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## Using Survey Data in the Estimation of Learning and Rational Expectations Models

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# Using Survey Data in the Estimation of Learning and Rational Expectations Models

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## Abstract

Does survey data contain useful information for the estimation of macroeconomic models? In this study we address this question by using data from the Survey of Professional Forecasters in the estimation of a dynamic stochastic general equilibrium model under rational expectations and learning. Exploiting the moments of this data allows us to identify the parameters of these models better, particularly those related to the formation of expectations and inflation dynamics. When using survey data, inflation persistence under rational expectations depends crucially on the persistence of the structural shocks while under learning price indexation and the learning process itself play an important role.

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# 1. Introduction

Expectations are a key element in economic analysis because they affect the decisions of households and firms. One way to model expectations that has gained popularity in recent years is learning. This way to model expectations departs from the benchmark assumption of perfect information, namely Rational Expectations (RE), by assuming that people's expectations about future variables are generated by forecasting models that reflect their imperfect understanding about how the economy works. These models are obtained by analyzing historical data and are updated whenever new data becomes available (see Evans and Honkapohja 2001). Learning is particularly attractive because, where present, it increases the propagation mechanisms of the model and renders the addition of ad hoc elements like persistent exogenous shocks unnecessary. However, learning has also been subjected to criticism as it relies heavily on the researcher's (arbitrary) assumptions about the forecasting model used by the agents to generate their expectations.

In this study we use survey data on inflation expectations in three related ways. Firstly, and in order to address the criticism previously mentioned, survey data is employed to determine the forecasting model that agents most likely use to predict inflation. Secondly, we include survey information to implement model-comparison analysis between RE and learning. In particular, similar to Del Negro and Eusepi (2010), we want to determine how the assessment as to which model fits the data better changes with the inclusion of survey data on inflation expectations in the estimation. Finally, by exploiting the moments derived from survey information, we are also able to identify the parameters of the model better, particularly those related to the formation of expectations.

We find that survey data of inflation expectations can be matched relatively well by small forecasting models and a high speed of learning, much higher than those obtained in previous

studies (e.g. Milani 2007 and Slobodyan and Wouters 2009a). We also provide evidence of the importance of the use of survey data for model-comparison analysis and parameter identification. When survey data is ignored there is no clear support based on the data used in this study in favor of RE or learning. This situation changes once survey data is incorporated: learning receives more support because it is flexible enough to match better the increases and decreases in inflation expectations during the second half of the 1970s and the beginnings of the 1980s. Moreover, the use of survey data highlights the differences in parameter estimates between RE and learning. Under RE, the persistence of the exogenous shocks plays an important role while under learning both price indexation and the learning process itself are the main sources of inflation persistence. Additionally, under learning we find that the use of survey data lets most of the stronger and more persistent responses of inflation be concentrated in the 1970s. In particular, it is observed that unexpected monetary policy shocks have more destabilizing effects on inflation during the 1970s than afterwards as in Boivin and Giannoni (2008).

The estimations implemented in this study are derived from the medium-size New Keynesian DSGE model developed by Smets and Wouters (2009a), one of the benchmark models for empirical analysis. In this sense, this study is related to the work of Slobodyan and Wouters (2009a and 2009b), who are the first to estimate learning in the context of a medium-size DSGE model. Working with this type of model reduces the risk that some omitted variable could distort the contribution of learning to the estimated dynamics.

Few studies have incorporated survey data into the estimation of a DSGE model so far. Del Negro and Eusepi (2010) use survey data on inflation expectations in order to discriminate between a model with imperfect information about the time-varying inflation targets of policy-makers, as in Erceg and Levin (2003), and a model where agents have perfect information about these targets. Additionally, Carboni and Ellison (2009) incorporate the Greenbook

unemployment forecast in the estimation of the model of Sargent et al. (2006) to explain the rise and fall of U.S. inflation during the second half of the twentieth century. The addition of this survey data removes the volatile and unrealistic beliefs of the Federal Reserve about unemployment-inflation dynamics. Our study shares some features of both references. As in Carboni and Ellison's work, we use survey data to improve the identification of the parameters of the model. However, survey data is used as a proxy of private agents' expectations and not of the Federal Reserve's forecasts. We implement the same model-comparison exercise as in Del Negro and Eusepi (2010), but consider a higher-order imperfect information setup, learning, in which the use of survey data plays an important role in the model setting.

The remainder of the paper is organized as follows. In the next section we describe the model used for the estimation. Section 3 discusses the setup of the learning process, while Section 4 presents the series of macroeconomic indicators used in the estimation as well as their relationship with the variables of the model. Section 5 contains the results of estimating the model under RE and learning with and without the use of survey data on inflation expectations. In particular, we show how the parameter estimates differ among the different expectations formation specifications and their effects on the relative importance of the sources of inflation persistence. Section 6 contains some robustness check exercises and Section 7 concludes and outlines possible avenues for future research.

## 2. The Model

Our estimation is based on a New Keynesian model similar to Smets and Wouters (2007).

This model incorporates several frictions affecting both nominal as well as real decisions of households and firms. Households maximize their utility over an infinite life-time horizon. Their utility function depends on the consumption of goods, which is considered relative to a time-varying external habit variable, and on the amount of labor supplied. They also own the

stock of capital in the economy, which they can either rent to firms or accumulate, subject to an adjustment cost. Differentiation in the household's labor provides households with monopolistic power in wage setting. Firms, on the other hand, produce differentiated goods, decide on the amount of labor and capital services and finally set their prices. Both, prices and wages are affected by nominal rigidities *à la* Calvo and additionally incorporate partial indexation with respect to past inflation. Finally, the model features a deterministic growth rate driven by labor-augmenting technological progress.

The version used in this study departs from the original specification by Smets and Wouters (2007) in only two respects. First, monetary policy rule does not adjust to the output gap (i.e. the difference between the output obtained under nominal rigidities and under flexible prices). Instead, monetary policy reacts to changes in the level of output (produced by the economy with rigidities) from one period to the next. This modification allows us to avoid the estimation of a parallel economy under flexible prices, which reduces the number of forward variables in the model considerably (as in Slobodyan and Wouters 2009b)<sup>1</sup>. Second, the stochastic shocks that affect wages and prices directly, namely price and wage mark-up shocks, are assumed to be autoregressive processes that do not incorporate past perturbations (in other words they are AR(1) processes, not ARMA(1,1)).

The model contains the following thirteen endogenous variables: output,  $y$ ; consumption,  $c$ ; investment,  $i$ ; the value of the capital stock,  $Q^k$ ; the installed stock of capital,  $\bar{k}$ ; stock of capital,  $k$ ; inflation,  $\pi$ ; the capital utilization rate,  $u$ ; the real rental rate on capital,  $r^k$ ; the real marginal cost,  $mc$ ; real wages,  $w$ ; hours worked,  $L$ ; and the interest rate,  $R$ . In addition, the stochastic part of the model is characterized by seven exogenous autoregressive processes, each of them including an *iid*-normally distributed error term. After detrending the model with

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<sup>1</sup> This modification, however, does not affect the results obtained by Smets and Wouters (2007).

respect to the deterministic growth rate of the labor-augmenting technological progress and linearizing it around the steady-state of the detrended variables, the model can be summarized in the following set of equations (where  $\hat{\cdot}$  represents detrended variables and  $*$  their steady state values)<sup>2,3</sup>:

$$\begin{aligned}
(1) \quad \hat{y}_t &= \hat{g}_t + \frac{c_*}{y_*} \hat{c}_t + \frac{i_*}{y_*} \hat{i}_t + \frac{r_*^k k_*}{y_*} \hat{u}_t \\
(2) \quad \hat{c}_t &= \frac{1}{(1+h)} (E_t \hat{c}_{t+1} + h \hat{c}_{t-1}) - c_1 (E_t \hat{L}_{t+1} - \hat{L}_t) - c_2 (\hat{R}_t - E_t \hat{\pi}_{t+1}) + \hat{b}_t \\
(3) \quad \hat{i}_t &= \frac{1}{1+\bar{\beta}\gamma} (\hat{i}_{t-1} + \bar{\beta}\gamma E_t \hat{i}_{t+1} + \frac{1}{\gamma^2 S} \hat{Q}_t^k) + \hat{q}_t \\
(4) \quad \hat{Q}_t^k &= -(\hat{R}_t - E_t \hat{\pi}_{t+1}) + \frac{r_*^k}{r_*^k + (1-\delta)} E_t \hat{r}_{t+1} + \frac{(1-\delta)}{r_*^k + (1-\delta)} E_t \hat{Q}_{t+1}^k + \tilde{b}_t \\
(5) \quad \hat{y}_t &= \Phi(\alpha \hat{k}_t + (1-\alpha) \hat{L}_t + \hat{A}_t) \\
(6) \quad \hat{k}_t &= \hat{u}_t + \hat{k}_{t-1} \\
(7) \quad \hat{u}_t &= \frac{1-\psi}{\psi} r_t^k \\
(8) \quad \hat{k}_t &= (1-i_*/k_*) \hat{k}_{t-1} + \frac{i_*}{k_*} \hat{i}_t + \frac{i_*}{k_*} (1+\bar{\beta}\gamma) \gamma^2 S \hat{q}_t \\
(9) \quad \hat{\pi}_t &= \frac{1}{(1+\bar{\beta}\gamma l_p)} (l_p \hat{\pi}_{t-1} + \bar{\beta}\gamma E_t \hat{\pi}_{t+1} + c_{mc} \hat{m}c_t) + \hat{\lambda}_{p,t} \\
(10) \quad \hat{m}c_t &= (1-\alpha) \hat{w}_t + \alpha r_t^k - \hat{A}_t \\
(11) \quad \hat{k}_t &= \hat{w}_t - \hat{r}_t + \hat{L}_t \\
(12) \quad \hat{w}_t &= \frac{1}{(1+\bar{\beta}\gamma)} (\hat{w}_{t-1} + \bar{\beta}\gamma (E_t \hat{w}_{t+1} + E_t \hat{\pi}_{t+1})) - (1+\bar{\beta}\gamma l_w) \hat{\pi}_t + l_w \hat{\pi}_{t-1} + \\
&\quad + c_w \left( \frac{1}{1-h} (\hat{c}_t - h \hat{c}_{t-1}) + \sigma_l \hat{L}_t - \hat{w}_t \right) + \hat{\lambda}_{w,t} \\
(13) \quad \hat{R}_t &= \rho_R \hat{R}_{t-1} + (1-\rho_R) (r_\pi \hat{\pi}_t + r_{\Delta y} (\hat{y}_t - \hat{y}_{t-1})) + \hat{r}_t
\end{aligned}$$

<sup>2</sup> The optimization problem of the households, firms and government as well as the equilibrium conditions are shown in the online appendix at <http://xxx> (see extra pdf file).

<sup>3</sup> The exact representation of the exogenous stochastic process is presented in Appendix A.

Equations (1) to (4) represent the demand side of the economy. Equation (1) is the aggregate resource constraint of the economy and indicates that output is spent on consumption, investment or absorbed via capital-utilization costs that are a function of the capital utilization rate, and exogenous spending  $\hat{g}_t$ . Equation (2) represents the Euler equation for consumption

where  $h = \eta / \gamma$ ,  $c_1 = \frac{(\sigma_c - 1)w_*^h L_* / c_*}{\sigma_c(1+h)}$ ,  $c_2 = \frac{1-h}{\sigma_c(1+h)}$ . Note that  $\eta$  captures the external

habit formation,  $\gamma$  is the growth rate of the labor-augmenting technological process,  $\sigma_c$  is the inverse of the elasticity of intertemporal substitution and  $w_*^h$  is the nominal wage received by households in the steady state. This equation implies that current consumption depends on a weighted average of past and expected future consumption, on the expected growth in hours worked, the ex-ante real interest rate  $(\hat{R}_t - E_t \hat{\pi}_{t+1})$ , and a disturbance term  $(\hat{b}_t)$ . The Euler equation for investment is represented by Equation (3), where  $\bar{\beta} = \beta / \gamma^{\sigma_c}$ .  $\beta$  represents the discount factor applied by households and  $S'$  stands for the steady-state elasticity of the capital adjustment cost function. The impact of the real value of existing capital stock  $(\hat{Q}^k)$  on investment depends on this elasticity.  $\hat{q}_t$  is a disturbance to the investment-specific technology process. Equation (4) represents the arbitrage equation for the value of capital. It states that the current value of the capital stock depends positively on its expected future value and the expected real rental rate on capital, but negatively on the ex-ante real interest rate and the risk premium disturbance,  $\tilde{b}_t = c_2 \hat{b}_t$ .  $\delta$  represents the depreciation rate.

The supply side of the economy is characterized by Equations (5) to (12). The aggregate production function, Equation (5), indicates that output is produced using capital and labor services as inputs and is affected by the total factor productivity  $\hat{A}_t$ .  $\alpha$  captures the share of capital in production and  $\Phi$  equals one plus the share of fixed costs in production. Current

capital used in production is assumed to be a linear function of the degree of capital utilization  $\hat{u}_t$  and the installed capital in the previous period,  $\hat{k}_{t-1}$ , Equation (6). The latter argument reflects the assumption that new capital becomes effective only with a one-quarter lag. The positive relationship between the degree of capital utilization and the rental rate of capital is represented by Equation (7). In this equation  $\psi$  is a positive function of the elasticity of the capital utilization adjustment cost function and is normalized to a value between zero and one. Equation (8) represents the accumulation of installed capital as a function of the flow of investment and the relative efficiency of the investment expenditures captured by the investment-specific technology disturbance. The New-Keynesian Phillips curve is represented by Equation (9) and incorporates partial indexation of lagged inflation, where  $l_p$  represents the degree of indexation to past inflation,  $\varepsilon_p$  the curvature of the Kimball (1995) goods market aggregator,  $\phi_p - 1$  the share of the fixed cost in production,  $\xi_p$  the degree of price stickiness and  $c_{mc}$  the slope related to marginal cost, where

$$c_{mc} = \frac{(1 - \xi_p \bar{\beta} \gamma)(1 - \xi_p)}{\xi_p ((\phi_p - 1)\varepsilon_p + 1)}.$$

Finally,  $\hat{\lambda}_{p,t}$  stands for the price mark-up disturbance and follows an AR(1) process. The marginal cost  $\widehat{mc}_t$  is defined by Equation (10). Equation (11) signifies that the rental rate of capital is positively related with the capital-labor ratio, but negatively with the real wage. In the same way that nominal rigidities affect the price level determination, real wages can only adjust gradually to their optimal level. Equation (12) shows how the real wage is determined, where  $l_w$  represents wage indexation,  $\varepsilon_w$  the curvature of the Kimball (1995) aggregator of labor,  $\phi_w - 1$  the steady state labor market mark-up; and  $c_w$  represents

$$c_w = \frac{(1 - \xi_w \bar{\beta} \gamma)(1 - \xi_w)}{\xi_w ((\phi_w - 1)\varepsilon_w + 1)}.$$

Analogously to above,  $\hat{\lambda}_{w,t}$  stands for the wage mark-up disturbance and it follows an AR(1) process.

Finally, Equation (13) represents the monetary policy rule where  $\rho_R$  captures the degree of smoothing over the policy instrument and  $r_\pi$  and  $r_{\Delta y}$  represent the responses of this instrument to deviations of inflation and output growth from their targets.  $\hat{r}_t$  represents the non-systematic component of the interest rate and is assumed to follow an AR(1) process.

### 3. Learning mechanism of expectations formation

The model of the previous section incorporates expectations of several forward variables. When dealing with expectations, researchers have traditionally adopted the rational expectations (RE) assumption. This assumption implies that agents have perfect knowledge of the true stochastic process of the economy. Arguably such a high level of cognitive abilities and computational skills is implausible in practice, and therefore researchers have developed models of imperfect knowledge and associated learning processes. One of the most popular learning mechanisms used in macroeconomics is a form of adaptive learning. Under this approach agents use historical data to update their perceptions about how the economy works and form their expectations about future variables using forecasting models that are updated whenever new data becomes available (see Evans and Honkapohja 2001).

It is common in the literature on learning to assume that agents update the coefficients of their forecasting models using constant-gain least squares (CG-LS). Under CG-LS, the most recent observations receive higher weights in the least square estimation. More precisely, the weight decreases geometrically depending on the distance in time to the most recent

observation. This learning mechanism implies that agents are concerned about structural changes of the economy.

In the remainder of this section we provide details of the algorithm followed by the representative agent to update her expectations and characterize the resulting dynamics of the economy.

### 3.1 Ordinary Least Squared with constant gain

The forecasting model that the representative agent uses to generate her one-period-ahead expectations of the set of variable  $Y^f$  can be represented as:

$$(14) \quad Y_t^f = \beta' X_{t-1},$$

the recursive expression for the estimate of  $\beta$  under CG-LS is:

$$(15a) \quad \hat{\beta}_t = \hat{\beta}_{t-1} + g (R_t)^{-1} X_t (Y_t^f - \hat{\beta}_{t-1}' X_t),$$

$$(15b) \quad R_t = R_{t-1} + g (X_t X_t' - R_{t-1})$$

where  $g$  represents the *constant-gain parameter* and  $R_t$  the variance-covariance matrix of the regressors included in the forecasting model. The gain refers to the relative weight of the most recent observation and  $1 - g$  is the discount factor over less recent observations (in ordinary least squares, the gain is not a constant value but equals  $1/t$ , where  $t$  is the position of the observation since the beginning of the sample).

Using  $\hat{\beta}$ , we can generate the forecast of the variables included in  $Y_t^f$ :

$$(16) \quad \hat{E}_t Y_{t+1}^f = \hat{\beta}_{t-1}' [X_t]$$

Employing  $\hat{\beta}_{t-1}$  instead of  $\hat{\beta}_t$  in equation (16) is a standard procedure in learning estimation in order to avoid the simultaneous determination of  $\hat{\beta}_t$  and the variables included in the solution of the model.

### 3.2 Model expectations augmented by learning

Using equations (1) to (13) described in Section 2, we can derive the equilibrium conditions describing the dynamics and the interactions of all endogenous variables under RE. For ease of representation, it is useful to use a generic form of the solution under RE:

$$(17) \quad \begin{bmatrix} Y_t \\ Z_t \end{bmatrix} = AA^{re} \begin{bmatrix} Y_{t-1} \\ Z_{t-1} \end{bmatrix} + BB^{re} [\vartheta_t]$$

where  $Y_t$  contains all the endogenous variables of the model,  $Z_t$  contains all the exogenous variables and  $\vartheta_t$  contains all their iid-normal perturbations. The matrices  $AA^{re}$  and  $BB^{re}$  are nonlinear functions of the structural parameters of the DSGE model.

When estimated under learning, the set of equations (1) – (13) is augmented by equation (16), by equations (15a) and (15b) which describe the estimation procedure for the  $\beta$ , and by some initial conditions for the CG-LS algorithm (which we describe later). The only parameter that is added to the set of structural parameters is the gain parameter,  $g$ .

Under learning, we can rewrite the system containing all endogenous variables of the model and replace those in expectations in the following compact way:

$$(18) \quad \begin{bmatrix} Y_t \\ Z_t \end{bmatrix} = AA_{t-1}^{learning} \begin{bmatrix} Y_{t-1} \\ Z_{t-1} \end{bmatrix} + BB_{t-1}^{learning} [\vartheta_t]$$

Matrices  $AA_{t-1}^{learning}$  and  $BB_{t-1}^{learning}$  can vary over time, as they contain not only the parameters of the structural model, but also the time-varying coefficients of the forecasting models ( $\hat{\beta}_{t-1}$ ). The time-variation of these coefficients depends on the value of the gain parameter  $g$ . If this parameter is equal to zero, the matrices  $AA^{learning}$  and  $BB^{learning}$  are constant. However, even in this situation they might not be equal to the corresponding matrices under RE, because  $AA^{learning}$  and  $BB^{learning}$  depend on the selection of the forecasting model and the initial conditions of the CG-LS.

### 3.3 Learning setting used in this study

We use survey data on inflation expectations to determine the forecasting model for inflation used by agents. Due to lack of availability for the sample of interest in this study, surveys can unfortunately not be applied in the selection of the forecasting model for the other variables that appear in expectations in the model<sup>4</sup>. Therefore, in these cases, we use as forecasting models specifications compatible with the RE solution, which implies that each of the variables included in  $Y^f$  has the same set of regressors as in the right-hand side of the equation (17). Moreover, the initial value of  $\beta$  is the corresponding rows of the matrices  $AA^{re}$  and  $BB^{re}$  and the matrix  $R_0$  can be obtained using the unconditional second moments implied by the RE solution (see Slobodyan and Wouters 2009a and Del Negro et al 2007).

The forecasting model for inflation has the following form:

$$(19) \quad dlP_t = \beta^\pi \begin{bmatrix} 1 & X_{t-1}^\pi \end{bmatrix}$$

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<sup>4</sup> These variables are consumption ( $\hat{c}_t$ ), investment ( $\hat{i}_t$ ), the value of the capital stock ( $\hat{Q}_t^k$ ), the real rental rate on capital ( $\hat{r}_t^k$ ), real wages ( $\hat{w}_t$ ) and hours worked ( $\hat{L}_t$ ).

where  $dIP$  refers to the actual series of inflation while  $X^\pi$  is a subset of all the actual series used in the estimation of the DSGE model (the precise composition of  $X^\pi$  is explained in section 5). Equation (19) indicates that for generating her forecast on inflation, the representative agent uses the same information that we use to estimate the DSGE model. The initial values of  $\beta^\pi$  and the second moment matrix related to the set of regressors of equation (19) are obtained using actual data for a pre-sample.

Considering the relationships between the actual series ( $dIP, X^\pi$ ) and the variables in the model ( $Y$ ) (summarized in the measurement equations in the following section), it is possible to rewrite (19) in terms of the latter ones, and therefore have a forecast model for inflation,  $\hat{\pi}$ , in a state-space form such as in equation (18).

Thus, our specification of learning is composed of two blocks: one related to the inflation process and the other to the processes of the remaining variables which appear with expectations in the model. Therefore, we have not one but two gain parameters.

Finally, it is important to mention that the equilibrium achieved by the model is no longer compatible with the equilibrium under RE because we use a forecasting model (for inflation) that is misspecified in terms of the solution under RE. The relevant equilibrium concept in this case is the *restricted perceptions equilibrium* (RPE)<sup>5</sup> and is motivated by agents that, because they are unaware of the “true” structure of the economy, find it optimal to use small (misspecified) forecasting models.

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<sup>5</sup> The name of Restricted Perceptions Equilibrium (RPE) was given by Evans and Honkapohja (2001). Branch (2004) discusses the generality of RPE as it encompasses many forms of misspecified equilibria such as the Self-Confirming Equilibrium in Sargent (1999) and the Consistent Expectations Equilibrium in Hommes and Sorger (1998).

## 4. Data and priors

The model is estimated using the same quarterly macroeconomic indicators for the US as in Smets and Wouters (2007), but in addition we also use survey data on inflation expectations provided by the Survey of Professional Forecasters (SPF). In particular, for each quarter, we calculate the median value of the reported one-period-ahead forecast of the percentage increase of the GDP deflator. The resulting series is henceforth referred to as “*exp\_dIP*”. As this data is available only from 1968Q4 onward, this date marks the starting point for our sample. The sample covers all quarters until 2008Q2. The other macroeconomic indicators are the log difference of real GDP (“*dIGDP*”), of real consumption (“*dICons*”), of real investment (“*dIInv*”) and of the real wage (“*dIWage*”), as well as the log of hours worked (“*lHours*”), the log difference of the GDP deflator (“*dIP*”) and the federal funds rate (“*FedFunds*”). Appendix B contains a description of the data.

The way in which these macroeconomic indicators are related to the variables of the model under RE and learning when survey data on inflation expectations is not included is summarized by the following measurement equations:

$$\begin{bmatrix} dIGDP_t \\ dICons_t \\ dIInv_t \\ dIWage_t \\ lHours_t \\ dP_t \\ FedFunds_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{i}_t - \hat{i}_{t-1} \\ \hat{w}_t - \hat{w}_{t-1} \\ \hat{l}_t \\ \hat{\pi}_t \\ \hat{R}_t \end{bmatrix}$$

where  $\bar{\gamma} = 100(\gamma - 1)$  represents the common quarterly trend growth rate,  $\bar{l}$  the steady state hours worked,  $\bar{\pi} = 100(\Pi_* - 1)$  the quarterly steady state inflation rate and  $\bar{r} = 100(\gamma^{\sigma_p} \Pi_* / \beta - 1)$  the quarterly steady state nominal interest rate.

When survey data is incorporated in the estimation, the measurement equations are:

$$\begin{bmatrix} dGDP_t \\ dCons_t \\ dInv_t \\ dWage_t \\ lHours_t \\ dP_t \\ FedFunds_t \\ exp\_dP_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \\ \bar{\pi} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{i}_t - \hat{i}_{t-1} \\ \hat{w}_t - \hat{w}_{t-1} \\ \hat{l}_t \\ \hat{\pi}_t \\ \hat{R}_t \\ E_t \hat{\pi}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \zeta_t \end{bmatrix}$$

where  $\zeta_t$  represents the measurement error (*iid*) related to the surveys on inflation expectations. Hence, survey data is taken as a noisy measure of actual expectations.

The model contains 38 structural parameters. 33 of them are estimated while the remaining 5 are fixed at the same values as in Smets and Wouters (2007)<sup>6</sup>. The learning estimation adds another two parameters (the gains for inflation and for the other variables that appear in expectations). When estimating the model with survey data, we consider one extra parameter, namely the standard deviation of the measurement error of the surveys ( $\zeta_t$ ). The prior distributions of the structural parameters are as in Smets and Wouters; for the gains, uniform distributions over the [0,1] domain; and for the standard error of  $\zeta_t$ , an inverse gamma distribution with zero mean and standard deviation of 2. The prior distributions for all the parameters are presented in Appendix C.

The estimation of the DSGE model is performed using Bayesian estimation methods. Employing the random walk Metropolis-Hastings algorithm I obtain 250 000 draws from each model's posterior distribution. The first half of these draws is discarded and 1 out of every 10 is selected in order to estimate the moments of the posterior distributions.

<sup>6</sup> These parameters are the depreciation rate (fixed at 0.025), the exogenous spending-GDP ratio (0.18), the steady state mark-up in the labor market (1.5) and the curvature parameters of the Kimball (1995) aggregators in the goods and labor market (both set at 10).

## 5. Results

We start this section by using survey data on inflation expectations to determine the forecasting model that people most likely use to generate their expectations on future inflation. As indicated in section 3.3, the resulting forecasting model defines the setup of learning used in this section. Afterwards, we implement a model-comparison exercise between RE and learning, and determine how the assessment as to which model fits the data better changes with the inclusion of survey data on inflation expectations. Then we discuss how survey information affects the identification of the structural parameters of the models, the relative importance of the sources of inflation persistence, the composition of inflation expectations, and the Impulse-Response functions and variance-covariance decomposition analyses.

### 5.1 Forecasting models for inflation

We select the forecasting model for inflation among all the linear specifications that can be obtained using the same information that is employed in the estimation of the DSGE model. More precisely, we estimate linear models for inflation where the regressors consist of (besides an intercept) all possible combinations of the lagged series of  $dIGDP$ ,  $dICons$ ,  $dIInv$ ,  $dIWage$ ,  $dIP$ ,  $FedFunds$  and  $IHours$ . We rank these models (127 in total) according to the similarities between the one-period-ahead inflation forecast series obtained by each of them and the survey data on inflation expectations. The statistic used to rank these models is the Mean Squared Error (MSE). Table 1 shows the five best-performing forecasting models for the periods 1968Q4 – 2008Q2<sup>7</sup> and Figure 1 represents the one-period-ahead inflation forecast

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<sup>7</sup> The ranking is constructed following four steps. First, each model is estimated using a recursive CG-LS, which allows for compatibility with the algorithm behind the inflation expectations formation under learning. Second, recursive CG-LS requires the definition of the initial values for the coefficients to be estimated and of the variance-covariance matrix of the regressors. These values are obtained using Ordinary Least Squares (OLS) over a pre-sample. Third, different values of the constant gain are

series of the best three models and the survey data. In general, the one-period-ahead forecasting series are very similar among them and they track relatively well the increases of survey expectations during the 1970s and its reduction at the beginning of the 1980s. However, during the 1980s and 1990s the forecast series underestimate the survey data while during the 2000s they overestimate it.

[Add Table 1]

We choose as the forecasting model for inflation for our benchmark learning specification the model that includes as regressors only lagged inflation and an intercept (the first model in the ranking in Table 1). The other forecasting models contained in the previous table are considered in the robustness analysis in section 6.

[Add Figure 1]

## 5.2 Model comparison

In this subsection we analyze which of the two specifications of expectations formations (RE and learning) receives more support from the data. In particular, and similar to Del Negro and Eusepi (2010), we want to determine how the assessment as to which model fits the data better changes with the inclusion of survey data on inflation expectations in the estimation.

[Add Table 2]

Table 2 shows the log marginal likelihoods of the two specifications when survey data is included in the estimation and when it is not. When survey data is not included, both specifications show similar log marginal likelihoods (see column 1). The log marginal likelihood

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employed to produce forecasts for each of the models (these values are taken from a grid of points that goes from 0 to 1, with steps of 0.0125). The ranking is then established taking into account for each of the models the value of the constant gain that results in the best MSE. Finally, given that the ordering could vary depending on the choice of the pre-sample, different pre-samples are considered and we select the one with the lowest MSE among the top models.

difference of 3.96 points can not be taken as a clear evidence in favor or learning, especially because this difference can be reduced to less than 1 by choosing different priors<sup>8</sup>. However, this result changes significantly when survey data is included in the estimation (see column 2). Now learning clearly outperforms the RE specification with a difference of 64.36 points in the log marginal likelihood, which implies posterior odds of 8.93E+27 in favor of the former specification.

Does learning provide a better description of survey data on inflation expectations than the rational expectations specification? The answer is affirmative as shown by the difference of the log marginal likelihoods when survey data is and is not included (column (3)). As indicated by Del Negro and Eusepi (2010), this measure represents how well each of the models fits the survey data conditional on the parameter distribution that delivers the best fit for the rest of the dataset<sup>9</sup>. Using this measure, learning clearly outperforms the rational-expectations model in describing the evolution of the survey data.

A graphical evaluation of the model-implied series of inflation expectations also shows that learning describes the evolution of survey expectations better, even for the case when survey data is not included in the estimation (see Figure 2). In particular, the RE specification underpredicts survey expectations during the last years of the 1970s and during most of the 1980s, and overpredicts surveys during the beginning and end of the sample. On the contrary,

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<sup>8</sup> Using the uniform prior distributions indicated in section 6, we find log marginal likelihood values for the RE and learning specifications of -120 and -119.2, respectively. Moreover, Del Negro and Schorfheide (2008) shows that even 5 points in the log marginal likelihood can be overturned by choosing a slightly different prior.

<sup>9</sup> The values of the log marginal likelihoods in column 1 of Table 2 can be represented as  $\ln p(Y_{1,T}^0)$ , where  $Y_{1,T}^0$  is the dataset that does not include the survey data, with observations that go from 1 until T. The log marginal likelihood values in column 2 can be represented as  $\ln p(Y_{1,T}^0, Y_{1,T}^1)$ , where  $Y_{1,T}^1$  represents the series of survey data on inflation expectations. Therefore, the difference between column 2 and 1 can be expressed as  $\ln p(Y_{1,T}^1 | Y_{1,T}^0)$  and represents how well the model fits the survey data conditional on the parameter distribution that delivers the best fit for the rest of the dataset.

learning is flexible enough to match more closely the fluctuations registered in the survey data. This is especially true when survey data is included in the estimation, which eliminates the overestimation of the increase of inflation expectations observed in 1974Q4, after the first oil crisis.

The comparatively strong performance of learning in matching the survey data on inflation expectations can also be measured by the correlation between surveys and the model-implied series of inflation expectations. As Table 3 shows, learning tracks the survey data more closely than RE not only when the series are in levels but also in first differences.

[Add Figure 2]

[Add Table 3]

### 5.3 Posterior estimates

In this subsection we use survey data on inflation expectations in the identification of the parameters of the model. In particular, we compare the parameter estimates obtained under RE and learning, and determine how these estimates change with the inclusion of survey data in the estimation of the DSGE model. Given that the parameters that change the most are also the ones that determine the degree of inflation persistence, we provide a measure of their relative importance. Additionally, for the learning estimates we show how the addition of survey data affects the composition of inflation expectations and the expected inflation target.

To start with, we find that different specifications of inflation expectations formation imply different results in terms of posterior estimates (see Table 4). Taking the estimations under RE as the benchmark case (column 1), the estimation under learning (column 2) results in a lower autocorrelation coefficient of the price mark-up shock, lower price stickiness, and higher price indexation. These results are compatible with Slobodyan and Wouters (2009b), but not with

the ones of Milani (2007). In his study, Milani finds that the introduction of learning makes the degree of habits in consumption and inflation indexation drop closer to zero, while the autocorrelation coefficient of the supply shocks increases significantly (from a posterior mean of 0.02 in his rational expectation version (Table 3) to 0.854 in his benchmark learning version (Table 2)). As in Slobodyan and Wouters (2009b), we use a small forecasting model for inflation while Milani uses forecasting models specifications that are compatible with the rational expectation solution of the model<sup>10</sup>. The differences in the forecasting model specifications and the use by Milani of internal habits in consumption, instead of the external type as we do, explain our discrepancies.

When estimating both the RE and learning specifications using survey data on inflation expectations, we find that the most important changes in the parameter estimates come from the RE specification. In particular, we find that the price indexation significantly decreases (from a posterior median of 0.327 to 0.052), the autocorrelation coefficient of the price mark-up shocks increases (from a posterior median of 0.448 to 0.726), and the wage stickiness is slightly lower (from 0.554 to 0.468) (see Table 4, column 3). With respect to the learning estimation, the only significant change in the parameter estimates is concentrated in the gain parameter of inflation, which decreases from 0.188 to 0.141 (see Table 4, column 4). This reduction in the gain parameter implies a reduction in the time-variability of the coefficients of the forecasting model of inflation, with consequences over the composition of inflation expectations and the implied expected inflation target as we show below. This reduction in the gain parameter can also be interpreted as a decline in the speed of learning or, in other words, that the past information used in the estimation of the forecasting model of inflation is

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<sup>10</sup> When we refer to “small” forecasting model in this study, it implies forecasting models with fewer regressors than the rational expectation solution of the DSGE model.

discarded at a slower pace<sup>11</sup>. These changes in the parameter estimates give a more important role to the persistence of the stochastic shocks as a source of inflation persistence under RE and reduce the importance of the time-variability of the coefficients of the forecasting model of inflation under learning.

[Add Table 4]

The posterior estimates of the gain parameter for inflation require a particular comment because they are higher than the estimates reported by previous studies. For instance, Orphanides and Williams (2005) considers a baseline calibrated value of the gain parameter of 0.02 and Milani (2007) and Slobodyan and Wouters (2009a) find posterior mean estimates that range between 0.0161 and 0.0247, and between 0.002 and 0.02, respectively. The high values for this parameter obtained in our study are related to the specification of the forecasting model. As the econometric exercise implemented in the subsection 5.1 shows, the forecasting models for inflation that best fit the survey data require significant time variation of their coefficients (the gain parameters are equal or higher than 0.10). Moreover, the fewer variables included in the forecasting model, the smaller the impact of the time-variation of their coefficients on the stability of the DSGE model. Thus, not only does the forecasting model for inflation require high levels of time-variability in its coefficients but its specification actually makes it possible to estimate the DSGE model for these levels of time-variability. Finally, it is important to mention that Slobodyan and Wouters (2009b) also find a high degree of time-variability in the coefficients of the small forecasting models employed in their learning estimation. However, this result is not reflected in a high gain parameter because they do not use constant-gain but Kalman-filter learning.

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<sup>11</sup> Constant-gain parameter values of 0.188 and 0.141 imply that 75 percent of the information that people employ to generate their inflation expectations is contained in the 6.7 and 9.1 most recent quarterly data observations, respectively.

The reduction in the posterior estimates of the gain parameter of inflation once survey data is included has some interesting repercussions in terms of the evolution of the coefficients of the forecasting model of inflation, the composition of inflation expectations, and the evolution of the implied inflation target. In order to facilitate the discussion of the following results, we rewrite the forecasting model used to generate inflation expectations under learning:

$$dIP_t = \hat{\beta}_{0,t-1}^\pi + \hat{\beta}_{1,t-1}^\pi dIP_{t-1},$$

from which conditional inflation expectations can be expressed as:

$$E_t(dIP_{t+1}) = \hat{\beta}_{0,t}^\pi + \hat{\beta}_{1,t}^\pi dIP_t$$

As shown by Figure 3, when survey data is not included in the estimation, the perceived inflation persistence ( $\hat{\beta}_1^\pi$ ) shows a sharp decline at the end of the 1970s with a subsequent increase. When survey data is included, the evolution of the intercept does not exhibit this decline but remains high through all the second half of the 1970s and all the 1980s. Additionally, the peaks observed in the intercept of the forecasting model ( $\hat{\beta}_0^\pi$ ) are less important. Thus, the use of survey data implies a restriction on the way in which beliefs fluctuate under learning.

[Add Figure 3]

These differences in the evolution of the coefficients of the forecasting model of inflation affect the way inflation expectations are composed. When survey data is included in the estimation, these expectations are closely related to the evolution of the perceived persistence of inflation up to the beginning of the 1990s (see Figure 4b). However, this is not the case when surveys data is absent (Figure 4a). Notice that since the 1990s, both estimations

show that inflation expectations are no longer related to the perceived persistence of inflation but to the perceived inflation mean<sup>12</sup>.

[Add Figure 4]

Finally, considering the form of the forecasting model and the evolution of its coefficients it is possible to represent the long-run inflation target at each point in time ( $E(dIP) = \frac{\hat{\beta}_0^\pi}{1 - \hat{\beta}_1^\pi}$ ).

Figure 5 shows that agents started the 1970s with an expected inflation target of 0.59 percent, but they kept increasing their expectations about this target until the level of 2.44 percent in 1981Q2. Afterwards, there is an important reduction that locates the target between 0.6 and 1 during the 1980s. The timing and the magnitude of the reduction in expected inflation target are consistent with the notion that the Volcker recession at the beginning of the 1980s reduced inflation expectations. During the 1990s, the target steadily decreases until the beginning of the 2000s (in 2000Q1 the target is situated at 0.37). After this point, the inflation target starts a positive trend that is interrupted by the beginning of the financial crisis in 2007. The use of survey data in the estimation avoids the presence of some outliers observed in the evolution of the inflation target.

[Add Figure 5]

#### 5.4 Impulse-Response and forecast error variance decomposition analyses

In this subsection we analyze how the use of survey data affects the Impulse-Response functions (IRFs) and variance-covariance decomposition analyses for each of the specifications of expectations formation<sup>13</sup>.

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<sup>12</sup> Given the structure of the forecasting model for inflation, if the perceived persistence coefficient is close to zero, the intercept can be interpreted as the perceived mean of inflation.

Under RE, there are no significant differences in the IRFs obtained with and without the use of survey data with the exception of the response of inflation to the wage mark-up shock. The reduction of wage stickiness results in a less persistent response of inflation to this type of shocks (see Figure 6a). Interestingly, the impact of a price mark-up shock on inflation does not change significantly despite the large reduction in the degree of inflation indexation (see Figure 6b). This result is obtained because the decline in price indexation is compensated by an increase in the autocorrelation coefficient of the price mark-up shock.

[Add Figure 6]

The addition of survey data under learning reduces some of the time-variability of the IRFs, letting most of the stronger and more persistent responses of inflation be concentrated in the 1970s. In particular, it is observed that unexpected monetary policy shocks have more destabilizing effects on inflation during the 1970s than afterwards (see Figure 7). This result is compatible with Boivin and Giannoni (2008), who find weaker responses of inflation to unexpected changes in interest rate for the period post-1979 in comparison with the previous period<sup>14</sup>.

[Add Figure 7]

Finally, we decompose the forecast error variance-covariance of inflation to determine its main driving forces and how they are affected by the use of survey data. In order to keep the comparison as simple as possible we are displaying the variance decomposition in the long-run (after 100 quarters). We find that the relative importance of the shocks depends on the expectations specification adopted rather than on the use of survey data.

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<sup>13</sup> The results we show are only related to inflation and not to the other variables of the model because the introduction of survey data on inflation expectations affects this variable most significantly.

<sup>14</sup> These authors indicate that these weaker responses reflect the fact that monetary policy had successfully managed to moderate the effects of exogenous shocks in the early 1980s.

Under the RE specification and with the use of survey data, the price and wage mark-up shocks and the investment-specific technology shock are the most important driving forces of inflation: jointly the first two account for 57 percent of the forecast error variance while the investment-specific shock explains 26 percent (see Figure 8). Once survey data is included the investment-specific technology shock slightly increases its importance to 34 percent while the wage mark-up shock decreases from 30 percent to 20 percent. The reduction in the importance of the latter type of shock is related to the reduction of wage stickiness while the increase in the importance of the investment-specific technology shock is related to the reduction in the cost of adjustment of capital and the increase in the persistence and volatility of this shock once survey data is included.

Under learning we find that the price mark-up shock is the most important determinant of the variance of the forecast error of inflation (it accounts for approx. 45 percent), despite the low estimate of the autocorrelation coefficient of this shock. This result is due to the fact that the high value of price indexation and the learning process add to the importance of this shock. The wage mark-up shock and the investment-specific technology shock are also important and both account for 28 percent and 20 percent, respectively, of the variance decomposition. The previous results do not change when survey data is included in the estimation.

[Add Figure 8]

## 6. Robustness exercises

We evaluate the robustness of the results presented in the previous section to the following three variations. First, we use alternative specifications of the forecasting model for inflation under learning. Second, we analyze how our results change when loose uniform priors are

used<sup>15</sup>. Finally, we replace the constant-gain least square algorithm of the estimation of the forecasting models under learning by ordinary least squares (OLS).

[Add Table 5]

The use of alternative forecasting models for inflation does not change significantly the posterior statistics obtained under learning (see Table 5, columns 2 to 5)<sup>16</sup>. In particular, the median values of the posterior distribution of price indexation and the autocorrelation coefficient of the price mark-up shock are close to 0.60 and 0.161, respectively; while the gain parameter of inflation is close to 0.135. These numbers are very similar to those obtained under the benchmark specification of the forecasting model for inflation (column 1).

With respect to the introduction of loose uniform prior distributions, we find that learning does not display a major change in the parameter estimates with the exception of the increases in wage stickiness and the coefficient of output growth in the Taylor rule (see Table 6). The latter result is also observed under RE. Although not reported here, the introduction of survey data in the estimations under RE reduces the changes in the parameter estimates observed by the change in the prior distributions. Despite these variations in the parameter estimates, our calculation of the relative importance of the different sources of inflation persistence does not change significantly.

[Add Table 6]

Finally, the use of the OLS algorithm instead of constant-gain least squares generates very similar log marginal likelihoods under RE and learning when survey data is not included (Table 7, column 1). However, when the estimations include the data on inflation expectations,

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<sup>15</sup> The online appendix contains the list of prior distributions used for these estimations.

<sup>16</sup> The specifications of the forecasting model for inflation that are considered are those presented in Table 1, which are the ones that generate the closest series of one-period-ahead inflation expectations with respect to the series of inflation expectations of the SPF.

learning performs worse than RE (see column 2). This result is explained by the inability of this specification to match the evolution of the survey data. The use of the OLS algorithm keeps the coefficients of the forecasting model very close to the initial conditions. Given that the initial conditions are obtained during a period of low and not persistent inflation (period 1950Q1-1968Q3), the model fails to replicate the increases in the expectations during the 1970s and beginning of the 1980s (see Figure 9). However, as indicated in section 5.2, RE also does not match the evolution of the survey data as well as learning when the constant-gain algorithm is in place (compare Figures 9 and 2).

[Add Table 7]

[Add Figure 9]

## 7. Conclusions

In this paper we show that survey data provide useful information when estimating macroeconomic models. In particular, we use survey data on inflation expectations in the following three ways. First, we employ surveys to determine the forecasting model that is most likely used by agents under learning. Second, we add surveys into the estimation of a DSGE model under RE and learning, and evaluate which of these specifications the data support more. Third, the use of survey data helps to identify the parameters of the models better, and therefore, to highlight the differences between RE and learning.

We find that small forecasting models with time-varying coefficients match the evolution of the survey data on inflation expectations. In particular, we use a forecasting model where inflation is explained by an intercept and only its first lag in our benchmark learning specification. When ignoring surveys, we do not find clear support from the data used in the estimations for either the rational expectations or the learning specifications. However,

learning receives more support from the data once survey information is incorporated in the estimation because under this specification the model can better track the important variations of inflation expectations during the 1970s and 1980s. There are also some important variations in the parameter estimates once survey data on inflation expectation is incorporated in the estimation. In particular, under the RE specification, the price indexation estimate is close to zero while the persistence of the exogenous shocks plays an important role generating inflation persistence. This is not the case under learning: both price indexation and the learning process itself are the main sources of inflation persistence. Under learning we also find that inflation responds stronger and more persistently during the 1970s to different structural shocks.

There are some important issues not addressed in this study. First, we only use the median value of inflation forecasts from all those which forecasters included in the SPF report at each point in time. However, information about other moments – such as the dispersion – can be exploited to evaluate issues like the credibility of the central bank or how periods of high disagreement in expectations affect the conduct of monetary policy. Second, survey data is also available for a variety of other macroeconomic indicators, and the use of this information may change existing results regarding the identification of the sources of the business cycle, among other things. It is true that surveys about inflation expectations have been a more common subject of academic studies and have attracted less criticism concerning their quality than other variables. Nevertheless, this should not imply that survey data of other indicators do not contain any useful information at all. Finally, survey data on inflation expectations can also be employed to evaluate models specially designed to better explain the low frequency movement of inflation observed during the second half of the 1970s and beginning of the 1980s in many developed countries. In light of our results, it is interesting to ask if other

perfect information setups (such as Sbordone 2007 or Ireland 2007) can provide better descriptions of the survey data than learning.

To conclude, this study is one of the first to show that information about expectations formation contained in survey data can have significant macroeconomic implications. So far, however, the information collected by surveys such as the Survey of Professional Forecasters (SPF), but also the Livingstone and Michigan surveys or the Greenbook, have been largely neglected by empirical macroeconomic studies. The use of this information could improve our understanding of the workings of the economy and might change some pre-established ideas.

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## Tables and Graphs

Table 1  
Ranking of forecasting models  
Sample 1968Q4-2008Q2

Rank	Model	Gain	MSE
1	dIP	0.125	0.0294
2	dIP IHours	0.113	0.0300
3	dIP dICons	0.100	0.0302
4	dIP dICons IHours	0.125	0.0303
5	dIP dIGDP	0.125	0.0315

Note: the models are estimated by recursive CG-LS. The initial conditions are obtained from the period 1950Q1-1968Q3.

Regression:  $dIP_t = \text{intercept} + \text{model}_{t-1}$

Table 2

Model comparison

Log Marginal Likelihood	Dataset	Dataset	(3) =(2)-(1)
	without survey data (1)	with survey data (2)	
RE	-146.78	-19.14	127.64
Learning	-142.82	45.22	188.04

Notes: This table shows the log marginal likelihood for RE and Learning. Survey data on inflation expectations come from the SPF one-quarter-ahead median forecast of the GDP deflator.

Table 3

Correlation between model-implied and SPF inflation expectations

	Database WITHOUT survey data		Database WITH survey data	
	RE	Learning	RE	Learning
Levels	0.7952 [ 0.7633; 0.8222]	0.908 [0.9004; 0.9214]	0.8704 [ 0.8581; 0.8809]	0.9381 [0.9346; 0.9407]
1st difference	0.166 [0.1412; 0.1905]	0.3048 [0.2845; 0.3212]	0.2009 [0.1815; 0.2137]	0.2757 [0.2560; 0.2875]

Notes: This table reports the correlation between the one-quarter-ahead model-implied and SPF inflation expectations. We report the median and, in brackets, the 16th and 84th quintile of the posterior distribution.

Table 4  
Posterior distribution statistics

	Symbol	(1)		(2)		(3)		(4)	
		WITHOUT survey data				WITH survey data			
		RE		Learning		RE		Learning	
		Median	Std	Median	Std	Median	Std	Median	Std
Wage stickiness	$\xi_w$	0.554	0.045	0.547	0.049	0.468	0.043	0.563	0.049
Price stickiness	$\xi_p$	0.648	0.044	0.481	0.035	0.629	0.058	0.480	0.035
Wage indexation	$\iota_w$	0.482	0.131	0.314	0.107	0.442	0.124	0.319	0.107
Price indexation	$\iota_p$	0.327	0.155	0.544	0.108	0.052	0.025	0.515	0.119
TR: inflation	$r_\pi$	1.666	0.130	1.396	0.116	1.711	0.114	1.398	0.104
TR: lag interest rate	$\rho_R$	0.760	0.028	0.763	0.028	0.706	0.030	0.777	0.029
TR: change in output	$r_{\Delta y}$	0.199	0.046	0.203	0.046	0.187	0.044	0.210	0.047
aut. Price Mk up shock	$\rho_p$	0.448	0.195	0.140	0.070	0.726	0.078	0.173	0.087
std. Price mkup shock	$\sigma_p$	0.145	0.026	0.213	0.017	0.112	0.013	0.204	0.014
gain - inflation	$g^\pi$			0.188	0.014			0.141	0.009
gain - others	$g^{\text{non}\pi}$			0.031	0.042			0.019	0.031
Measurement exp error	$\sigma_{\text{exp}}$					0.265	0.016	0.176	0.010
Log. Mg. Likelihood			-146.8		-142.8		-19.1		45.2

Notes: this table shows the median and standard deviation of the posterior distributions of those parameters most closely related to the dynamics of inflation. The online appendix contains the same statistics for the complete list of parameters of the model, their prior and posterior distributions and a convergence check of the random walk Metropolis-Hasting (see online appendix at <http://xxx> (see extra pdf file)).

Table 6

Posterior distribution statistics: different specifications of the forecasting model for inflation

Estimations include survey data on inflation expectations

	Symbol	(1)		(2)		(3)		(4)		(5)	
		dIP (benchmark)		dIP lHours		dIP dlCons		dIP dlCons lHours		dIP dlGDP	
		Median	Std	Median	Std	Median	Std	Median	Std	Median	Std
Wage stickiness	$\xi_w$	0.563	0.049	0.555	0.048	0.548	0.046	0.548	0.046	0.551	0.048
Price stickiness	$\xi_p$	0.480	0.035	0.446	0.033	0.467	0.037	0.453	0.035	0.462	0.037
Wage indexation	$\iota_w$	0.319	0.107	0.314	0.101	0.342	0.110	0.326	0.106	0.319	0.109
Price indexation	$\iota_p$	0.515	0.119	0.584	0.118	0.553	0.118	0.618	0.112	0.674	0.101
TR: inflation	$r_\pi$	1.398	0.104	1.432	0.111	1.404	0.121	1.468	0.111	1.423	0.115
TR: lag interest rate	$\rho_R$	0.777	0.029	0.775	0.029	0.767	0.029	0.778	0.023	0.776	0.031
TR: change in output	$r_{\Delta y}$	0.210	0.047	0.210	0.045	0.203	0.046	0.214	0.044	0.206	0.046
aut. Price Mk up shock	$\rho_p$	0.173	0.087	0.180	0.083	0.168	0.083	0.175	0.080	0.124	0.062
std. Price mkup shock	$\sigma_p$	0.204	0.014	0.221	0.014	0.206	0.015	0.223	0.013	0.221	0.014
gain - inflation	$g^\pi$	0.141	0.009	0.132	0.006	0.137	0.007	0.130	0.005	0.140	0.008
gain - others	$g^{\text{non}\pi}$	0.019	0.031	0.023	0.022	0.028	0.049	0.019	0.021	0.019	0.032
Measurement exp error	$\sigma_{\text{exp}}$	0.176	0.010	0.177	0.011	0.175	0.009	0.173	0.009	0.184	0.011
Log. Mg. Likelihood		45.2		38.6		43.3		40.4		24.1	

Notes: this table shows the median and standard deviation of the posterior distributions of those parameters most closely related to the dynamics of inflation. Each of the columns indicates the use of different specifications of forecasting models for inflation. These specifications are the ones that generate the series of one-period-ahead inflation forecasts closer to the series of survey data on inflation expectations.

Table 7

Posterior distribution statistics: DSGE estimations using loose uniform prior distributions

Estimations include survey data on inflation expectations

	Symbol	RE		Learning	
		Median	Std	Median	Std
Wage stickiness	$\xi_w$	0.234	0.075	0.891	0.046
Price stickiness	$\xi_p$	0.799	0.048	0.388	0.042
Wage indexation	$\iota_w$	0.246	0.205	0.349	0.168
Price indexation	$\iota_p$	0.010	0.013	0.626	0.154
TR: inflation	$r_\pi$	1.492	0.172	1.667	0.199
TR: lag interest rate	$\rho_R$	0.734	0.038	0.853	0.019
TR: change in output	$r_{\Delta y}$	0.937	0.180	1.107	0.123
aut. Price Mk up shock	$\rho_p$	0.567	0.049	0.073	0.074
std. Price mkup shock	$\sigma_p$	0.137	0.015	0.207	0.014
gain - inflation	$g^\pi$			0.144	0.008
gain - others	$g^{\text{non}\pi}$			0.016	0.023
Measurement exp error	$\sigma_{\text{exp}}$	0.219	0.017	0.177	0.010
Log. Mg. Likelihood		11.6		67.6	

Notes: this table shows the median and standard deviation of the posterior distributions of those parameters most closely related to the dynamics of inflation.

Table 8

Model comparison: estimation using OLS learning

	Dataset	Dataset	
Log Marginal	without	with	
Likelihood	survey data	survey data	
	(1)	(2)	(3)=(2)-(1)
RE	-146.78	-19.14	127.64
Learning	-148.35	-77.20	71.14

Notes: This table shows the log marginal likelihood for RE and Learning. Survey data on inflation expectations come from the SPF one-quarter-ahead median forecast of the GDP deflator.

Figure 1

Inflation forecasts and survey data on inflation expectations

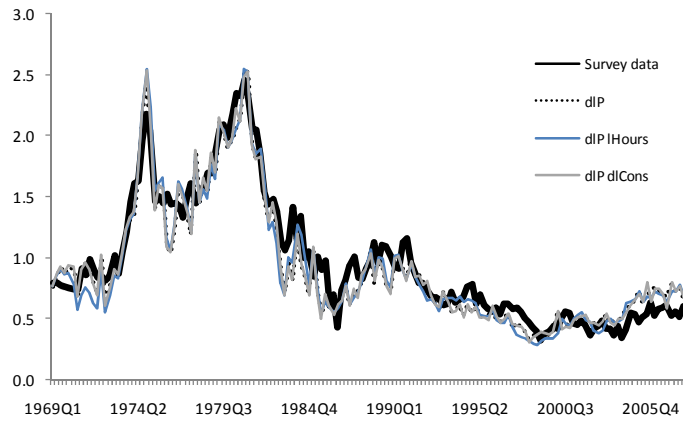
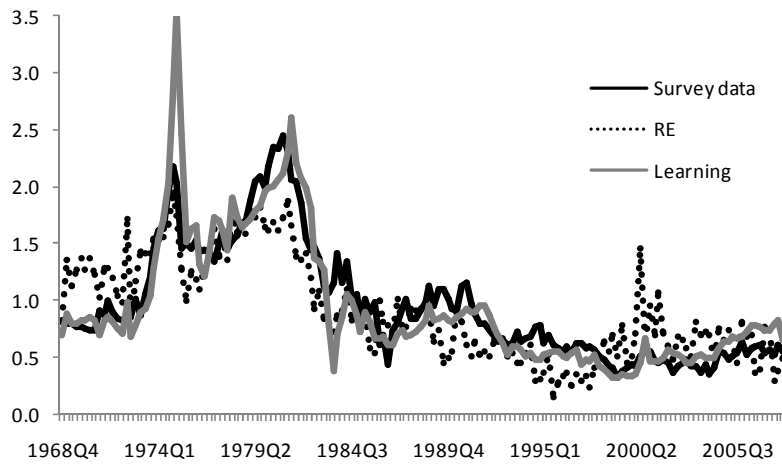


Figure 2

Inflation expectations: survey data and model-implied expectations

(a) Database WITHOUT survey data



(b) Database WITH survey data

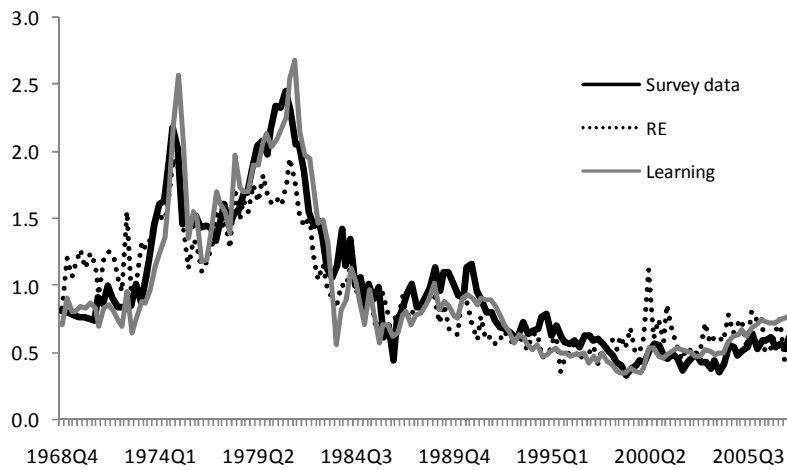
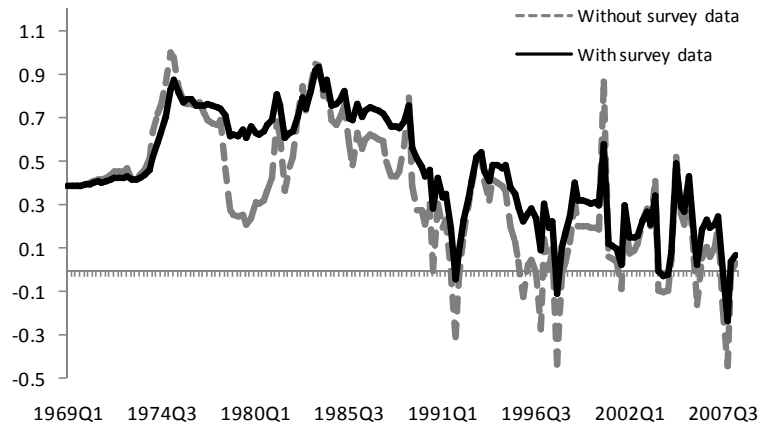


Figure 3

Evolution of the coefficients of the forecasting model for inflation

(a) Perceived persistence of inflation



(b) Intercept

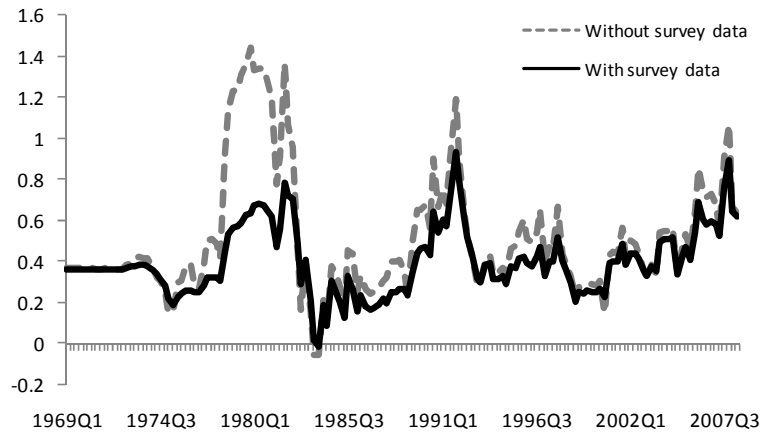
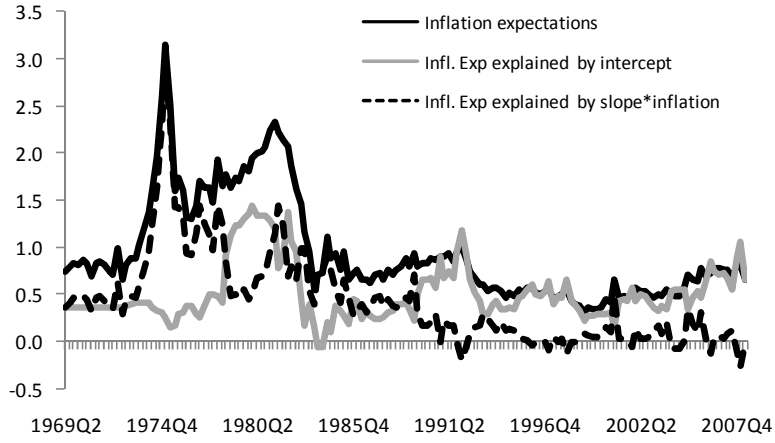


Figure 4

Composition of inflation expectations under learning  
(a) Not using survey data



(b) Using survey data

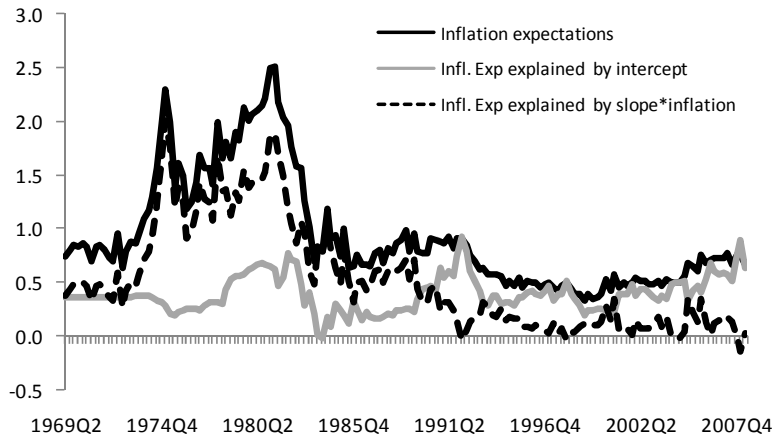


Figure 5

Evolution of the inflation target under learning

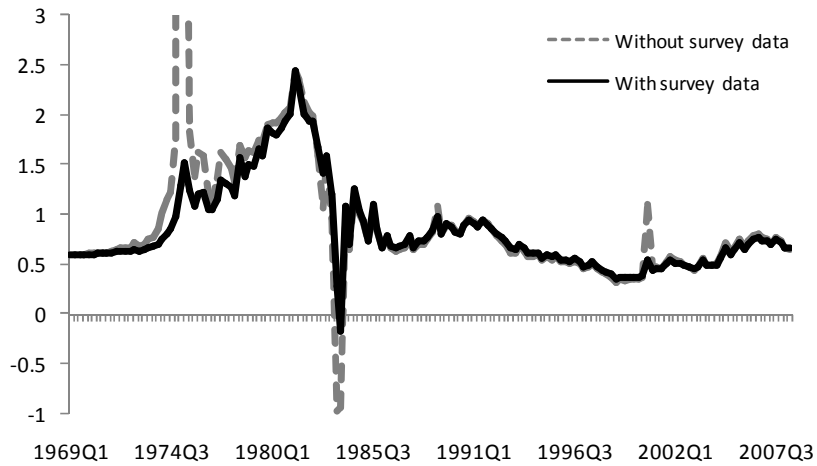
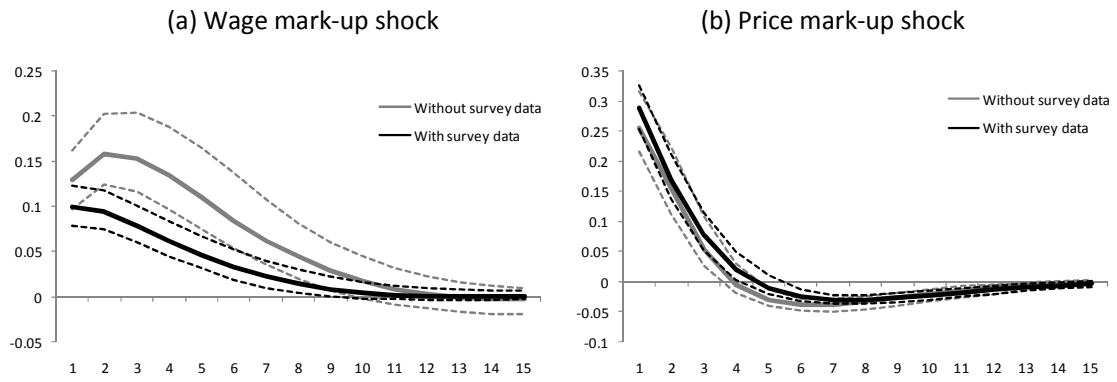


Figure 6

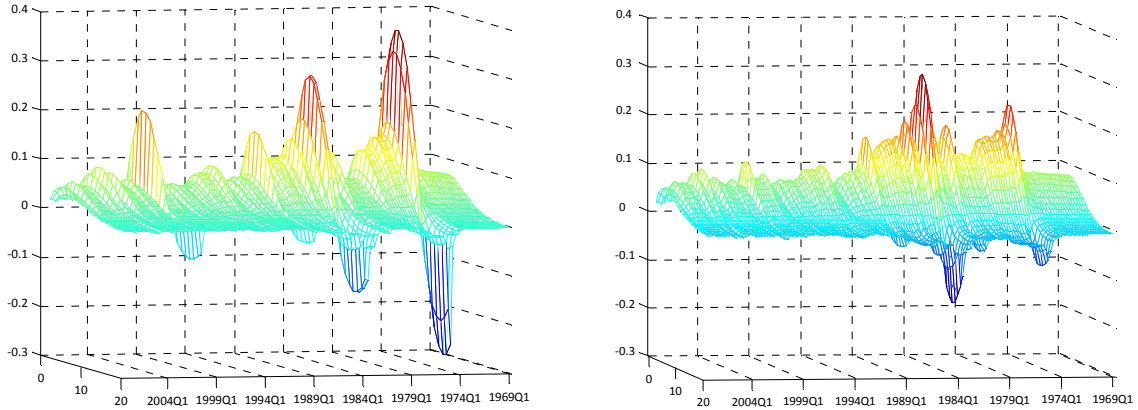
IRFs under RE: response of inflation to price and wage mark-up shocks



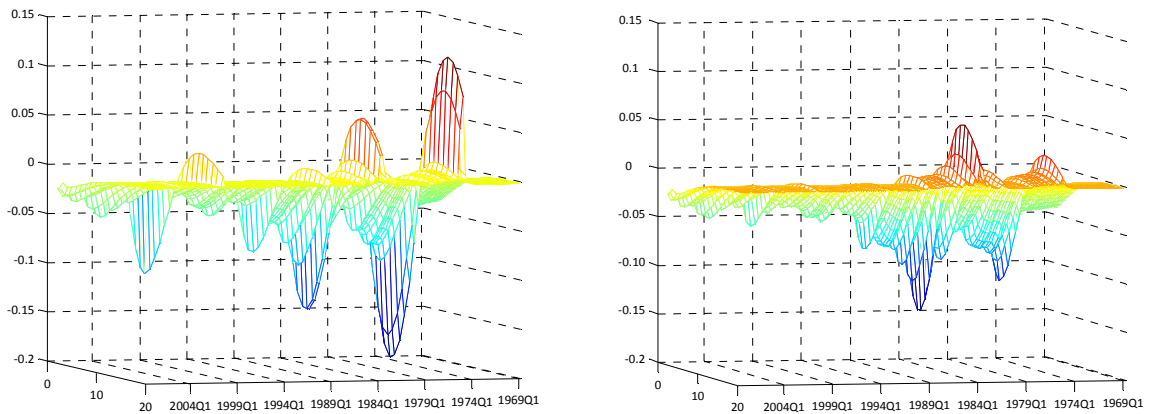
Notes: This figure shows the responses of inflation to a price and a wage mark-up shocks. Dotted lines are the 90% confidence intervals.

Figure 7  
 IRFs under learning: Response of inflation to wage mark-up, productivity  
 and monetary policy shocks  
 Without using survey data                      Using survey data

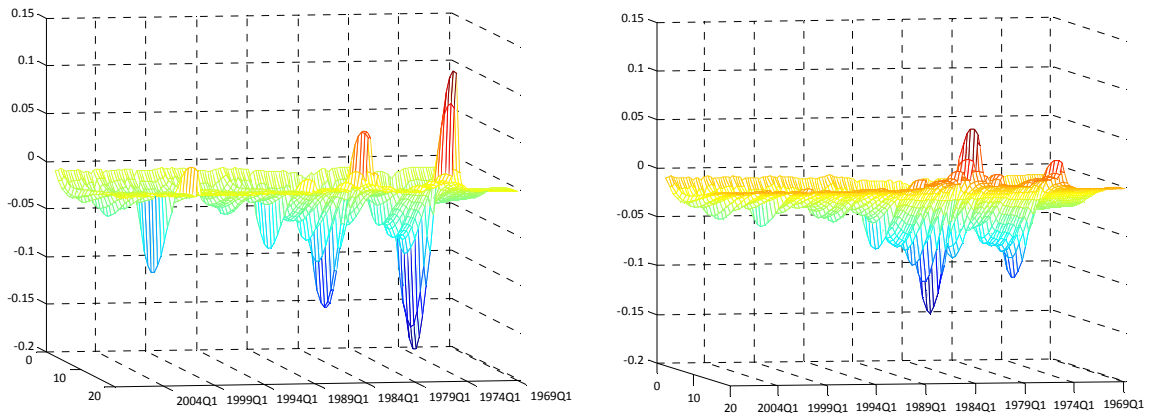
(a) Wage mark-up shock



(b) Productivity shock



(c) Monetary policy shock

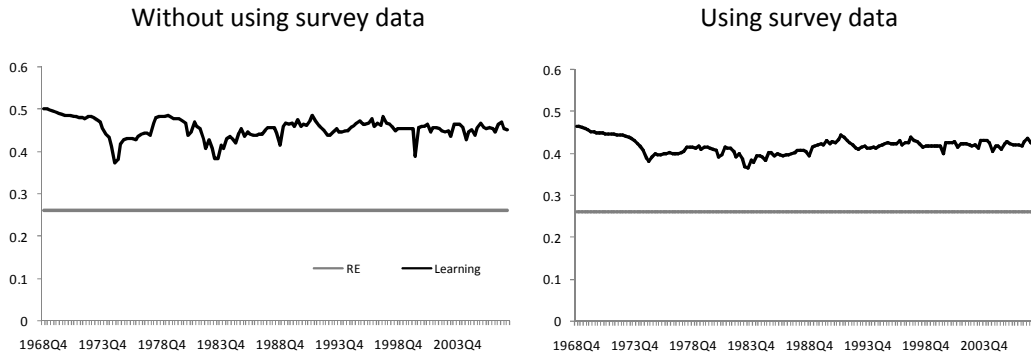


Notes: This figure shows the responses of inflation to a wage mark-up, productivity and monetary policy shocks using the structure of the economy at every point in time.

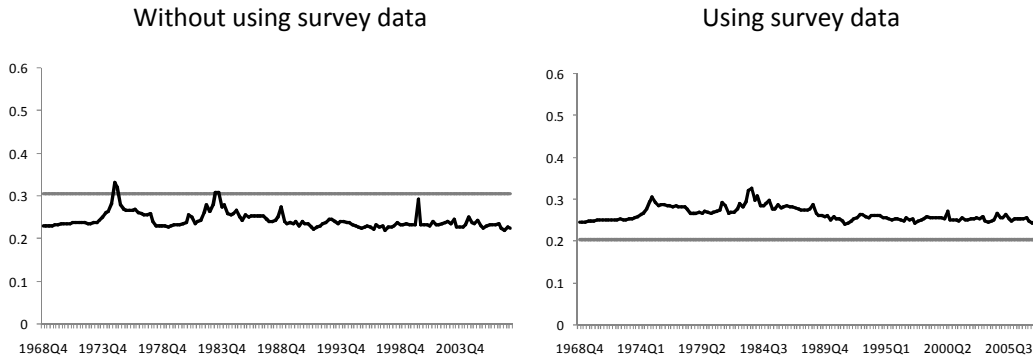
Figure 8

Forecast error variance decomposition (long-run analysis)

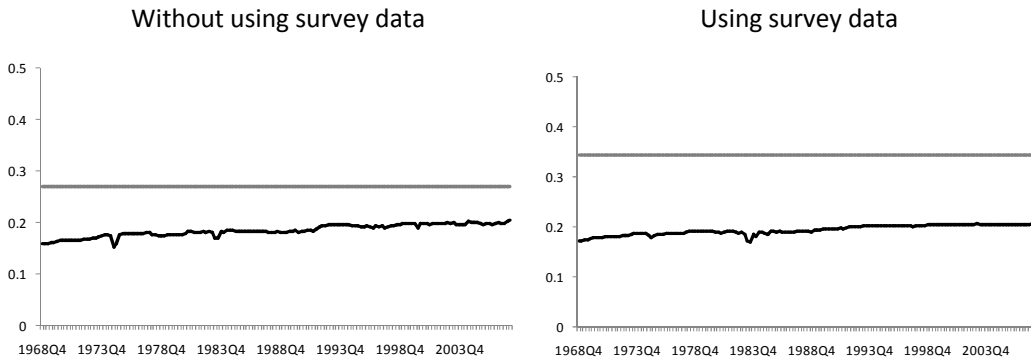
(a) Price mark-up shock



(b) Wage mark-up shock



(c) Investment-specific technology shock

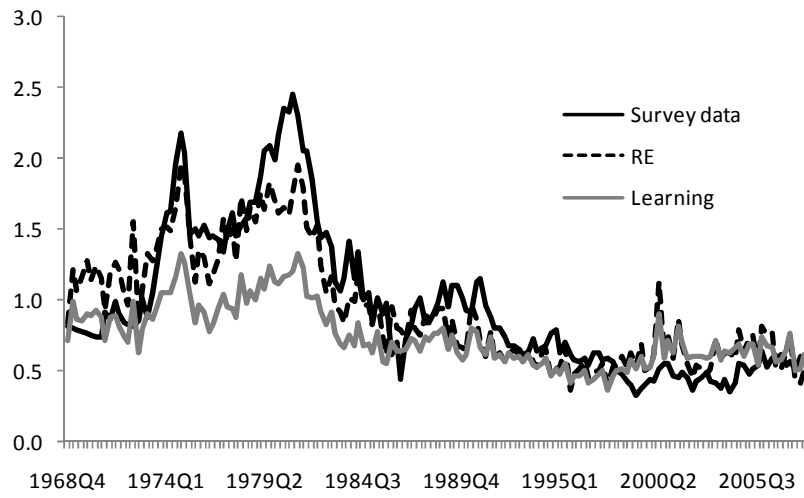


Notes: This figure shows the forecasting error variance decomposition in the long-run (after 100 quarters) for the price and wage mark-up shocks and investment-specific technology shock.

Figure 9

Inflation expectations

Survey data and model-implied expectations using OLS learning



## Appendix A: Stochastic part of the model

The stochastic part of the model is characterized by seven exogenous processes: two which affect the intertemporal margin, such as the risk premium shocks,  $\widehat{b}_t$ , and the investment-specific technology shocks,  $\widehat{q}_t$ ; a further two that affect the intratemporal margin, such as the wage mark-up shocks,  $\widehat{\lambda}_{w,t}$ , and the price mark-up shocks,  $\widehat{\lambda}_{p,t}$ ; another two policy shocks, the exogenous spending,  $\widehat{g}_t$ , and the monetary policy shocks,  $\widehat{r}_t$ ; and last the total factor productivity shocks,  $\widehat{A}_t$ .

Each of these processes is characterized as first-order autoregressive (AR(1)) process with and *iid*-normal distributed error term. Their representations are the following:

- $\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \rho_{ga} \varepsilon_t^a + \varepsilon_t^g$
- $\widehat{b}_t = \rho_b \widehat{b}_{t-1} + \varepsilon_t^b$
- $\widehat{q}_t = \rho_q \widehat{q}_{t-1} + \varepsilon_t^q$
- $\widehat{A}_t = \rho_a \widehat{A}_{t-1} + \varepsilon_t^a$
- $\widehat{\lambda}_{p,t} = \rho_p \widehat{\lambda}_{p,t-1} + \varepsilon_t^p$
- $\widehat{\lambda}_{w,t} = \rho_w \widehat{\lambda}_{w,t-1} + \varepsilon_t^w$
- $\widehat{r}_t = \rho_R \widehat{r}_{t-1} + \varepsilon_t^r$

## Appendix B: Definition of the dataset<sup>17</sup>

### Definition of data variables

- consumption =  $\text{LN} \left( \frac{\text{PCEC}}{\text{GDPDEF}} \right) / \text{LNSindex} * 100$
- investment =  $\text{LN} \left( \frac{\text{FPI}}{\text{GDPDEF}} \right) / \text{LNSindex} * 100$
- output =  $\text{LN} \left( \frac{\text{GDPC96}}{\text{LNSindex}} \right) * 100$
- hours =  $\text{LN} \left( \frac{\text{PRS85006023} * \text{CE16OV}}{100} \right) / \text{LNSindex} * 100$
- inflation =  $\text{LN} \left( \frac{\text{GDPDEF}}{\text{GDPDEF}(-1)} \right) * 100$
- real wage =  $\text{LN} \left( \frac{\text{PRS85006103}}{\text{GDPDEF}} \right) * 100$
- interest rate = Federal Funds Rate / 4

### Source of the original data:

GDPC96 : Real Gross Domestic Product - Billions of Chained 1996 Dollars, Seasonally Adjusted Annual Rate. Source: U.S. Department of Commerce, Bureau of Economic Analysis

GDPDEF : Gross Domestic Product - Implicit Price Deflator - 1996=100, Seasonally Adjusted Source: U.S. Department of Commerce, Bureau of Economic Analysis

PCEC : Personal Consumption Expenditures - Billions of Dollars, Seasonally Adjusted Annual Rate. Source: U.S. Department of Commerce, Bureau of Economic Analysis

FPI : Fixed Private Investment - Billions of Dollars, Seasonally Adjusted Annual Rate. Source: U.S. Department of Commerce, Bureau of Economic Analysis

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<sup>17</sup> Taken from the data documentation of Smets and Wouters (2007).

CE16OV : Civilian Employment: Sixteen Years & Over, Thousands, Seasonally Adjusted. Source:  
U.S. Department of Labor: Bureau of Labor Statistics

CE16OV index : CE16OV (1992:3)=1

Federal Funds Rate : Averages of Daily Figures – Percent. Source: Board of Governors of the  
Federal Reserve System (Before 1954: 3-Month Treasury Bill Rate, Secondary Market Averages  
of Business Days, Discount Basis)

LFU800000000 : Population level - 16 Years and Older - Not Seasonally Adjusted. Source: U.S.  
Bureau of Labor Statistics

LNS100000000 : Labor Force Status : Civilian noninstitutional population - Age : 16 years and  
over . Seasonally Adjusted - Number in thousands. Source: U.S. Bureau of Labor Statistics  
(before 1976: LFU800000000 : Population level - 16 Years and Older)

LNSindex : LNS100000000(1992:3)=1

PRS85006023 - Nonfarm Business, All Persons, Average Weekly Hours Duration : index, 1992 =  
100, Seasonally Adjusted. Source : U.S. Department of Labor

PRS85006103 - Nonfarm Business, All Persons, Hourly Compensation Duration : index, 1992 =  
100, Seasonally Adjusted. Source : U.S. Department of Labor

## Appendix C: Prior distributions of structural parameters

	Symbol	Distribution	Mean	Std.
Share of capital in production	$\alpha$	Normal	0.30	0.05
Inv. Elasticity of Intertemporal substitution	$\sigma_c$	Normal	1.50	0.38
Fix cost in production	$\Phi$	Normal	1.25	0.13
Adjust cost of investment	$S''$	Normal	4.00	1.50
Habits in consumption	$\eta$	Beta	0.70	0.10
Wage stickiness	$\xi_w$	Beta	0.50	0.10
inv. Elast. labor supply	$\sigma_l$	Normal	2.00	0.75
Price stickiness	$\xi_p$	Beta	0.50	0.10
Wage indexation	$\iota_w$	Beta	0.50	0.15
Price indexation	$\iota_p$	Beta	0.50	0.15
Capital utilization elasticity	$\psi$	Beta	0.50	0.15
Taylor rule: response to inflation	$r_\pi$	Normal	1.50	0.25
Taylor rule: response to lagged interest rate	$\rho_R$	Beta	0.75	0.10
Taylor rule: response to changes in output	$r_{\Delta y}$	Normal	0.13	0.05
Trend growth rate	$\gamma$	Normal	0.40	0.10
Steady state of inflation	$\pi\_bar$	Gamma	0.63	0.10
Steady state of hours worked	$l\_bar$	Normal	0.00	2.00
Steady state of nominal int rate	$r\_bar$	Gamma	1.15	0.30
Autocorrelation coef. Price Mk up shock	$\rho_p$	Beta	0.50	0.20
Autocorrelation coef. Wage Mk up shock	$\rho_w$	Beta	0.50	0.20
Autocorrelation coef. Product. Shock	$\rho_a$	Beta	0.50	0.20
Autocorrelation coef. Risk premium shock	$\rho_b$	Beta	0.50	0.20
Autocorrelation coef. Government shock	$\rho_g$	Beta	0.50	0.20
Autocorrelation coef. Investment-Specific shock	$\rho_q$	Beta	0.50	0.20
Autocorrelation coef. Monet policy shock	$\rho_r$	Beta	0.50	0.20
Correlation Government and productivity shocks	$\rho_{ga}$	Normal	0.50	0.25
Std Price Mk up innovation	$\sigma_p$	Inv. Gamma	0.10	2.00
Std. Wage Mk up innovation	$\sigma_w$	Inv. Gamma	0.10	2.00
Std. Product. Innovation	$\sigma_a$	Inv. Gamma	0.10	2.00
Std. Risk premium innovation	$\sigma_b$	Inv. Gamma	0.10	2.00
Std. Government innovation	$\sigma_g$	Inv. Gamma	0.10	2.00
Std. Inv. Specific innovation	$\sigma_q$	Inv. Gamma	0.10	2.00
Std. Monet policy innovation	$\sigma_r$	Inv. Gamma	0.10	2.00
Gain - no inflation	$g^\pi$	Uniform	0.00	0.30
Gain - inflation	$g^{non\pi}$	Uniform	0.00	0.30
Std. measurement error on expectations	$\sigma_{exp}$	Inv. Gamma	0.10	2.00

Note: for uniform distributions the values assigned as mean and standard deviation correspond to the range of the domain.