(\cdot)$ and $f(\cdot)$ denote the common marginal distribution function and density function, respectively. Let a be the common reservation utility in each dimension. In particular, with symmetric products, we have $f(u_1, u_2) = f(u_2, u_1)$ and the reservation frontier satisfies $\phi(\cdot) = \phi^{-1}(\cdot)$, i.e., it is symmetric around the 45-degree line in the match utility space. If p is the equilibrium price of each product, then both (11) and (12) simplify to

$$\begin{aligned} \frac{1}{p} = & \underbrace{2 \int_B f(u_i) f(u_i, u_j) d\mathbf{u}}_{\text{standard effect: } \alpha} + \underbrace{\int_a^{\bar{u}} [1 - F(\phi(u))] f(u, \phi(u)) du}_{\text{joint search effect: } \beta} \\ & + \int_a^{\bar{u}} [1 - F(u)] f(u, \phi(u)) du . \end{aligned} \quad (13)$$

Before proceeding to the comparative statics analysis, let us first study two examples.

The uniform example: Suppose u_1 and u_2 are independent, and $u_i \sim U[0, 1]$. Then $\zeta_i(x) = (1 - x)^2/2$. So $a = 1 - \sqrt{2s}$ and the search cost condition (4) requires $s \leq 1/2$. The reservation frontier satisfies

$$(1 - u)^2 + (1 - \phi(u))^2 = 2s ,$$

so the stopping region A is a quarter of a disk with a radius $\sqrt{2s}$. Then (13) implies²¹

$$p = \frac{1}{2 - (\pi/2 - 1)s} ,$$

where $\pi \approx 3.14$ is the mathematical constant.

it is rather complicated to investigate the second-order condition in general. However, in the uniform and exponential examples (which we will often use below), we can verify that a firm's profit function is locally concave around the equilibrium prices. In addition, one can also show that the first-order conditions are sufficient for the equilibrium prices at least when the search cost is close to zero (i.e., when the pricing problem is almost separable between the two products) and $1 - F_i$ is logconcave.

²¹The standard effect is $\alpha = 2 - s\pi/2$: the first term in (13) is $2 \int_B d\mathbf{u}$, so it equals two times the area of region B , i.e., $2(1 - s\pi/2) = 2 - s\pi$; and the second term is $\int_a^1 [1 - \phi(u)] du$, which is the area of region A and so equals $s\pi/2$. The joint search effect is $\beta = \int_a^1 (1 - u) du = s$ according to the definition of a .

The exponential example: Suppose u_1 and u_2 are independent, and $f_i(u_i) = e^{-u_i}$ for $u_i \in [0, \infty)$. Then $\zeta_i(x) = e^{-x}$. So $e^{-a} = s$ and the search cost condition (4) requires $s \leq 1$. The reservation frontier satisfies

$$e^{-u} + e^{-\phi(u)} = s ,$$

so $\phi(u)$ is one branch of a hyperbola. Then (13) implies²²

$$p = \frac{1}{1 + s^3/6} .$$

The prices in these two examples are depicted as the thick solid curves in Figure 3 below. The price increases with search costs in the uniform example, but it decreases with search costs in the exponential example. As we will see below, the result that prices can decline with search costs is not exceptional in our multiproduct search model.

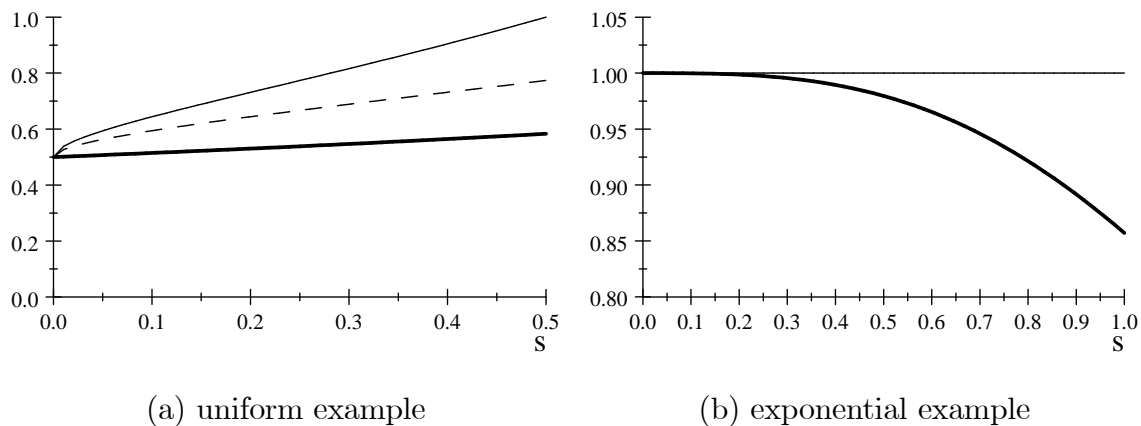


Figure 3: Prices and search costs (symmetric products)

3.3 Search cost and price

This section investigates how search costs affect prices in our multiproduct search model. We first introduce a regularity condition:

$$\frac{h_i(u_i|u_j)}{1 - F_i(u_i)} \text{ increases with } u_i \text{ for any given } u_j. \quad (14)$$

If the two products have independent match utilities, this is just the standard increasing hazard rate condition.

When search costs rise, fewer consumers will sample both firms and become fully informed, which always induces firms to raise their prices. At the same time, when search costs rise, the mass of marginal consumers who distribute on the reservation

²²One can check that the standard effect is $\alpha = 1$ and the joint search effect is $\beta = s^3/6$.

frontier also changes. This is another determinant for prices. In particular, the latter can work against the former if the mass of marginal consumers increases with search costs. In multiproduct search, this occurs more likely than in single-product search because the reservation frontier often becomes “longer” as search costs rise (in the permitted range). Moreover, the marginal-consumer effect is also stronger in multiproduct search due to the joint search effect (i.e., stopping a marginal consumer from searching can boost demand for both products).

Our first observation is that if the joint search effect were absent, the marginal-consumer effect would be usually insufficient to outweigh the first. (Recall \tilde{p}_i is the price of product i if both firms ignore the joint search effect. All omitted proofs can be found in Appendix A.)

Lemma 2 *Suppose the search cost condition (4) and the regularity condition (14) hold. Then \tilde{p}_i , $i = 1, 2$, weakly increase with the search cost.*

That is, without the joint search effect, the relationship between prices and search costs would be actually similar to that in the single-product scenario. For instance, in the uniform example, we have $\tilde{p}_i = 1/(2 - \pi s/2)$ which increases with s ; and in the exponential example which has a constant hazard rate, we have $\tilde{p}_i = 1$ which is independent of s (so p_i decreasing with s in this case is purely due to the joint search effect).

However, taking into account the joint search effect can qualitatively change the picture. We pursue this issue by considering two cases.

Symmetric products. Suppose first the two products are symmetric, and so the equilibrium price p is given in (13). Lemma 2 implies that the standard effect α indicated in (13) usually decreases with s . However, the joint search effect β can vary with s in either direction even under the regularity condition. If β also decreases with s , then the joint search effect will make the price increase with search costs even faster. Conversely, if β increases with s , then the joint search effect will mitigate or even overturn the usual relationship between price and search costs. As shown in the proof,

$$\frac{d\beta}{ds} = f(a, \bar{u}) - \int_a^{\bar{u}} h'(u|\phi(u))f(\phi(u))du . \quad (15)$$

This derivative is positive, for example, when the (conditional) density function is weakly decreasing. This is true in both the uniform and exponential example.

As a result, the standard hazard rate condition is no longer enough to ensure that prices increase with search costs in our model. The following result gives a new condition.

Proposition 2 *Suppose the search cost condition (4) holds, and the two products are symmetric.*

(i) The equilibrium price p defined in (13) increases with the search cost if and only if

$$\int_a^{\bar{u}} \frac{f(\phi(u))}{1 - F(\phi(u))} \{f(u)h(u|\phi(u)) + [2 - F(u) - F(\phi(u))]h'(u|\phi(u))\} du - f(a, \bar{u}) > 0 \quad (16)$$

for all a .

(ii) If the two products further have independent valuations, a sufficient condition for (16) is that the marginal density $f(u)$ is (weakly) increasing.

Condition (16) can be easily violated by some distributions having a decreasing or non-monotonic density (but still having an increasing hazard rate).²³ By continuity, this is true for sufficiently small departures from the exponential case. Other relatively simple examples include: the distribution with a decreasing density $f(u) = 2(1 - u)$ for $s \in [0, 1/3]$; and the logistic distribution $f(u) = e^u/(1 + e^u)^2$ for s less than about 1.

On the other hand, if firms supply (and consumers need) more products, the joint search effect could have an even more pronounced impact such that prices fall with search costs more likely. We can extend our two-product model to the case with m products (see the details in Appendix A). In particular, in the uniform case, the equilibrium price p is given by

$$\frac{1}{p} = 2 - \underbrace{\frac{V_m(\sqrt{2s})}{2^m}}_{\text{standard effect}} + \underbrace{\frac{(m-1)V_m(\sqrt{2s})}{2^{m-1}\pi}}_{\text{joint search effect}}, \quad (17)$$

where $s \in [0, 1/2]$ and $V_m(\sqrt{2s})$ is the volume of an m -dimensional sphere with a radius $\sqrt{2s}$.²⁴ One can see that p increases with s if and only if $m < 1 + \pi/2 \approx 2.6$. Then we have the following result.

Proposition 3 *Suppose the search cost condition (4) holds, and each firm supplies m symmetric products with independent valuations $u_i \sim U[0, 1]$. Then the equilibrium price p is defined in (17), and it increases with s if $m \leq 2$ and decreases with s if $m \geq 3$.*

In this example, if the joint search effect were absent, the price would increase with the search cost for any m . But its presence makes the price decline with search costs whenever consumers are searching for more than two products.

Asymmetric products. Another force which could influence the relationship between prices and search costs is product asymmetry. Intuitively, when one product has a lower

²³One may wonder, if $f(a, \bar{u})$ is bounded away from zero, whether the condition always fails to hold as $a \rightarrow \bar{u}$ (i.e., as $s \rightarrow 0$). This is not true because $\frac{f(\phi(u))}{1 - F(\phi(u))}$ may converge to infinity at the same time. For example, in the uniform case, the left-hand side is equal to $\frac{\pi}{2} - 1 > 0$, independent of a .

²⁴The volume formula for an m -dimensional sphere with a radius r is $V_m(r) = \frac{(r\sqrt{\pi})^m}{\Gamma(1+m/2)}$, where $\Gamma(\cdot)$ is the Gamma function. One can show that for any fixed r , $\lim_{m \rightarrow \infty} V_m(r) = 0$. Then as m goes to infinity, p will approach the perfect information price $1/2$. This is simply because for a fixed search cost, if each consumer is searching for a large number of products, almost all of them will sample both firms.

profit margin than the other, the joint search effect from adjusting its price is stronger (i.e., reducing its price can induce consumers to buy the more profitable product). Then this product’s price may go down with the search cost. We confirm this possibility in a uniform example in which product 1 is a “small” item and has match utility uniformly distributed on $[0, 1]$, and product 2 is a “big” item and has match utility uniformly distributed on $[0, 4]$. Figure 4 below depicts how p_1 (in the left panel) and p_2 (in the right panel) vary with search costs. This example suggests that when the two products are asymmetric, search costs can affect their prices in different directions. (This discussion also opens up the possibility that loss leading might occur in a multiproduct search model. We further discuss this issue in section 6.)

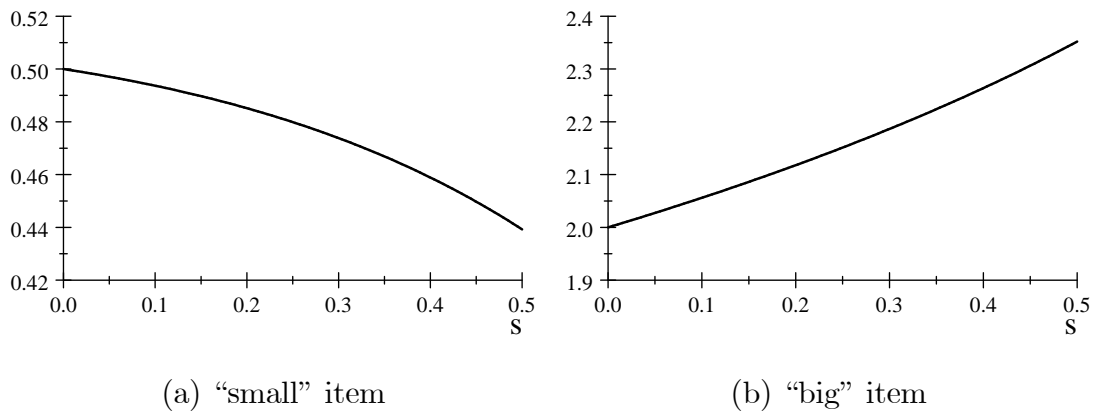


Figure 4: Prices and search costs (asymmetric products)

Discussion: larger search costs. Our analysis so far has been restricted to relatively small search costs such that it is even worthwhile to search for one good alone. In some circumstances, however, consumers conduct multiproduct search just because it is not worthwhile to search for each good separately. We now discuss this case. (As we shall see later, this discussion will also be useful for understanding the results in the bundling case.) For simplicity, let us focus on the case of symmetric products. Suppose s is beyond the condition (4), so $s > \zeta_i(\underline{u})$ and $a < \underline{u}$. (But s cannot exceed $2\zeta_i(\underline{u})$ in order to ensure an active search market.) Then the reservation frontier is shown in Figure 5 below, where $c = \phi(\underline{u})$. The key difference between this case and the case of small search costs is that now the frontier becomes “shorter” as search costs go up. This feature has a significant impact on how prices vary with search costs. For example, in the uniform case, a higher search cost now leads to fewer marginal consumers on the reservation frontier, which provides firms with a greater incentive to *raise* prices. (In this case, the joint search effect strengthens the usual relationship between prices and search costs.)

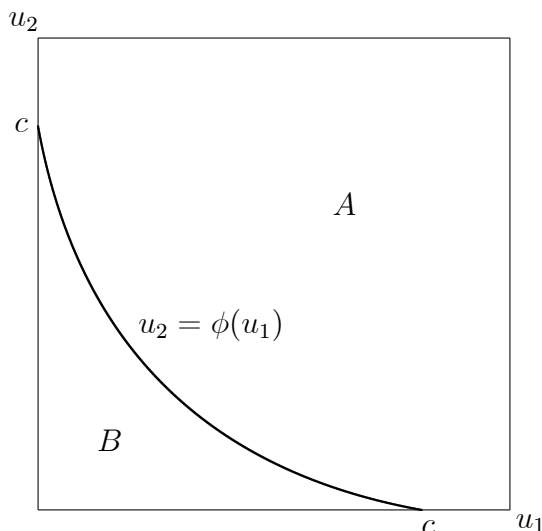


Figure 5: The optimal stopping rule for a large search cost

In general, the following result suggests that prices often increase with search costs when they are beyond the condition (4). (The equilibrium price formula is given in the proof.)

Proposition 4 *Suppose the two products are symmetric and have independent match utilities, and search costs are relatively high such that $\zeta_i(\underline{u}) < s < 2\zeta_i(\underline{u})$. Then the equilibrium price p increases with search costs if each product's match utility has a monotonic density and an increasing hazard rate.*

For example, in the exponential example, we now have $p = 1/(4/3 - s^3/6)$ whenever $s \in (1, 2)$, which increases with s .

3.4 Preference correlation and price

In some circumstances, consumers may exhibit significantly correlated preferences for the same firm's products. For instance, when each multiproduct firm has its own brand image and consumers have diverse brand preferences, a consumer who prefers a firm's product 1 may also prefer its product 2. Preference correlation might affect price competition in a subtle way, because it may influence both consumers' stopping rule and the extent of differentiation between firms. For tractability, we focus on the case where the marginal distribution of each product's match utility is fixed and only the extent of correlation between them can vary. As we have shown, the reservation frontier will then remain unchanged. But the correlation still affects the number of consumers who sample both firms and so can influence firms' pricing.²⁵

²⁵Chen and Riordan (2009) have studied preference correlation and pricing in a multiproduct monopoly and a single-product duopoly, but in their models each consumer only buys one product.

To this end, we adopt the copula approach. For any two marginal distributions $F_1(u_1)$ and $F_2(u_2)$, we can generate a joint cdf $F(u_1, u_2) = C(F_1(u_1), F_2(u_2))$, where $C(x, y)$ is a copula which is itself a joint cdf defined on $[0, 1]^2$ and has uniform marginal distributions. To be specific, consider the parametrized copula

$$C(x, y) = xy + \rho xy(1-x)(1-y)$$

defined on $[0, 1]^2$. Here ρ represents the degree of dependence between u_1 and u_2 , and takes values in the range $-1 \leq \rho \leq 1$, where the positive (negative) values correspond to positive (negative) correlation.²⁶ We can obtain analytical solutions in both the uniform example with $F_i(u_i) = u_i$ and the exponential example with $F_i(u_i) = 1 - e^{-u_i}$. Table 2 below summarizes the impact of preference correlation when ρ increases from -1 to 1 , where search intensity is defined as the expected number of searches (i.e., one plus the measure of region B).

	Uniform ($s \in [0, 0.5]$)	Exponential ($s \in [0, 1]$)
Search intensity	\downarrow if $s \in [0, 0.39]$; \uparrow otherwise	\downarrow
Price	\uparrow	\downarrow if $s \in [0, 0.4]$; \uparrow otherwise

Table 2: The impact of preference correlation

In both examples, as ρ increases, fewer consumers will sample both firms at least when the search costs are relatively small, and this will typically drive firms to increase their prices. (In the exponential example, price can go down with ρ when the search costs are relatively small, but the change is insignificant compared to the price rise when the search costs are relatively large.) These examples suggest that multiproduct firms may benefit from creating firm-level differentiation which makes consumers' preferences over the same firm's products more positively correlated.

3.5 An application: consumer search and countercyclical pricing

As the end of this section, we discuss an application of our multiproduct search model to an empirical phenomenon that has received widespread attention. As documented in Warner and Barsky (1995), MacDonald (2000), Chevalier, Kashyap, and Rossi (2003) and others, prices of many retail products fall during demand peaks such as holidays and weekends. (All these paper use data from multiproduct retailers such as supermarkets

(They introduced the copula approach we adopt below.) Armstrong and Vickers (2010) discussed preference correlation and pricing in the bundling scenario with perfect information.

²⁶This is a copula of the Fairlie-Gumbel-Morgenstern class; see Chen and Riordan (2009) for further discussion. The extent of correlation cannot be extreme in this family, and the standard correlation coefficient ranges from $-1/4$ to $1/4$.

and department stores.) This phenomenon is termed *countercyclical pricing*.²⁷ A slightly extended version of our multiproduct search model can offer an explanation for this empirical fact.²⁸

As a preparation, we first compare the multiproduct search prices in section 3.2 with the single-product search prices in section 3.1.

Proposition 5 *Suppose the search cost condition (4) and the regularity condition (14) hold. Then $p_i \leq p_i^0$, $i = 1, 2$, i.e., each product's price is lower in multiproduct search than in single-product search.*

This result is intuitive. In our model, there are economies of scale in search, i.e., searching for two products is as costly as searching for only one, so more consumers are willing to sample both firms in multiproduct search, which intensifies the price competition. On top of that, the joint search effect gives rise to a complementary pricing problem and induces firms to further lower their prices. This result is illustrated in Figure 3 where the thin solid curves represent p_i^0 and the thick solid curves represent p_i (they coincide only when $s = 0$). For example, in the uniform case with $s = 0.1$, the multiproduct search price is 0.51, lower than the single-product search price 0.64 by 20%. We want to emphasize that even if economies of scale in search are weak (e.g., when multiproduct search is more costly than single-product search), the joint search effect can still induce substantial price reduction. For instance, in the uniform case, if single-product search is half as costly as two-product search (i.e., if its search cost is $s/2$), then the single-product search price becomes $1/(2 - \sqrt{s})$, depicted as the dashed curve in Figure 3(a). The multiproduct search price is still significantly lower than that. For example, when $s = 0.1$, the new single-product search price is 0.59, so the multiproduct search price is still lower than it by 13.5%.

We now extend our basic model by allowing for both single-product and multiproduct searchers in the market. Then if more consumers become multiproduct searchers during peak-demand periods such as holidays,²⁹ Proposition 5 implies that prices may go down.

²⁷In macroeconomics, countercyclical pricing means that markups in the market go down in economic booms and go up in economic recessions, which has important implications for aggregate economic fluctuations. See Rotemberg and Woodford (1999) for a survey.

²⁸There are of course other possible explanations for countercyclical pricing. From the supply side, it may be due to the dynamic interaction among competing retailers who are more likely to have a price war during demand booms (Rotemberg and Saloner, 1986). From the demand side, it may be a consequence of a rise of demand elasticity. Possible reasons include: retailers may advertise price information more intensely when the demand is high (Lal and Matutes, 1994); or more new consumers, who have not learned their valuations for products and so are more price sensitive, may enter the market during peak-demand periods (Bils, 1989). Our search model provides an alternative justification.

²⁹It can even endogenously arise that during holidays and weekends more consumers are multiproduct searchers and firms hold sales. Expecting firms' pricing patterns, consumers may strategically delay their less urgent demand for some products arising in high-price periods. So more consumers will be multiproduct searchers during low-price periods, which in turn justifies firms' pricing strategies.

To illustrate this, consider a simple case with two symmetric products, and assume that each product is needed by a consumer with a probability $\theta \in (0, 1)$. (Our basic model corresponds to $\theta = 1$.) Suppose the need for each product occurs independently across products and consumers. Then there are three groups of consumers in the market: a fraction of θ^2 of consumers are searching for both products, a fraction of $2\theta(1 - \theta)$ of consumers are searching for only one product, and the rest need none of them. A demand rise can be reflected by an increase of θ .

Recall that α_0 is the right-hand side of (5), the standard effect in single-product search, and α and β are indicated in (13) as the standard effect and the joint search effect in multiproduct search, respectively. Then the equilibrium price for each product in this extended model satisfies

$$\frac{1}{p} = (1 - \theta)\alpha_0 + \theta(\alpha + \beta) .$$

(Conditional on a consumer buying one product, this consumer is a single-product searcher with probability $1 - \theta$ and a multiproduct searcher with probability θ .) Proposition 5 implies that $\alpha_0 < \alpha + \beta$ if search is costly and the regularity condition holds, so p decreases with θ . Note that this result is due to both $\alpha_0 < \alpha$ (which reflects the economies of scale in search) and $\beta > 0$ (which reflects the joint search effect).

Our paper is not the first to suggest a consumer search based explanation for countercyclical pricing. In effect, Warner and Barsky (1995) have much earlier suggested such an explanation (though they did not formalize the idea in a real search model):

“Customers for whom it does not pay to search or travel very much when only one item is to be purchased will invest more in information and transportation to obtain the lowest possible price when purchasing a number of units of the same good or a number of different items for which search and travel costs can be at least partly ‘shared’. Because consumers are more vigilant and better informed in the high demand states, individual retailers perceive their demand to be more elastic in such periods.” (p. 324)

Their explanation is wholly based on economies of scale in search. Our model further suggests that even if economies of scale in search are weak, the joint search effect can still induce multiproduct firms to reduce their prices substantially. In effect, one argument in Chevalier, Kashyap, and Rossi (2003) against Warner and Barsky’s explanation is that they did not find clear evidence that consumers become significantly more price elastic during peak-demand periods. However, they only consider each product (category)’s own price elasticity. According to our model, the cross price elasticity due to the joint search effect may play an important role in multiproduct retailers’ pricing decisions. Taking that into account may enhance the explanatory power of a search model for countercyclical pricing.

4 Bundling in Search Markets

Bundling is a widely used multiproduct pricing strategy. In practice, the most often adopted form is that alongside each separately priced product, a package of more than one product is sold at a discount relative to the components. For example, retailers such as electronic stores, travel agencies and online book shops often offer a customer a discount or reward (e.g., free delivery) if she buys more than one product from the same store.³⁰ This is termed *mixed bundling*. Another less often adopted form, termed *pure bundling*, is that the firm only sells a package of all its products, and no product is available for individual purchase.

Consumer search is clearly relevant in various circumstances where firms use bundling strategies, and could have a significant influence on firms' incentive to bundle and the welfare impacts of bundling. However, the existing literature has not explored this issue. This section intends to fill this gap by allowing firms to adopt bundling strategies in the multiproduct search model presented in section 3. We continue to focus on the duopoly setting with two products. The rest of this section is organized as follows. We will first consider how bundling affects consumers' search incentive, which is the driving force behind the main result in this section. We will then show that starting from the linear pricing equilibrium, each firm does have an incentive to introduce bundling. After that, we will characterize bundling equilibrium and examine the welfare impacts of bundling relative to linear pricing.

4.1 Bundling and consumer search incentive

We first examine how bundling might affect consumers' search incentive. In the linear pricing case, given match utilities (u_1, u_2) at firm 1, the expected benefit from sampling firm 2 is

$$\mathbb{E} \left[\max \left(0, \sum_{i=1}^2 (v_i - u_i), v_1 - u_1, v_2 - u_2 \right) \right] . \quad (18)$$

(This merely rewrites the left-hand side of (6). The expectation operator is over (v_1, v_2) .) If both products at firm 2 are a worse match, the consumer will return and buy at firm 1 and so the gain from the extra search will be zero; if both products at firm 2 are a better match, the consumer will buy at firm 2 and gain $\sum_{i=1}^2 (v_i - u_i)$; if only product i at firm 2 is the better match, she will mix and match and the gain will be $v_i - u_i$.

Suppose now both firms adopt the mixed bundling strategy and charge the same prices. Let \hat{p}_i denote the stand-alone price for product i and \hat{P} denote the bundle price. In the meaningful case, the bundle should be cheaper than buying the two products separately (i.e., $\hat{P} \leq \hat{p}_1 + \hat{p}_2$). (Otherwise, no consumers will opt for the package and we go back to the linear pricing case.) Let $\delta \equiv \hat{p}_1 + \hat{p}_2 - \hat{P}$ denote the *joint-purchase*

³⁰In a multiproduct environment, a two-part tariff—an access fee plus separate prices for each product—can also be interpreted as a mixed bundling strategy.

discount. The bundle is also usually more expensive than each single product (i.e., $\hat{P} > \hat{p}_i$ for $i = 1, 2$). (If the bundle is cheaper than either single product, firms are in effect using the pure bundling strategy.) Then, conditional on (u_1, u_2) , the expected benefit of sampling firm 2 becomes

$$\mathbb{E} \left[\max \left(0, \sum_{i=1}^2 (v_i - u_i), v_1 - u_1 - \delta, v_2 - u_2 - \delta \right) \right]. \quad (19)$$

If the consumer buys both products from firm 2, the gain is the same as before (since the bundle price is the same across firms); but if she sources supplies from both firms, she must forgo the joint-purchase discount δ , which is the cost of mixing-and-matching. (Note that for consumers this tariff-intermediated cost plays the same role as an exogenous cost involved in dealing with multiple firms.) This benefit is clearly lower than (18), i.e., mixed bundling reduces a consumer's search incentive.³¹ In other words, when both firms bundle, consumers become more likely to stop at the first sampled firm. As we will demonstrate below, this may induce firms to compete less aggressively and reverse the usual welfare impacts of competitive bundling in a perfect information setting.

4.2 Incentive to bundle

Before proceeding to the equilibrium analysis with bundling, we first investigate, starting from the linear pricing equilibrium, whether a firm has a unilateral incentive to bundle. We suppose that firms choose their bundling strategies and prices simultaneously, and both choices are unobservable to consumers until they reach the store. We will focus on the incentive to employ mixed bundling.³² First of all, given the rival's linear prices, introducing mixed bundling cannot make a firm worse off since it can at least set linear prices (by setting $\delta = 0$). What we will show below is that each firm has a strict incentive to choose $\delta > 0$.

Suppose firm 2 sticks to the linear equilibrium prices (p_1, p_2) . Consider the following deviation for firm 1: $\hat{p}_1 = p_1 + \varepsilon$, $\hat{p}_2 = p_2 + \varepsilon$, and $\hat{P} = p_1 + p_2$, where $\varepsilon > 0$. That is, firm 1 raises each stand-alone price by ε , but keeps the bundle price unchanged. We will examine the impact of such a deviation on firm 1's profit as ε approaches zero. First, the consumers who originally bought a single product from firm 1 now pay more, which of course brings firm 1 a benefit.

³¹Pure bundling will lead to an even lower search incentive. The exact search incentive with pure bundling depends on whether consumers can buy both bundles and mix and match. If this is permitted, the expected benefit of sampling firm 2 is $\mathbb{E}[\max(0, \sum_{i=1}^2 (v_i - u_i), v_1 - u_1 - P, v_2 - u_2 - P)]$, since it now costs a bundle price P for a consumer to mix and match. If this is not permitted (e.g., if pure bundling introduces the compatibility problem), the expected benefit is $\mathbb{E}[\max(0, \sum_{i=1}^2 (v_i - u_i))]$, since the consumer has totally lost the opportunity of mixing-and-matching.

³²A similar result as below can be established for pure bundling if it is the only possible bundling strategy, and if, once a firm bundles, consumers cannot mix and match.

There are also two demand effects. First, for those consumers who sample firm 1 first, due to the joint-purchase discount, more of them will stop searching and buy both products immediately. More precisely, given (u_1, u_2) , the expected benefit of sampling firm 2 now becomes $\mathbb{E}[\max(0, \sum_{i=1}^2 (v_i - u_i), v_1 - u_1 - \varepsilon, v_2 - u_2 - \varepsilon)]$, since buying products from both firms involves an extra outlay ε . This is clearly lower than the search incentive in the linear pricing case. (Note that in our model even *increasing* prices can reduce consumers' search incentive.³³) Consumers who switch from keeping searching to buying immediately will make a positive contribution to firm 1's profit.

Second, for those consumers who eventually sample both firms, the introduced joint-purchase discount will make them buy from the same firm with a higher probability. Firm 1 gains from those consumers who switch from two-stop shopping to buying both products from it, but suffers from those who switch to buying both products from firm 2. We show in the proof that as $\varepsilon \approx 0$ the pros and cons just cancel out each other, such that this second demand effect does not affect firm 1's profit. Therefore, the proposed deviation is strictly profitable at least when ε is small.

Proposition 6 *Starting from the linear pricing equilibrium, each firm has a strict incentive to introduce mixed bundling.*

Note that our argument works even if the search cost is zero (the first positive effect is still present, though the second one disappears). That is, each firm has a strict incentive to bundle even in a perfect information scenario.³⁴ Costly search provides firms with an extra incentive to do so.

4.3 The welfare impacts of bundling

We now investigate the welfare impact of bundling relative to linear pricing in a search environment. The first observation is that total welfare—defined as the sum of industry profit and consumer surplus—must fall with bundling. With the assumption of full market coverage, consumer payment is a pure transfer and only the match efficiency (including search costs) matters. Bundling reduces efficiency because it not only results in insufficient consumer search (i.e., too few consumers search beyond the first sampled firm due to the joint-purchase discount) but also induces too many consumers who have sampled both firms to buy both products from the same firm than is efficient. This result holds no matter whether information frictions exist or not.

In the following, we focus on the impacts of bundling on industry profit and consumer surplus, which, however, depend on information frictions. To do that, we first need to

³³The increased stand-alone price is paid only when a consumer returns to firm 1 and buys a single product, so it actually generates a returning cost for consumers who want to mix and match.

³⁴Armstrong and Vickers, 2010 have shown a similar result in the Hotelling setting with perfect information.

characterize equilibrium prices when both firms bundle. However, as we shall see later, the equilibrium analysis in the mixed bundling case is fairly intricate. By contrast, pure bundling is easier to analyze and can capture the key insight. Therefore, we will start with analyzing the pure bundling case. (Schmalensee, 1984 uses a similar approach in his study on monopoly bundling.)

4.3.1 Pure bundling

As a pricing strategy, pure bundling is less often observed than mixed bundling. But it may become relevant when implementing the mixed bundling strategy is rather costly for firms (e.g., when the number of products is large). In the following analysis, we assume that when both firms bundle, consumers buy only one of the two bundles, i.e., they will not buy both bundles to mix and match. (Nalebuff, 2000, makes the same assumption in studying competitive pure bundling.) This is the case, for instance, when pure bundling introduces the compatibility problem, or when the equilibrium bundle price is so high that it is not worthwhile to buy both bundles.³⁵

Equilibrium prices with pure bundling. When both firms bundle, consumers face a single-product search problem: firm 1 offers a composite product with a match utility $U = u_1 + u_2$ and firm 2 offers another one with an independent match utility $V = v_1 + v_2$. Both U and V belong to $[\underline{U} = \underline{u}_1 + \underline{u}_2, \bar{U} = \bar{u}_1 + \bar{u}_2]$. Let $G(\cdot)$ denote their common cdf, and let $g(\cdot)$ be the associated pdf. Denote by b the reservation utility level in this search problem. It satisfies

$$\int_b^{\bar{U}} (U - b) dG(U) = s . \quad (20)$$

The left-hand side is the expected benefit from sampling the second bundle given the first one has a match utility b . Hence, in a symmetric equilibrium a consumer will visit the second firm if and only if the first bundle has a match utility below b . Since pure bundling reduces consumers' search incentive, the acceptance set expands, i.e., $b < u_1 + \phi(u_1)$ for $u_1 \in [a_1, \bar{u}_1]$. Figure 6 below illustrates this change in the consumer stopping rule, where the linear line is the reservation frontier in the pure bundling case and the new acceptance set is A plus the shaded area.

Let P be the equilibrium bundle price. Then, similar to (5), P is determined in the first-order condition

$$\frac{1}{P} = g(b)[1 - G(b)] + 2 \int_{\underline{U}}^b g(U)^2 dU . \quad (21)$$

P increases with the search costs provided that U has an increasing hazard rate, which is true if each u_i has an increasing hazard rate and is independent from u_j (see, for instance, Miravete, 2002).

³⁵For example, in the uniform example below, when the search cost is relatively high, the bundle price is greater than 1. Then even for a consumer who values firm 1's products at $(1, 0)$ and firm 2's products at $(0, 1)$, it is not worthwhile to buy both bundles.

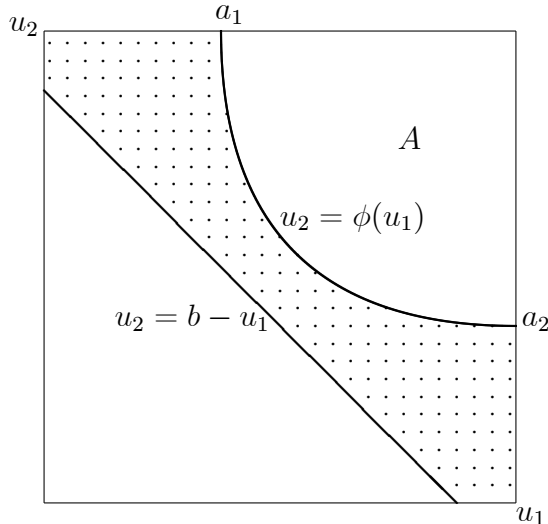


Figure 6: The optimal stopping rule—linear pricing *vs* pure bundling

Comparison with linear pricing. When information is perfect, Matutes and Regibeau (1988), Economides (1989), and Nalebuff (2000) have shown in the two-dimensional Hotelling setting (with full market coverage) that pure bundling typically lowers price (and profit) and boosts consumer welfare. This is mainly because pure bundling makes a price reduction doubly profitable, thereby intensifying price competition.³⁶

The same argument applies in our setting when the search cost is zero.³⁷ Suppose the two products are symmetric. Then from (13) and (21) we can see that at $s = 0$ (so $a = \bar{u}$ and $b = \bar{U}$) pure bundling results in a lower bundle price ($P < 2p$) if and only if

$$\int_{\underline{u}}^{\bar{u}} f(u)^2 du < 2 \int_{\underline{U}}^{\bar{U}} g(U)^2 dU . \quad (22)$$

If the two products' valuations are independent, one can check that this condition holds for a variety of distributions such as uniform, normal and logistic. But it does not always hold. For instance, as we will see below, in the exponential case *equality* in (22) holds.³⁸

When search is costly, the pro-competitive effect of pure bundling still applies among the consumers who sample both firms. However, pure bundling also weakens consumers'

³⁶This intuition is, however, incomplete because bundling also affects the extent of product differentiation (see also Economides, 1989). For example, in our random utility setting, the bundle's match utility has a greater variance than a single product, which usually softens price competition. Therefore, even with perfect information, whether pure bundling increases or decreases market price depends on a delicate interplay of these two effects. This accounts for why pure bundling does not always lead to lower prices even in the perfect information setting (see, for example, our exponential example below).

³⁷In effect, with perfect information our random utility model can be converted into a two-dimensional Hotelling model.

³⁸The opposite can also occur, for example, for a Weibull distribution $f(u) = ku^{k-1}e^{-u^k}$ with k less than but close to one.

search incentive and so reduces the number of informed consumers, which has a tendency to soften price competition. The net effect hinges on the relative importance of these two forces. Intuitively, when the search cost is higher, there will be fewer fully informed consumers and the first effect will appear less important. Then pure bundling may lead to a higher bundle price. This intuition is confirmed in the following examples.³⁹

The uniform example: Suppose u_1 and u_2 are independent, and $u_i \sim U[0, 1]$. To facilitate the comparison with linear pricing, we keep the search cost condition $s \leq 1/2$. One can show that $G(U) = U^2/2$ and $g(U) = U$ if $U \in [0, 1]$, and $G(U) = 1 - (2 - U)^2/2$ and $g(U) = 2 - U$ if $U \in [1, 2]$. According to (20), the reservation utility b satisfies $(2 - b)^3/6 = s$ (so $b \geq 1$) if $s \in [0, 1/6]$, and $1 - b + b^3/6 = s$ (so $b < 1$) if $s \in [1/6, 1/2]$. Then (21) implies

$$P = \begin{cases} \frac{1}{4/3 - s} & \text{if } s \in [0, 1/6) \\ \frac{1}{b^3/6 + b} & \text{if } s \in [1/6, 1/2] \end{cases} .$$

One can check that P increases with s , but the speed is much faster when $s > 1/6$. (The upward sloping curve in Figure 7(a) below depicts how $P - 2p$ varies with search costs.) This is because in the range of $s \in [0, 1/6)$, $b > 1$ and so as s increases, the reservation frontier gets “longer” (i.e., there are more marginal consumers), which mitigates firms’ incentive to raise prices. By contrast, after s exceeds $1/6$, $b < 1$ and so the reservation frontier gets “shorter” as s increases, which strengthens firms’ incentive to raise prices. In other words, when the reservation frontier is still getting longer in the linear pricing case, it already starts to get shorter in the bundling case. In particular, when the search cost exceeds roughly 0.26, the bundle price is higher in the pure bundling case than in the linear pricing case.

The exponential example: Suppose u_1 and u_2 are independent, and $f_i(u_i) = e^{-u_i}$ for $u_i \in [0, \infty)$. Then $G(U) = 1 - (1 + U)e^{-U}$ and $g(U) = Ue^{-U}$. (Note that U has a strictly increasing hazard rate, though u_i has a constant one.) According to (20), the reservation utility b satisfies $(2 + b)e^{-b} = s$. Substituting G and g into (21) yields

$$P = \frac{2}{1 - e^{-2b}} ,$$

which increases with s and is *always* greater than the bundle price $2p$ in the linear pricing case (except $P = 2p$ at $s = 0$). (The upper curve in Figure 7(b) depicts how $P - 2p$ varies with search costs in this example.) With pure bundling, as s increases the reservation frontier always gets “shorter” in the exponential case,

³⁹We can verify in both examples that (21) is also sufficient for the equilibrium price.

which explains why pure bundling reverses the relationship between price and search costs.

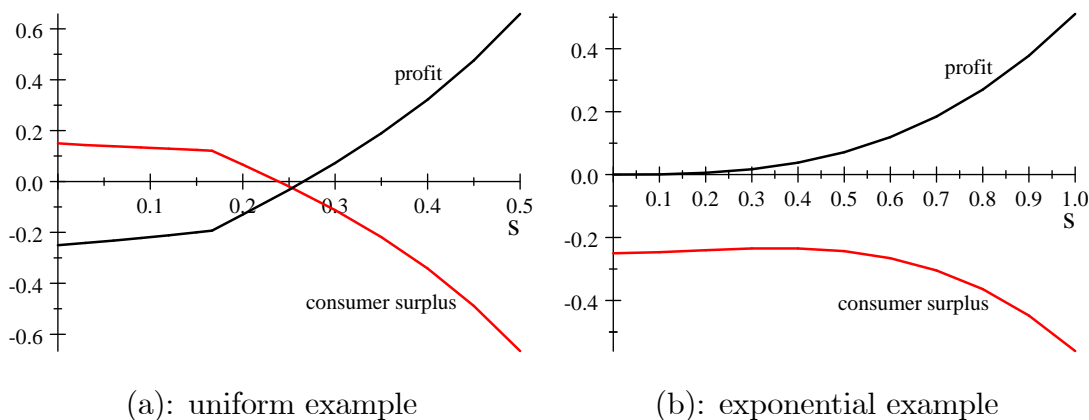


Figure 7: The impacts of pure bundling

Let us turn to welfare impacts. First, each firm earns a higher profit whenever pure bundling leads to a higher bundle price (given the assumption of full market coverage). Hence, given that total welfare always falls with bundling, consumers must become worse off if the bundle price rises in the pure bundling case. But things are less clear when the bundle price falls because consumers also end up consuming less well matched goods. In the uniform example, as indicated by the downward sloping curve in Figure 7(a) which represents the impact of pure bundling on consumer surplus relative to linear pricing, pure bundling benefits consumers when the search costs are lower than about 0.24; while it harms consumers when the search costs exceed that threshold. In the exponential case, pure bundling always harms consumers since it (weakly) raises the bundle price for any search cost level. This is indicated by the lower curve in Figure 7(b). (See Appendix B for how to calculate consumer surplus in our search framework.)

In sum, in a search environment pure bundling can generate a significant competition-relaxing effect such that relative to linear pricing it can benefit firms and harm consumers, in contrast to the perfect information case.⁴⁰

Nevertheless, this search-based effect is most pronounced when the number of goods a consumer is looking for is relatively small. For a given search cost, if a consumer is looking for a large number of goods, she will almost surely sample both firms and the

⁴⁰A more extreme example is when the two products are symmetric and have perfectly negatively correlated valuations. Then in the pure bundling case, the two bundles are in effect homogenous. With perfect information, we have the Bertrand competition and price will be equal to marginal cost, which is often better than linear pricing for consumers; while with costly search, we have the Diamond paradox in which all consumers stop at the first sampled firm (if the first search is costless) and the price will be the monopoly price (in our setting the consumer's willingness to pay), which is of course worse than linear pricing for consumers.

situation will then be close to the perfect information case. In that case, as the following result shows, the pro-competitive effect of pure bundling will dominate.

Proposition 7 *For given search costs, if each firm supplies (and each consumer needs) a large number of symmetric products with independent valuations, then compared to linear pricing, pure bundling leads to a lower bundle price (and lower industry profit), and it also benefits consumers if f is logconcave.*

This result is not trivial (remember in the two-product case, even with perfect information we did not get a general result concerning the welfare impact of pure bundling). Though pure bundling leads to lower prices, it also lowers match efficiency. What we show in the proof is that the bundle price increases with the number of products much slower in the pure bundling case than in the linear pricing case, such that the price effect eventually dominates the match effect and consumers become better off.

4.3.2 Mixed bundling

We now turn to mixed bundling. We aim to deliver a similar message as in the pure bundling case: since mixed bundling also reduces consumers' search incentive (and expands the stopping region), the reservation frontier starts to get shorter with search costs earlier than in the linear pricing case. As a result, for relatively high search costs prices may increase much faster than in the linear pricing case. This in turn may lead to higher profits and lower consumer surplus.

Let us first characterize symmetric equilibrium $(\hat{p}_1, \hat{p}_2, \delta)$. For expositional convenience, we need to introduce several pieces of notation. For a consumer who samples firm 1 first, let $u_2 = \phi_\delta(u_1)$ be her equilibrium reservation frontier which satisfies

$$\mathbb{E} \left[\max \left(0, \sum_{i=1}^2 (v_i - u_i), v_1 - u_1 - \delta, v_2 - u_2 - \delta \right) \right] = s. \quad (23)$$

It is easy to see that $\phi_\delta(u_1)$ decreases with δ , which reflects the search-discouraging effect of the joint-purchase discount, and $\phi_\delta(u_1)$ also decreases with u_1 as usual. Let $A(\delta)$ be the acceptance set and $B(\delta)$ be its complement.

If a consumer finds out match utilities (u_1, u_2) at firm 1 and continues on to sample firm 2, her demand pattern is depicted in Figure 8 below. $r_{12}(\delta) \equiv r_{12}(u_1, u_2; \delta)$ measures the probability that the consumer will return and buy both products at firm 1 (i.e., when the zero term in (23) dominates), $r_0(\delta) \equiv r_0(u_1, u_2; \delta)$ is the probability that she will not return and so buy both products at firm 2 (i.e., when the second term in (23) dominates), and $r_i(\delta) \equiv r_i(u_1, u_2; \delta)$ measures the probability that she will return to buy only product i at firm 1 (i.e., when the third or fourth term in (23) dominates). (Notice that if the supports of match utilities are bounded, then $r_1(\delta)$ or $r_2(\delta)$ or both may disappear when the u_i are close to their bounds.) In particular, for a consumer on the frontier with $(u_1, \phi_\delta(u_1))$, if she chooses to visit firm 2, we denote the corresponding returning probabilities by $\hat{r}_i \equiv r_i(u_1, \phi_\delta(u_1); \delta)$.

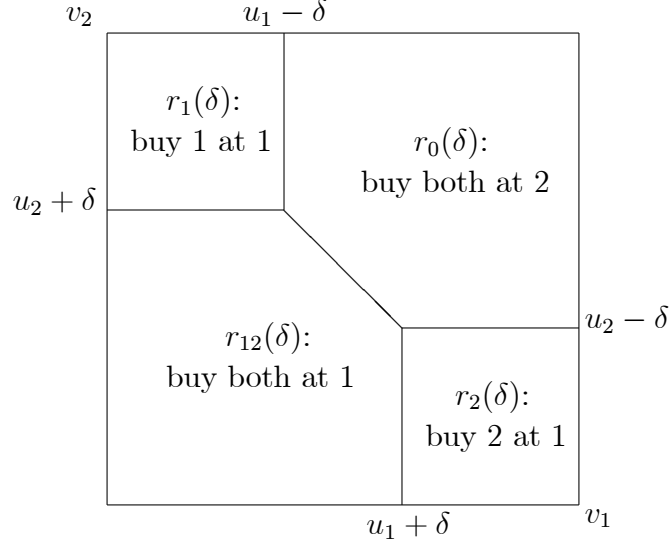


Figure 8: Demand pattern of fully informed consumers (conditional on (u_1, u_2))

Let $Q_1(\delta)$ denote the number of consumers who buy only product 1 from firm 1 in equilibrium. A consumer will buy a single product from some firm only after she samples both firms. So

$$\begin{aligned} Q_1(\delta) &= \frac{1}{2} \int_{B(\delta)} r_1(u_1, u_2; \delta) dF(\mathbf{u}) + \frac{1}{2} \int_{B(\delta)} r_2(v_1, v_2; \delta) dF(\mathbf{v}) \\ &= \frac{1}{2} \int_{B(\delta)} [r_1(\delta) + r_2(\delta)] dF(\mathbf{u}) . \end{aligned}$$

(The first part is from the consumers who sample firm 1 first and the second part is from the consumers who sample firm 2 first. The second step uses the symmetry of firms.) Since the joint-purchase discount δ is the cost of sourcing supplies from both firms, a greater δ will reduce the number of consumers who mix and match. That is, both $r_i(\delta)$ and $B(\delta)$ shrink with δ , and so $Q_1'(\delta) < 0$. Similarly, we can define $Q_2(\delta)$, the number of consumers who buy a single product 2 from firm 1. Then $Q_1(\delta) = Q_2(\delta) = 1/2 - Q_{12}(\delta)$, where $Q_{12}(\delta)$ is the number of consumers who buy both products from firm 1. This is because in equilibrium an equal number of consumers buy a product from each firm.

By adopting a similar methodology as in the linear pricing case, one can derive the first-order conditions for \hat{p}_i and δ . (The details are provided in the online supplementary document.) Here we report the first-order conditions when the two products are symmetric:

$$\begin{cases} 2Q_1(\delta) + \delta Q_1'(\delta) + J_1(\hat{p}, \delta) = 0 \\ (2\hat{p} - \delta)R(\delta) + J_2(\hat{p}, \delta) = 1 \end{cases} , \quad (24)$$

where

$$J_1(\hat{p}, \delta) = \frac{1}{2} \int_{\alpha(\delta)}^{\bar{u}} \frac{\hat{r}_1 + \hat{r}_2}{\hat{r}_1 + \hat{r}_0} [\hat{r}_0(2\hat{p} - \delta) + (\hat{r}_1 + \hat{r}_2)\hat{p}] f(u, \phi_\delta(u)) du$$

and

$$J_2(\hat{p}, \delta) = \int_{a(\delta)}^{\bar{u}} [\hat{r}_0(2\hat{p} - \delta) + (\hat{r}_1 + \hat{r}_2)(\hat{p} - \delta)] f(u, \phi_\delta(u)) du$$

are effects associated with costly search,⁴¹ and

$$R(\delta) = \int_{B(\delta)} \frac{\partial}{\partial u_2} [r_{12}(\delta) - r_0(\delta)] dF(\mathbf{u}) .$$

Note that both equations are linear in \hat{p} but nonlinear in δ . (In particular, if $\delta = 0$, the second equation in (24) degenerates to (13), the first-order condition in the linear pricing case.)

The system (24) is in general complicated. Unlike the linear pricing or pure bundling case, no analytical solution appears to be available even in the uniform or exponential example. In the following, we will first consider the case with zero search costs, which has some independent interest, and we will then discuss the case with costly search by largely resorting to numerical calculation.

Perfect information. When $s = 0$, both search effects J_1 and J_2 vanish and the system (24) simplifies to

$$\begin{cases} 2Q_1(\delta) + \delta Q_1'(\delta) = 0 \\ (2\hat{p} - \delta)R(\delta) = 1 \end{cases} ,$$

where the first equation solely pins down δ (which must be positive as $Q_1'(\delta) < 0$), and the second one determines the bundle price. (Armstrong and Vickers, 2010 have derived a similar result in the two-dimensional Hotelling model.) In this special case, both the uniform and exponential examples are analytically solvable.⁴² The results are reported in the tables below (including, for reference, the pure bundling case).

	Linear Pricing	Mixed Bundling	Pure Bundling
Prices	$p = 0.5$	$\hat{p} \approx 0.57, \hat{P} = 0.81$	$P = 0.75$
Profit	1	0.84	0.75
Consumer surplus	0.33	0.44	0.48
Total welfare	1.33	1.28	1.23

Table 3(a): Uniform example with perfect information

⁴¹Notice that even with the search cost condition (4), now $a(\delta)$, the lower bound of the domain of $\phi_\delta(\cdot)$, can be below \underline{u} due to the joint-purchase discount. In that case, the effective domain of $\phi_\delta(\cdot)$ becomes $[\underline{u}, \phi_\delta(\underline{u})]$ where $\phi_\delta(\underline{u}) < \bar{u}$. But our formulas still apply since $f(u, \phi_\delta(u))$ will be zero when u is beyond the domain of $\phi_\delta(u)$.

⁴²One can show that in the uniform example, $Q_1(\delta) = (1 - \delta)^4/4$ and $R(\delta) = 1 + \delta - \delta^2 + \delta^3/3$; and in the exponential example, $Q_1(\delta) = e^{-2\delta}/4$ and $R(\delta) = 1/2$.

	Linear Pricing	Mixed Bundling	Pure Bundling
Prices	$p = 1$	$\hat{p} = 1.5, \hat{P} = 2$	$P = 2$
Profit	2	2.07	2
Consumer surplus	1	0.78	0.75
Total welfare	3	2.85	2.75

Table 3(b): Exponential example with perfect information

In the uniform example, compared to linear pricing, the stand-alone price becomes higher and the bundle price becomes lower. But the second effect dominates such that mixed bundling harms firms and benefits consumers. This is similar to the finding in Matutes and Regibeau (1992), Thanassoulis (2007), and Armstrong and Vickers (2010). The intuition is that the joint-purchase discount leads to more one-stop shopping consumers, and for them a price reduction is doubly profitable, which intensifies the (bundle) price competition.

The exponential example, which is less familiar in the literature, gives a different picture. Compared to linear pricing, mixed bundling leads to the same bundle price but a higher stand-alone price, and so the welfare assessment is the opposite—firms benefit and consumers suffer from mixed bundling.⁴³ So at least when information is perfect, mixed bundling and pure bundling tend to have similar welfare impacts relative to linear pricing. This appears to be true even if information frictions exist, as the following analysis suggests.

Costly search. The case with costly search is more involved. However, when the search costs are sufficiently small and the match utility distribution has a finite upper bound, the equilibrium has the following properties.

Proposition 8 *Suppose the two products are symmetric and $\bar{u} < \infty$. Let $\delta_0 > 0$ be the equilibrium joint-purchase discount when $s = 0$, and recall b is the reservation utility in the pure bundling case. Then for any small search cost such that $b > 2\bar{u} - \delta_0$, the equilibrium joint-purchase discount is $\delta = \delta_0$, independent of the search cost, and the reservation frontier is $\phi_\delta(u) = b - u$, the same as in the pure bundling case.*

For example, in the uniform case, the joint-purchase discount is always equal to $\delta_0 = 1/3$ when $b > 2 - \delta_0 = 5/3$ or $s < 1/162$. In this range, marginal consumers behave like in the pure bundling case, so they always buy both products from the same firm, i.e., $\hat{r}_1 = \hat{r}_2 = 0$. Then one can show from the second equation in (24) that

⁴³In the two-dimensional Hotelling setting, Proposition 4 in Armstrong and Vickers (2010) shows that if the distribution of consumer position in each dimension satisfies $\frac{d F_i(x_i)}{x_i f_i(x_i)} \geq \frac{1}{4}$ (where $x_i = u_i - v_i$ in our setting), then compared to linear pricing, mixed bundling lowers profit and increases consumer surplus. Our uniform example satisfies this condition but the exponential one does not.

$1/\hat{P} = b^3/6 - b^2 + 2b - 8/81$, and this bundle price (and the stand-alone price) is increasing in s .

Nevertheless, the scope of search costs permitted in Proposition 8 is usually very small. Beyond that scope, we have to resort to numerical calculation. We are only able to work out the uniform example (see the details in the online supplementary document). Two main observations from the numerical exercise are: (i) The joint-purchase discount δ weakly decreases with search costs (remember it is a constant for $s < 1/162$), but δ does not vary too much. When s increases from 0 to 0.5, δ decreases from $1/3$ to about 0.294. Both the stand-alone price and the bundle price increase with s . (ii) As depicted in Figure 9 below, relative to linear pricing, mixed bundling has a qualitatively similar impact on industry profit (the upward sloping curve) and consumer surplus (the downward sloping curve) as pure bundling. That is, when the search costs are relatively small, mixed-bundling harms firms but benefits consumers, while the opposite is true when the search costs are relatively high. (See Appendix B for how to calculate consumer surplus in the mixed bundling case.) However, since mixed-bundling is less able to deter consumers from searching than pure bundling, as we can see (by comparing Figures 7(a) and 9) even higher search costs are needed to reverse the welfare impacts (and the magnitude of impacts is also smaller).

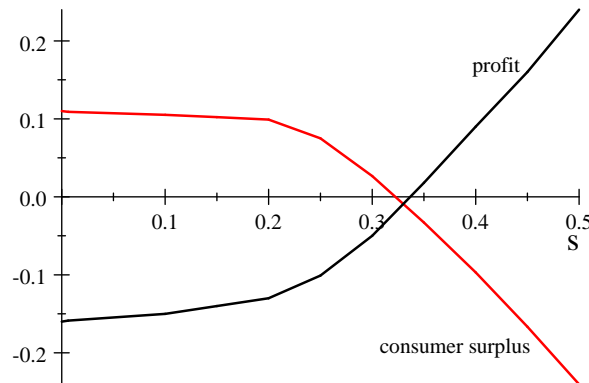


Figure 9: The impact of mixed bundling (uniform example)

5 Discussions

5.1 More firms

Considering an arbitrary number of firms entails a more intricate analysis (see the online supplementary document for the details). But the main insights from the duopoly case can survive.

Search cost and price. In the linear pricing case, when a firm lowers one product's price, more consumers who are currently visiting it for the first time will stop searching, which boosts the demand for its both products. So the joint search effect is still present.

However, a subtle difference emerges: for those consumers who stop searching at some firm (except at the first one), they now do not necessarily buy both products from that firm. Instead, some of them may go back to a previous firm to buy one product. This tends to weaken the joint search effect, but does not eliminate it. For instance, in the exponential example with more firms prices can still decline with s (though not necessarily everywhere on $[0, 1]$).⁴⁴

Bundling. Bundling still reduces the anticipated benefit from mixing-and-matching after sampling more firms, and so restrains consumers' search incentive. However, a new opposite force will come into play when $n \geq 3$ —bundling now also restricts mixing-and-matching among previous offers and so lowers the maximum utility so far (except at the first firm), which can increase consumers' search incentive.⁴⁵ We can compare the expected search times and the bundle price between linear pricing and pure bundling in the uniform example. (Analyzing mixed bundling with more than two firms appears rather intractable.) We find that consumers search more intensively in the pure bundling case only if n is sufficiently large and s is sufficiently small. In particular, even if $n = \infty$, we need s to be lower than about 0.03. This suggests that the new force may be relatively weak most of the time. Consequently, pure bundling can still lead to a higher bundle price (and lower consumer surplus). For example, when $n = \infty$, this is true at least when s is greater than about 0.38.

The number of firms and price. The general model with n firms also allows us to examine how the number of firms affects market prices in a multiproduct search environment. In the single-product case, Anderson and Renault (1999) have shown that the equilibrium price decreases with n under the regularity condition. But this is no longer true in our multiproduct case. Although an analytical investigation is infeasible, numerical simulations suggest that prices can *increase* with n . For instance, in the uniform example with $s = 0.5$, the duopoly price is 0.583 while the price for $n = \infty$ is 0.602. (More examples are provided in the supplementary document.) The intuition is that when there are more firms, it becomes more likely that a consumer, when she stops searching, will return to previously visited firms to buy some products. This weakens the joint search effect and so the complementary pricing problem such that firms may raise their prices.

⁴⁴When $n = \infty$, if prices tend to zero at $s \approx 0$, then they cannot decrease with s at least when s is small. However, the perfect-information prices for $n = \infty$ may not be equal to zero, for example, in the exponential case.

⁴⁵For example, when the first firm offers $(0, 1)$ and the second firm offers $(1, 0)$, linear pricing obviously leads to higher maximum utilities so far than pure bundling.

5.2 Costly recall

When recall is costly, the optimal stopping rule has a new feature: when one product is a good match and the other is a bad match, a consumer will buy the well matched product first (to avoid paying the returning cost) and then continue to search for the other. As a result, each firm will (endogenously) face both single-product searchers (who have bought one product from some previous firm) and multiproduct searchers. The joint search effect survives, but the effect of bundling on consumer search can be different. For instance, in the polar case with no recall, since consumers cannot return to mix and match anyway, bundling does not reduce consumers' search incentive any more. However, in a more reasonable case where recall is costly but not totally banned, the search-discouraging effect of bundling, though reduced, will persist.

A complete analysis with costly recall is beyond the scope of this paper. In effect, when the returning cost is mild (such that returning consumers exist), the optimal stopping rule does not have a simple characterization even in the duopoly case.

5.3 Search costs *vs* shopping costs

Search costs usually mean the costs incurred to find and evaluate a new option. The literature sometimes also considers shopping costs. Literally, shopping costs should include all costs except payment involved in a shopping process, so search costs (if they exist) should be part of it. In a single-product case, these two terms are often used interchangeably, because if there are any shopping costs they are usually related with search activity. However, in a multiproduct case, even if information is perfect, there may still exist substantial shopping costs (e.g., the costs of conducting extra transactions) when the customer sources supplies from more than one firm (see, for instance, Klemperer, 1992, and Armstrong and Vickers, 2010). This type of shopping costs can cause a similar effect as our joint search effect, i.e., it renders two independent products complements and so has a tendency to intensify price competition. Nevertheless, there is an essential difference between search costs and this kind of shopping costs. Search costs always have their own anti-competition effect since they reduce consumers' incentive to shop around, while shopping costs in a perfect information setting are usually pro-competitive. In effect, shopping costs are similar to the joint-purchase discount in the mixed bundling scenario, or to the restriction of no mixing-and-matching in the pure bundling scenario. If information is initially perfect, shopping costs tend to intensify competition and reduce market prices. While if information is initially imperfect and consumers need to conduct costly search, shopping costs can work in the opposite way by reducing consumers' search incentive. Therefore, how shopping costs affect competition may crucially depend on whether search costs are present or not in the same time.

6 Conclusion

This paper has explored a multiproduct search model and shown how consumers and firms may behave differently compared to a single-product search framework. In particular, the presence of the joint search effect may induce prices to decline with search costs and to rise with the number of firms. The developed framework has also been used to address other economic issues such as countercyclical pricing and bundling, and new insights emerged. For instance, we find that compared to the perfect information scenario, the welfare assessment of competitive bundling can be reversed in a search environment.

Our multiproduct search framework has other possible applications including the following.

Multiproduct vs single-product shops. In the market, large multiproduct sellers often coexist with smaller competitors (e.g., specialist shops). We can modify our basic model to investigate this kind of market structure. Consider a market with three asymmetric firms: firm 1 supplies two products (say, clothes and shoes), while firms 2 and 3 are two single-product shops (say, firm 2 is a clothes shop and firm 3 is a shoe shop). Suppose the costs of reaching any firm are identical for all firms, and consumers visit firm 1 first (which can be rational in equilibrium). After visiting the multiproduct firm 1, a consumer will continue to visit firm 2 (firm 3) if and only if firm 1 offers unsatisfactory clothes (shoes). In this simple setting, changing the clothes price will no longer affect a consumer's decision whether to visit the shoe shop, so the joint search effect disappears and firms have two separate competitions for each product. However, other interesting insights will emerge. Given all consumers visit firm 1 first, their search order reveals information about their preferences: a consumer will visit a single-product shop only if she is unsatisfied with the product in the multiproduct shop. This gives the single-product shop extra monopoly power and induces it to charge a higher price.⁴⁶ Therefore, this variant can explain why multiproduct shops often set lower prices than their smaller competitors, without appealing to other exogenous reasons such as the multiproduct shop's economies of scale in operations or its advantage in bargaining with manufacturers.

Advertising and loss leaders. In the case of asymmetric products, we have found that firms have an incentive to sacrifice the profit from some small item to induce more consumers to buy the more profitable big item. This opens up the possibility of using the loss-leading strategy, but we have not found an example with a real loss leader (whose price is below its marginal cost). Allowing for price advertising, however, may generate real loss leaders (see Lal and Matutes, 1994, for instance). Reducing a product's price (privately) can only induce some consumers who are already in the store from searching on, but advertising this price cut can increase the store traffic in the first place. This

⁴⁶See a similar logic in Armstrong, Vickers, and Zhou (2009) where a prominent firm which is always sampled first by consumers in a single-product search scenario charges a lower price than its non-prominent rivals.

suggests that firms may compete intensely via advertised prices to attract consumers, and compensate the possibly resulted loss by charging high prices for unadvertised products (which can be sustained because of costly search). Compared to Lal and Matutes (1994), our richer setting may better predict which products will be sold as loss leaders. This remains another interesting future research topic.

Appendix A

Proof of Lemma 2: We only prove the result for \tilde{p}_2 (the proof for \tilde{p}_1 is similar). The price \tilde{p}_2 , when both firms ignore the joint search effect, is given by

$$\begin{aligned} \frac{1}{\tilde{p}_2} &= 2 \int_B f_2(u_2) dF(\mathbf{u}) + \int_{a_1}^{\bar{u}_1} [1 - F_2(\phi(u_1))] f(u_1, \phi(u_1)) du_1 \\ &= \int_{\underline{u}_1}^{\bar{u}_1} \left\{ 2 \int_{\underline{u}_2}^{\phi(u_1)} f_2(u_2) h_2(u_2|u_1) du_2 + [1 - F_2(\phi(u_1))] h_2(\phi(u_1)|u_1) \right\} dF_1(u_1) \end{aligned} \quad (25)$$

(Note that for $u_1 < a_1$, $\phi(u_1)$ is independent of s and $1 - F_2(\phi(u_1)) = 0$.) The regularity condition (14) implies $f_2(x)h_2(x|u_1) + [1 - F_2(x)]h_2'(x|u_1) \geq 0$. Then the bracket term in (25) is an increasing function of $\phi(u_1)$. Moreover, $\phi(u_1)$ is the only element related with s and it decreases with s .⁴⁷ Therefore, (25) decreases with s , i.e., \tilde{p}_2 increases with s .

Proof of Proposition 2: In the case of symmetric products, from (13) we know the standard effect is

$$\begin{aligned} \alpha &= 2 \int_B f(u_i) dF(\mathbf{u}) + \int_a^{\bar{u}} [1 - F(\phi(u))] f(u, \phi(u)) du \\ &= \int_{\underline{u}}^{\bar{u}} \left\{ 2 \int_{\underline{u}}^{\phi(u)} f(u_i) h(u_i|u) du_i + [1 - F(\phi(u))] h(\phi(u)|u) \right\} dF(u) . \end{aligned}$$

(Note that for $u < a$, $\phi(u)$ is independent of a and $1 - F(\phi(u)) = 0$.) Using the notation

$$\lambda(x|u) \equiv f(x)h(x|u) + [1 - F(x)]h'(x|u) , \quad (26)$$

we have

$$\begin{aligned} \frac{d\alpha}{ds} &= \int_a^{\bar{u}} \frac{d\phi(u)}{ds} \lambda(\phi(u)|u) dF(u) \\ &= \int_a^{\bar{u}} \frac{\phi'(u)}{1 - F(u)} \lambda(\phi(u)|u) dF(u) \\ &= - \int_a^{\bar{u}} \frac{f(\phi(x))}{1 - F(\phi(x))} \lambda(x|\phi(x)) dx . \end{aligned}$$

⁴⁷From the definition of $\phi(\cdot)$ in (6), we have $\frac{d\phi(u_1)}{ds} = -\frac{1}{1 - F_2(\phi(u_1))} < 0$ for $u_1 > a_1$, i.e., the reservation frontier moves downward as the search cost rises; and $\phi(u_1) > \bar{u}_2$ for $u_1 < a_1$ and is independent of s .

The second step used

$$\frac{d\phi(u)}{ds} = -\frac{1}{1-F(\phi(u))}, \quad \phi'(u) = -\frac{1-F(u)}{1-F(\phi(u))}, \quad (27)$$

which are both derived from the definition of $\phi(\cdot)$ in (6). The last step is from changing the integral variable from u to $x = \phi(u)$ and using the symmetry of $\phi(\cdot)$.

The joint search effect is $\beta = \int_a^{\bar{u}} [1-F(u)]f(u, \phi(u))du$, and so

$$\begin{aligned} \frac{d\beta}{ds} &= f(a, \bar{u}) - \int_a^{\bar{u}} \frac{d\phi(u)}{ds} [1-F(u)]h'(\phi(u)|u)f(u)du \\ &= f(a, \bar{u}) - \int_a^{\bar{u}} [-\phi'(u)]h'(\phi(u)|u)f(u)du \\ &= f(a, \bar{u}) - \int_a^{\bar{u}} h'(x|\phi(x))f(\phi(x))dx . \end{aligned}$$

The first step used $\frac{da}{ds} = -1/[1-F(a)]$, the second step used (27), and the last step is again from changing the integral variable from u to $x = \phi(u)$. Therefore, $p = 1/(\alpha + \beta)$ increases with s if and only if $\frac{d\alpha}{ds} + \frac{d\beta}{ds} \leq 0$ or the condition (16) in the main text holds.

Now suppose the two products have independent valuations and the marginal density satisfies $f'(u) \geq 0$. Then

$$\begin{aligned} -\frac{d\alpha}{ds} &= \int_a^{\bar{u}} \frac{f(\phi(x))}{1-F(\phi(x))} \{f(x)^2 + [1-F(x)]f'(x)\}dx \\ &\geq \int_a^{\bar{u}} \frac{f(\phi(x))}{1-F(\phi(x))} f(x)^2 dx \\ &\geq \frac{f(a)}{1-F(a)} \int_a^{\bar{u}} f(x)^2 dx \\ &\geq \frac{f(a)^2}{1-F(a)} \int_a^{\bar{u}} f(x) dx = f(a)^2 , \end{aligned}$$

and

$$\begin{aligned} -\frac{d\beta}{ds} &= \int_a^{\bar{u}} f'(x)f(\phi(x))dx - f(a)f(\bar{u}) \\ &\geq f(a)[f(\bar{u}) - f(a)] - f(a)f(\bar{u}) = -f(a)^2 . \end{aligned}$$

Therefore, $\frac{d\alpha}{ds} + \frac{d\beta}{ds} \leq 0$, i.e., p increases with s .

Proof of Proposition 3: We first derive the first-order conditions for the linear pricing case with m products. Let $\mathbf{u}_{-i} \equiv (u_j)_{j \neq i} \in \mathbb{R}^{m-1}$. In a symmetric equilibrium, without loss of generality the reservation frontier can be defined as $u_m = \phi(\mathbf{u}_{-m})$, where $\phi(\mathbf{u}_{-m})$ satisfies

$$\sum_{i=1}^{m-1} \zeta_i(u_i) + \zeta_m(\phi(\mathbf{u}_{-m})) = s .$$

As in the two-product case, let A denote the acceptance set and B denote its complement. Suppose firm 2 sticks to the equilibrium prices, and firm 1 lowers p_m by a small ε . Following the same logic as in the two-product case, we get the first-order condition for p_m :

$$1 = \underbrace{2p_m \int_B f_m(u_m) dF(\mathbf{u}) + p_m \int_{A_{-m}} [1 - F_m(\phi(\mathbf{u}_{-m}))] f(\mathbf{u}_{-m}, \phi(\mathbf{u}_{-m})) d\mathbf{u}_{-m}}_{\text{standard effect}} + \underbrace{\sum_{i=1}^{m-1} p_i \int_{A_{-m}} [1 - F_i(u_i)] f(\mathbf{u}_{-m}, \phi(\mathbf{u}_{-m})) d\mathbf{u}_{-m}}_{\text{joint search effect}}. \quad (28)$$

This can be understood as follows. First of all, the price reduction leads to a loss $\varepsilon/2$ since the half consumers who buy product m from firm 1 now pay less. The gain from this price reduction consists of three parts. (i) The consumers who sample both firms will buy product m from firm 1 more likely, and this benefit is $\varepsilon/2$ times the first term in (28). (ii) Some consumers who sample firm 1 first will switch from searching on to buying at firm 1 immediately. More precisely, the reservation frontier moves downward by ε along the dimension of u_m . Denote by A_{-m} the projection of A on an $(m-1)$ -dimensional hyperplane with a fixed u_m . Then the measure of these marginal consumers is

$$\frac{\varepsilon}{2} \int_{A_{-m}} f(\mathbf{u}_{-m}, \phi(\mathbf{u}_{-m})) d\mathbf{u}_{-m}.$$

For a marginal consumer with $(\mathbf{u}_{-m}, \phi(\mathbf{u}_{-m}))$, she would come back to buy product m with a probability $F_m(\phi(\mathbf{u}_{-m}))$ even if she searched on. So the net benefit from the increased demand for product m is $\varepsilon/2$ times the second term in (28). (iii) Similarly, the net benefit from the increased demand for all other products is $\varepsilon/2$ times the third term in (28), which is the joint search effect.

Now consider the uniform case with m symmetric products and independent valuations. Then the first integral in (28) measures the volume of solid B , and so it equals one minus the volume of solid A . Since A is $1/2^m$ of an m -dimensional sphere with a radius $\sqrt{2s}$, we get

$$1 - \frac{V_m(\sqrt{2s})}{2^m}.$$

(See the expression for $V_m(\cdot)$ in footnote 24.) The second integral equals

$$\int_{A_{-m}} [1 - \phi(\mathbf{u}_{-m})] d\mathbf{u}_{-m} = \frac{V_m(\sqrt{2s})}{2^m},$$

since it just measures the volume of A . Finally, the third integral equals

$$\int_{A_{-m}} (1 - u_1) d\mathbf{u}_{-m} = \frac{V_m(\sqrt{2s})}{2^{m-1}\pi}. \quad (29)$$

(This equality has no straightforward geometric interpretation. See its proof below.) Then (17) in the main text follows.

Proof of (29): For $m = 2$, $A_{-m} = [a, 1]$ and (29) is easy to be verified. Now consider $m \geq 3$. Let $A_{-1,m}(u_1)$ be a “slice” of A_{-m} at u_1 . Then we have

$$\int_{A_{-m}} (1 - u_1) d\mathbf{u}_{-m} = \int_a^1 (1 - u_1) \left(\int_{A_{-1,m}(u_1)} d\mathbf{u}_{-1,m} \right) du_1 .$$

Since $A_{-1,m}(u_1)$ is $1/2^{m-2}$ of an $(m-2)$ -dimensional sphere with a radius $r = \sqrt{2s - (1 - u_1)^2}$, the internal integral term equals

$$\frac{V_{m-2}(r)}{2^{m-2}} = \frac{\pi^{(m-2)/2} \cdot r^{m-2}}{2^{m-2}\Gamma(m/2)} ,$$

where $\Gamma(\cdot)$ is the Gamma function. Hence,

$$\begin{aligned} \int_{A_{-m}} (1 - u_1) d\mathbf{u}_{-m} &= \frac{\pi^{(m-2)/2}}{2^{m-2}\Gamma(m/2)} \times \int_a^1 (1 - u_1) \left(\sqrt{2s - (1 - u_1)^2} \right)^{m-2} du_1 \\ &= \frac{\pi^{(m-2)/2}}{2^{m-2}\Gamma(m/2)} \times \frac{(\sqrt{2s})^m}{m} \\ &= \frac{V_m(\sqrt{2s})}{2^{m-1}\pi} . \end{aligned}$$

The second step used $a = 1 - \sqrt{2s}$ and the fact that the integrand is the derivative of $\frac{1}{m}(\sqrt{2s - (1 - u_1)^2})^m$ with respect to u_1 . The last step used the expression for $V_m(\cdot)$ and the fact $x\Gamma(x) = \Gamma(x + 1)$.

Proof of Proposition 4: In the case with two symmetric products, if the search costs satisfy $\zeta_i(\underline{u}) < s < 2\zeta_i(\underline{u})$, the equilibrium price p is given by

$$\frac{1}{p} = 2 \underbrace{\int_B f(u_i) dF(\mathbf{u}) + \int_{\underline{u}}^c [1 - F(\phi(u))] f(u, \phi(u)) du}_{\text{standard effect: } \alpha} + \underbrace{\int_{\underline{u}}^c [1 - F(u)] f(u, \phi(u)) du}_{\text{joint search effect: } \beta} ,$$

where $c = \phi(\underline{u})$. This is the same as (13), except the domain of $\phi(\cdot)$ is now different (see Figure 5). Following the same logic as in the proof of Proposition 2, one can verify that

$$\frac{d\alpha}{ds} = -\frac{f(c, \underline{u})}{1 - F(c)} - \int_{\underline{u}}^c \frac{f(\phi(u))}{1 - F(\phi(u))} \lambda(u|\phi(u)) du ,$$

and

$$\frac{d\beta}{ds} = -f(c, \underline{u}) - \int_{\underline{u}}^c h'(u|\phi(u)) f(\phi(u)) du .$$

We aim to show $\frac{d\alpha}{ds} + \frac{d\beta}{ds} \leq 0$ under the proposed conditions. Independent valuations and increasing hazard rate imply $\lambda(u|\phi(u)) \geq 0$. So it suffices to show

$$f(c) f(\underline{u}) \frac{2 - F(c)}{1 - F(c)} + \int_{\underline{u}}^c f'(u) f(\phi(u)) du \geq 0 . \quad (30)$$

If $f' \geq 0$, (30) is obviously true. Now suppose $f' < 0$. Since $f' \geq -\frac{f^2}{1-F}$ (which is implied by the increasing hazard rate condition), the second term in (30) is greater than

$$\begin{aligned} - \int_{\underline{u}}^c \frac{f(u)^2}{1-F(u)} f(\phi(u)) du &\geq - \frac{f(c)}{1-F(c)} \int_{\underline{u}}^c f(u) f(\phi(u)) du \\ &\geq - \frac{f(c)f(\underline{u})}{1-F(c)} \int_{\underline{u}}^c f(u) du \\ &= - \frac{f(c)f(\underline{u})}{1-F(c)} F(c). \end{aligned}$$

The first inequality used increasing hazard rate, and the second one used $f' < 0$. Hence, we only need to have $[2 - F(c)] - F(c) \geq 0$, which is always true.

Proof of Proposition 5: Recall \tilde{p}_i is product i 's price if both firms ignore the joint search effect, and $p_i \leq \tilde{p}_i$. So it suffices to show $\tilde{p}_i \leq p_i^0$. Let us consider product 2 (the proof for product 1 is similar). The price \tilde{p}_2 is defined in (25). We have known that under the regularity condition (14), the bracket term in (25) is an increasing function of $\phi(u_1)$. Since $\phi(u_1) \geq a_2$, it is greater than

$$2 \int_{\underline{u}_2}^{a_2} f_2(u_2) h_2(u_2|u_1) du_2 + [1 - F_2(a_2)] h_2(a_2|u_1).$$

Realizing $\int_{\underline{u}_1}^{\bar{u}_1} h_2(x|u_1) dF_1(u_1) = f_2(x)$, we obtain

$$\begin{aligned} \frac{1}{\tilde{p}_2} &\geq \int_{\underline{u}_1}^{\bar{u}_1} \left\{ 2 \int_{\underline{u}_2}^{a_2} f_2(u_2) h_2(u_2|u_1) du_2 + [1 - F_2(a_2)] h_2(a_2|u_1) \right\} dF_1(u_1) \\ &= 2 \int_{\underline{u}_2}^{a_2} f_2(u_2)^2 du_2 + [1 - F_2(a_2)] f_2(a_2) = \frac{1}{p_2^0}. \end{aligned}$$

Proof of Proposition 6: Following the argument in the main text, we only need to show that the second demand effect has no first-order impact on firm 1's profit. Given that firm 2 sticks to the linear prices (p_1, p_2) and firm 1 deviates to $\hat{p}_i = p_i + \varepsilon$ and $\hat{P} = p_1 + p_2$, a consumer's demand pattern after sampling both firms (conditional on the match utilities at the first sampled firm) is depicted in Figure 10 below, where the dashed lines indicate the boundaries in the linear pricing equilibrium and the solid lines indicate the boundaries after firm 1 deviates. It is clear that more consumers now buy both products from the same firm. For a consumer who samples firm 1 first and then visits firm 2 (so she must have $\mathbf{u} \in B(\varepsilon)$ which converges to B as $\varepsilon \rightarrow 0$), the shaded areas in Figure 10(a) represent the probability (conditional on (u_1, u_2)) that she switches from two-stop shopping to buying from the same firm. The pros and cons of such a change for firm 1 are also indicated in the figure. For example, firm 1 gains p_2 from a consumer who originally only bought product 1 from firm 1 but now buys both products from it,

indicated by “ $+p_2$ ” in the left shaded strip. A similar change occurs to a consumer who samples firm 2 first and then visits firm 1 (so she must have $\mathbf{v} \in B$). This is depicted in Figure 10(b). Due to the symmetry of firms (i.e., for every $\mathbf{u} \in B$ in Figure 10(a), there is a corresponding $\mathbf{v} \in B$ in Figure 10(b)) and the random search order, one can see that all effects in these two figures just cancel out each other.

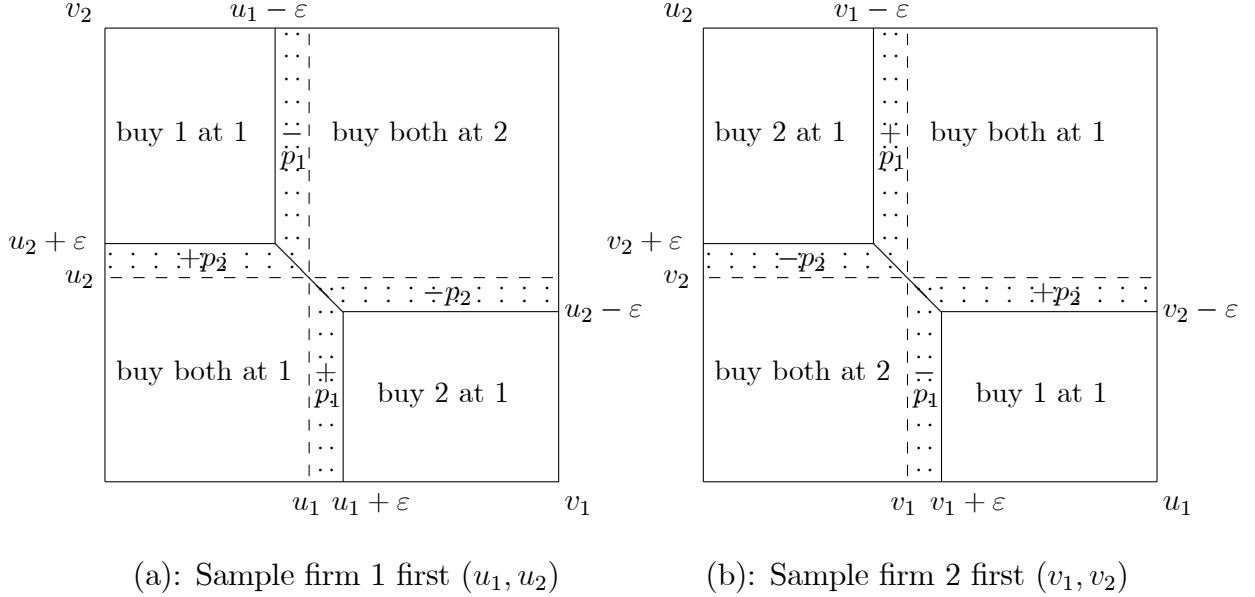


Figure 10: Demand pattern after sampling both firms

Proof of Proposition 7: For given search costs, when the number of products goes to infinity, consumers will always sample both firms. Therefore, we only need to prove the result in the perfect information scenario.

Suppose each firm supplies m products, and each product’s match utility distributes independently according to a cdf $F(\cdot)$ and has a mean μ and variance σ^2 . When m is large, by applying the *central limit theorem*, the match utility of the bundle distributes (approximately) according to a normal distribution $N(m\mu, m\sigma^2)$, so

$$g(U) \approx \frac{1}{\sqrt{2m\pi\sigma}} \exp \left[-\frac{1}{2} \frac{(U - m\mu)^2}{m\sigma^2} \right].$$

When $s = 0$, one can check that (21) implies

$$\frac{1}{P} = 2 \int_{-\infty}^{\infty} g(U)^2 dU \approx \frac{1}{\sqrt{m\pi\sigma}}.$$

That is, the bundle price P rises at the speed of \sqrt{m} . However, in the linear pricing case with $s = 0$ we have

$$\frac{1}{mp} = \frac{2}{m} \int_{-\infty}^{\infty} f(u)^2 du.$$

So the bundle price mp rises at the speed of m . Hence, when $s = 0$, $P < mp$ for a sufficiently large m . (This generalizes Nalebuff, 2000's observation in the Hotelling model with a uniform distribution.)

Now turn to consumer surplus. Denote by $v(0)$ the consumer surplus in the linear pricing case with $s = 0$. Then the expected surplus from each product is

$$\frac{v(0)}{m} = \mathbb{E}[\max(u_i, v_i)] - p .$$

Denote by $V(0)$ the consumer surplus in the pure bundling case with $s = 0$. Then

$$\frac{V(0)}{m} = \mathbb{E} \left[\max \left(\frac{1}{m} \sum_{i=1}^m u_i, \frac{1}{m} \sum_{i=1}^m v_i \right) \right] - \frac{P}{m} .$$

As P rises with m at the speed of \sqrt{m} , P/m tends to zero as $m \rightarrow \infty$. On the other hand, the expectation term tends to μ . So

$$\lim_{m \rightarrow \infty} \frac{V(0)}{m} = \mu .$$

Therefore, when $s = 0$ and m is large, pure bundling improves consumer welfare if

$$\mathbb{E}[\max(u_i, v_i)] - \mu < p .$$

With linear pricing, consumers enjoy better matched goods (which is reflected the left-hand side) but they also pay more (which is reflected the right-hand side). Using $1/p = 2 \int_{\underline{u}}^{\bar{u}} f(u)^2 du$, this condition can be written as $\int_{\underline{u}}^{\bar{u}} u dF(u)^2 - \int_{\underline{u}}^{\bar{u}} u dF(u) < 1/(2 \int_{\underline{u}}^{\bar{u}} f(u)^2 du)$. By integration by parts, it simplifies to

$$\int_{\underline{u}}^{\bar{u}} F(u)[1 - F(u)] du \int_{\underline{u}}^{\bar{u}} f(u)^2 du < \frac{1}{2} .$$

This is further equivalent to

$$\int_0^1 \frac{t(1-t)}{f(F^{-1}(t))} dt \int_0^1 f(F^{-1}(t)) dt < \frac{1}{2} \quad (31)$$

by changing the integral variable from u to $t = F(u)$. (31) holds if f is logconcave by invoking the following lemma.⁴⁸

Lemma 3 *Suppose $\varphi : [0, 1] \rightarrow \mathbb{R}$ is a nonnegative function such that*

$$\int_0^1 \frac{\varphi(t)}{t(1-t)} dt < \infty ,$$

and $h : [0, 1] \rightarrow \mathbb{R}$ is a concave pdf. Then

$$\int_0^1 \frac{\varphi(t)}{h(t)} dt \leq \max \left(\int_0^1 \frac{\varphi(t)}{2t} dt, \int_0^1 \frac{\varphi(t)}{2(1-t)} dt \right) .$$

⁴⁸I am grateful to Tomas F. Mori in Budapest for helping me prove this lemma.

Let $\varphi(t) = t(1 - t)$ and

$$h(t) = \frac{f(F^{-1}(t))}{\int_0^1 f(F^{-1}(t))dt}.$$

Since $f(F^{-1}(t))$ is concave if and only if f is logconcave, the defined $h(t)$ is indeed a concave pdf. (The integral in the denominator is finite since $f(F^{-1}(t))$ is nonnegative and concave.) Then the lemma implies that the left-hand side of (31) is no greater than $1/4$.⁴⁹ (For the exponential density $f(x) = e^{-x}$, it equals $1/4$.)

Proof. Since h is a concave pdf, it is a mixture of triangular distributions and admits a representation of the form

$$h(t) = \int_0^1 h_\theta(t)\mu(\theta)d\theta,$$

where $\mu(\cdot)$ is a pdf defined on $[0, 1]$, $h_1(t) = 2t$, $h_0(t) = 2(1 - t)$, and for $0 < \theta < 1$

$$h_\theta(t) = \begin{cases} 2\frac{t}{\theta} & \text{if } 0 \leq t < \theta \\ 2\frac{1-t}{1-\theta} & \text{if } \theta \leq t \leq 1 \end{cases}.$$

(See, for instance, Example 5 in Csiszár and Móri, 2004.)

By Jensen's inequality we have

$$\frac{1}{h(t)} = \frac{1}{\int_0^1 h_\theta(t)\mu(\theta)d\theta} \leq \int_0^1 \frac{1}{h_\theta(t)}\mu(\theta)d\theta.$$

Then

$$\int_0^1 \frac{\varphi(t)}{h(t)}dt \leq \int_0^1 \varphi(t) \left(\int_0^1 \frac{1}{h_\theta(t)}\mu(\theta)d\theta \right) dt = \int_0^1 \left(\int_0^1 \frac{\varphi(t)}{h_\theta(t)}dt \right) \mu(\theta)d\theta \leq \sup_{1 \leq \theta \leq 1} \int_0^1 \frac{\varphi(t)}{h_\theta(t)}dt.$$

Notice that

$$\int_0^1 \frac{\varphi(t)}{h_\theta(t)}dt = \frac{\theta}{2} \int_0^\theta \frac{\varphi(t)}{t}dt + \frac{1-\theta}{2} \int_\theta^1 \frac{\varphi(t)}{1-t}dt.$$

This is a convex function of θ , because its derivative is

$$\frac{1}{2} \int_0^\theta \frac{\varphi(t)}{t}dt - \frac{1}{2} \int_\theta^1 \frac{\varphi(t)}{1-t}dt,$$

which is increasing in θ . Hence, its maximum is attained at one of the endpoints of the domain $[0, 1]$. This completes the proof. ■

Proof of Proposition 8: Let δ be the equilibrium joint-purchase discount. For a consumer who samples firm 1 first, if she finds out $(u_1, u_2) \in [\bar{u} - \delta, \bar{u}]^2$ and keeps

⁴⁹Our result is not tight. However, if f is non-logconcave, it is easy to find counterexamples. For instance, (31) fails to hold for a power distribution $F(x) = x^k$ with k close to $1/2$, or for a Weibull distribution $F(x) = 1 - e^{-x^k}$ with a small $k \in (0, 1)$.

searching, then she will never mix and match (i.e., both $r_1(\delta)$ and $r_2(\delta)$ are zero) as the potential benefit $\max(v_1 - u_1, v_2 - u_2)$ can never exceed the joint-purchase discount δ . Thus, if the reservation frontier lies within the region of $[\bar{u} - \delta, \bar{u}]^2$, the marginal consumers must behave as in the pure bundling case and buy both products from the same firm, i.e., the reservation frontier should satisfy $u + \phi_\delta(u) = b$. It is indeed within the region of $[\bar{u} - \delta, \bar{u}]^2$ if $b > 2\bar{u} - \delta$. Then, since $\hat{r}_1 = \hat{r}_2 = 0$ and so J_1 in the first equation of (24) equals zero, δ is determined by $2Q_1(\delta) + \delta Q_1'(\delta) = 0$, and so $\delta = \delta_0$.

Appendix B: Calculating Consumer Surplus

In our search model (especially in the case of linear pricing or mixed bundling), it is complicated to calculate consumer surplus directly. Here we develop a more efficient indirect method (which also carries over to the case with more than two firms).

Linear pricing. Consider first the linear pricing case with equilibrium prices (p_1, p_2) . Let Σ be the set of all possible stopping rules. If a consumer adopts a stopping rule $\sigma \in \Sigma$, her expected surplus can be written as $U(\sigma) - st(\sigma)$, where $U(\sigma)$ is the expected match utility minus the expected payment when the stopping rule σ is implemented, and $t(\sigma)$ is the expected search times. Then consumer surplus is $v(s) = \max_{\sigma \in \Sigma} [U(\sigma) - st(\sigma)]$ (note that prices are given and so the impact of s on equilibrium prices is not reflected in this expression), and the expected search times is $t(s) = t(\sigma(s))$, where $\sigma(s)$ is the optimal stopping rule given a search cost s . $v(s)$ is convex since the objective function is linear in s , and so it is differentiable almost everywhere and $v'(s) = -t(s)$. Therefore, we have

$$v(s) = v(0) - \int_0^s t(x) dx ,$$

where $v(0) = \sum_{i=1}^2 (\mathbb{E}[\max(u_i, v_i)] - p_i)$ is the expected consumer surplus when $s = 0$ (but given the equilibrium prices associated with the search cost s), and $t(x) = 2 - A(x)$ (where $A(x)$ is the probability measure of the acceptance set in the linear pricing case when the search cost is x). This result indicates that given the equilibrium prices, consumer surplus can be decomposed into two parts: the surplus when information is perfect minus the expected information gathering cost. Both $v(0)$ and $t(x)$ are straightforward to calculate.

Pure bundling. A similar logic applies in the pure bundling case. Given equilibrium price P , a consumer's expected surplus is

$$V(s) = V(0) - \int_0^s T(x) dx ,$$

where $V(0) = \mathbb{E}[\max(U, V)] - P$ is the expected consumer surplus when $s = 0$ (but given the equilibrium price P associated with the search cost s), and $T(x) = 1 + G(b(x))$ is the expected search times when search cost is x (where $b(x)$ is the reservation utility in the pure bundling case when the search cost is x).

Mixed bundling. In the mixed-bundling case, we have

$$v_\delta(s) = v_\delta(0) - \int_0^s t_\delta(x) dx ,$$

where $v_\delta(0)$ is consumer surplus when $s = 0$ but given prices $(\hat{p}_1, \hat{p}_2, \delta)$ associated with the search cost s , and $t_\delta(x)$ is the expected search times when the search cost is x and the joint-purchase discount is δ .

How to calculate $v_\delta(0)$ needs more explanation. We have

$$v_\delta(0) = \underbrace{\mathbb{E}[\max(u_1 + u_2 + \delta, v_1 + v_2 + \delta, u_1 + v_2, v_1 + u_2)]}_{w(\delta)} - (\hat{p}_1 + \hat{p}_2) .$$

The expectation is taken over all random variables u_i and v_i . $w(0) = \sum_{i=1}^2 \mathbb{E}[\max(u_i, v_i)]$ is straightforward to calculate but $w(\delta)$ for $\delta > 0$ is not. Realize that

$$w'(\delta) = 2\bar{Q}_{12}(\delta) = 1 - 2\bar{Q}_1(\delta) ,$$

where $\bar{Q}_{12}(\delta)$ is the probability that a consumer buys both products from firm i when the search cost is *zero* (i.e., the probability that the first two terms in $w(\delta)$ dominate), and $\bar{Q}_1(\delta)$ is the probability that a consumer buys product 1 alone from firm i when the search cost is *zero* (i.e., the probability that the third or fourth term in $w(\delta)$ dominates). The first equality is because when δ is increased by a small ε , a consumer will benefit ε when she buys both products from the same firm, which occurs with a probability $2\bar{Q}_{12}(\delta)$. (Of course, the change of δ also affects which term dominates in $w(\delta)$, but that effect on $w(\delta)$ is of second order when ε is small.) Thus, we obtain

$$w(\delta) = w(0) + \int_0^\delta [1 - 2\bar{Q}_1(z)] dz .$$

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