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# Estimating Incentive and Welfare Effects of Non-Stationary Unemployment Benefits

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The distribution of unemployment duration in an equilibrium matching model with spell-dependent unemployment benefits displays a time-varying exit rate. Building on Semi-Markov processes, we obtain an expression for the aggregate unemployment rate under endogenous heterogeneous search effort. Structural estimation using a German micro-data set (SOEP) allows us to discuss the effects of a recent unemployment benefit reform (Hartz IV). The reform reduced unemployment by only 0.3%. Contrary to general beliefs, we find that both employed and unemployed workers gain (the latter from an intertemporal perspective). The reason is the rise in the net wage caused by more vacancies per unemployed worker.

JEL Codes: E24, J64, J68, C13

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## 1 Introduction

Continental European unemployment is notorious for its persistence. France, Italy and Germany have had rising unemployment rates from the 1960s up to 2000 and even onward. There seems to be a consensus now that a combination of shocks and institutional arrangements lies at the origin of these high unemployment rates (Ljungqvist and Sargent, 1998, 2007a, b; Mortensen and Pissarides, 1999; Blanchard and Wolfers, 2000). Neither institutions nor shocks alone explain the rise in unemployment: institutions have always been there but unemployment has not (at least not at this level) and shocks have hit many countries but not all countries have high unemployment rates. The step from this shock-institutions insight towards finding a solution to the European unemployment problem seems to be short: As shocks will not go, we need to address the institutions.

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A common suggestion to fight unemployment is to reduce long and generous unemployment benefits. This raises other questions, however: Should one reduce the length and level of unemployment benefits in order to reduce unemployment? One seems to be faced by a classic efficiency-equity trade-off. While reducing unemployment per se is beneficial, income of the unemployed and the insurance mechanism provided by unemployment benefits should not be neglected.

We examine qualitatively and quantitatively the employment and welfare effects of a policy reform which reduces the length and level of unemployment benefits. We use Germany as an example of a continental European country for three reasons. First, the unemployment rate in Germany has been rising for many decades, just as e.g. in France or Italy. Second, the German unemployment benefit system has a two-tier structure which is typical for many OECD countries. Third, the so-called “Hartz IV reform” implemented in January 2005 comprises both the reduction of benefit levels and the cut of the duration of entitlement. Reforms of this type were undertaken in many other OECD countries as well (OECD, 2004).

The reform did not change the level of unemployment insurance (UI) payments. Unemployment assistance (UA) payments, formerly proportional to net earnings before the job loss, were replaced by a uniform benefit level, however. The effect of this new rule on the income of long-term unemployed workers was ambiguous. There are unemployed workers, mainly from low wage groups, whose benefit payments were lower before the reform than afterwards. Those were the “winners” of the reform (47 percent of long-term unemployed) - in a static sense. On the other hand, there were also long-term unemployed with relatively high wages before entering unemployment. These were affected negatively by the new law and their income has dropped (53 percent of long-term unemployed). Despite the fraction of “winners” and “losers” is roughly equal, the gain of the winners has turned out to be lower than the loss of the losers leading to a loss of the average long-term unemployed worker of a bit more than 7% due to Hartz IV (Blos and Rudolph, 2005; OECD, 2007). The reform also adjusted, for workers who entered unemployment from February 2006 onwards, the maximum duration of entitlement to UI payments. It was (almost) uniformly reduced to 12 months (formerly, 15 months was the average).

At first sight, the reforms seem to have worked.<sup>2</sup> The reported unemployment rate dropped from an annual average of 10.5% in 2004 (Bundesagentur für Arbeit, 2009) to 9.0% in 2007. On the other hand, growth rates in Germany were (for German standards) fairly high. While the German economy shrank in 2003, it has recovered since then and probably also created new jobs. The real GDP grew by 0.8% in 2005, by 3.0% in 2006 and by 2.5% in 2007 (Bundesagentur für Arbeit, 2009). The question therefore arises whether the drop in unemployment can be credited to the reform. It is also a priori unclear how strongly various groups were affected by the reform. Did utility of the (short- and long-term) unemployed or employed workers rise or fall? Did firms gain from the reform? What about social welfare?

We provide answers by using a model which combines various strands of the literature and adds some new and essential features. We employ an equilibrium matching framework and extend the standard text-book model for time-dependent unemployment benefits, endoge-

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<sup>2</sup>We talk about reforms as the measures we highlight are part of a series of reforms (Hartz I to Hartz IV). Our central measures were those who probably had the strongest effect and were one of the most discussed in the public.

nous effort, risk-averse households, an exogenous “spell-effect” and Semi-Markov features. Each of these extensions is crucial. Unemployment benefits in our model need to depend on the length of the unemployment spell as this is a feature of basically all OECD unemployment benefit systems. Letting agents optimally choose their effort to find a job, we can analyze the incentive effects of (reforms of) the unemployment benefit system on the search intensity. Risk-averse households are required as we also want to evaluate insurance effects. The spell-effect allows us to obtain - depending on how fast it sets in - rising (up to some upper limit), falling or hump-shaped exit rates. Finally, tools from the Semi-Markov literature are required as they allow us to deduce aggregate (un)employment from individual search. We can thereby compute macro efficiency effects resulting from micro incentives. Without these Semi-Markov tools, we would not be able to formulate an equilibrium model.

We solve this model numerically by looking at Bellman equations as differential equations. This gives us solutions which are as accurate as numerical precision and which do not require us to approximate the model in any way. Optimal behaviour implies an exit rate into employment which is a function of the time spent in unemployment. We thereby obtain a flexible enough endogenous distribution of unemployment duration which we employ for structural estimation of model parameters. Estimation by maximum likelihood is then (relatively) straightforward.

The main theoretical contribution of our analysis is the explicit treatment of the Semi-Markov nature of optimal individual behaviour due to the presence of spell-dependent unemployment benefits: Optimal exit rates not only depend on whether the individual is unemployed (the current state of the worker) but also on how long an individual has been unemployed. While this Semi-Markov aspect has been known for a while, it has not been fully exploited so far in the search literature. Using results from the applied mathematics literature, we obtain analytic expressions for individual employment probabilities contingent on current employment status and duration of unemployment - equations of the so-called Volterra type. They allow us to compute aggregate unemployment rates using a law of large numbers in our pure idiosyncratic risk economy. Given this link from optimal individual behaviour to aggregate outcomes, we can analyze the distribution and efficiency effects of changes in level and length of unemployment benefits.

The main empirical contribution is the careful modelling of exit rates into employment. Individual incentives due to falling unemployment benefits imply more search effort and therefore higher exit rates over time. Empirical evidence shows, however, that exit rates tend to fall - at least after some initial increase over the first 3-4 months of unemployment. We therefore combine individual incentive effects with an exogenous time-decreasing spell-effect and with unobserved heterogeneity. As is well known, the latter implies inter alia falling aggregate exit rates even though individual exit rates are rising. Structural estimation then establishes the importance of the time-effect and the unobserved heterogeneity effect. We find that the model can replicate empirical stylized facts of first rising and then falling exit rates.

The main policy contribution is our emphasis and structural estimation of the trade-off between insurance and incentive effects of labour market policies. The degree of risk-aversion - crucial for understanding the insurance effect - is jointly estimated with exit rates and the spell-effect (and other model parameters). A comparative static analysis using the estimated version of the theoretical model then allows us to derive precise predictions about

the employment and distribution effects of changes in the length and level of unemployment benefits.

Providing a short preview of our results, we find that the reform did decrease the unemployment rate - which is the desirable effect - but only by 0.3% - which is disappointing. Very much to our surprise, we also find that the reform increased wages and decreased profits. Usually, one would expect that a reduction of alternative income reduces wages. While this channel is present in our setup as well, the equilibrium effect of a rise in the number of vacancies per unemployed worker overcompensates the first effect - wages rise when benefits fall. The mechanism is very simple: lower benefits induce the unemployed to search harder which in turn induces firms to open more vacancies per unemployed worker. As wages rise, profits fall.

This rise in wages is also the reason why the reform is social welfare increasing. The uninterested worker (behind the veil of ignorance) experiences a rise in expected utility. This is *not* due to a better insurance mechanism cause by the reform. In fact, the insurance mechanism is worsened when unemployment benefits are reduced. The rise in expected utility is entirely due to the equilibrium increase in the wage. The most surprising finding of our analysis is that even the long-term unemployed workers gain in an intertemporal perspective. Of course, this group loses instantaneously due to the drop of  $UA$  payments. But given the rise in the wage and the probability that they will find a job at some later point, they gain. Given our setup, the Hartz IV reform is Pareto-improving.

Our paper is related to various strands in the literature. From a theoretical perspective, we build on the search and matching framework of Diamond (1982), Mortensen (1982) and Pissarides (1985), recently surveyed by Rogerson et al. (2005). Time-dependent unemployment benefits and endogenous effort have been originally analyzed by Mortensen (1977) in a one-sided job search model. Equilibrium search and matching models include Cahuc and Lehman (2000), Fredriksson and Holmlund (2001).<sup>3</sup> These models, however, are less powerful than our model in explaining the anticipation effect of the reduction in benefits, as exit rates within each benefit regime are constant. There also exists a substantial literature that studies optimal insurance allowing for an arbitrary time path of unemployment benefit payments (Shavell and Weiss, 1979; Hopenhayn and Nicolini, 1997, Shimer and Werning, 2007). Our focus is more of a positive nature trying to understand the welfare effects of existing systems which have a simpler benefit structure than the ones resulting from an optimization approach. We also allow for an unlimited number of transitions between employment and unemployment and take equilibrium effects of wages, vacancies and tax rates into account.<sup>4</sup>

From an empirical perspective, we estimate a parametric duration model (Lancaster, 1990) in which time dependence of the hazard function due to time-dependent benefits is fully described by the equilibrium solution of our theoretical model. Econometric models with time-dependent benefits were originally estimated by van den Berg (1990) and Ferrall

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<sup>3</sup>Albrecht and Vroman (2005) and Coles and Masters (2007) also have time-dependent unemployment payments but they do not analyze the implications for individual effort. Albrecht and Vroman focus on the equilibrium wage dispersion and inefficient job rejection. Coles and Masters model aggregate uncertainty implying implicit transfers between firms and the stabilizing effect this has on the unemployment rate over the cycle.

<sup>4</sup>Acemoglu and Shimer (1999) and Moscarini (2005) use a general equilibrium model, but their setting is restricted to time-invariant benefits only.

(1997).<sup>5</sup> Van den Berg et al. (2004) and Abbring et al. (2005) extend the setting by introducing time dependence due to monitoring and sanctions. In contrast to our model, this literature deals with one-sided job search, which makes application of its estimates in an equilibrium analysis rather difficult. In addition to that, focus on the incentive effect in is only partial (van den Berg et al., 2004; Abbring et al., 2005) and insurance effect remains largely unaddressed. There also exists a larger empirical equilibrium search literature that deals with unemployment benefit heterogeneity (Bontemps et al., 1999), heterogeneity in workers abilities (Postel-Vinay and Robin, 2002) and heterogeneity in workers value of nonparticipation (Flinn, 2006). Unlike in our model, however, neither of these contributions takes time-dependent unemployment benefits or equilibrium wage, vacancy and tax effects into account.<sup>6</sup>

Finally, Semi-Markov methods are taken from the applied mathematical literature, see e.g. Kulkarni (1995) or Corradi et al. (2004). Economic papers which allowed for Semi-Markov features (e.g. Burdett et al. 1985, Aase, 1990, Magnac et al. 1995) focused on time-varying exit rates but did not exploit their full potential, i.e. they did not use Volterra equations which we need here for equilibrium considerations.<sup>7</sup>

The structure of our paper is as follows. In section 2 we present the theoretical model, institutional setting, behaviour of supply and demand sides and the combination of both in economic welfare. Section 3 describes the equilibrium properties of the model. Section 4 illustrates the structural estimation and the underlying data. The simulation results and the evaluation of the institutional reforms are presented in section 5. Section 6 concludes.

## 2 The model

We use a Diamond-Mortensen-Pissarides type matching model and extend it for time-dependent unemployment benefits, endogenous effort, risk-averse households and an exogenous spell-effect. To solve it, we use Semi-Markov tools. The separation rate for jobs is constant and there is no search on the job. We focus on steady states in our analysis. Households are ex-ante identical but endogenously heterogenous in their unemployment duration.

### 2.1 Production, benefits and employment

The economy has a work force of exogenous constant size  $N$ . Employment is endogenous and given by  $L$  and the number of unemployed amounts to  $N - L$ . Firms produce under perfect competition on the goods market and each worker-firm match produces output  $A$ , which is constant. The production process of the worker and the firm can be interrupted

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<sup>5</sup>See also Eckstein and van den Berg (2007) for literature review on nonstationary empirical models. Paserman (2008) provides a structural estimation taking hyperbolic discounting into account.

<sup>6</sup>A general equilibrium model (of economic growth) is estimated by Lentz and Mortensen (2008).

<sup>7</sup>There is a very small academic literature which discusses the Hartz reforms. Heer (2006) provides a “tentative analysis” which does not explicitly look at the effects of a two-tier system. Fahr and Sunde (2009) focus on aspects of the Hartz reforms (Hartz I-III) which do not affect unemployment benefits. Franz et al. (2007) study the effects of Hartz IV in a CGE model focusing on the impact on various household types.

by exogenous causes which occur according to a time-homogenous Poisson process with a constant arrival rate  $\lambda$ .

Unemployed workers receive UI benefits  $b_{UI}$  and UA benefits  $b_{UA}$ . Benefits are modelled to reflect institutional arrangements in many European countries. One of the most important features is the dependence of UI benefits on the unemployment spell. Workers with a spell  $s$  shorter than  $\bar{s}$  (say one year) receive UI benefits  $b_{UI}$ , afterwards, they receive  $b_{UA}$ ,

$$b(s) = \begin{cases} b_{UI} & 0 \leq s \leq \bar{s} \\ b_{UA} & \bar{s} < s \end{cases} . \quad (1)$$

We assume  $b_{UI} > b_{UA} \geq 0$ . Benefits can be paid either at a fixed level or proportional to previous income.

An unemployed worker finds a job according to a time-inhomogeneous Poisson process with arrival rate  $\mu(\cdot)$ . This rate will also be called the job-finding rate, hazard rate or exit rate into employment. We allow this rate to depend on effort  $\phi(s)$  an individual exerts to find a job. Effort depends on the length  $s$  this individual has been spending in unemployment since his last job. If  $s > \bar{s}$ , the individual will be called long-term unemployed.

In addition to effort, the exit rate of an individual will also depend on aggregate labour market conditions and on something which, for simplicity, we call a spell-effect. Labour market conditions are captured by labour market tightness  $\theta$ , i.e. the ratio of the number of vacancies  $M$  divided by the number of unemployed,

$$\theta \equiv M / (N - L) . \quad (2)$$

We assume that effort and tightness are multiplicative: no effort implies permanent unemployment and no vacancies imply that any effort is in vain. The spell-effect captures all factors exogenous to the individual which affects her exit rate into employment. This can include stigma (Vishvanath, 1989), ranking (Blanchard and Diamond, 1994) and gains or losses in individual search productivity. We denote this effect by  $\eta(s)$ . Assuming that a stigma becomes worse the longer  $s$ , we would expect  $\eta(s)$  to fall in  $s$ . Summarizing, the exit rate will be of the form  $\mu(\phi(s(t))\theta, \eta(s))$ .<sup>8</sup>

There is a long discussion in the literature whether aggregate falling exit rates are due to a time effect (as modeled here by  $\eta(s)$ ) or due to unobserved heterogeneity (Kiefer and Neumann, 1981, Flinn and Heckman, 1982 and, non-parametrically, Heckman and Singer, 1984, van den Berg and van Ours, 1996). We take unobserved heterogeneity into account in our empirical part and discuss its effects there.

The outcome of our time-varying exit rate will be an endogenous distribution of unemployment duration. Its density is given by (e.g. Ross, 1996, ch. 2)

$$f(s) = \mu(\phi(s)\theta, \eta(s)) e^{-\int_0^s \mu(\phi(u)\theta, \eta(u)) du} . \quad (3)$$

This density will be crucial later for various purposes including the estimation of model parameters. It is endogenous to the model, as exit rate  $\mu(\phi(s(t))\theta, \eta(s))$  follows from the optimizing behaviour of workers and firms.

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<sup>8</sup>Given our focus on individual search behaviour, we start at the individual level and then derive a matching function (see the discussion following (20)) rather than the other way round. Both ways are of course equivalent.

Unemployment benefit payments to short- and long-term unemployed are financed by a tax rate  $\kappa$  on gross wages. The labour tax  $\kappa$  implies that the net wage is  $w = (1 - \kappa) w^{gross}$ . The number of short-term unemployed workers is  $\int_0^{\bar{s}} f(s) ds (N - L)$  and  $\int_{\bar{s}}^{\infty} f(s) ds (N - L)$  is the number of the long-term unemployed. The budget constraint of the government therefore reads

$$\left( b_{UI} \int_0^{\bar{s}} f(s) ds + b_{UA} \int_{\bar{s}}^{\infty} f(s) ds \right) (N - L) = \kappa \frac{w}{1 - \kappa} L. \quad (4)$$

The government adjusts the wage tax  $\kappa$  such that this constraint holds at each point in time.

## 2.2 Optimal behaviour

Households are infinitely lived and do not save. They have a strictly positive time preference rate  $\rho$ . The present value of having a job is given by  $V(w)$  and depends on the current endogenous wage  $w$  only. Employed workers enjoy instantaneous utility  $u(w)$ . The value  $V(w)$  is constant in a steady state as the wage is constant, but differs across steady states. Whenever a worker loses his job, he enters the unemployment benefit system by obtaining insurance payments  $b_{UI}$  for the full length of  $\bar{s}$ . Workers are immediately granted full benefit entitlements, i.e. unemployment payments are not experience rated. See the wage setting setup for further discussion. Hence, the value of being unemployed when just having lost the job is given by  $V(b_{UI}, 0)$  where 0 stands for a spell of length zero. This leads to a Bellman equation for the employed worker of

$$\rho V(w) = u(w) + \lambda [V(b_{UI}, 0) - V(w)]. \quad (5)$$

The Bellman equation for the unemployed worker reads

$$\rho V(b(s), s) = \max_{\phi(s)} \left\{ u(b(s), \phi(s)) + \frac{dV(b(s), s)}{ds} + \mu(\phi(s)\theta, \eta(s)) [V(w) - V(b(s), s)] \right\}. \quad (6)$$

The instantaneous utility flow of being unemployed,  $\rho V(b(s), s)$ , is given by three components. The first component shows the instantaneous utility resulting from consumption of  $b(s)$  and effort  $\phi(s)$ . The second component is a deterministic change of  $V(b(s), s)$  as the value of being unemployed changes over time. The third component is a stochastic change that occurs at job-finding rate  $\mu(\phi(s)\theta, \eta(s))$ . When a job is found, an unemployed gains the difference between the value of being employed  $V(w)$  and  $V(b(s), s)$ .

An optimal choice of effort  $\phi(s)$  for (6) requires

$$u_{\phi(s)}(b(s), \phi(s)) + \mu_{\phi(s)}(\phi(s)\theta, \eta(s)) [V(w) - V(b(s), s)] = 0, \quad (7)$$

where subscripts denote (partial) derivatives. It states that the expected utility loss resulting from increasing search effort must be equal to expected utility gain due to higher effort.

We require that the value of unemployment an instant before becoming a long-term unemployed is identical to the value of being long-term unemployed at  $\bar{s}$ , i.e.

$$V(b_{UI}, \bar{s}) = V(b_{UA}, \bar{s}). \quad (8)$$

The value of a job  $J$  to a firm is given by instantaneous profits  $A - w/(1 - \kappa)$ , which is the difference between revenue  $A$  and the gross wage  $w/(1 - \kappa)$ , reduced by the risk of being driven out of business

$$\rho J = A - w/(1 - \kappa) - \lambda J, \quad (9)$$

where  $\rho > 0$  stands for the interest rate (being identical to the discount rate of households) and where we anticipate that the value of a vacancy is zero.

Given that individual arrival rates are functions of the individual unemployment spell, the expected rate of exit out of unemployment is just the mean over individual arrival rates, given the endogenous distribution of the unemployment spell  $f(s)$  from (3),

$$\bar{\mu} = \int_0^\infty \mu(\phi(s), \eta(s)) f(s) ds. \quad (10)$$

As a consequence, the vacancy filling rate is  $\theta^{-1}\bar{\mu}$ . The value of a vacant job is  $\rho J_0 = -\gamma + \theta^{-1}\bar{\mu}[J - J_0]$ . With free entry, the value of holding a vacancy is  $J_0 = 0$ , leading to

$$J = \gamma\theta/\bar{\mu}. \quad (11)$$

Concerning wages, we assume that the wage is determined by Nash bargaining. The total surplus is the gain of the firm plus the gain of the worker from the match where the latter depends crucially on the outside option of the worker. We use  $V(b_{UI}, 0)$  as the outside option of the worker implying that all workers (even if only working for an instant or, in the limit, if only bargaining) are entitled to full unemployment benefits, i.e.  $b_{UI}$  over the full length  $\bar{s}$  and  $b_{UA}$  for  $s > \bar{s}$ .<sup>9</sup> An alternative would consist in specifying  $V(b(s), s)$  as the outside option: if the bargain fails, the unemployed worker remains unemployed and continues to receive benefits she received before the unsuccessful bargaining. This would be theoretically interesting as an endogenous wage distribution would arise (see Albrecht and Vroman, 2005) where the distinguishing determinant across workers is the previous unemployment spell. Using an identical outside option for all individuals, however, has the advantage that all workers are homogenous. Once an unemployed finds a job, all history is deleted, all workers are the same and, independently of their employment history, earn the same wage.<sup>10</sup>

Following the steps as in Pissarides (1985) or Lehmann and van der Linden (2007), we end up with a generalized wage equation that reads (see app. B.1.1)

$$(1 - \beta)u(w) + \beta u_w(w)w = (1 - \beta)u(b_{UI}, \phi(0)) + \beta(1 - \kappa)u_w(w) \left[ A + \gamma\theta \frac{\mu(\phi(0), 0)}{\bar{\mu}} \right]. \quad (12)$$

The left hand side corresponds to what in models with risk-neutrality is simply the wage rate  $w$ . On the right hand side, benefits for the unemployed (for risk-neutral households and

<sup>9</sup>In the quantitative part, the “full length”  $\bar{s}$  will be provided by the data. In this sense, entitlement is taken into account.

<sup>10</sup>Our assumption that all workers, even if they have worked only for a second, are entitled to  $b_1$  for the full period of length  $\bar{s}$  is identical to saying that benefit payments are not experience rated. While the absence of experience rating is generally distorting the firms decision to lay off workers (see e.g. Mongrain and Roberts, 2005), this does not play a role in our setup as the separation rate is exogenous. It would be interesting to study the impact of endogenous separation decisions but we leave this for future research.

no time-dependence of effort), are replaced by instantaneous utility from being unemployed,  $u(b_{UI}, \phi(0))$ . The contribution of the production side is translated into “utils” by multiplying with marginal utility and takes tax effects into account. The contribution itself is corrected for the ratio of the instantaneous exit rate out of unemployment  $\mu(\phi(0), 0)$  and the mean exit rate  $\bar{\mu}$ .

Instead of specifying the outside option differently, one could also allow for strategic bargaining. Many recent papers have used strategic bargaining given that either payoffs change over time and Nash bargaining would correspond to myopic behaviour (Coles and Wright, 1998; Coles and Muthoo, 2003), that a careful analysis of on-the-job search makes strategic bargaining more appropriate (Cahuc, Postel-Vinay and Robin, 2006) or that unemployment does not have such a strong effect on bargaining as generally thought (Hall and Milgrom, 2008).<sup>11</sup> Brügemann and Moscarini (2009) find (for a different question) that the quantitative differences between different wage-setting rules are small. Given that we want to focus here on the direct incentive effects of non-stationary unemployment benefits on search effort, we feel justified to “switch off” the strategic channel and leave this for future work.

### 2.3 The social welfare function

In addition to the incentive effect of reforms, we would also like to understand the insurance effect. In a world without moral hazard, optimal unemployment insurance would require unemployment benefits equal to the net wage. With moral hazard, i.e. with effort being a function of unemployment benefits, insurance consideration must take into account that effort decreases in unemployment benefits.

We can easily understand whether the insurance effect was taken into account in an appropriate way by computing expected utility of an individual which is “behind the veil of ignorance”. This is similar in spirit to social welfare functions employed by Hosios (1990) or Flinn (2006). One can alternatively look at this expected utility as average utility over all (employed and unemployed) workers. Expected utility  $EU$  is given by

$$EU \equiv \frac{L}{N}V(w) + \frac{N-L}{N} \left( \int_0^{\bar{s}} V(b_{UI}, s) f(s) ds + \int_{\bar{s}}^{\infty} V(b_{UA}, s) f(s) ds \right). \quad (13)$$

It is computed by the share  $L/N$  of employed workers times their welfare  $V(w)$  plus the share  $(N-L)/N$  of unemployed workers times the average welfare of an unemployed. This average is obtained by integrating over all spells  $s$ , where  $f(s)$  is the endogenous density (3), with exit rates  $\mu(\phi(s)\theta, \eta(s))$  that follow from the steady state solution of the model, and the  $V(b_i, s)$  are the values of being unemployed with a spell  $s$  and benefit payments  $b_i$  from (1).<sup>12</sup>

<sup>11</sup>Coles and Masters (2004) analyse wage setting by strategic bargaining in a matching setup with non-stationary unemployment benefits. They do not consider endogenous search intensity, however.

<sup>12</sup>Our social welfare function does not take the value  $J$  of a firm into account. The resource cost  $\gamma$  for financing a vacancy are also neglected. We could transfer the balance  $\pi L - \gamma M$  to households. As this balance rises in the reform in all of our quantitative analyses, our quantitative results below would not change if the balance was taken into account.

### 3 Equilibrium properties

#### 3.1 Individual (un)employment probabilities

In models with constant job-finding and separation rates, employment can easily be derived by assuming a law of large numbers. Aggregate employment then follows  $\dot{L} = \mu [N - L] - \lambda L$ . With spell-dependent effort, individual arrival rates  $\mu(\cdot)$  are heterogeneous and employment needs to be derived using techniques from the literature on Semi-Markov or renewal processes, e.g. Kulkarni (1995) or Corradi et al. (2004). We need employment in order to compute the unemployment rate and for computing income and expenditure in the government budget constraint. These Semi-Markov tools are therefore essential for any equilibrium model with time-dependent unemployment benefits.

The generalization of Semi-Markov processes compared to continuous time Markov chains consists in allowing the transition rate from one state to another to depend on the time an individual has spent in the current state. We apply this here and let the transition rate from unemployment to employment depend on the time  $s$  the individual has been unemployed. Hence, switching from a constant job-finding rate  $\mu$  to a spell-dependent rate  $\mu(s)$  implies switching from Markov to Semi-Markov processes. Processes are called “semi” as the history-dependence of the job finding rate  $\mu(s)$  is not Markov. Processes are still called “Markov” as once an individual has found a job, history no longer counts. This is also why these processes are called renewal processes: whenever a transition to a new state occurs, the system starts from the scratch, it is “renewed” and history vanishes.

We start by looking at individual employment probabilities. Let  $p_{ij}(\tau, s(t))$  describe the probability with which an individual, who is in state  $i$  (either  $e$  for employed or  $u$  for unemployed) today in  $t$ , will be in state  $j \in \{e, u\}$  at some future point in time  $\tau$ , given that his current spell is now  $s(t)$ . These expressions read, starting with  $s(t) = 0$  and taking into account that the separation rate  $\lambda$  remains constant (see app. A.5),

$$p_{uu}(\tau, 0) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) p_{eu}(\tau - v) dv, \quad (14a)$$

$$p_{eu}(\tau) = \int_t^\tau e^{-\lambda[v-t]} \lambda p_{uu}(\tau - v, 0) dv. \quad (14b)$$

Expressions for complementary transitions are given by  $p_{ue}(\tau) = 1 - p_{uu}(\tau)$  and  $p_{ee}(\tau) = 1 - p_{eu}(\tau)$ , respectively.

These equations have a straightforward intuitive meaning. Consider first the case of  $\tau$  being not very far in the future. Then all integrals (for  $\tau = t$ ) are zero and the probability of being unemployed at  $\tau$  is, if unemployed at  $t$ , one from (14a) and, if employed at  $t$ , zero from (14b). For a  $\tau > t$ , the part  $e^{-\int_t^\tau \mu(s(y))dy}$  in (14a) gives the probability of remaining in unemployment for the entire period from  $t$  to  $\tau$ . An individual unemployed today can also be unemployed in the future if he remains unemployed from  $t$  to  $v$  (the probability of which is  $e^{-\int_t^v \mu(s(y))dy}$ ), finds the job in  $v$  (which requires multiplication with the exit rate  $\mu(s(v))$ ) and then moves from employment to unemployment again over the remaining interval  $\tau - v$  (for which the probability is  $p_{eu}(\tau - v)$ ). As this path is possible for any  $v$  between  $t$  and  $\tau$ , the densities for these paths are integrated. The sum of the probability of remaining unemployed all of the time and of finding a job at some  $v$  but being unemployed again at

$\tau$  gives then the overall probability  $p_{uu}(\tau, 0)$  of having no job in  $\tau$  when having no job in  $t$ . Note that there can be an arbitrary number of transitions in and out of employment between  $v$  and  $\tau$ . The interpretation of (14b) is similar. The probability of remaining employed from  $t$  to  $v$  is simpler,  $e^{-\lambda[v-t]}$ , as the separation rate  $\lambda$  is constant.

As we can see, these equations are interdependent: The equation for  $p_{uu}(\tau)$  depends on  $p_{eu}(\tau - v)$  and the equation for  $p_{eu}(\tau)$ , in turn, depends on  $p_{uu}(\tau - v)$ . Formally speaking, these equations are integral equations, sometimes called Volterra equations of the first type (14b) and of the second type (14a). Integral equations can sometimes be transformed into differential equations, which will simplify their solution in practice. In our case, however, no transformation into differential equations is known.

After having computed the probability of being unemployed in  $\tau$  when being unemployed in  $t$  for individuals that just became unemployed in  $t$ , i.e. who have a spell of length  $s(t) = 0$ , we will need an expression for  $p_{uu}(\tau, s(t))$ . This means, we will need the transition probabilities for individuals with an arbitrary spell  $s(t)$  of unemployment. Luckily, given the results from (14a and b), this probability is straightforwardly given by

$$p_{uu}(\tau, s(t)) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) p_{eu}(\tau - v) dv. \quad (15)$$

An unemployed with spell  $s(t)$  in  $t$  has different exit rates  $\mu(s(y))$  which, however, are known from our analysis of optimal behaviour at the individual level. Hence, only the integrals in (15) are different, the probabilities  $p_{eu}(\tau - v)$  can be taken from the solution of (14a and b).

### 3.2 Aggregate unemployment

Given our finding in (14) and (15) on  $p_{eu}(\tau)$  and  $p_{uu}(\tau, s(t))$ , we can now compute the expected number of unemployed for any distribution of spell  $F(s)$ ,

$$E_t[N - L_\tau] = [N - L_t] \int_0^\infty p_{uu}(\tau, s(t)) dF(s(t)) + p_{eu}(\tau) L_t. \quad (16)$$

Starting at the end of this equation, given there are  $L_t$  employed workers in  $t$ , the expected number of unemployed workers at some future point  $\tau$  out of the group of those currently employed in  $t$  is given by  $p_{eu}(\tau) L_t$ . Again, one should keep in mind that the probability  $p_{eu}(\tau)$  allows for an arbitrary number of switches between employment and unemployment between  $t$  and  $\tau$ , i.e. it takes the permanent turnover into account.

For the unemployed, we compute the mean over all probabilities of being unemployed in the future, if unemployed today, by integrating over  $p_{uu}(\tau, s(t))$  given the current distribution  $F(s(t))$ . Multiplying this by the number of unemployed today,  $N - L_t$ , gives us the expected number of unemployed at  $\tau$  out of the pool of unemployed in  $t$ . The sum these two expected quantities gives the expected number of unemployed at some future point  $\tau$ .

The expected unemployment rate at  $\tau$  is simply the expression (16) divided by  $N$ . When we focus on a steady state, we let  $\tau$  approach infinity. In order to obtain a simple expression for the aggregate unemployment rate and to show the link to the textbook expression, we assume a pure idiosyncratic risk model where micro-uncertainty cancels out at the aggregate level. Hence, we assume a law of large numbers holds and the population

share of unemployed workers equals the average individual probability of being unemployed. This “removes” the expectations operators, so that (16) for a steady state becomes  $N - L = [N - L] \int_0^\infty p_{uu}(s(t)) dF(s(t)) + p_{eu}L$ . We have replaced  $L_\tau = L_t$  by the steady state employment level  $L$  and the individual probabilities by the steady state expressions  $p_{uu}(s(t))$  and  $p_{eu}$ . The probability  $p_{eu}$  is no longer a function of  $\tau$  as this probability will not change in steady state, while there will always be a distribution of  $p_{uu}(s)$ , even in steady state.

Solving for the unemployment rates gives

$$\frac{N - L}{N} = \frac{p_{eu}}{p_{eu} + [1 - \int_0^\infty p_{uu}(s(t)) dF(s(t))]} = \frac{p_{eu}}{p_{eu} + \int_0^\infty p_{ue}(s(t)) dF(s(t))}, \quad (17)$$

where the second expression is more parsimonious. If we assumed a constant job arrival rate here, we would get  $p_{eu} = p_{uu} = \lambda / (\lambda + \mu)$  and  $p_{ue} = \mu / (\lambda + \mu)$ . Inserting this into our steady state results would yield the standard expression for the unemployment rate,  $(N - L) / N = \lambda / (\lambda + \mu)$ . In our generalized setup, the long-run unemployment rate is given by the ratio of individual probability  $p_{eu}$  to be unemployed when employed today divided by this same probability plus  $1 - \int_0^\infty p_{uu}(s(t)) dF(s(t))$ .

### 3.3 Functional forms and steady state

For estimation purposes and for the numerical solution, we need functional forms for the instantaneous utility function and for the arrival rate. We assume that the instantaneous utility function of an unemployed worker used e.g. in (6) is

$$u(b(s), \phi(s)) = \frac{b(s)^{1-\sigma} - 1}{1 - \sigma} - \phi(s). \quad (18)$$

Effort is measured in utility terms.<sup>13</sup> The utility function of an employed worker has the same structure only that consumption is given by  $w$  and there is no explicit effort. One could therefore look at  $\phi$  as a measure of the difference between disutility from searching and disutility from work.

For reasons that will be discussed in the next section, let us express the spell-effect as

$$\eta(s) = \eta_0 g(s). \quad (19)$$

The arrival rate of jobs  $\mu(\phi(s)\theta, \eta(s))$  is assumed to obey

$$\mu(\phi(s)\theta, \eta(s)) = \eta(s) [\phi(s)\theta]^\alpha, \quad (20)$$

This specification can easily be made plausible when linking it to a matching function. The matching function represents the aggregate arrival rate and equals the sum over individual arrival rates,  $m(N - L, V) = (N - L) \int \mu(s) f(s) ds$ . Defining  $\Omega \equiv \int \eta(s) \phi(s)^\alpha f(s) ds$

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<sup>13</sup>The term “-1” is essential here. While omitting “-1” would still yield risk aversion, including it makes the utility function continuous in  $\sigma$ , insuring that the likelihood function satisfies necessary regularity conditions.

and using (2), we find  $m(N - L, V) = \Omega [N - L]^{1-\alpha} V^\alpha$ .<sup>14</sup> This shows that we succeed in identifying the elasticity  $\alpha$  of vacancies as we assume that both effort and tightness have the same power  $\alpha$  in (20). The more appropriate interpretation for  $\alpha$  is therefore the elasticity of effort.

In a steady state, all aggregate variables are constant and there will be a stationary distribution for unemployment spells. The solution of the steady state can most easily be found in two steps. Taking the wage  $w$  and labour market tightness  $\theta$  as exogenous, one can use expressions related to the unemployed for effort, the value of being unemployed and the value of a job,  $\phi(b(s), s)$ ,  $V(b(s), s)$  and  $V(w)$ . Once these quantities are known, one can use the remaining equations of the model to solve for the wage rate and tightness,  $w$  and  $\theta$ . In doing so, all other endogenous variables (exit rate  $\mu(s)$  and the implied density  $f(s)$ , instantaneous utilities  $u(\cdot)$ , the tax rate  $\kappa$ , individual employment probabilities  $p_{ij}$  and the implied number of short- and long-term unemployed and the unemployment rate  $(N - L)/N$ , the number of vacancies, the value function  $J$  for the firm and social welfare  $EU$ ) are determined as well.<sup>15</sup>

## 4 Structural estimation

### 4.1 Exit rates out of unemployment

Before we estimate the model using the data from the German Socio-Economic Panel (SOEP), we need to specify the functional forms for our spell-effect (19).<sup>16</sup> In order to do so, we consider the distributional aspects of our data on the observed unemployment duration. Fig. 1 shows the nonparametric estimates of the hazard functions from the entire sample of unemployment durations (solid line) and the subsample of individuals with the entitlement length equal to 12 months (dashed line).<sup>17</sup> The illustration focuses on the first 2.5 years of unemployment.

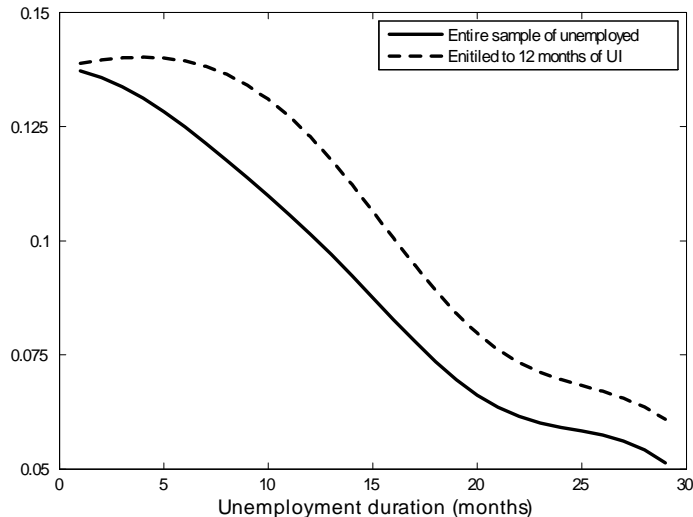
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<sup>14</sup>Note that one could argue (see e.g. Cahuc and Lehman, 2000 or Fredriksson and Holmlund, 2001) that the individual arrival rate is a function of the ratio  $V/(\Omega U)$  and not of the ratio  $\theta = U/V$  as used here. The former specification would assume a negative externality: If some other unemployed workers search harder, the arrival rate of an individual - ceteris paribus - decreases. Computing the aggregate matching function would then yield  $m(U, V) = (\Omega U)^{1-\alpha} V^\alpha$ . We do not believe that this will make a major quantitative difference and we therefore stick to our specification. For details, see app. B.1.3.

<sup>15</sup>Appendix A.2 provides an explicit presentation of all equations (which above in the model description are given implicitly) and describes the solution procedure.

<sup>16</sup>For more background on the SOEP, our sampling method and for descriptive statistics, see app. A.1.

<sup>17</sup>We use the Tanner and Wong (1983) smoothed nonparametric hazard function estimator with bias correction. Kernel function is Gaussian. Optimal bandwidth is estimated by cross-validation discussed in Tanner and Wong (1984).



**Figure 1** *Non-parametric hazard functions (entire sample and  $\bar{s} = 12$ )*

From this figure we can see a clear downward time dependence of the exit risk. On the one hand this may be the evidence of the true downward state dependence of an individual hazard rate (see e.g. van den Berg and van Ours, 1996, or Eckstein and Wolpin, 1995, for the evidence of this). On the other hand, this may be due to some neglected heterogeneity. Generally, individuals may differ with respect to certain observed and unobserved characteristics that condition the unemployment duration (see e.g. Heckman and Singer, 1984a,b). Furthermore, as far as German benefit system is concerned, an additional source of unobserved heterogeneity is the eligibility to UA benefits, since not all individuals receiving UI benefits may be eligible to UA benefits once the entitlement period expires. Eligibility to UA benefits is determined at the “means test”, where an individual has to provide lengthy information about income sources of the household, number and age of dependents etc. If the means are sufficient, the person becomes ineligible to UA benefits, but might still claim social assistance, which eventually may or may not be provided. Unobservability, in this context, is in the fact that once exit out of unemployment occurs before the expiration of entitlement, an econometrician cannot know about the outcome of the means test. The individuals themselves, however, are very likely to know what the result of the test will be. So, once they do not expect to pass the test, they would search harder and therefore exit faster. This behaviour, if uncontrolled for, results in a decreasing nonparametric estimate of the hazard rate, just as it would be under the influence of some individual-specific unobserved component. Clearly the true individual exit rate in this case may as well be constant, or increasing up to the expiration of entitlement and constant thereafter, as in Mortensen (1977), van den Berg (1991) and also in our theoretical model. Finally, both true individual state dependence and unobserved heterogeneity may manifest themselves simultaneously (see e.g. van den Berg and van Ours, 1996, 1999, for evidence of this in U.S. and French data respectively).<sup>18</sup>

<sup>18</sup>Here we also need to note that in what follows we would rather view as a failure at the means test only a situation in which a complete ineligibility to either UA benefits or social assistance obtains.

Thus the individual exit rate derived from the theoretical model must be sufficiently flexible to capture both of these aspects. It needs to have a parametric form flexible enough to replicate the observed downward pattern even if no unobserved heterogeneity is present. Simultaneously it needs to make provision for unobserved heterogeneity with respect to the outcome of the means test and unmeasured individual characteristics, so that it could match the observed downward pattern even if the true individual exit risks are increasing or constant. Our aim is to provide a fully structural econometric model that estimates the deep parameters of the theoretical model of Section 2 adequately addressing both above possibilities.

## 4.2 Econometric model

- Specification

Our data are sampled as a flow of entrants into unemployment and employment. Therefore, exit rates contain all information necessary for the construction of the likelihood function. The exit rate from our theoretical model is given by (20). The effort level  $\phi(s)$  needs to be replaced by the optimal value implied by the first-order condition (7), i.e.  $\phi(s)$  is a function of the duration  $s$  given the particular benefit environment (insurance and assistance benefits together with the entitlement length), the spell-effect  $\eta(s)$ , the wage  $w$  and the labour market density  $\theta$ . To stress this dependence but to keep notation short, we group these explanatory variables into a vector  $\mathbf{z} \equiv \{b_{UI}, b_{UA}, \bar{s}, w, \theta\}$  and write  $\phi = \phi(s; \mathbf{z})$ . With respect to this group of variables it is finally important to notice that even though  $w$  and  $\theta$  are endogenous to the theoretical model they are exogenous to the duration of unemployment which is our dependent variable. Therefore one can either substitute them out using their theoretical solutions, which depend on the productivity  $A$  and vacancy costs  $\gamma$  among others, or one can use the data on  $w$  and  $\theta$  directly. Using the data on  $w$  and  $\theta$  directly simplifies an already complex numerical task of fitting the nonstationary model considerably, making it faster by a factor of about 4. Moreover, it lifts the necessity of having employer-side data which are unavailable in SOEP as well as in any typical panel household survey.

Clearly, there also exist other variables that may potentially influence both the search effort and the spell-effect, affecting thereby the exit rate to employment (20). These variables may be either observable, such as gender, education level etc., or unobservable, like for instance individual ability of performing a job or looking for jobs. We group the observed explanatory variables in a vector  $\mathbf{x}$  and assume that  $\mathbf{x}$  enters the spell-effect and the separation rate with parameters  $\zeta_\eta$  and  $\zeta_\lambda$ , respectively. Likewise, we represent the unobserved component by  $\nu$  and assume that it influences the spell-effect and the separation rate via the parameters  $\xi_\eta$  and  $\xi_\lambda$ , respectively. Hence, the spell-effect (19) now reads  $\eta(s; \mathbf{x}, \nu) = \eta_0(\mathbf{x}, \nu) g(s)$  and the separation rate reads  $\lambda(\mathbf{x}, \nu)$ .<sup>19</sup>

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<sup>19</sup>In principle, it would always be desirable to integrate some aspects of the described ex ante heterogeneity into the theoretical model right from the beginning. Our objective, however, is to focus on the implications of a two-tier benefit system under optimal effort with anticipated end date of the entitlement. We therefore leave theoretical treatment of complementary ex ante heterogeneity for future research. It is standard in the literature to model the basic feature of optimal behaviour one is interested in and capture the rest of heterogeneity in the econometric part (see e.g. van den Berg and Ridder, 1998, or Flinn, 2006).

Having introduced observed and unobserved characteristics, the exit rate from (20) can be written as

$$\mu_j(s; \mathbf{x}, \mathbf{z}, \nu) = \eta(s; \mathbf{x}, \nu) [\phi(s; \mathbf{x}, \mathbf{z}, \nu) \theta]^\alpha, \quad j = UI, UA, \quad (21)$$

where for ease of distinction  $j$  indicates either the UI or the UA regime.<sup>20</sup> In (21) also note that since the optimal search effort depends on both the spell-effect and the separation rate (see app. A.2),  $\mathbf{x}$  and  $\nu$  will always implicitly enter the  $\phi(s; \mathbf{z})$  via  $\eta(s; \mathbf{x}, \nu)$  and  $\lambda(\mathbf{x}, \nu)$ .

We have four different types of labour market trajectories in our data set. The first group consists of individuals who enter unemployment with the right to claim UI benefits and exit unemployment before the expiration of entitlement period ( $s \leq \bar{s}$ ). As argued in Section 4.1, for these individuals we do not observe the outcome of the means test for eligibility to  $b_{UA}$ . We do assume, however, that individuals know about the outcome even before applying for  $b_{UA}$ . Let  $\phi(s; \mathbf{z}|0)$  indicate the search effort given that  $b_{UA} = 0$ , which corresponds to the hypothetical failure at the test. Similarly, let  $\phi(s; \mathbf{z}|b_{UA})$  stand for the hypothetical case in which the test will be passed (and so,  $b_{UA} > 0$ ). Finally, let  $\pi$  denote the fraction of the individuals that pass the test. Then, given the observed and unobserved characteristics, the individual contribution to the likelihood in this group is

$$\begin{aligned} \ell(s; \mathbf{x}, \mathbf{z}, \nu) &= \pi [\mu_{UI}(s; \mathbf{x}, \mathbf{z}, \nu|b_{UA})]^{d_u} e^{-\int_0^s \mu_{UI}(u; \mathbf{x}, \mathbf{z}, \nu|b_{UA}) du} \\ &\quad + (1 - \pi) [\mu_{UI}(s; \mathbf{x}, \mathbf{z}, \nu|0)]^{d_u} e^{-\int_0^s \mu_{UI}(u; \mathbf{x}, \mathbf{z}, \nu|0) du}, \end{aligned} \quad (22a)$$

where  $d_u$  is a dummy variable such that  $d_u = 1$  if unemployment spell is uncensored.<sup>21</sup>

The second group comprises individuals who enter unemployment with the right to claim UI benefits, fail to find a job before entitlement expires, transit to either UA or zero benefit level and thereby reveal the outcome of the means test, and exit unemployment (or not) only after the expiration of entitlement ( $s > \bar{s}$ ). For such individuals, the contribution to the likelihood is given by

$$\ell(s; \mathbf{x}, \mathbf{z}, \nu) = e^{-\int_0^{\bar{s}} \mu_{UI}(u; \mathbf{x}, \mathbf{z}, \nu) du} \pi^{d_t} (1 - \pi)^{1-d_t} [\mu_{UA}(s; \mathbf{x}, \mathbf{z}, \nu)]^{d_u} e^{-\int_s^s \mu_{UA}(u; \mathbf{x}, \mathbf{z}, \nu) du}, \quad (22b)$$

where  $d_t$  is a dummy variable such that  $d_t = 1$  if we observe that an individual passes the means test.

The third group embarks all individuals who do not have the right to claim UI benefits and enter unemployment receiving lower UA benefits from the very beginning ( $d_t = 1$ ) or not receiving benefits at all ( $d_t = 0$ ). Their contribution to the likelihood is therefore

$$\ell(s; \mathbf{x}, \mathbf{z}, \nu) = \pi^{d_t} (1 - \pi)^{1-d_t} [\mu_{UA}(s; \mathbf{x}, \mathbf{z}, \nu)]^{d_u} e^{-\int_0^s \mu_{UA}(u; \mathbf{x}, \mathbf{z}, \nu) du}. \quad (22c)$$

For our final group, which consists of entrants to employment, conditional on observed and unobserved characteristics the contribution is simply

$$\ell(l; \mathbf{x}, \nu) = [\lambda(\mathbf{x}, \nu)]^{d_j} e^{-\lambda(\mathbf{x}, \nu)l}, \quad (22d)$$

<sup>20</sup>Appendix A.3 shows the differential equation that solves for  $\mu_j(s)$  once optimal search effort is substituted out.

<sup>21</sup>In case of failure at the means test and ineligibility to  $b_{UA}$  it is possible to argue that the person still gets some sort of subsistence, e.g. due to within family transfers. One can express this subsistence level by, say, a fraction of  $b_{UI}$  and try to estimate this fraction along with the rest of the parameters. We also follow this way. When trying to estimate this fraction, however, we find that it converges to zero.

where  $d_j$  is a dummy variable such that  $d_j = 1$  if employment spell is uncensored and  $l$  is the duration of employment.

The parameterization of  $\eta_0$  in the spell-effect is standard, namely  $\eta_0(\mathbf{x}, \nu) = \exp\{\mathbf{x}'\boldsymbol{\zeta}_\eta + \xi_\eta \nu\}$ , where  $\mathbf{x}$  contains an intercept. Similarly, the conditional separation rate is parameterized as  $\lambda(\mathbf{x}, \nu) = \exp\{\mathbf{x}'\boldsymbol{\zeta}_\lambda + \xi_\lambda \nu\}$ . Since  $\nu$  is unobservable, identification requires that either  $\xi_\eta$  or  $\xi_\lambda$  should be normalized. We set  $\xi_\lambda = 1$ . Finally, our parametric assumptions about the shape of  $g(s)$  are

$$g(s) = S(s, \delta) + 1, \quad (23)$$

where  $S(s, \delta)$  denotes a survivor function of the chi-square distribution with  $\delta$  degrees of freedom (see app. B.3.2). Intercept in  $\mathbf{x}$  plays a role of a scale parameter in the distribution of unemployment duration, adjusting  $g(s)$  appropriately. Our choice of this particular functional form in (23) is motivated by the nonparametric estimates in fig. 1, where even in absence of unobserved heterogeneity the individual exit risk is well-matched by the reverse S-shaped curve. Sensitivity analysis with other parametric alternatives, e.g. the survivor function of the Weibull distribution, which has one parameter more, have shown no significant improvement in the model fit.

At this end it is only left to specify the distribution of the unobserved component. As common in the reduced-form literature starting from Heckman and Singer (1984b), as well as in the structural estimation literature (see e.g. van den Berg and Ridder, 1998 or Frijters and van der Klaauw, 2006), we assume that unobserved heterogeneity has a discrete distribution with  $K$  points of support,  $\{\nu_k\}_{k=1}^K$ . Let  $p_k$  denote the probability mass attached to the  $k$ -th point of support. With this assumption the likelihood contribution of an unemployed individual in our sample finally reads

$$\ell(s; \mathbf{x}, \mathbf{z}) = \sum_{k=1}^K p_k \ell(s; \mathbf{x}, \mathbf{z}, \nu_k), \quad (24a)$$

where  $\ell(s; \mathbf{x}, \mathbf{z}, \nu_k)$  is any of the contributions in (22a)-(22c), depending on a particular type of the unemployed worker. Similarly, the likelihood contribution of the employed worker reads

$$\ell(l; \mathbf{x}) = \sum_{k=1}^K p_k \ell(l; \mathbf{x}, \nu_k), \quad (24b)$$

where  $\ell(l; \mathbf{x})$  is shown in (22d).

- Estimation procedure

Estimation of model parameters uses a part of the numerical solution method for the steady state. As described in app. A.2, for a given wage  $w$  and vacancy to unemployment ratio  $\theta$ , the individual exit rate can be computed at any moment of the unemployment spell. Using the individual survey data implies that the wage  $w$  for each individual is known and the corresponding  $\theta$  can be taken from macro data. Thus for any given parameter vector individual exit rates immediately follow. Details on the implementation can be found in app. A.3 and app. B.3.1.

Note that the model is estimated without explicitly specifying the wage setting mechanism. If we used linked employer-employee data, the model could be estimated by using the observable productivity data. This would also allow us to estimate the bargaining power parameter  $\beta$  as well as provide more information on the discrepancy between the observed

wage and an endogenous wage solution implied by the model. For the rest of the parameters unrelated to wage setting mechanism, however, both approaches must be equivalent (assuming that wage setting in the second one is correctly specified).

- Identification

The set of model parameters to estimate comprises  $\alpha, \sigma, \delta, \pi, \zeta_\lambda, \zeta_\eta, \xi_\eta$  and  $\{\nu_k, p_k\}_{k=1}^K$ . As just mentioned, bargaining power parameter  $\beta$  drops out once we use the household survey data only. Furthermore, we keep the rate time of preference  $\rho$  fixed.<sup>22</sup>

Identification of the model parameters comes from different sources. From (22d) and (24b) we can see that separation rate parameters  $\zeta_\lambda$  and the distribution of the unobserved component  $\{\nu_k, p_k\}_{k=1}^K$  in  $\lambda(\mathbf{x}, \nu_k) = \exp\{\mathbf{x}'\zeta_\lambda + \nu_k\}, \forall k$ , are always identified from the data on the job duration  $l$  and observed individual characteristics  $\mathbf{x}$ , since the distribution of job duration is simply a finite mixture of exponential distributions which is known to be identifiable. Since  $\mathbf{x}$  contains an intercept, the first mass point in the distribution of unobserved component is normalized to zero (i.e.  $\nu_1 = 0$ ). Finally, since  $\{p_k\}_{k=1}^K$  is a probability mass function, we set  $p_K = 1 - \sum_{k=1}^{K-1} p_k$ .

Next we demonstrate identifiability of  $\eta_0, \alpha$  and  $\sigma$  in a model, where  $\eta_0$  is a single parameter. To do so, we need to consider the endpoint condition that pins down the path of the optimal exit rate at infinity. This condition is shown in equation (A.9) in app. A.3 and corresponds to the exit rate in a completely stationary environment, where the two-step system and the exogenous spell-effect have no impact anymore. From equation (A.9) we see that  $\sigma$  is identified through the variation in the benefit level. For a fixed rate of time preference, conditioning on  $\lambda$ , the middle term in this equation further suggests that variation in the value of employment, which itself stems from the variation in wages and benefits, will identify  $\alpha$  and  $\eta_0$  up to scaling, even if  $\theta$  is a constant. From this follows that in the the solution for the exit rate  $\mu_{UA}$  the unemployment duration, wage and benefit data will identify  $\eta_0, \alpha$  and  $\sigma$ . Clearly, once  $\eta_0$  is identified from the endpoint condition, given that  $\{\nu_k\}_{k=1}^K$  are identified from the data on the employment duration and assuming for the moment that  $\xi_\eta$  is known, the parameters  $\zeta_\eta$  in  $\eta_0(\mathbf{x}, \nu_k) = \exp\{\mathbf{x}'\zeta_\eta + \xi_\eta \nu_k\}, \forall k$ , are always identified from the observed individual characteristics  $\mathbf{x}$ .

Once endpoint condition is identified, identification of the exogenous spell-effect comes from the optimal solution for the time path of the exit rate  $\mu_j(s)$  itself. This path is shown in equation (A.8), app. A.3. From the second term in this equation we see that the spell-effect will always be identified as long as  $\dot{\eta}(s)/\eta(s)$  remains a time-dependent function, which is true for our specification. Identification of the particular parameters in  $g(s)$  will depend on its parametric form. With our specification in (23),  $\delta$  is always identified being just a single parameter.

Finally, the fraction  $\pi$  of those who pass the means test is always identified at least via observability of the outcome of this test in the subsample of individuals that fail to exit unemployment before the entitlement period is over, transiting to a lower-benefit regime, and in the subsample of individuals with no entitlement to UI benefits. This is seen from (22b) and (22c).

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<sup>22</sup>We set  $\rho$  to 0.002, which corresponds to an annual interest rate of 2.4%.

Returning to the coefficient  $\xi_\eta$  that multiplies the unobserved component  $\nu$  in  $\eta_0(\mathbf{x}, \nu)$ , its identification in practice is less clear. This parameter should be identified from the unobserved component  $\nu$  just by the data on unemployment duration that insure the appropriate scaling of  $\nu$  in  $\eta_0(\mathbf{x}, \nu)$ . However, all our attempts to fit the model keeping  $\xi_\eta$  as a free parameter were permanently plagued by very poor convergence and unreliable Hessian matrices evaluated at the maximizer. This has finally made us setting  $\xi_\eta$  to zero. Despite setting this parameter to zero the influence of unobserved component on the exit rate to employment is still preserved, because this exit rate, as seen in (21), is a function of the optimal search effort and the optimal effort is determined by  $\lambda(\mathbf{x}, \nu)$ . Thus, the exit rate retains the conditioning on  $\nu$  and the mixture model (24a) is still valid for all three types of unemployed workers. This makes us believe that setting  $\xi_\eta$  to zero is not restrictive.

### 4.3 Estimation results

The table below reports estimation results for three specifications. Specification I does not condition on the observed and unobserved individual characteristics, specification II conditions only on the observed characteristics, and specification III takes into account both observed and unobserved components. All specifications make provision for unobservability of the results of the means test.

Numerical complexity of the model makes us restricting attention on only a selected number of key characteristics, which are *sex* (=1 if male), *region* (=1 if an individual comes from East Germany), two education dummies: *medium-skilled* (=1 if middle vocational education) and *high-skilled* (=1 if higher vocational or higher education), and two *age* dummies that define the groups of workers up to 30 years old and 31 to 45 years old.

As for the estimation results alone, our main finding is the significance of  $\alpha$ . This means that changes in the optimal effort levels in response to any unemployment benefit reform, be it the reform of  $b_{UI,UA}$  or of  $\bar{s}$ , will have a significant impact on the exit rate out of unemployment. This finding in particular contributes to the empirical reduced form literature that analyses the dependence between unemployment benefits and the probability of leaving unemployment. Evidence on this dependence are somewhat conflicting with earlier Hujer and Schneider (1989) and Arulampalam and Stewart (1995) finding mostly no significant influence of benefits and later Carling et al. (2001) and Røed and Zhang (2003) stating the opposite.<sup>23</sup> Rather than conducting a reduced form analysis, we address the same question from an entirely structural perspective, providing thereby an alternative view. While we do not rule out that for certain types of heterogeneous agents changing benefits may play no role, our result on the significance of  $\alpha$  shows that in sufficiently aggregate terms there exists a positive significant relationship between the reemployment probability and

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<sup>23</sup>Application is to different countries. Moreover, later studies, notably Røed and Zhang (2003), provide more sophisticated treatment of unobserved heterogeneity in comparison to earlier ones.

		(I)				(II)				(III)			
		Coeff.	SE	z-Stat.	p-Value	Coeff.	SE	z-Stat.	p-Value	Coeff.	SE	z-Stat.	p-Value
$\zeta_\lambda$ :	intercept	-4.4976	0.0563	-79.9007	0.0000	-4.4567	0.1805	-24.6865	0.0000	-5.6674	0.4370	-12.9688	0.0000
	sex					0.5403	0.1186	4.5552	0.0000	0.7117	0.1802	3.9494	0.0001
	region					0.8707	0.1140	7.6370	0.0000	1.1316	0.2054	5.5103	0.0000
	medium-skilled					-0.3855	0.1513	-2.5480	0.0108	-0.5102	0.2324	-2.1956	0.0281
	high-skilled					-0.7512	0.1889	-3.9773	0.0001	-0.8449	0.2594	-3.2566	0.0011
	age (up to 30)					-0.3134	0.1556	-2.0139	0.0440	-0.1687	0.2460	-0.6858	0.4928
	age (31 to 45)					-0.4102	0.1548	-2.6500	0.0080	-0.2583	0.2425	-1.0654	0.2867
$\zeta_\eta$ :	intercept	-3.5421	0.5414	-6.5427	0.0000	-3.8200	0.3455	-11.0553	0.0000	-4.0233	0.4737	-8.4936	0.0000
	sex					0.0650	0.1301	0.5000	0.6171	0.0897	0.1245	0.7206	0.4712
	region					-0.0201	0.1310	-0.1531	0.8783	0.0777	0.1395	0.5571	0.5774
	medium-skilled					0.2832	0.1627	1.7408	0.0817	0.2298	0.1737	1.3230	0.1858
	high-skilled					0.4758	0.2115	2.2499	0.0245	0.3937	0.2185	1.8016	0.0716
	age (up to 30)					0.7751	0.1726	4.4901	0.0000	0.7872	0.1711	4.6002	0.0000
	age (31 to 45)					0.6647	0.1581	4.2053	0.0000	0.6597	0.1616	4.0812	0.0000
$\alpha$	0.4945	0.0783	6.3122	0.0000	0.4102	0.0762	5.3848	0.0000	0.4562	0.0750	6.0836	0.0000	
$\sigma$	0.6538	0.1253	5.2196	0.0000	0.7748	0.1010	7.6678	0.0000	0.7022	0.1367	5.1378	0.0000	
$\delta$	11.8035	1.7599	6.7067	0.0000	14.5479	2.9778	4.8855	0.0000	14.8408	3.0935	4.7975	0.0000	
$\pi$	0.2462	0.0297	8.2840	0.0000	0.2448	0.0294	8.3331	0.0000	0.2450	0.0295	8.3182	0.0000	
$\nu$									2.4717	0.1798	13.7493	0.0000	
$p$									0.6404	0.0753	8.5083	0.0000	
$\log \mathcal{L}$			-2920.36				-2853.94				-2819.37		

**Table 1** *Estimation results*

a change in the level of unemployment benefit payments. Consequently, a change in the design of the unemployment benefit mechanism will induce a significant response on the macro level.

Our next important finding is on the role of unobserved heterogeneity. As for heterogeneity with respect to the outcome of the means test, from tab. 1 we can see that  $\pi$  is always different from zero and unity at 5% significance level. Together with the significance of  $\alpha$  this implies that the prospect of not passing the means test significantly increases search effort and exit probability. Moreover, looking at the results of specification III, we find that heterogeneity with respect to further unobserved characteristics also has an impact.<sup>24</sup> Our estimates show a two-point distribution of the unobserved component with the probability mass value  $p$  different from both zero and unity at 5% level. Along with the statistically significant estimate of the mass point this shows a significant improvement of specification III over the specification II.

While importance of unmeasured heterogeneity is frequently highlighted in the literature on the structural estimation of search models, much fewer attempts have been made to explicitly address the exogenous spell dependence within this framework. Since our model is nonstationary from the very beginning, accounting for such spell-dependence is particularly easy. From tab. 1 we can see that the estimate of  $\delta$  is significantly different from zero. However, absence of exogenous spell dependence is not nested in our model, because (23) does not include time-invariance as a special case. Therefore the appropriate test for exogenous spell-dependence is the Vuong (1989) likelihood ratio test for model selection where spell-effect in the competing model is absent, i.e.  $\eta(s; \mathbf{x}, \nu) = \eta_0(\mathbf{x}, \nu)$ .<sup>25</sup> Performing this test for the richest specification, which is specification III, it turns out that the null hypothesis of the equivalence of these two models is clearly rejected in favour of the model with the spell-effect. This means that once key observed and unobserved components are accounted for, there still remains a significant degree of downward state dependence among long-term unemployed. Coexistence of unobserved heterogeneity and duration dependence aligns with the similar finding of van den Berg and van Ours (1996, 1999), who use an entirely different modelling approach.

Finally, our estimate of the utility parameter  $\sigma$  is also very comforting. First, the degree of risk aversion is high enough to reject the hypothesis of risk neutrality. Second, significance of  $\sigma$  provides the empirical evidence on the existence of a significant insurance effect of a benefit reform.

Regarding the fit of the model, from tab. 1 we see a clear improvement from specification I to specification III, the latter being the best of all three. To illustrate the fit of our best specification, for each of the four different subsamples (East Germany, West Germany,

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<sup>24</sup>The estimated number of mass points in the distribution of the unobserved component is equal to two ( $K = 2$ ). Adding the third mass point would always make us converge to any of the estimated two. With  $\nu_1 = 0$  and  $p_2 = 1 - p_1$  the estimates shown in tab. 1 are the estimates of  $\nu_2$  and  $p_1$ . As there is no ambiguity, subscripts are omitted.

<sup>25</sup>This is the test for strictly non-nested models as both models have no conditional distribution in common (see Vuong, 1989, Definition 2). The appropriate test statistic is given in Vuong (1989), Theorem 5.1. We use the Schwarz correction factor to adjust for the difference in the number of parameters in the competing models (see Vuong, 1989, p.318). The calculated value of the adjusted test statistic is 56.82. Asymptotic distribution of the test statistic is standard normal.

males and females) we predict the survivor function and plot it against the Kaplan-Meier nonparametric estimate of the survival probability in unemployment. These plots can be seen in app. A.4, fig. 6. We find that with the exception of the 3rd and 4th months of unemployment duration, where model predictions tend to lie outside the 95% confidence interval for the Kaplan-Meier estimate, our model provides quite an accurate fit to the data. This assures that all our simulations based on the estimated parameters will be well-grounded.

After having estimated all the parameters we are left with determining labour productivity  $A$  and vacancy cost  $\gamma$ . The wage  $w$  and tightness  $\theta$  were taken as exogenous in this first part of the estimation which was built on the household side of the model only. As the wage and tightness are endogenous in equilibrium, we now take the estimated parameters and compute parameters  $A$  and  $\gamma$  using the full equilibrium structure of our economy in the steady state (see app. B.2.2). We compute  $A$  and  $\gamma$  such that the equilibrium endogenous variables  $w$  and  $\theta$  equalize with the average wage and labour market tightness from our data. See tab. 2 for results.

## 5 Evaluating the labour market reforms

In this chapter we use the structurally estimated parameters in order to describe the steady state equilibrium of 2004 and to evaluate the reforms effective as of January 2005.

### 5.1 The pre-reform steady state

Data is heterogeneous in many respects and we have vectors of  $b_{UI,i}$ ,  $b_{UA,i}$ ,  $\bar{s}_i$  and the wage  $w_i$ . Building on the mean wage (used above e.g. to predict  $A$  and  $\gamma$ ), UI payment for our representative agent is given by the replacement rate  $\rho_{UI}$  times the mean wage, corrected for the share  $\omega$  of individuals who are entitled to UI payments,  $b_{UI} = \omega\rho_{UI}w$ . As only an estimated share of  $\pi = 24.4\%$  pass the means test (see tab. 1), UA payments for our representative agent are the product of the replacement rate  $\rho_{UA}$ , the previous wage and the share  $\pi$ ,  $b_{UA} = \pi\rho_{UA}w$ . The replacement rates we use are the averages over statutory replacement rates weighted by population size.<sup>26</sup> Average sample entitlement to UI payments is  $\bar{s} = 12.22$  months, again for those entitled to UI payments. With these means for  $b_{UA}$  and  $\bar{s}$ , our representative agent receives the same amount of benefits at each point in time  $s$  as the mean in the data over a cohort of unemployed who all have an unemployment spell of  $s$  (see app. B.4.1).

All parameters used in this paper apart from the ones presented in the tab. 1 plus some selected endogenous variables are provided in tab. 2. As in the estimation part, the time preference rate is chosen to fit an annual interest rate of 2.4%. The bargaining power  $\beta$  is set equal to .5. (Increasing the time preference rate to 3% or 4% or changing  $\beta$  to .3 or .7 in robustness checks had basically no effects.) As the conditional model dominates the unconditional model, we use these estimates from tab. 1 to predict the average separation

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<sup>26</sup>When  $x\%$  in our sample receive a  $\rho_{UI}$  of 67% and the rest receives 60%, the replacement rate we use is  $x\%67\% + (1 - x\%)60\%$ .

rate  $\lambda$  and the parameter  $\eta_0$  in (20) and (21) for the spell-effect. By representative we understand the average individual in our data set in 2004 (for descriptive statistics of individual characteristics in 2004 see tab. 3 in the Appendix). The corresponding separation rate  $\lambda$ ,  $\eta_0$  and the implied mean exit rate  $\bar{\mu}$  can be seen in tab. 2.<sup>27</sup>

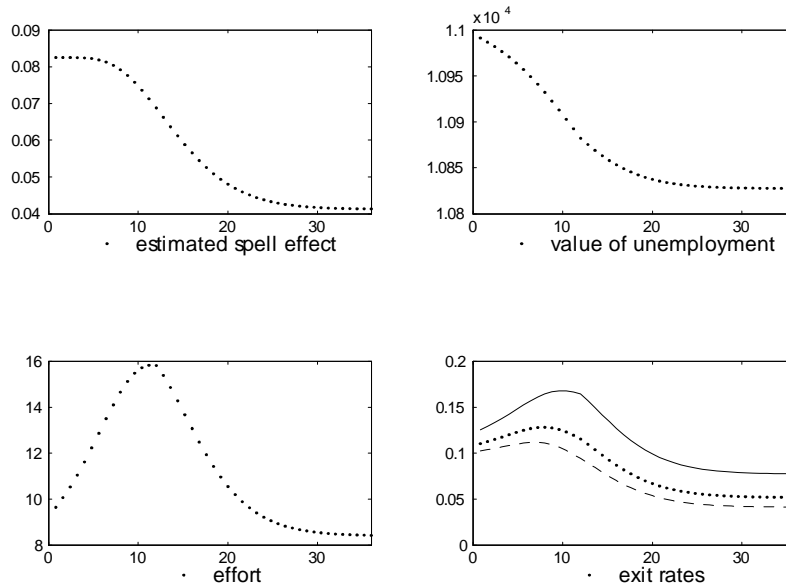
exogenous parameters				predicted parameters				
$\rho$	$\beta$				$\lambda$	$\eta_0$	$A$	$\gamma$
.002	.5				.010	.041	1453.6	11925.1
policy parameters				equilibrium values				
$\rho_{UI}$	$\rho_{UA}$	$\bar{s}$	$\omega$	$w$	$\theta$	$\kappa$	$\bar{\mu}$	$(N - L) / N$
.6	.53	12.22	.56	1172.6€	.20	2.4%	0.11	8.7%

**Table 2** *Parameters and selected equilibrium values*

Our equilibrium values fit perfectly by construction for the wage and labour market tightness. Our tax rate  $\kappa$  is sufficiently close to the actual social security contribution rate (this is the only purpose of taxes in our model). The average matching rate of 0.11 means that the probability to lose a job per month (as we use monthly data) is given by (see Wälde, 2010, ch. 10.5.2)  $e^{-\bar{\mu} * 1} \bar{\mu} * 1 = 9.8\%$ , which is reasonable as well. The only quantity which does not match aggregate statistics at first sight is the unemployment rate. While it is estimated at 8.7% in our model using micro-data, the official aggregate number is 10.5% (Bundesagentur für Arbeit, 2009). This is mainly caused by the different datasources used for our estimation and for official unemployment rates. We do not consider this to be a major problem as our main interest lies in understanding the *change* of the unemployment rate after the reform and not in its level and as the official unemployment rate lies within the 95% confidence intervall of our estimated 8.7%.

For comparative statics below, we will take the exogenous parameters, the estimated and the predicted parameters and the replacement rate for short-term unemployed  $\rho_{UI}$  as given. We will then change long-term benefits  $b_{UA}$  and the entitlement period  $\bar{s}$  to understand the effects on equilibrium values.

<sup>27</sup>Results for the unconditional model or for male and female workers differ by around 0.1%.



**Figure 2** *The spell effect  $\eta(s)$ , the value of being unemployed  $V(b(s), s)$ , effort  $\phi(s)$  and the exit rate  $\mu(b(s), s)$  ( $\cdots b_{UA}$  as in model,  $- b_{UA} = 0$ ,  $- - b_{UA} = b_{UI}$ ) as a function of the spell  $s$  (in months)*

Although the economy is in the steady state, there are still dynamics on the micro level as illustrated in fig. 2. At any point in time individuals find and lose jobs. The upper left panel shows that and how the estimated exogenous spell-effect from (19) with (23) falls over time. The value of being unemployed thereby unambiguously falls over time. This is shown by the upper right panel and needs to hold generally as (A.2) in the appendix shows. The intuition is simple: If there was no spell-effect ( $\eta(s)$  is constant), a long-term unemployed would live in a stationary world and the value of being a long-term unemployed would be stationary as well. With a negative spell-effect, the job finding rate - taking optimally chosen effort into account - goes down and the value of being unemployed approaches a lower limit determined by the lower limit of  $\eta(s)$ .

The optimal reaction of the unemployed worker is shown by the lower left panel. Effort increases during the first 12 months and then starts falling when entitlement to unemployment insurance ceases at  $\bar{s} = 12.2$  months.<sup>28</sup> Note that optimal effort could rise longer or fall earlier than at  $\bar{s}$  as it is the outcome of the interplay of the spell-effect (lower  $\eta(s)$  reduces optimal effort) and the incentive-effect, i.e. the potential gain from finding a job. As gains increase due to a falling value of being unemployed, this second effect tends to increase effort. This can be seen from the first-order condition in (7) or, more directly, from (A.1) in the

<sup>28</sup>Here and in what follows, numerical solutions allow for a certain precision (given time constraints for solving the programme). In our implementation, entitlement length was set to  $\bar{s} = 12$  months and the step length was set to 0.8 months. All robustness checks down to a step length of 0.2 months indicated that the effects of using 12 and 0.8 can be quantitatively neglected indeed.

appendix. The initial increase of effort clearly reflects the rising incentive to search harder the closer  $\bar{s}$ . For our estimation, after around 12 months, the increase in the gain of finding a job is no longer strong enough to compensate the “discouraging” spell-effect. Search effort falls and approaches a constant. The fact that unemployed workers finally “give up” is ultimately the effect of the exogenous negative spell-effect.

The exit rate shows paths for  $b_{UA} = 0$ , average  $b_{UA}$  and  $b_{UA} = b_{UI}$ . Average  $b_{UA}$  is the one in our pre-reform steady state, we will return to the other ones in our analysis of the reform.

## 5.2 The effects of the reform

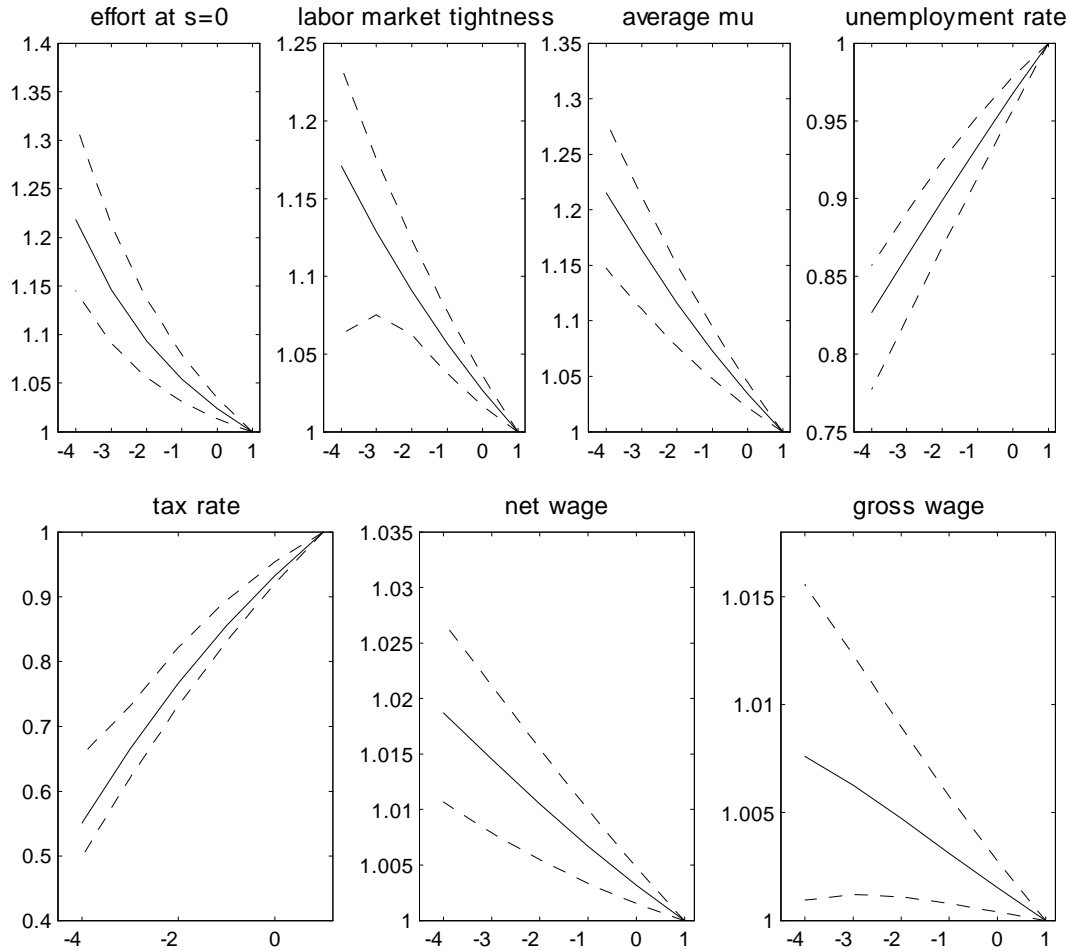
Labour market reforms were characterized by a reduction in UA benefits  $b_{UA}$  and entitlement length  $\bar{s}$ . Benefits decreased on average by 7%, average entitlement length dropped from 12.2 to 10.9 months, i.e. by 10.7%. We present here the effect of the reform, i.e. a joint decrease of both  $b_{UA}$  and  $\bar{s}$ .<sup>29</sup>

The horizontal axis of fig. 3 plots “Hartz-units”. The “1” represents the situation before the reform. The “0” represents the situation after the reform and “-1” to “-4” shows the effects of stronger reforms, i.e. of reducing  $b_{UA}$  by another 7% and  $\bar{s}$  by another 10.7%. The vertical axes plot changes relative to the pre-reform steady state which is normalized (levels were reported in tab. 2 above) to 1. The pre-reform steady state is therefore always given by the point (1, 1). The solid line shows the changes as predicted by our estimated model. The dashed lines above and below provide a 95% confidence interval for the statistic in the middle. This confidence interval is constructed using parametric bootstrap where draws are made from the multivariate normal distribution of the maximum likelihood estimates. For every new draw we recompute the equilibrium solution at each Hartz-unit. Due to the high numerical complexity of the equilibrium solution, confidence intervals at each Hartz-unit are based on 80 replications.<sup>30</sup>

Concerning results, we find - generally speaking - very weak effects of the reform. The qualitative effects of the reform are as intended by policy makers. Effort  $\phi(0)$  when becoming unemployed rises the stronger the reform, i.e. the further we are to the right of the upper left figure. Labour market tightness  $\theta$  and the average job finding rate  $\bar{\mu}$  from (10) rise as well. The increase in  $\theta$ , i.e. the number of vacancies per unemployed worker, is crucial for our interpretation further below. Understanding this effect is simple, however: More effort by unemployed workers makes opening a vacancy more attractive. Hence, lower benefits induce a higher number of vacancies per unemployed worker. The quantitatively weak effect of the reform becomes visible when looking at the unemployment rate. It decreases from the pre-reform steady state “1” to the reform level at “0” to (slightly more than) 97% only. Starting at an unemployment rate of around 10.4%, the effect of the reform would be to decrease the unemployment rate to 10.1%, i.e. by 0.3 percentage points only.

<sup>29</sup>When reducing  $b_{UA}$  and  $\bar{s}$  individually, the results are qualitatively similar. Quantitatively, the individual effects are smaller than the joint effects. The only qualitative difference consists in the value of being a long-term unemployed. While it falls when  $b_{UA}$  is reduced, it rises when  $\bar{s}$  is reduced.

<sup>30</sup>To the best of our knowledge, we are the first to explicitly show confidence intervals for our policy analysis and experiments - even though it turns out that not all of our experiments have a significant effect.



**Figure 3** *Aggregate effects of UA payments  $b_{UA}$  and entitlement length  $\bar{s}$*

The lower figures show that the tax rate falls. This has various reasons: Lower benefit payments and a lower number of recipients reduce overall expenditure. This reduced expenditure is paid by more workers who earn higher gross wages.

One of the most surprising results is the increase in the net and gross wage. This increase is also the basis for the rise in social welfare which we will discuss later. The increase in the net wages becomes clear from the wage equation (12) when taking into account that the positive tightness effect dominates the negative effort effect. This is an interesting feature of this wage bargaining setup with endogenous effort and is in strong contrast to perfectly competitive setups, to bargaining setups with exogenous effort and to search setups where the reservation wage is a simple decreasing function of the outside option. Here, the outside option (utility from being an unemployed worker) decreases as well but this is overcompensated by the positive effect of more vacancies per unemployed worker.

Confidence intervals for all the statistics in fig. 3 show that the changes induced by the reform are statistically significant, as both the upper and the lower bound lie strictly above (or strictly below) unity. Summarizing and ignoring distributional effects, the reform has

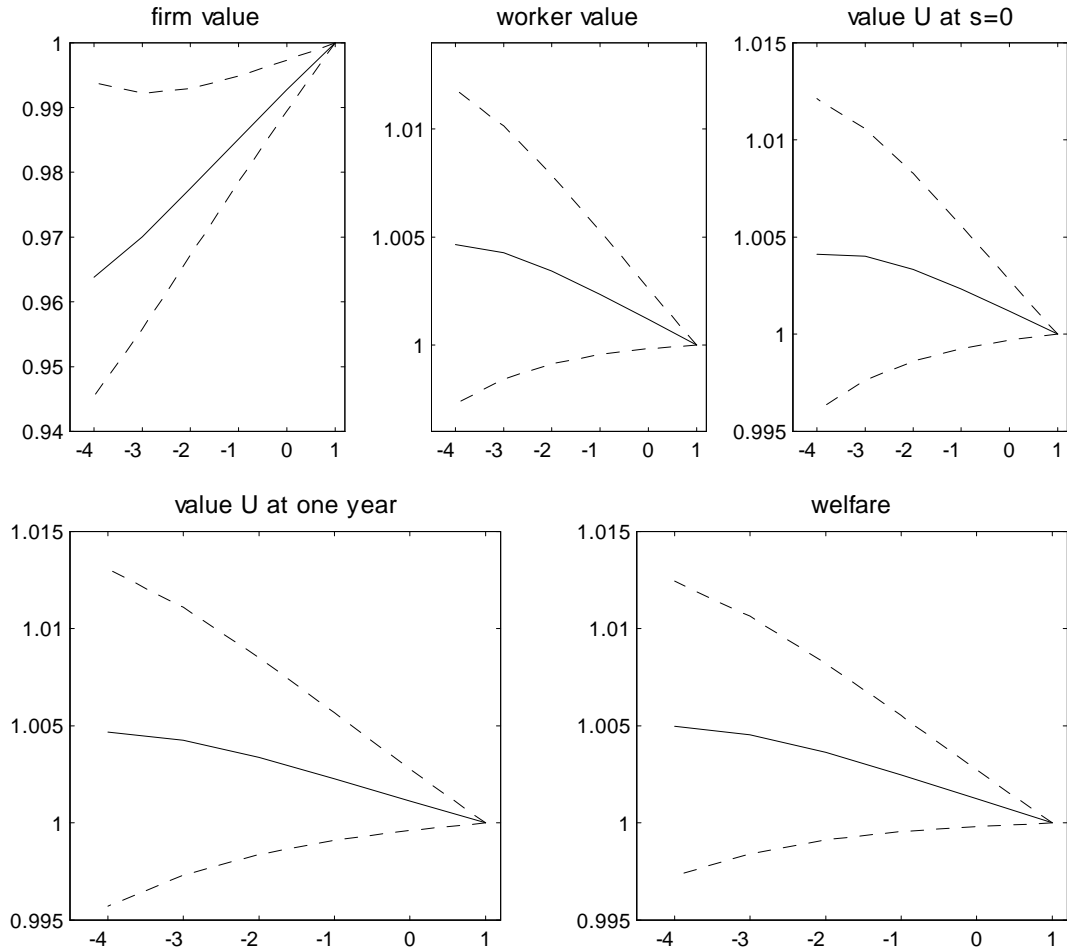
the qualitatively desired effects but is quantitatively of hardly any importance: While the reduction of unemployment is statistically significant, economically, it is not.

When we look at welfare measures in fig. 4, we see the distributional nature of the reform. Employed workers gain from the reform. This might not appear surprising given that the net wage rises. Employed workers do also anticipate, however, the potential loss from the reform as they have a certain risk of becoming unemployed. If the value of being unemployed falls, the value of having a job could fall as well.

When we look at the value of being unemployed at  $s = 0$ , i.e. right after having lost the job, this value slightly *increases* as well due to the reform. This is entirely due to the anticipation effect of having a job. The short-term unemployed clearly loses as benefits will fall earlier and by more due to the reform. But a short-term unemployed worker also gains in an expected intertemporal sense as he will gain more due to the reform in case he finds a job. More formally, the value of unemployment depends negatively on search effort and positively on exit rate, which can be seen from equations (6) and (18). As both search effort and the exit rate go up, each effect could dominate. In our case, the positive exit-rate effect is the stronger one for the short-term unemployed. Our most stunning result is that even the long-term unemployed workers gain when we define long-term as those with an unemployment spell of 12 months. Looking at unemployed workers with a spell of two or three years, we also find that they still gain. Looking at the reason analytically (see app. B.4.2) confirms the intuition: The rise in the wage is so large that any temporary drop in consumption is overcompensated by expected future consumption..

Considering the 95% confidence intervals for each of these three statistics, though, we can see that despite the predicted wins and losses, all these wins and losses are not statistically significant. The confidence bounds lie above and below unity. What is significant, however, are the losses of the firms, i.e. a drop of  $J$  when benefits  $b_{UA}$  are reduced.

The loss of firms is a surprising qualitative result of our analysis and is a direct consequence of the rise in the gross wage. Rewriting (9) slightly shows that the value of the firm  $J$  decreases in the gross wage, hence firms dislike the reform. Is this a strong disadvantage of our model given that employers generally were in favour of the reform in public discussions? We do not think so. The wage can rise or fall in our theoretical setup. Whether the worse outside option of employed workers reduces the wage more than the higher job-finding rate due to higher  $\theta$  is theoretically an open question. Given our estimates, the “ $\theta$ -effect” dominates. While firms urged the unemployed to search harder and the government to pay them lower benefits, they might not have seen that higher vacancies per unemployed worker finally makes workers stronger, wages rise and profits fall.



**Figure 4** *Distributional effects of UA payments  $b_{UA}$  and entitlement length  $\bar{s}$*

What is also surprising - despite the lack of statistical significance - is that overall welfare increases. We measure welfare as in (13). It is identical to expected utility of a worker who does not yet know in which state he is. Apparently, the gains of employed workers and of short-term unemployed workers are stronger than the losses of long-term unemployed workers.

### 5.3 Understanding the effects of the reform

- The magnitude of the effects

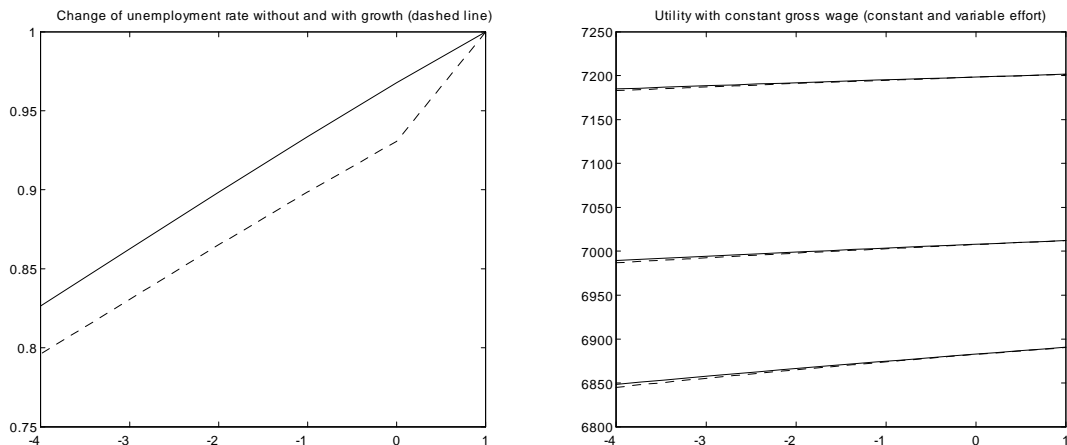
Generally speaking, we find very weak effects of any type of reform. This can be made plausible with a back-of-the-envelope calculation. As the unemployment rate is approx. 10% and only 1/3 becomes long-term unemployed, only 3.3% of the entire labour force are affected. Of these 3.3% only  $\pi = 24.5\%$  pass the means test. In an intertemporal sense, income of the representative household is reduced only during  $24.5\% * 3.3\% \approx 1\%$  of ones' lifetime. The duration of unemployment insurance payments is reduced by 10.7%, the level of the

payments by 7%. Let this add up - to make this simple and high - to 18%. If 1% of lifetime income is reduced by 18%, overall lifetime income reduces by  $1\% \cdot (100-18)\% \approx 0.8\%$ . No surprise that quantitative effects are weak.

- The reform and economic growth

Real GDP in Germany grew by 0.8%, 3.0% and 2.5% in 2005, 2006 and 2007, respectively, i.e. by 6.4% over these three years. TFP grew by 4.0% in the period 2005 to 2007. The question arises whether the drop in unemployment in Germany is not due mainly to TFP growth. From 2004 to 2007 (annual averages of) unemployment in Germany fell from 10.5% to 9.0%. When the growth effect is taken into account here by letting  $A$  increase by 4.0% (Ameco, 2010) when evaluating the effects of the reform, the unemployment rate falls to 93.1% of its initial level (instead of 96.8% without TFP growth), as shown in fig. 5. If we attribute the reduction to 96.8% to the reform and the remaining reduction (amounting to  $96.8\% - 93.1\% = 3.7\%$ ) to economic growth, economic growth was slightly more successful in reducing unemployment than the reform.

- Insurance and social welfare



**Figure 5** *The effect of economic growth on the change of the unemployment rate (left panel) and the pure insurance effect of the reform for employed, short-term and long-term unemployed workers (right panel)*

The most surprising result of our analysis is that workers as a whole gain. This is surprising as one would expect that the insurance mechanism of unemployment benefit payments is reduced following the reduction in  $b_{UA}$ . Remember that a world without moral hazard unemployment benefits should equal the net wage. Given that a reduction in the length  $\bar{s}$  and the level  $b_{UA}$  of unemployment benefits moves our economy further away from this setup, one should expect that the insurance mechanism is reduced. Why is it then that social welfare increases?

To understand this, we divide the effects of the reform into two partial effects. The first partial effect is the insurance effect, i.e. the effect of the reform on social welfare when

the gross wage is being held constant. What would happen if a reduction in  $b_{UA}$  implies a reduction in the tax  $\kappa$  and thereby an increase in the net wage  $w$ ? The result of this thought-experiment is given in the right panel of fig. 5.

The horizontal axis plots the same Hartz-steps as in previous figures. We clearly see that expected utility falls for worker groups when effort is not affected by benefit levels. This clearly reflects that in a world with constant effort, unemployment benefits should equal the net wage. Any departure from this equality reduces expected utility. The same result holds when we allow effort to be a function of benefits, i.e. we allow incentives of the reform to play a role. The figure shows (the three dashed lines) that welfare decreases less (as incentives are now improved) but welfare still falls.

As we know that welfare rises in the equilibrium, we can conclude that all beneficial effects of the reform come from the second partial effect, i.e. the increase in the wage due to the higher number of vacancies per unemployed worker. This stresses the importance of two points: Academically speaking, an evaluation of labour market reforms should be undertaken in equilibrium to provide a complete picture of the effects. More importantly, the Hartz reforms did decrease the insurance mechanism implied by unemployment benefits but - in the end - this was beneficial to the average worker.

## 6 Conclusion

Our project started by inquiring about the effects of the Hartz IV labour market reforms on incentives and insurance mechanisms for the workforce. On a macro level, these questions inquire into the effects on the unemployment rate and social welfare. We have developed an estimable search and matching model with endogenous effort under time-dependent unemployment benefits. The main extension compared to the existing search and matching literature is the endogenous distribution of unemployment duration that arises due to individual choice of search intensity in a nonstationary environment. A link between these micro-dynamics and macro quantities like the unemployment rate was developed using tools from the literature on Semi-Markov processes. The theoretical model provides the density of unemployment duration of an individual being a function of various model parameters. This density provides the basis for structural estimation via maximum likelihood. Equilibrium policy analyses were performed using the parameter estimates of the best fitting specification.

We find that the unemployment rate did decrease due to the reform. Unemployed workers have stronger incentives to search hard. The reduction of the unemployment rate was quantitatively very small, however. While statistically significant, the reduction amounts to 0.3% only. Concerning insurance mechanisms, we find that employed workers and short-term unemployed win but long-term unemployed lose. These results are not statistically significant, however. We also find - much to our own surprise and in contrast to the perception of the public - that generally speaking social welfare increases! As our economy is populated by workers only, this means that workers as a whole gain through the reform.

Looking into the mechanisms in more detail, we find that the gain in social welfare is not due to an improvement in the insurance mechanism. The Hartz IV reform clearly reduced expected utility of a worker - keeping the gross wage constant. As Hartz IV reforms did have equilibrium effects as well, however, expected utility of a worker did increase as the

wages increased. Summarizing, while workers need to search harder, this higher search effort induces firms to open new vacancies. Firms do this so much that the net and gross wage, overall utility for workers and social welfare increases.

These findings clearly give rise to new research questions of which one of the more pressing ones concerns the issue of ex-ante identical individuals. What if we had ex-ante heterogeneity in skill levels? One would expect that social welfare gains would not be as large. Long-term unemployed close to retirement age or with little educational background have low exit rates into employment and they might lose from such a reform. Could a reform be designed in a different way such that it is Pareto-improving? If, by contrast, individuals were allowed to self-insure through savings, consumption drops would be smoother and welfare gains could be even larger than what was found here. If, however, low-income groups hardly hold any wealth, this would not be true for this group. Finally and maybe most importantly, the wage setting mechanism needs more empirical investigation. Given that our results are driven by the positive wage effect, one should inquire into the empirical plausibility of this channel. This can be done by using matched employer-employee data with reliable firm productivity measures and by adding macro channels which might also influence wage setting. Exploring this in more detail is left for future research.

## A Appendix

### A.1 Data

The data comes from the German Socio-Economic Panel (SOEP). The SOEP is a panel surveying households on an annual basis. The survey is coordinated by the Deutsche Institut für Wirtschaftsforschung (Berlin, see [www.gsoep.de](http://www.gsoep.de)).

We draw a flow sample of entrants to employment and unemployment at each month of years 1997-98. The choice of the year of sampling is determined by the fact that no changes to either benefit level or entitlement length were made between the 1st of January 1997 and the 1st of January 2005, when Hartz IV reform came into power. With December 2004 being the latest month of our observation period we end up with a sample that describes a stationary entitlement-benefit environment and provides a fairly reliable information on long-term unemployment (only 9.12% of unemployment durations in our sample are right-censored). For each entrant we retrieve the duration of stay in the current state since the moment of entry. Following van den Berg and Ridder (1998), p.1194, we refrain from considering individuals that allow transitions to the states other than full-time employment or unemployment within the above defined 01.97-12.98 window.

Units of measurement are months for the duration data and Euros for the wage and benefits data. Wage is the average monthly wage for the months of employment within a year prior to job loss, as these are the wage bases that conform with the observed benefit levels. Price base is that of 2004. Descriptive statistics can be found in tab. 4.

It is important to notice that GSOEP data do not contain information on the length of entitlement to UI benefits. There exist, however, strict and relatively simple rules that allow computing the length of entitlement once we know the length of previous job durations and the age of an individual. For this reason, for every person that enters unemployment we

also have to retrieve his/her previous job history. In addition to that, previous job history provides us with the record of the latest wages earned.

The mean of the vacancy-unemployment ratio  $\theta$  between 1997 and 1999 in Germany is 0.2435 (data of the Institute for Employment Research, IAB, adjusted for underreporting).

	Unemployment		Employment		Sample characteristics			
	Mean	StD	Mean	StD	Mean	StD		
Duration ( $s$ )	8.81	13.16	Duration ( $l$ ), censored	57.55	25.73	Males	0.5380	0.4988
UI benefits ( $b_{UI}$ )	727.46	294.94	Duration ( $l$ ), all sample	40.68	30.13	East Germans	0.4227	0.4942
Entitlement ( $\bar{s}$ )	12.18	5.48	obs: total/cens.	694	392	Medium-skill	0.5961	0.4909
Wage ( $w$ )	1166.26	538.07				High-skill	0.2090	0.4068
Share of entitled to UI ( $\omega$ )	0.5657	0.4963				Age up to 30	0.3843	0.4866
Share of $\bar{s} = 12$ among entitled	0.4882	0.5010				Age 31 to 45	0.4264	0.4948
Observed share of passing the test	0.2850	0.4525				obs: total		1067
obs: total/cens.		373 / 34						

**Table 3** *Descriptive statistics (months and EUR)*

Finally, in the predictions for the pre-reform steady state we use the average observed characteristics of the individuals sampled immediately before the reform, i.e. in 2004. For that we similarly draw a flow sample with entry between 01.2004-12.2004 and record personal characteristics. Descriptive statistics for these characteristics is shown in tab. 3.

Sample characteristics		
	Mean	StD
Males	0.4957	0.5005
East Germans	0.3848	0.4870
Medium-skill	0.5109	0.5004
High-skill	0.2565	0.4372
Age up to 30	0.3413	0.4747
Age 31 to 45	0.4152	0.4933
obs: total		460

**Table 4** *Descriptive statistics: Individual characteristics in 2004*

## A.2 Steady state solution

We solve for the steady state of the model by separating the model into two “blocks”.

- Block 1: Household behaviour

Given the functional forms for utility and the spell-effect in (18) and (20), the first-order condition for effort (7) reads

$$\phi(s) = \{\alpha\eta(s)\theta^\alpha [V(w) - V(b(s), s)]\}^{\frac{1}{1-\alpha}}. \quad (\text{A.1})$$

It holds for both short- and long-term unemployed. Plugging this into the Bellman equation for the unemployed (6), using (20) and expressing it as a differential equation in  $s$  gives (see app. B.2.1)

$$\dot{V}(b(s), s) = \rho V(b(s), s) - \frac{b(s)^{1-\sigma} - 1}{1-\sigma} - \frac{1-\alpha}{\alpha} [\alpha\eta(s)\theta^\alpha]^{\frac{1}{1-\alpha}} [V(w) - V(b(s), s)]^{\frac{1}{1-\alpha}}, \quad (\text{A.2})$$

which is again valid for both short- and long-term unemployed. As the value of being unemployed an instant before and an instant after becoming a long-term unemployed is identical, we impose  $V(b_{UI}, \bar{s}) = V(b_{UA}, \bar{s})$  when solving this differential equation. Finally, since for an infinite unemployment spell, the spell-effect in (19) with (23) becomes a constant,  $\lim_{s \rightarrow \infty} \eta(s) = \eta_0$  and all other quantities are stationary as well, we get the terminal condition for (A.2) by using  $\lim_{s \rightarrow \infty} \dot{V}(b_{UA}, s) = 0$ ,

$$\rho V(b_{UA}) = \frac{b_{UA}^{1-\sigma} - 1}{1-\sigma} + \frac{1-\alpha}{\alpha} [\alpha\eta_0\theta^\alpha]^{\frac{1}{1-\alpha}} [V(w) - V(b_{UA})]^{\frac{1}{1-\alpha}}. \quad (\text{A.3})$$

The Bellman equation for the employed worker (5) can be written with the explicit utility function as

$$V(w) = \frac{1}{\rho + \lambda} \left( \frac{w^{1-\sigma} - 1}{1-\sigma} + \lambda V(b_{UI}, 0) \right). \quad (\text{A.4})$$

Now imagine we insert  $V(w)$  from (A.4) into (A.2) and (A.3). Imagine further that we know all parameters and assume, for the time being, some values for  $w$  and  $\theta$ . Then we can solve the differential equation (A.2) starting from some initial value  $V(b_{UI}, 0)$  and see whether the solution for  $s \rightarrow \infty$  is identical to  $V(b_{UA})$  from (A.3). If it does not, we need to adjust our initial guess  $V(b_{UI}, 0)$  until it does. Hence, with some exogenous  $w$  and  $\theta$ , we have obtained the time path of effort over the unemployment spell,  $\phi(b(s), s)$ , the spell-path of the value of being unemployed,  $V(b(s), s)$ , and the value of a job  $V(w)$ .

- Block 2: Wage, tightness and vacancy filling rate

Given the equilibrium values  $\{\phi(b(s), s), V(b(s), s), V(w)\}$  as a function of  $w$  and  $\theta$ , we now endogenize  $w$  and  $\theta$ .

The Bellman equation for the firm and the free entry result, (9) and (11), gives us

$$\frac{A - \frac{w}{1-\kappa}}{\rho + \lambda} = \gamma \frac{\theta}{\bar{\mu}}. \quad (\text{A.5})$$

The bargaining equation (12) can be written as

$$u(w) + \frac{\beta}{1-\beta} u_w(w) w = u(b_{UI}, \phi(0)) + \frac{\beta}{1-\beta} (1-\kappa) u_w(w) \left[ A + \gamma \theta \frac{\mu(\phi(0), 0)}{\bar{\mu}} \right], \quad (\text{A.6})$$

where  $\phi(0)$  is the optimal search effort at the instant of entry into unemployment, which is given from (A.1). The above two equations require the average exit rate  $\bar{\mu}$  and the tax rate  $\kappa$ .

The average rate  $\bar{\mu}$  is given by (10) which can easily be computed given that, after having solved block 1, the exit rates  $\mu(\cdot)$  are known from (20) and the density  $f(s)$  can therefore be computed from (3). The tax rate  $\kappa$  makes the government budget constraint (4) hold and solves

$$b_{UI} U_{short} + b_{UA} U_{long} = \kappa \frac{w}{1-\kappa} L + \Pi. \quad (\text{A.7})$$

Given the density  $f(s)$ , one can compute the number of short-term and long-term unemployed on the right-hand side of this expression from  $U_{short} = (N-L) \int_0^{\bar{s}} f(s) ds$  and  $U_{long} = N-L-U_{short}$ . The number of unemployed  $N-L$  in turn follows from (17), using (14a,b) and (15) which we can now solve, given again that exit rates are known from block 1.

Hence, we are basically left with (A.5) and (A.6) to determine the missing endogenous variables  $w$  and  $\theta$ . After having solved block 1 with a guess of  $w$  and  $\theta$ , we verify whether this guess fulfills (A.5) and (A.6). If not, we (matlab) adjusts the guess until we find a solution.

### A.3 Transition rates to employment

Transition rates to employment are fully described by the optimal search effort. Using first order condition for search effort (7) and the definition of the exit rate (20) we can therefore express value of unemployment as a function of the optimal exit rate. Inserting this value of unemployment into the Bellman equation for the unemployed (6) and expressing it as a differential equation in  $s$  we obtain the time path of the optimal exit transition rate to employment as a result (see app. B.3.1 for derivation). Omitting observed and unobserved characteristics for brevity, the differential equation that describes the exit rate to employment both for the short-term and the long-term unemployed workers is

$$\begin{aligned} \dot{\mu}_j(s) = & \alpha [\mu_j(s)]^2 + \left( \frac{\dot{\eta}(s)}{\alpha \eta(s)} + \rho \right) \frac{\alpha}{1-\alpha} \mu_j(s) \\ & - \frac{\alpha^2}{1-\alpha} [\eta(s) \theta^\alpha]^\frac{1}{\alpha} [\mu_j(s)]^{2-\frac{1}{\alpha}} \left[ \rho V(w) - \frac{b_j^{1-\sigma} - 1}{1-\sigma} \right], \end{aligned} \quad (\text{A.8})$$

where, as before,  $j$  indicates the regime before ( $j = UI$ ) and after ( $j = UA$ ) expiration of unemployment insurance payments. It can be further shown (see app. B.3.1) that the relevant endpoint conditions are

$$(1-\alpha) \mu_{UA} - \alpha [\eta_0 \theta^\alpha]^\frac{1}{\alpha} [\mu_{UA}]^{1-\frac{1}{\alpha}} \left[ \rho V(w) - \frac{b_{UA}^{1-\sigma} - 1}{1-\sigma} \right] + \rho = 0 \quad (\text{A.9})$$

for the second regime at  $s \rightarrow \infty$ , and

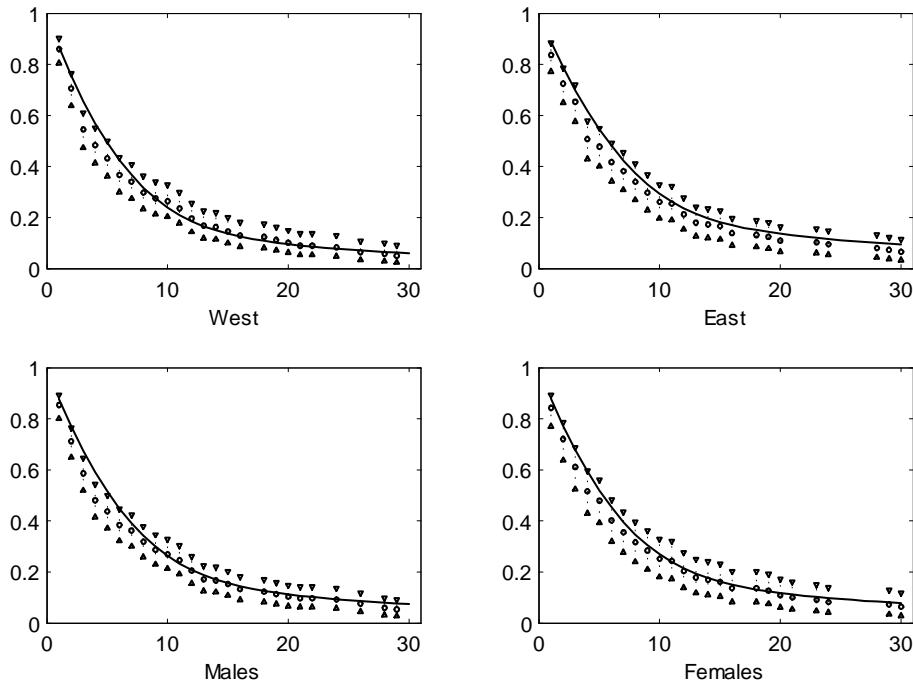
$$\mu_{UI}(\bar{s}; b_{UI}) = \mu_{UA}(\bar{s}; b_{UA}) \quad (\text{A.10})$$

for the first regime at  $s = \bar{s}$ .

Once wages and market tightness are observed from the data,  $V(w)$  can be obtained from the solution to block 1 in app. A.2. Consequently, under assumption that observed  $w$  and  $\theta$  are the direct results of the solution to block 2 in app. A.2, we can compute the equilibrium exit rates to without requiring employer-side data.

## A.4 Estimated and predicted survivor functions

Here we show the predicted survivor functions for heterogeneous population groups (solid line everywhere). Along with model predictions we plot the Kaplan-Meier survivor probabilities (circles) with corresponding 95% confidence intervals (dash lines connecting the triangles).



**Figure 6** *Kaplan-Meier and predicted survivor functions*

## A.5 A Semi-Markov process

This section provides a short introduction into Semi-Markov processes. Technically, it follows Kulkarni (1995) and Corradi et al. (2004). The original work is by Pyke (1961a,b).<sup>31</sup> Due to their technical nature, these papers are less accessible and we hope that this appendix helps

<sup>31</sup>We are grateful to Ludwig Fahrmeir for comments on Semi-Markov processes. For an excellent introduction in German, see Fahrmeir et al. (1981).

that these very useful methods become more widely used. To the best of our knowledge, this is the first application of these concepts in economics. For more details and the numerical implementation, see Schumm (2010, ch. 4). The first subsection describes the general approach to Semi-Markov processes while the second adapts it to our question.

### A.5.1 The general approach

Let  $Y_n$  denote the state of a system after the  $n$ th transition. Let this state be  $i$ . Let the point in time of the  $n$ th transition be denoted by  $S_n$ . Define the probability that the system after the next transition is in  $j$  and that this transition takes place within a period of length  $x$  or shorter, conditional on the system being in  $i$  after the  $n$ th transition, as

$$Q_{ij}(x) \equiv P \{Y_{n+1} = j, S_{n+1} - S_n \leq x | Y_n = i\}.$$

The probability that any transition takes place is then given by summing up the probabilities for each  $j$ ,  $Q_i(x) = \sum_{j \neq i} Q_{ij}(x)$ , not taking into account transitions from  $i$  to  $i$ .<sup>32</sup> The probability that the system will be in  $j$  in  $\tau$  is given by

$$p_{ij}(\tau) = (1 - Q_i(\tau)) \delta_{ij} + \sum_{k \neq i} \int_0^\tau p_{kj}(\tau - x) dQ_{ik}(x). \quad (\text{A.11})$$

The interpretation of this integral equation is as follows: the first part of the right hand side gives the probability that the system, being currently in state  $i$ , never leaves state  $i$  until  $\tau$ . In this case  $j = i$  and  $\delta_{ij} = 1$ , so  $1 - Q_i(\tau)$  is the survival probability in state  $i$ . If  $j \neq i$ ,  $\delta_{ij} = 0$ . The second part of the right hand side collects all cases in which the transition from  $i$  to  $j$  (which includes  $i$ ) occurred via another state  $k \neq i$ . First, we take the probability that the process stayed in state  $i$  for a period of length  $x$  and passed to state  $k$  then (captured by  $Q_{ik}(x)$ ). Then we need the probability that the process which is in state  $k$  after  $x$  will be in state  $j$  at  $\tau$  (captured by  $p_{kj}(\tau - x)$ ). As the transition from  $i$  to  $k$  can be anywhere between 0 and  $\tau$ , we have to integrate over  $x$  in order to cover all possible transitions.

Equation (A.11) can slightly be rewritten, provided that  $Q_{ik}(x)$  is once differentiable (which holds for our case), as

$$p_{ij}(\tau) = (1 - Q_i(\tau)) \delta_{ij} + \sum_{k \neq i} \int_0^\tau p_{kj}(\tau - x) \frac{dQ_{ik}(x)}{dx} dx. \quad (\text{A.12})$$

The derivative  $dQ_{ik}(x)/dx$  now gives the density of going from  $i$  to  $k$  after duration  $x$ . Multiplied by the probability of subsequently going from  $k$  to  $j$  gives the density of ending up in  $j$  after having gone to  $k$  after  $x$ . Integrating over all durations  $x$  gives the probability of starting in  $i$  and being in  $j$  at  $\tau$ .

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<sup>32</sup>We differ from the notation in the cited literature in that we explicitly write  $j \neq i$  here or  $k \neq i$  below. This is equivalent to setting the transition rate from  $i$  to  $i$  to zero. As our application does not have transitions from  $i$  to  $i$  (i.e. transition rates from  $i$  to  $i$  are zero), we find using  $j \neq i$  explicitly more intuitive for our purpose.

### A.5.2 Our two-state system

We now need to adjust the notation such that it suits our purposes. We look at a worker who just moved in  $t$  (like today) into either employment  $e$  or unemployment  $u$ . Define  $Q_{eu}(\tau)$  as the probability that a worker who just found a job in  $t$  “jumps” to  $u$  in a period of time shorter or equal to  $\tau - t$ . With a duration  $s$  dependent arrival rate  $\lambda(s(v))$ , this is then simply given by

$$Q_{eu}(\tau|t_e) = 1 - e^{-\int_t^\tau \lambda(s(v))dv}, \quad (\text{A.13})$$

where  $s(v) = v - t$  is the duration in her current state. In perfect analogy and using a spell-dependent arrival rate  $\mu(s(v))$ , we get  $Q_{ue}(\tau) = 1 - e^{-\int_t^\tau \mu(s(v))dv}$ . For the complementary events - remaining in a given state - the probabilities are simply  $Q_{ee}(\tau) = 1 - Q_{eu}(\tau)$  and  $Q_{uu}(\tau) = 1 - Q_{ue}(\tau)$ . The probabilities that a transition takes place at all in this two state process are

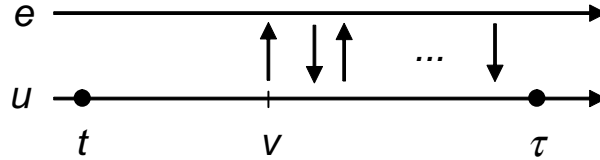
$$Q_e(\tau) \equiv Q_{eu}(\tau), \quad Q_u(\tau) \equiv Q_{ue}(\tau). \quad (\text{A.14})$$

With two possible states, we have four transition probabilities for the future: an unemployed (employed) person can either be unemployed or employed at some future point in time  $\tau$ . Two are redundant as the probability of e.g. an unemployed worker of being employed is complementary to the probability of being unemployed,  $p_{ue}(\tau) = 1 - p_{uu}(\tau)$ , and similarly  $p_{ee}(\tau) = 1 - p_{eu}(\tau)$ . Hence, we only focus on  $p_{uu}(\tau)$  and  $p_{eu}(\tau)$ . These probabilities are, using the general equation (A.12),

$$p_{uu}(\tau) = 1 - Q_u(\tau) + \int_t^\tau p_{eu}(\tau - v) \frac{dQ_{ue}(v)}{dv} dv, \quad (\text{A.15a})$$

$$p_{eu}(\tau) = \int_t^\tau p_{uu}(\tau - v|t_u) \frac{dQ_{eu}(v)}{dv} dv. \quad (\text{A.15b})$$

These equations can be most easily be understood by looking at the following figure.



**Figure 7** *Illustrating transition probabilities*

Let’s consider  $p_{uu}(\tau)$ : An individual unemployed in  $t$  can be unemployed in  $\tau$  by always remaining unemployed. This is the term  $1 - Q_u(\tau)$ . The individual can be unemployed in  $\tau$  by remaining unemployed until  $v$  where she jumps into employment, the density for which is  $dQ_{ue}(v|t_u)/dv$ . After  $v$ , the probability of returning to unemployment in the remaining time span of  $\tau - v$  is  $p_{eu}(\tau - v)$ . Note that this probability includes an arbitrary number of transitions larger than zero in this remaining period  $\tau - v$ . In contrast to integrating over  $x$  as in (A.12), we integrate over the point in time  $v$  here simply as this is the more intuitive way.

As a last step, we need to determine the two derivatives  $dQ_{ue}(v)/dv$  and  $dQ_{eu}(v)/dv$ . Given duration-dependent arrival rates, the derivatives of (A.13) are,

$$\frac{dQ_{ue}(v)}{dv} = e^{-\int_t^v \mu(s(y))dy} \frac{d}{dv} \int_t^v \mu(s(y)) dy = e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) \quad (\text{A.16a})$$

$$\frac{dQ_{eu}(v)}{dv} = e^{-\int_t^v \lambda(s(y))dy} \frac{d}{dv} \int_t^v \lambda(s(y)) dy = e^{-\int_t^v \lambda(s(y))dy} \lambda(s(v)). \quad (\text{A.16b})$$

Given (A.14) and the derivatives, the equations (A.15) become

$$p_{uu}(\tau) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau p_{eu}(\tau - v) e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) dv,$$

$$p_{eu}(\tau) = \int_t^\tau p_{uu}(\tau - v) e^{-\int_t^v \lambda(s(y))dy} \lambda(s(v)) dv.$$

The final adjustment we need to make is to replace  $\lambda(s(v))$  by  $\lambda$  as the separation rate is assumed to be constant. This then gives equations (14) in the main text.

## B Appendix

All references to appendices starting with *B* are available upon request.

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