A joint initiative of Ludwig-Maximilians University's Center for Economic Studies and the Ifo Institute for Economic Research

CESifo Area Conference

on

Macro, Money and International Finance

20-21 February 2009 CESifo Conference Centre, Munich

Microfoundations of Staggered Wage Contracts

Leif Danziger

CESifo GmbH
Poschingerstr. 5
81679 Munich
Germany

Phone: +49 (0) 89 9224-1410
Fax: +49 (0) 89 9224-1409
E-mail: office@cesifo.de
Web: www.cesifo.de

Microfoundations of Staggered Wage Contracts

Leif Danziger

Department of Economics, Ben-Gurion University
Beer-Sheva 84105, Israel

Abstract

This paper provides microfoundations of staggered wage contracts. In the model, short and long contracts as well as long contracts concluded in different periods are strategic substitutes, and this affects the choice of contract duration and provides a strong incentive for staggering. We show that short and long wage contracts may coexist, in which case the long contracts are always uniformly staggered. Further, the proportion of long contracts increases with the contracting cost and decreases with the variance of the monetary policy shocks. If only long contracts exist, then uniform staggering is just one out of a continuum of possible equilibria with different degrees of staggering.

JEL Classifications: E24, E31, E32, J41
Keywords: microfoundations, staggered wage contracts, monetary policy shocks

1 Introduction

Staggered wage contracts play a prominent role in recent dynamic stochastic general equilibrium models that study the output effects of monetary policy shocks. Andersen (1998), Huang and Liu (2002), and Christiano et al. (2005) establish that a staggered wage setting (in contrast to a staggered price setting) is a powerful mechanism for creating a critical nominal friction that makes monetary policy disturbances capable of generating persistent output effects. Olivei and Tenreyro (2007, 2008) further show that output responses may be markedly different depending on the proportion of wage contracts that are reset at the same time as the monetary policy shock.[1] This is important since Matsukawa (1986), Fethke and Policano (1990), and Olivei and Tenreyro (2007, 2008) provide empirical evidence that wage contracts are nonuniformly staggered in several countries including the US, but uniformly staggered in other countries.

The theoretical models in the above papers build on either Taylor-style contracts that have a fixed duration (Taylor, 1980) or Calvo-style contracts that have a constant probability of being reset in each period (Calvo, 1983). There is, however, no modelling of the economic forces that determine the extent of contract staggering which is taken as exogenous. Accordingly, in this paper we provide microfoundations of Taylor-style staggered wage contracts, and determine analytically how monetary policy shocks and other variables influence the duration and staggering of contracts. In our framework, firms and workers may conclude either short contracts (covering one period) or long contracts (covering two periods), and the economy is exposed to both monetary policy shocks and productivity shocks. We assume that firms produce a homogenous product that is sold in a competitive product market, that all shocks are aggregate, and that the monetary authority follows a passive monetary policy. Hence, in our approach staggering of wage contracts cannot be caused by monopolistic competition, informational asymmetries, idiosyncratic shocks, or systematic interventions by the

[1] See, however, Smets and Wouters (2007) who find that wage and price stickiness are equally responsible for the slow transmission of monetary policy shocks. Other contributions in which the timing pattern of wage contracts play a central role include Blanchard (1986), Phaneuf (1990), Fuhrer and Moore (1995), Cho et al. (1997), Clarida et al. (1999), Ascari (2000), Erceg et al. (2000), Edge (2002), and Woodford (2003).

monetary authority.[2]

In the model, there is strategic substitutability between short and long contracts, as well as between long contracts that are concluded in different periods, and this affects the choice contract length and provides a powerful rationale for staggering. To explain what causes the strategic substitutability, we note that given its own real wage rate, any firm benefits from the other firms paying higher real wage rates as this increases the equilibrium price of output. In fact, given its own real wage rate, the proportion of firms with an existing contract has an equi-proportional positive effect on each firm's real profit if the monetary policy shock is negative, and an equi-proportional negative affect on each firm's real profit if the monetary policy shock is positive. Now, the real wage rate in an existing wage contract is higher than the real wage rate in a new contract if the monetary policy shock is negative, but lower if the monetary policy shock is positive. It follows that the real profit with a continuing contract increases less with the proportion of firms that have an existing contract if the monetary policy shock is negative, and decreases more with the proportion of firms that have an existing contract if the monetary policy shock is positive. As a consequence, an increase in the proportion of firms that have an existing contract leads to a decrease in the expected real profit from an existing contract relative to the expected real profit from a new contract, and the upshot is that contracts of different lengths and contracts concluded in different periods are strategic substitutes.

We show that if the contracting cost is small enough, it is possible that all contracts are short and the question of staggering is moot. For a somewhat higher contracting cost, short and long contracts coexist and the latter are always uniformly staggered. For a still higher contracting cost, all contracts are long and there is a continuum of equilibria with different

[2] Matsukawa (1986) and Fethke and Policano (1987) have shown that an activist monetary authority may support a nonuniform contracting pattern, and Fethke and Policano (1990) that if there are both competitive and monopolistically competitive sectors, then informational asymmetries together with a real balance effect may lead to a nonuniform contracting pattern. For the related problem of price adjustments, staggered price setting may be an equilibrium if, as in Ball and Cecchetti (1988), firms learn from contracts others have written, or, as in Ball and Romer (1989), firms fix their prices for two periods and half of the firm receive idiosyncratic shocks in even-numbered periods and the other half in odd-numbered periods. See also Ball (1987) and Fethke and Policano (1984, 1987).

degrees of staggering. There then exists a positive lowest degree of staggering such that there is an equilibrium for any contracting pattern with at least this degree of staggering (and hence with uniformly staggered contracts), but not for any contracting pattern with less staggering (and hence ruling out completely synchronized contracts). Finally, if the contracting cost is even higher, all contracts are long and there is a continuum of equilibria with the contracts being anything from uniformly staggered to completely synchronized.

The comparative-static analysis reveals that in the case that short and long contracts coexist, the proportion of long contracts increases with the contracting cost and decreases with the variance of the monetary policy shocks.[3] In the case that all contracts are long and the lowest possible degree of staggering is positive, then that lowest degree of staggering decreases with the contracting cost and increases with the variance of the monetary policy shocks. Thus, the impact of a higher contracting cost and of a higher variance of the monetary policy shocks are always in opposite directions, for which the intuition is as follows: On the one hand, a higher contracting cost reduces the attractiveness of concluding short rather than long contracts, and thereby also of deviating from a given pattern of staggered long contracts by concluding a short contract. On the other hand, increased monetary uncertainty has a more negative impact in the second period of a long contract than in a short contract covering the same period, and it therefore increases the attractiveness of concluding short rather than long contracts, and thereby also of deviating from a given pattern of staggered long contracts.

Other results include that if short and long contracts coexist, then the proportion of long contracts is independent of the discount rate, while if all contracts are long contracts, then the lowest possible positive degree of staggering decreases with the discount rate. Interestingly, neither the proportion of long contracts nor the lowest possible positive degree of staggering

[3] Hence, the average contract duration also increases with the contracting cost and decreases with the variance of the monetary policy shocks. A similar result has been obtained by Gray (1978) in a macroeconomic setting that assumes complete synchronization of all contracts, and by Dye (1985), Harris and Holmstrom (1987), and Danziger (1988) in a partial equilibrium setting. Empirical support can be found in Christofides (1990), Murphy (1992), Wallace (2001), Rich and Tracy (2004), and Christofides and Peng (2006).

are affected by the uncertainty of the aggregate productivity.

2 The Model

We consider an economy with a unit continuum of firms that produce a homogeneous output that is sold in a competitive market. The firms are labelled by $i \in [0,1]$. The production function of the ith firm in period t is $y_{it} = a_t \ell_{it}^x$, where y_{it} is the firm's output, a_t is the level of aggregate productivity, ℓ_{it} is the firm's labor input, and $x \in (0,1)$. Since $\ell_{it} = (y_{it}/a_t)^{1/x}$ of labor is required to produce y_{it}, the ith firm's nominal profit from production in period t is

$$p_t y_{it} - w_{it} \left(\frac{y_{it}}{a_t} \right)^{1/x}, \tag{1}$$

where p_t is the nominal price of output and w_{it} is the nominal wage rate paid by the ith firm. Assuming that firms can freely choose their labor input, the ith firm's profit-maximizing production is

$$y_{it} = a_t^{1/(1-x)} \left(\frac{p_t x}{w_{it}} \right)^{x/(1-x)}. \tag{2}$$

It follows that the aggregate production is

$$\begin{aligned} \int_0^1 y_{it} di &= \int_0^1 a_t^{1/(1-x)} \left(\frac{p_t x}{w_{it}} \right)^{x/(1-x)} di \\ &= a_t^{1/(1-x)} \left(\frac{p_t x}{W_t} \right)^{x/(1-x)}, \end{aligned}$$

where

$$W_t \equiv \left[\int_0^1 \frac{1}{w_{it}^{x/(1-x)}} di \right]^{(x-1)/x} \tag{3}$$

is an index of the wage rates in the economy.

The demand function in period t equals the real money balances M_t/p_t, where M_t is the money supply. In order for demand to equal supply, we have that

$$\frac{M_t}{p_t} = a_t^{1/(1-x)} \left(\frac{p_t x}{W_t} \right)^{x/(1-x)},$$

which implies that the equilibrium price is

$$p_t = \frac{M_t^{1-x} W_t^x}{a_t x^x}.$$

Substituting the equilibrium price in eq. (2) shows that the ith firm will produce

$$y_{it} = \frac{a_t M_t^x W_t^{x^2/(1-x)} x^x}{w_{it}^{x/(1-x)}}.$$

Then, by dividing the nominal profit in eq. (1) by p_t and substituting p_t and y_{it}, we obtain that the ith firm's real profit from production in period t is

$$\frac{a_t M_t^x W_t^{x^2/(1-x)} x^x (1-x)}{w_{it}^{x/(1-x)}}. \tag{4}$$

The future money supply and aggregate productivity are uncertain since both are exposed to shocks between the periods. Thus, the money supply changes according to $M_t/M_{t-1} = 1 + \gamma_t$, where γ_t is the monetary policy shock between period $t-1$ and period t. The monetary policy shocks are non-degenerate i.i.d. random variables with zero mean. The aggregate productivity changes according to $a_t/a_{t-1} = 1 + \alpha_t$, where α_t is the productivity shock between period $t-1$ and period t. The productivity shocks are i.i.d. random variables with zero mean.

Each firm concludes consecutive wage contracts with its workers, such that when one contract expires, the next is concluded. A wage contract may cover one period, which is called a short contract, or it may cover two periods, which is called a long contract.[4] We assume that the bargaining process leads to a nominal wage rate which is $\kappa > 0$ times the nominal value of the average firm's output in the bargaining period. Since the latter equals M_t in period t, if a firm concludes a new contract in period t, the nominal wage rate will be κM_t for the duration of the contract.

The wage index in period t depends on how many contracts are concluded in period t and how many were concluded in period $t-1$. Let n_t denote the proportion of firms that

[4] A contracts cannot cover more than two periods since the cumulative effect of adverse shocks might then make it worthwhile for workers to leave the firm.

conclude long contracts in period t; then $n_t \in [0, 1-n_{t-1}]$, and $1-n_t-n_{t-1}$ is the proportion of firms that conclude short contracts in period t. Since the wage rate in period t equals κM_t for the $1-n_{t-1}$ firms that conclude contracts in period t, and equals $\kappa M_{t-1} = \kappa M_t/(1+\gamma_t)$ for the n_{t-1} firms that concluded long contracts in period $t-1$, we obtain from eq. (3) that the wage index in period t is

$$\begin{aligned} W_t &= \left\{ \frac{1-n_{t-1}}{(\kappa M_t)^{x/(1-x)}} + \frac{n_{t-1}}{[\kappa M_t/(1+\gamma_t)]^{x/(1-x)}} \right\}^{(x-1)/x} \\ &= \kappa M_t (1-n_{t-1}+n_{t-1}B_t)^{(x-1)/x}, \end{aligned} \tag{5}$$

where $B_t \equiv (1+\gamma_t)^{x/(1-x)}$ is the ratio of the labor demand of a firm that is in the second period of a long contract concluded in period $t-1$ to the labor demand of a firm that concludes a new contract in period t. Thus, if $n_{t-1} = 0$ and all the firms conclude new contracts in period t, the index in period t is κM_t. Conversely, if $n_{t-1} = 1$ and none of the firms conclude new contracts in period t, the index in period t is $\kappa M_t/(1+\gamma_t)$. More generally, since B_t increases with the monetary policy shock, unless all the contracts are concluded in period t, for a given money supply in period t the effect of the second-period wage rates in the long contracts causes the wage index in period t to decrease with the monetary policy shock between periods $t-1$ and t.

Now, let us suppose that the ith firm concludes a short contract in period t. By substituting $w_{it} = \kappa M_t$ and W_t from eq. (5) in eq. (4), we obtain that its real profit from production in period t is

$$\begin{aligned} & \frac{a_t x^x (1-x)}{(1-n_{t-1}+n_{t-1}B_t)^x \kappa^x} \\ = \; & a_t k A_{n_{t-1},t}, \end{aligned}$$

where

$$\begin{aligned} k &\equiv \frac{x^x(1-x)}{\kappa^x}, \\ A_{nt} &\equiv \frac{1}{(1-n+nB_t)^x}. \end{aligned}$$

In the absence of a monetary policy shock between period $t-1$ and period t, the real profit from production in period t would equal $a_t k$ and thus would fully incorporate the productivity shock between period $t-1$ and period t. The factor $A_{n_{t-1},t}$, which depends on n_{t-1}, embodies the effect of the monetary policy shock between period $t-1$ and period t on the real profit from production in period t. Thus, $A_{n_{t-1},t}$ is the ratio of the firm's real profit from production to what it would be if all firms would conclude their contracts in period t. Moreover, if $B_t = 1$ and the wage rate in the new contract equals the wage rate in a long contract concluded in the previous period, then $A_{n_{t-1},t} = 1$.

Suppose that instead the ith firm concludes a long contract in period t. Its real profit from production in the first period of the contract is the same as the real profit in a short contract. To determine the real profit from production in the second period of the contract, we first advance the period index from t to $t+1$ in eq. (4) and in eq. (5) to show that the real profit from production in period $t+1$ is

$$\frac{a_{t+1} M_{t+1}^x W_{t+1}^{x^2/(1-x)} x^x (1-x)}{w_{i,t+1}^{x/(1-x)}}, \tag{6}$$

while the wage index in period $t+1$ is

$$W_{t+1} = \kappa M_{t+1} (1 - n_t + n_t B_{t+1})^{(x-1)/x}.$$

We then substitute $w_{i,t+1} = \kappa M_t$, or equivalently, $w_{i,t+1} = \kappa M_{t+1}/(1+\gamma_{t+1})$, and W_{t+1} in expression (6) to obtain that the real profit from production in the second period of the contract concluded in period t is

$$\begin{aligned} & \frac{a_{t+1} B_{t+1} x^x (1-x)}{(1 - n_t + n_t B_{t+1})^x \kappa^x} \\ = \; & a_{t+1} k A_{n_t,t+1} B_{t+1}. \end{aligned}$$

The real profit from production would be $a_{t+1} k A_{n_t,t+1}$ in a short or long contract that is concluded in period $t+1$. Therefore, B_{t+1} is not only the ratio of the labor demand of a firm that is in the second period of a long contract concluded in period t to the labor demand of a firm that concludes a new contract in period $t+1$, but also the ratio of the real profit from production in the second period of a long contract concluded in period t to the real profit from production in a new contract concluded in period $t+1$. Of course, if $B_t = 1$ and the wage rate in the new contract is equal to the second-period wage rate in a long contract concluded in the previous period, then both $A_{n_t,t+1}$ and $A_{n_t,t+1}B_{t+1}$ would be equal to 1.

At each t in which a firm negotiates a new contract, it chooses whether the contract should be short (i.e., cover only t) or long (i.e., cover both t and $t+1$). In order to negotiate a contract, the firm incurs a real contracting cost that is equal to a proportion of the real profit in the absence of a monetary policy shock. That is, the real contracting cost in period t is cka_t, where $c > 0$. A firm's real profit in period t can therefore be written as $a_t k \left(A_{n_{t-1},t} B_t^{1-\lambda_t} - \lambda_t c\right)$, where $\lambda_t = 1$ if a new contract is negotiated in period t, and $\lambda_t = 0$ if a long contract was negotiated in period $t-1$.

A firm's objective is to maximize its discounted expected real profits

$$kE_t \sum_{\tau=t}^{\infty} \frac{a_\tau \left(A_{n_{\tau-1},\tau} B_\tau^{1-\lambda_\tau} - \lambda_\tau c\right)}{(1+\rho)^{\tau-t}},$$

where $\rho > 0$ is the discount rate, and the expectation at t is taken over the distribution of all future real profits. The firm's discounted expected real profits depend on the firm's and all the other firms' choices of contracts in the present and future, and each firm's contracting strategy maps all available information to a choice of a short or a long contract at each t that a new contract is negotiated. We consider only Markov contracting strategies, that is, strategies for which a firm's current choice between concluding a short or a long contract depends on only the available information that directly affects the firm's current or future real profits.

DEFINITION: *The economy is in a Markov perfect contracting equilibrium if in each period that a firm negotiates a new contract, the firm's Markov contracting strategy maximizes its discounted expected real profits given that all other firms follow their Markov contracting strategies.*

3 Contract Characteristics

Firms that always conclude short contracts are called short-contract firms, and firms that always conclude long contracts are called long-contract firms. The latter are subdivided into even firms that always conclude long contracts in even-numbered periods, and odd firms that always conclude long contracts in odd-numbered periods.

This section now proceeds to establish Lemma 1 which derives the expressions for short- and long-contract firms' discounted expected real profits; Lemma 2 which uses these expressions to determine the short- and long-contract firms' gains from a one-time deviation; and Lemma 3 which implies strategic substitutability between short and long contracts, as well as strategic substitutability between long contracts concluded in even-numbered periods and long contracts concluded in odd-numbered periods.[5]

3.1 The Discounted Expected Real Profits

Let T_e denote an arbitrary even-numbered period, and T_o denote an arbitrary odd-numbered period. We now derive the discounted expected real profits for a short-contract firm, an even firm, and an odd firm:

LEMMA 1: *Assume that there are $n_e \in [0,1]$ even firms, $n_o \in [0, 1-n_e]$ odd firms, and that the remaining firms are short-contract firms. The discounted expected real profits at T_e for a short-contract firm are*

$$a_{T_e} k \left[A_{n_o,T_e} + \frac{E_{t-1} A_{n_o,t} + (1+\rho) E_{t-1} A_{n_e,t}}{\rho(2+\rho)} - \frac{(1+\rho)c}{\rho} \right], \tag{7}$$

and for an even firm are

$$a_{T_e} k \left[A_{n_o,T_e} + \frac{E_{t-1} A_{n_o,t} + (1+\rho) E_{t-1} \left(A_{n_e,t} B_t \right) - (1+\rho)^2 c}{\rho(2+\rho)} \right]. \tag{8}$$

The discounted expected real profits at T_o for a short-contract firm are

$$a_{T_o} k \left[A_{n_e,T_o} + \frac{E_{t-1} A_{n_e,t} + (1+\rho) E_{t-1} A_{n_o,t}}{\rho(2+\rho)} - \frac{(1+\rho)c}{\rho} \right], \tag{9}$$

[5] The proofs of Lemmas 1-3 are in Appendix A.

and for an odd firm are

$$a_{T_o}k\left[A_{n_e,T_o}+\frac{E_{t-1}A_{n_e,t}+(1+\rho)E_{t-1}\left(A_{n_o,t}B_t\right)-(1+\rho)^2c}{\rho(2+\rho)}\right]. \qquad (10)$$

To understand Lemma 1, assume that $a_{T_e}k = 1$. Consider first expression (7) for a short-contract firm's discounted expected real profits. The first term in the brackets of expression (7) is the real profit from production in period T_e. To explain the next term in the brackets, we note that random changes in the aggregate productivity have no effect on expected future real profits. Thus, the expected real profit from production in any future even-numbered period equals $E_{t-1}A_{n_o,t}$ and the expected real profit from production in any future odd-numbered period equals $E_{t-1}A_{n_e,t}$. Accordingly, the second term in the brackets of expression (7) is the sum of the expected real profits from production in all future even- and odd-numbered periods, discounted the relevant number of periods. That is, the second term consists of $E_{t-1}A_{n_o,t}$ multiplied by $1/(1+\rho)^2 + 1/(1+\rho)^4 + ... = 1/[\rho(2+\rho)]$ plus $E_{t-1}A_{n_e,t}$ multiplied by $1/(1+\rho) + 1/(1+\rho)^3 + ... = (1+\rho)/[\rho(2+\rho)]$. The third term in the brackets of expression (7) are the discounted expected real cost of contracting in each period.

Consider next expression (8) for an even firm's discounted expected real profits. Since the real profit from production in the first period of any long contract (which is an even-numbered period) is the same as in a short contract, the real profit from production in period T_e and the discounted expected real profits from production in all future even-numbered periods are the same as with short contracts. The real profit from production in the second period of a long contract (which is an odd-numbered period) equals $E_{t-1}\left(A_{n_e,t}B_t\right)$. Hence, the discounted expected real profits from production in all odd-numbered periods are $(1+\rho)E_{t-1}\left(A_{n_e,t}B_t\right)/[\rho(2+\rho)]$. Finally, the sum of the discounted expected real cost of contracting is less for a long-contract firm than for a short-contract firm. Expressions (9) and (10) for the discounted expected real profits at T_o for a short-contract firm and for an odd firm have a similar interpretation.

3.2 The Gain from Deviating

We proceed to determine the gain to a deviating short-contract firm, by which we mean a short-contract firm that makes a one-time deviation by concluding a long contract and then reverts to always concluding short contracts. We also determine the gain to a deviating even (odd) firm, by which we mean an even (odd) firm that makes a one-time deviation by concluding a short-period contract in an even-numbered (odd-numbered) period and then reverts to always concluding long contracts. Let

$$\begin{aligned} D(n) &\equiv E_{t-1}\left[A_{nt}(B_t - 1)\right] + c, \\ G(n', n'') &\equiv -D(n') + \frac{D(n'')}{1+\rho}, \\ h &\equiv \frac{k(1+\rho)}{\rho(2+\rho)}. \end{aligned}$$

We then have:

LEMMA 2: *Assume that there are $n_e \in [0,1]$ even firms, $n_o \in [0, 1-n_e]$ odd firms, and that the remaining firms are short-contract firms. The gain of discounted expected real profits at T_e for a deviating short-contract firm is $a_{T_e} kD(n_e)/(1+\rho)$, and for a deviating even firm is $a_{T_e} hG(n_e, n_o)$. The gain of discounted expected real profits at T_o for a deviating short-contract firm is $a_{T_o} kD(n_o)/(1+\rho)$, and for a deviating odd firm is $a_{T_o} hG(n_o, n_e)$.*

Lemma 2 can be understood by observing that if a short-contract firm deviates in period T_e, then it gains the difference between the discounted expected real profit in the second period of the long contract concluded in period T_e, that is, $a_{T_e} kE_{t-1}(A_{n_e,t}B_t)/(1+\rho)$, and what the discounted expected real profit would be in a short contract concluded in period $T_e + 1$, that is, $a_{T_e} k\left(E_{t-1}A_{n_e,t} - c\right)/(1+\rho)$. By subtracting the latter from the former, one obtains that the gain from deviating is $a_{T_e} kD(n_e)/(1+\rho)$.

Similarly, if an even firm deviates in period T_e, then each of the first (second) periods of the long contracts that the firm will conclude at future odd-numbered (even-numbered) periods corresponds to a second (first) period of one of the long contracts that it would have concluded at even-numbered periods if it had not deviated. Hence, the gain of discounted expected real profit from deviating stemming from the odd-numbered periods $T_e+1, T_e+3, ...$ is $-a_{T_e}hD(n_e)$. The gain of discounted expected real profit from deviating stemming from the even-numbered periods $T_e+2, T_e+4, ...$ is $a_{T_e}hD(n_o)/(1+\rho)$. Adding the gains stemming from all the odd- and even-numbered periods, the even firm's total gain from deviating is $a_{T_e}hG(n_e, n_o)$.

The interpretations of the gains for a short-contract firm and for an odd firm from deviating in period T_o are similar.

3.3 Strategic Substitutability between Contracts

We now establish the fundamental monotonicity property of the $D(n)$ function:

LEMMA 3: $dD(n)/dn < 0$.

If $n_e \in [0, 1]$ of firms are even, $n_o \in [0, 1-n_e]$ of firms are odd, and the remaining firms are short-period, then subtracting (7) from (8) shows that the difference between the discounted expected real profits at T_e for an even firm and for a short-contract firm is $a_{T_e}hD(n_e)$. Similarly, the difference between the discounted expected real profits at T_o for an odd firm and for a short-contract firm is $a_{T_o}hD(n_o)$. Lemma 3 therefore implies that the value of always concluding long rather than short contracts decreases with the proportion of long contracts that are concluded in the same periods. Accordingly, long and short contracts are strategic substitutes.

The explanation for the strategic substitutability between long and short contracts is that given its own real wage rate, both a firm that is in the second period of a long contract and a firm that has just concluded a short contract benefit from a higher wage index, as other firms paying higher real wages is associated with a higher equilibrium price. Since the wage index in period t increases with n_{t-1} if the monetary policy shock is negative (as $\gamma_t < 0 \Rightarrow B_t < 1$) and decreases with n_{t-1} if the monetary policy shock is positive (as $\gamma_t > 0 \Rightarrow B_t > 1$), it follows that, given its own real wage rate, a higher n_{t-1} has an equi-proportional positive effect on each firm's real profit if $\gamma_t < 0$, and an equi-proportional negative effect on each firm's real profit if $\gamma_t > 0$. The real wage rate in the second period of a long contract is higher than the real wage rate in a short contract if $\gamma_t < 0$, but is lower if $\gamma_t > 0$. Hence, a higher n_{t-1} leads to a smaller increase in the real profit in period t of firms that that are in the second period of a long contract than for firms that have concluded a short contract if $\gamma_t < 0$, and to a larger decrease in the real profit of firms that are in the second period of a long contract than for firms that have concluded a short contract if $\gamma_t > 0$. The implication is that an increase in n_{t-1} decreases the expected real profit of firms that are in the second period of a long contract relative to the expected real profit of firms that have concluded a new contract; that is, $dD(n)/dn < 0$ so that long and short contracts are strategic substitutes.

The strategic substitutability between long and short contracts has important implications for the gains of a deviating short-contract firm. According to Lemma 2, if the deviation takes place in an even-numbered period, the gain of discounted expected real profits is proportional to $D(n_e)$ and hence decreases with n_e, while if the deviation takes place in an odd-numbered period, the gain of discounted expected real profits is proportional to $D(n_o)$ and hence decreases with n_o. In particular, if $D(\frac{1}{2}) < 0 < D(0)$, there exists a unique proportion of long-contract firms $m \in (0, \frac{1}{2})$ such that $D(m) \gtreqless 0$ as $n \lesseqgtr m$. Hence, a short-contract firm will find it profitable to deviate in an even-numbered period iff $n_e < m$, and to deviate in an odd-numbered period iff $n_o < m$.

Turning to the case in which there are only long-contract firms (and hence $n_e + n_o = 1$), the difference between the discounted expected real future profits at T_e for an even firm and for an odd firm is $a_{T_e} h \left[D(n_e) - D(1 - n_e)/(1 + \rho)\right]$. Now, $dD(n_e)/dn_e < 0$, which reflects that in any future even-numbered period an increase in n_e decreases the expected real profit of an even firm (that is in its second contract period) relative to the expected real profit of an odd firm (that is in its first contract period), and $-dD(1 - n_e)/dn_e < 0$, which reflects that in any future odd-numbered period an increase in n_e decreases the expected future real profit of an even firm (that is in its first contract period) relative to the expected real profit of an odd firm (that is in its second contract period). Lemma 3 therefore also implies that the value of always concluding long contracts in even-numbered periods rather than in odd-numbered periods decreases with the proportion of long contracts that are concluded in the even-numbered periods. Similarly, the value of always concluding long contracts in odd-numbered periods rather than in even-numbered periods decreases with the proportion of long contracts that are concluded in the odd-numbered periods. Accordingly, long contracts concluded in even- and odd-numbered periods are strategic substitutes.

The strategic substitutability between long contracts concluded in different periods has important implications for the gains of a deviating long-contract firm. Lemma 2 shows that if there are only long-contract firms and the deviation takes place in an even-numbered period, then the gain of discounted expected real profits is proportional to $G(n_e, 1-n_e) = -D(n_e) + D(1-n_e)/(1+\rho)$ and hence increases with n_e. If the deviation takes place in an odd-numbered period, then the gain of discounted expected real profits is proportional to $G(n_o, 1-n_o) = -D(n_o) + D(1-n_o)/(1+\rho)$, and hence increases with n_o. As a result, if $G(\frac{1}{2}, \frac{1}{2}) \leq 0 < G(1, 0)$, there exists a unique proportion of even firms $\hat{m} \in [\frac{1}{2}, 1)$ such that $G(\hat{m}, 1-\hat{m}) \gtreqless 0$ as $n \gtreqless \hat{m}$. Hence, a long-contract firm will find it profitable to deviate in an even-numbered period iff $n_e > \hat{m}$, and to deviate in an odd-numbered period iff $n_o > \hat{m}$.

4 Markov Perfect Contracting Equilibria

To prove the existence of Markov perfect contracting equilibria, we first define

$$
\begin{aligned}
c_1 &\equiv E_{t-1}\left[A_{0t}(1-B_t)\right], \\
c_2 &\equiv E_{t-1}\left[A_{1/2,t}(1-B_t)\right], \\
c_3 &\equiv \frac{1}{\rho}E_{t-1}\left[(1+\rho)A_{1t}(1-B_t) - A_{0t}(1-B_t)\right].
\end{aligned}
$$

Note that $c_1 < c_2 < c_3$, and that $c_1 \gtreqless 0$ as $x \lesseqgtr \frac{1}{2}$.[6] The properties of the equilibria depend on the value of c and are:[7]

THEOREM 1: *There exist Markov perfect contracting equilibria in which firms are either short-period, even, or odd. There are four different regimes:*

Regime 1: *If* $c \leq c_1$, *then* $n_1 = 1$ *and* $n_e = n_o = 0$;

Regime 2: *If* $c_1 < c < c_2$, *then* $n_1 = 1 - 2m$ *and* $n_e = n_o = m$;

Regime 3: *If* $c_2 \leq c < c_3$, *then* $n_1 = 0$, $n_e \in [1 - \hat{m}, \hat{m}]$, *and* $n_o = 1 - n_e$;

Regime 4: *If* $c_3 \leq c$, *then* $n_1 = 0$, $n_e \in [0, 1]$, *and* $n_o = 1 - n_e$.

We discuss each of the regimes in turn:

Regime 1 assumes that the contracting cost is so small that $c \leq c_1$. Since this implies that $D(0) \leq 0$, no short-contract firm can gain by deviating if all the other firms are short-contract firms. The equilibrium is illustrated in Figure 1(a) where the horizontal axis measures n_e from left to right, and n_o from right to left. The downward-sloping $D(n_e)$ curve shows the gain of a short-contract firm from deviating in an even-numbered period if $a_{T_e}k/(1+\rho) = 1$. This gain is always nonpositive. The $D(n_o)$ curve is the reflection of the $D(n_e)$ curve around $n_e = \frac{1}{2}$, and it shows the gain of a short-contract firm from deviating in an odd-numbered period if $a_{T_o}k/(1+\rho) = 1$. Consequently, if $n_1 = 1$ and all firms conclude short contracts, no firm has an incentive to deviate. The economy is therefore in a Markov perfect contracting equilibrium, which is illustrated by the thick dots at $n_e = 0$ and $n_o = 0$.

Regime 2 assumes a larger contracting cost satisfying $c_1 < c < c_2$. Since this implies that $D(\frac{1}{2}) < 0 < D(0)$, a short-contract firm gains from deviating if all the other firms are

[6] This can be seen by observing that c_1 can be written as $c_1 = 1 - E_{t-1}(1 + \gamma_t)^{x/(1-x)}$, and that $(1+\gamma_t)^{x/(1-x)}$ is concave in γ_t if $x < \frac{1}{2}$, linear in γ_t if $x = \frac{1}{2}$, and convex in γ_t if $x > \frac{1}{2}$. Calibrated dynamic stochastic general equilibrium models typically assume that $x = 2/3$. See, for example, Huang and Liu (2002).

[7] The proof of Theorem 1 is in Appendix B.

short-contract firms, but loses if half of the firms are even or if half of the firms are odd. The equilibrium is illustrated in Figure 1(b). The $D(n_e)$ curve intersects the horizontal axis at m, where the gain of a short-contract firm from deviating in an even-numbered period vanishes. The upward-sloping $G(n_e, n_e)$ curve is the gain of an even firm from deviating in an even-numbered period if $a_{T_e}h = 1$ and $n_o = n_e$. Since $G(n_e, n_e) = -\rho D(n_e, n_e)/(1+\rho)$, this curve likewise intersects the horizontal axis at m, where an even firm's gain from deviation also vanishes. If $n_e < m$, there are so few even firms that a short-contract firm would gain and an even firm would lose by deviating, while if $n_e > m$, there are so many even firms that a short-contract firm would lose and an even firm would gain by deviating. The $D(n_o)$ and $G(n_o, n_e)$ curves show the corresponding gains for a firm that deviates in an odd-numbered period. It is apparent that if $n_1 = 1 - 2m$ and $n_e = n_o = m$, then all the firms – whether short-period, even, or odd – obtain the same discounted expected real profits. No firm, therefore, has an incentive to deviate so that short- and long-contract firms coexist in the Markov perfect contracting equilibrium that is illustrated by the two thick dots at $n_e = m$ and $n_o = m$.[8] As an equal proportion of the long-contract firms conclude contracts in even and odd-numbered periods ($n_e = n_o$), the long contracts are uniformly staggered.

Regime 3 assumes a still larger contracting cost satisfying $c_2 \leq c < c_3$. As this entails that $G(\frac{1}{2}, \frac{1}{2}) \leq 0 < G(1, 0)$, a long-contract firm does not gain from deviating if half of the firms are even or half of the firms are odd, but gains from deviating if all the other firms have concluded long contracts in the previous period. The equilibrium is illustrated in Figure 1(c). The upward-sloping $G(n_e, 1-n_e)$ curve shows an even firm's gain from deviating in an even-numbered period if $a_{T_e}h = 1$ and $n_o = 1 - n_e$. The $G(n_e, 1 - n_e)$ curve intersects the horizontal axis at $\hat{m}$, where an even firm's gain from deviation vanishes. If $n_e < \hat{m}$, there are so many odd firms that an even firm would lose by deviating and becoming an odd firm, while if $n_e > \hat{m}$, there are so many even firms that an even firm would gain by deviating and becoming an odd firm. The $G(n_o, 1 - n_o)$ curve is the reflection of the $G(n_e, 1 - n_e)$ curve

[8] Note that if $x > \frac{1}{2} \Leftrightarrow c_1 < 0$, then some firms conclude long contracts even if there is no contracting cost ($c = 0$).

around $n_e = \frac{1}{2}$, and it shows an odd firm's gain from deviating in an odd-numbered period if $a_{T_o}h = 1$ and $n_e = 1 - n_o$. This curve intersects the horizontal axis at $n_o = \hat{m} \Leftrightarrow n_e = 1 - \hat{m}$, where an odd firm's gain from deviation vanishes. If $n_o < \hat{m} \Leftrightarrow n_e > 1 - \hat{m}$, an odd firm would lose by deviating and becoming an even firm, while if $n_o > \hat{m} \Leftrightarrow n_e < 1 - \hat{m}$, an odd firm would gain by deviating and becoming an even firm. Accordingly, for $n_1 = 0$ and any $n_e \in [1 - \hat{m}, \hat{m}]$ and $n_o = 1 - n_e$, there are no short-period firms and neither even nor odd firms have an incentive to deviate. The economy is therefore in a Markov perfect contracting equilibrium with all contracts being long. The range of possible equilibria are illustrated by the thick line between $n_e = 1 - \hat{m}$ and $n_e = \hat{m}$ in Figure 1(c). Uniform staggering of the contracts is possible, but is only one out of a continuum of possible equilibria. All other equilibria involve nonuniform staggering with different proportions of firms concluding long contracts in even- and odd-numbered periods (n_e,$n_o \neq \frac{1}{2}$).

Finally, Regime 4 assumes that the contracting cost is so large that $c_3 \leq c$. Since this entails that $G(1, 0) \leq 0$, a long-contract firm does not gain from deviating even if all the other firms are even or all the other firms are odd. The equilibrium is illustrated in Figure 1(d). The $G(n_e, 1 - n_e)$ curve is never above the horizontal axis, which indicates that no long-contract firm has an incentive to deviate in an even-numbered period. Since the $G(n_o, 1 - n_o)$ curve is similarly never above the horizontal axis, it follows that for $n_1 = 0$ and any $n_e \in [0, 1]$ and $n_o = 1 - n_e$, the economy is in a Markov perfect contracting equilibrium. The possible equilibria are illustrated by the thick line between $n_e = 0$ and $n_e = 1$. All contracts cover two periods. Uniform staggering as well as complete synchronization where all firms conclude their contracts in the same periods ($n_e = 1$ or $n_o = 1$) are possible, but are only two out of a continuum of possible equilibria.

To sum up: Starting with a c smaller than c_1, the economy will be in Regime 1 in which there are only short contracts; as c is increased and reaches c_1, the economy will move to Regime 2 in which there are both short and long contracts, and the long-contract firms are uniformly staggered; as c is increased further and reaches c_2, the economy will move to Regime 3 in which there are only long contracts and a positive lower bound for the degree of staggering; eventually, as c is increased even further and reaches c_3, the economy will move to Regime 4 in which there are only long contracts and any degree of staggering (including no staggering at all) can be an equilibrium.

5 Comparative Statics

A natural measure of the degree of staggering of long contacts is $\min\{n_e, n_o\}/\max\{n_e, n_o\}$. The degree of staggering decreases with the majority proportion of long-contract firms (i.e., with n_e if $n_e > n_o$, and with n_o if $n_o > n_e$). It equals one if the contracts are uniformly staggered (which necessarily happens in Regime 2 and may happen in Regimes 3 and 4), and equals zero if the contracts are completely synchronized (which may happen only in Regime 4). The lowest degree of staggering that is possible in Regime 3 is given by $s \equiv (1 - \hat{m})/\hat{m}$.

We now determine the comparative-static effects of changes in the parameter values on m in Regime 2, and on $\hat{m}$ and s in Regime 3.[9]

THEOREM 2: $dm/dc > 0$; $d\hat{m}/dc > 0$; $ds/dc < 0$.

In Regime 2, an increase in the contracting cost reduces the attractiveness of the short contracts for any n. This increases m and thereby the proportion of long-contract firms. In Figure 1(b), the $D(n_e)$ and $D(n_o)$ curves would shift upward, and the $G(n_e, n_e)$ and $G(n_o, n_o)$ curves would shift downward. A higher contracting cost is therefore associated with a longer average contract length.

In Regime 3, an increase in the contracting cost makes it less profitable to deviate for any n. This increases $\hat{m}$ and thus widens the equilibrium range of n's. In Figure 1(c), the $G(n_e, 1 - n_e)$ and $G(n_o, 1 - n_o)$ curves would shift downward. A higher contracting cost therefore reduces the lowest degree of staggering that is possible in equilibrium. Observe that it is the same logic that compels an increase in the contracting cost to increase the proportion of long-contract firms in Regime 2 and to decrease the lowest possible degree of staggering in Regime 3.

[9] The proofs of Theorems 2 and 3 are in Appendix C.

THEOREM 3: $dm/d\rho = 0$; $d\hat{m}/d\rho > 0$; $ds/d\rho < 0$.

In order for short and long contracts to coexist in Regime 2, the discounted expected real profits from any long contract must equal the discounted expected real profits from two consecutive short contracts that cover the same two periods. Since the real profit in the first period of a long contract equals the real profit in the corresponding first short contract, the expected real profit in the second period of a long contract must equal the expected real profit in the corresponding second short contract. Therefore, in equilibrium the gain from deviation is zero for both short- and long-contract firms, independently of the discount rate. It follows that in Figure 1(b) the point at which the $D(n_e)$ and $G(n_e, n_e)$ curves intersect the horizontal axis, and analogously the point at which the $D(n_o)$ and $G(n_o, n_o)$ curves intersect the horizontal axis, are independent of ρ. Hence, discounting has no bearing on the determination of m.

Turning to Regime 3 in which all firms conclude long contracts and there is a positive lower bound for the degree of staggering, observe that $D(1 - \hat{m}) > 0$ and $D(\hat{m}) > 0$ (since the $D(n_e)$ curve is downward sloping). At $n = \hat{m}$, therefore, a deviating even firm's gain of discounted expected real profits stemming from odd-numbered periods is negative while its gain of discounted expected real profits stemming from even-numbered periods is positive. In any long contract concluded after the deviation, the first contract period is odd-numbered and the second contract period is even-numbered, so a higher discount rate attaches relatively more weight to the negative gain of expected real profits stemming from the odd-numbered periods than to the positive gain of expected real profits stemming from the even-numbered periods. Accordingly, it becomes less attractive to deviate, and in Figure 1(c) the $G(n_e, 1 - n_e)$ and $G(n_o, 1 - n_o)$ curves would move down. A higher discount rate therefore leads to a higher $\hat{m}$ and thereby to a lower s.[10]

For low levels of monetary uncertainty, it is possible to obtain simple analytical approx-

[10] An increase in the discount rate does not affect c_1 and c_2, and hence does not affect the values of the contracting cost for which Regime 1 and Regime 2 occur. However, it reduces c_3 so that the economy will move from Regime 3 to Regime 4 at a smaller value of c.

imations for m, $\hat{m}$, and s, which are denoted by m^*, $\hat{m}^*$, and s^*. To derive m^*, we expand $D(n)$ around $\gamma_t = 0$. By ignoring terms with γ_t of a higher order than two and noting that $E_{t-1}\gamma_t = 0$, we can approximate $D(n)$ by

$$\frac{x(2x - 1 - 2nx^2)\sigma^2}{(1-x)^2} + c,$$

where $\sigma^2 \equiv E_{t-1}\gamma_t^2$ is the variance of the monetary policy shocks. Setting this expression equal to zero, we obtain that for low levels of monetary uncertainty, m is approximately

$$m^* = \frac{2x^2 - x + (1-x)^2 c/\sigma^2}{2x^3}.$$

To derive $\hat{m}^*$, we expand $G(n, 1-n)$ around $\gamma_t = 0$. By ignoring terms with γ_t of a higher order than two and again noting that $E_{t-1}\gamma_t = 0$, we can approximate $G(n, 1-n)$ by

$$\frac{[\rho x(1-2x) - 2x^3 + 2n(2+\rho)x^3]\sigma^2}{(1+\rho)(1-x)^2} - \frac{\rho c}{1+\rho}.$$

Setting this expression equal to zero, for low levels of monetary uncertainty $\hat{m}$ is approximately

$$\hat{m}^* = \tfrac{1}{2} + \frac{\rho(-x + c/\sigma^2)(1-x)^2}{2(2+\rho)x^3}.$$

Hence, s is approximately

$$s^* = \frac{1 - \hat{m}^*}{\hat{m}^*}.$$

It is clear from the expressions for m^*, $\hat{m}^*$, and s^* that c and ρ affect the approximate values of m, $\hat{m}$, and s in the same way as they affect the exact values. It is also apparent that the variance of the monetary policy shocks has a negative effect on m^* and $\hat{m}^*$, and a positive effect on s^*. Thus, we have shown

THEOREM 4: $dm^*/d\sigma^2 < 0$; $d\hat{m}^*/d\sigma^2 < 0$; $ds^*/d\sigma^2 > 0$.

As expected, an increase in the variance of the monetary policy shocks has the opposite effects of an increase in the contracting cost. The logic for this is straightforward in that firms with long contracts are more exposed to monetary uncertainty than firms with short contracts. In Regime 2, therefore, an increase in the monetary uncertainty would give a long-contract firm an incentive to deviate if the proportion of long-contract firms would remain unchanged. Since the gain of a long-contract firm from deviating decreases with the proportion of long-contract firms, that proportion – i.e., m^* – must decrease in order to eliminate the incentive to deviate.

The majority proportion of long-contract firms are more exposed to monetary uncertainty than the minority proportion of long-contract firms. In Regime 3, therefore, an increase in the monetary uncertainty would give a firm from the majority proportion of long-contract firms an incentive to deviate if the majority proportion would remain unchanged. Since the gain for a deviating firm from the majority proportion decreases with the size of the majority proportion, an increase in monetary uncertainty causes the majority proportion – i.e., $\hat{m}^*$ – to decrease in order to eliminate the incentive to deviate. Finally, an increase in the majority proportion of firms leads to a decrease in the lowest possible degree of staggering, i.e., in s^*.[11]

At last, we point out that the uncertainty of aggregate productivity has no effect on the Markov perfect contracting equilibrium, and a fortiori, on m, $\hat{m}$, and s. This follows from the fact that the real wages in both short and long contracts are proportional to aggregate productivity. Therefore, the uncertainty of aggregate productivity (as opposed to the realized productivity shocks) does not affect the current real wage or the expected future real wages in short or long contracts, and hence also not the firms' discounted expected real profits and their gains from deviation.

6 Conclusion

This paper presents a micro-founded model that endogenizes the duration and timing of wage contracts. We assume that firms sell a homogeneous product in a competitive product market, that all shocks are aggregate, and that the monetary policy is passive in order to

[11] The approximation for c_1 is $\frac{1}{2}x(1-2x)\sigma^2/(1-x)^2$, for c_2 it is $2^{x-1}x(1-2x+2x^2)\sigma^2/(1-x)^2$, and for c_3 it is $x[\rho(1-x+x^2)+2x^2]\sigma^2/\left[2\rho(1-x)^2\right]$, all of which increase with monetary uncertainty. Hence, with more monetary uncertainty, the economy will move from one regime to the next for higher values of the contracting cost.

emphasize that the staggering of wage contracts does not require monopolistic competition, idiosyncratic shocks, or an activist monetary policy. The crucial feature of the model is that short and long contracts as well as long contracts concluded in different periods are strategic substitutes, which has important implications for the choice of contract duration and provides a strong incentive for staggering. We show that short and long wage contracts may coexist, in which case the long contracts are always uniformly staggered. Further, the proportion of long contracts increases with the contracting cost and decreases with the variance of the monetary policy shocks. We also show that if only long contracts exist, then uniform staggering is just one out of a continuum of possible equilibria with different degrees of staggering. Unless the contracting cost is very large, the lowest possible positive degree of staggering increases with the contracting cost and decreases with the variance of the monetary policy shocks.

Appendix A

Proof of Lemma 1

Consider first a short-contract firm in period T_e. The firm's real profit in period T_e is $a_{T_e} k\left(A_{n_o,T_e} - c\right)$, and its discounted expected real profit at T_e from a future even-numbered period $t_e \in \{T_e + 2, T_e + 4, ...\}$ is

$$\frac{kE_{T_e}\left[a_{t_e}\left(A_{n_o,t_e} - c\right)\right]}{(1+\rho)^{t_e - T_e}}. \tag{A1}$$

Since $a_{t_e} = a_{T_e} \prod_{\tau=T_e+1}^{t_e}(1+\alpha_\tau)$, expression (A1) becomes

$$\frac{a_{T_e} k}{(1+\rho)^{t_e - T_e}} E_{T_e}\left[\prod_{\tau=T_e+1}^{t_e}(1+\alpha_\tau)\left(A_{n_o,t_e} - c\right)\right]. \tag{A2}$$

The independence of the productivity shocks implies that $E_{T_e}(1+\alpha_\tau) = 1$ for any T_e and τ. The independence of the monetary policy shocks implies that $E_{T_e} A_{n_o,t_e}$ is independent of T_e and t_e, so that $E_{T_e} A_{n_o,t_e} = E_{t-1} A_{n_o,t}$ for any T_e, t_e, and t. Accordingly, (11) can be written as

$$\frac{a_{T_e} k\left(E_{t-1} A_{n_o,t} - c\right)}{(1+\rho)^{t_e - T_e}}. \tag{A3}$$

The discounted expected real profit at T_e from a future odd-numbered period $t_o \in \{T_e + 1, T_e + 3, ...\}$ is

$$\frac{kE_{T_e}\left[a_{t_o}\left(A_{n_e,t_o} - c\right)\right]}{(1+\rho)^{t_o - T_e}}. \tag{A4}$$

Using that $a_{t_o} = a_{T_e} \prod_{\tau=T_e+1}^{t_o}(1+\alpha_\tau)$, that $E_{T_e}(1+\alpha_\tau) = 1$ for any T_e and τ, and that $E_{T_e} A_{n_e,t_o} = E_{t-1} A_{n_e,t}$ for any T_e, t_o, and t, expression (A4) becomes

$$\begin{aligned} & \frac{a_{T_e} k}{(1+\rho)^{t_o - T_e}} E_{T_e}\left[\prod_{\tau=T_e+1}^{t_o}(1+\alpha_\tau)\left(A_{n_e,t} - c\right)\right] \\ = \; & \frac{a_{T_e} k\left(E_{t-1} A_{n_e,t} - c\right)}{(1+\rho)^{t_o - T_e}}. \end{aligned} \tag{A5}$$

A short-contract firm's total expected real profits discounted to period T_e consists of $a_{T_e} k\left(A_{n_o,T_e} - c\right)$ from period T_e plus the sum of (A3) for all future even-numbered periods and the sum of (A5) for all future odd-numbered periods, and thus are

$$
\begin{aligned}
& a_{T_e} k \left(A_{n_o,T_e} - c\right) + \sum_{t_e > T_e}^{\infty} \left[\frac{a_{T_e} k \left(E_{t-1} A_{n_o,t} - c\right)}{(1+\rho)^{t_e - T_e}}\right] + \sum_{t_o > T_e}^{\infty} \left[\frac{a_{T_e} k \left(E_{t-1} A_{n_e,t} - c\right)}{(1+\rho)^{t_o - T_e}}\right] \\
= \;& a_{T_e} k \left(A_{n_o,T_e} - c\right) + \frac{a_{T_e} k \left(E_{t-1} A_{n_o,t} - c\right)}{\rho(2+\rho)} + \frac{(1+\rho) a_{T_e} k \left(E_{t-1} A_{n_e,t} - c\right)}{\rho(2+\rho)} \\
= \;& a_{T_e} k \left[A_{n_o,T_e} + \frac{E_{t-1} A_{n_o,t} + (1+\rho) E_{t-1} A_{n_e,t}}{\rho(2+\rho)} - \frac{(1+\rho)c}{\rho}\right]. \qquad \text{(A6)}
\end{aligned}
$$

Consider next an even firm in period T_e. The firm's real profit in period T_e and its discounted expected real profit at T_e from a future even-numbered period (i.e., in the first period of a future contract) are the same as for a short-contract firm. Its discounted expected real profit at T_e from a future odd-numbered period (i.e., in the second period of the present contract or of a future contract) is

$$
\frac{k E_{T_e} \left(a_{t_o} A_{n_e,t_o} B_{t_o}\right)}{(1+\rho)^{t_o - T_e}}. \qquad \text{(A7)}
$$

Since $a_{t_o} = a_{T_e} \prod_{\tau=T_e+1}^{t_o} (1+\alpha_\tau) = a_{T_e}$ for any T_e and τ, and $E_{T_e}\left(A_{n_e,t_o} B_{t_o}\right) = E_{t-1}\left(A_{n_e,t} B_t\right)$ for any T_e, t_o, and t, expression (A7) becomes

$$
\begin{aligned}
& \frac{a_{T_e} k}{(1+\rho)^{t_o - T_e}} E_{T_e} \left[\prod_{\tau=T_e+1}^{t_o} (1+\alpha_\tau) A_{n_e,t_o} B_{t_o}\right] \\
= \;& \frac{a_{T_e} k E_{t-1}\left(A_{n_e,t} B_t\right)}{(1+\rho)^{t_o - T_e}}.
\end{aligned}
$$

Consequently, an even firm's total expected real profits discounted to period T_e is

$$
\begin{aligned}
& a_{T_e} k \left(A_{n_o,T_e} - c\right) + \sum_{t_e > T_e}^{\infty} \left[\frac{a_{T_e} k \left(E_{t-1} A_{n_o,t} - c\right)}{(1+\rho)^{t_e - T_e}}\right] + \sum_{t_o > T_e}^{\infty} \left[\frac{a_{T_e} k E_{t-1}\left(A_{n_e,t} B_t\right)}{(1+\rho)^{t_o - T_e}}\right] \\
= \;& a_{T_e} k \left(A_{n_o,T_e} - c\right) + \frac{a_{T_e} k \left(E_{t-1} A_{n_o,t} - c\right)}{\rho(2+\rho)} + \frac{(1+\rho) a_{T_e} k E_{t-1} A_{n_e,t} B_t}{\rho(2+\rho)} \\
= \;& a_{T_e} k \left[A_{n_o,T_e} + \frac{E_{t-1} A_{n_o,t} + (1+\rho) E_{t-1}\left(A_{n_e,t} B_t\right) - (1+\rho)^2 c}{\rho(2+\rho)}\right]. \qquad \text{(A8)}
\end{aligned}
$$

We now turn to a short-contract firm in period T_o. The firm's real profit in period T_o is $a_{T_o} k \left(A_{n_e,T_o} - c\right)$, its discounted expected real profit at T_o from a future odd-numbered period is

$$
\frac{E_{T_o}\left(a_{t_o} k A_{n_e,t_o} - c a_{t_o}\right)}{(1+\rho)^{t_o - T_o}},
$$

and its discounted expected real profit at T_o from a future even-numbered period is

$$\frac{E_{T_o} k\left(a_{t_e} A_{n_o,t_e} - c a_{t_e}\right)}{(1+\rho)^{t_e - T_o}}.$$

It follows that the short-contract firm's total expected real profits discounted to period T_o can be obtained from expression (A6) for a short-contract firm's total expected real profits discounted to period T_e by substituting T_o for T_e and interchanging n_o and n_e, resulting in

$$a_{T_o} k\left[A_{n_e,T_o} + \frac{E_{t-1}A_{n_e,t} + (1+\rho)E_{t-1}A_{n_o,t}}{\rho(2+\rho)} - \frac{(1+\rho)c}{\rho}\right].$$

An odd firm's total expected real profits discounted to period T_o can be obtained from expression (A8) for an even firm's total expected real profits discounted to period T_e by substituting T_o for T_e and interchanging n_o and n_e, yielding

$$a_{T_o} k\left[A_{n_e,T_o} + \frac{E_{t-1}A_{n_e,t} + (1+\rho)E_{t-1}\left(A_{n_o,t}B_t\right) - (1+\rho)^2 c}{\rho(2+\rho)}\right].$$

Proof of Lemma 2

Consider first a short-contract firm that deviates in period T_e by concluding a long contract, and in period $T_e + 2$ reverts to always concluding short contracts. The deviating short-contract firm's real profit in period T_e and its discounted expected real profits from period $T_e + 2$ and later periods do not change. Its gain of discounted expected real profits is therefore given by the difference between the discounted expected real profit from the second period of an even contract concluded in period T_e, i.e., $a_{T_e} k E_{t-1}(A_{n_e,t}B_t)/(1+\rho)$, and the discounted expected real profit in a short contract concluded in period $T_e + 1$, i.e., $a_{T_e} k\left(E_{t-1}A_{n_e,t} - c\right)/(1+\rho)$; that is,

$$\begin{aligned} &\frac{a_{T_e} k E_{t-1}(A_{n_e,t}B_t)}{1+\rho} - \frac{a_{T_e} k\left(E_{t-1}A_{n_e,t} - c\right)}{1+\rho} \\ =\ &\frac{a_{T_e} k}{1+\rho} D(n_e). \end{aligned} \tag{A9}$$

Consider next an even firm that deviates in period T_e by concluding a short contract, and in period $T_e + 1$ reverts to always concluding long contracts (that from then on start in odd-numbered periods). The deviating even firm's real profit in period T_e is the same, i.e., $a_{T_e}k\left(A_{n_o,T_e}-c\right)$, and its discounted expected real profits at T_e from all periods after T_e is given by the expected value of concluding long contracts at odd-numbered periods starting in period T_e+1 and discounted one period, i.e., by the expected value at T_e of expression (10) after T_e+1 has been substituted for T_o and the expression multiplied by $1/(1+\rho)$. Hence, a deviating even firm's total discounted expected real profits at T_e is

$$\begin{aligned}&a_{T_e}k\left(A_{n_o,T_e}-c\right)\\&+\frac{k}{1+\rho}E_{T_e}\left\{a_{T_e+1}\left[A_{n_e,T_e+1}+\frac{E_{t-1}A_{n_e,t}+(1+\rho)E_{t-1}\left(A_{n_o,t}B_t\right)-(1+\rho)^2c}{\rho(2+\rho)}\right]\right\}.\end{aligned}$$

Since $E_{T_e}a_{T_e+1}=a_{T_e}$ and $E_{T_e}A_{n_e,T_e+1}=E_{t-1}A_{n_e,t}$, this can be written as

$$\begin{aligned}&a_{T_e}k\left(A_{n_o,T_e}-c\right)\\&+\frac{a_{T_e}k}{1+\rho}\left[E_{t-1}A_{n_e,t}+\frac{E_{t-1}A_{n_e,t}+(1+\rho)E_{t-1}\left(A_{n_o,t}B_t\right)-(1+\rho)^2c}{\rho(2+\rho)}\right]\\=\;&a_{T_e}k\left[A_{n_o,T_e}+\frac{(1+\rho)E_{t-1}A_{n_e,t}+E_{t-1}\left(A_{n_o,t}B_t\right)-(1+3\rho+\rho^2)c}{\rho(2+\rho)}\right].\end{aligned}$$

Subtracting the total discounted expected real profits for a non-deviating even firm (expression (8)), we obtain that an even firm's gain from deviating in period T_e is

$$\begin{aligned}&a_{T_e}k\left[A_{n_o,T_e}+\frac{(1+\rho)E_{t-1}A_{n_e,t}+E_{t-1}\left(A_{n_o,t}B_t\right)-(1+3\rho+\rho^2)c}{\rho(2+\rho)}\right]\\&-a_{T_e}k\left[A_{n_o,T_e}+\frac{E_{t-1}A_{n_o,t}+(1+\rho)E_{t-1}\left(A_{n_e,t}B_t\right)-(1+\rho)^2c}{\rho(2+\rho)}\right]\\=\;&a_{T_e}h\left\{E_{t-1}\left[A_{n_e,t}\left(1-B_t\right)\right]-\frac{E_{t-1}\left[A_{n_o,t}\left(1-B_t\right)\right]+\rho c}{1+\rho}\right\}\\=\;&a_{T_e}h\left[-D(n_e)+\frac{D(n_o)}{1+\rho}\right]\\=\;&a_{T_e}hG(n_e,n_o).\qquad\qquad\text{(A10)}\end{aligned}$$

The gain of discounted expected real profits for a short-contract firm that deviates in period T_o by concluding a long contract and in period T_o+2 reverts to always concluding short contracts can be obtained from expression (A9) by substituting T_o for T_e and n_o for n_e, yielding $a_{T_e}kD(n_o)/(1+\rho)$. Similarly, the gain of discounted expected real profits for an odd firm that deviates in period T_o by concluding a short-period contract and in period T_o+1 reverts to always concluding long contracts (that from then on start in even-numbered periods) can be obtained from expression (A10) by substituting T_o for T_e and interchanging n_o and n_e, yielding $a_{T_o} hG(n_o, n_e)$.

Proof of Lemma 3

Differentiating $D(n)$ with respect to n yields

$$\frac{dD(n)}{dn} = -xE_{t-1}\left[\frac{(B_t-1)^2}{(1-n+nB_t)^{1+x}}\right],$$

which is negative.

Appendix B

Proof of Theorem 1

Regime 1: If $c \leq c_1$, then $D(0) \leq 0$ and the candidate strategy for each firm is to always conclude short contracts. We need to show that if $n_1 = 1$, then a short-contract firm cannot gain from a one-time deviation from the candidate strategy by concluding a long contract and afterwards always concluding short contracts. According to Lemma 2, if $n_1 = 1$, then the gain of a short-contract firm that deviates from the candidate strategy is proportional to $D(0)$ and hence nonpositive. The candidate strategy is therefore optimal.

Regime 2: If $c_1 < c < c_2$, then $D(\frac{1}{2}) < 0 < D(0)$ and the candidate strategy for each firm is to always conclude a similar contract whenever the old contract expires. This makes each firm either a short-contract firm, an even firm, or an odd firm. We need to show that if $n_1 = 1-2m$ and $n_e = n_o = m$, then a short-contract firm cannot gain from a one-time deviation from the candidate strategy by concluding a long contract and afterwards always concluding short contracts, and, likewise, that a long-contract firm cannot gain from deviating from the candidate strategy by concluding a short contract and afterwards always concluding long contracts. According to Lemma 2, if $n_1 = 1-2m$ and $n_e = n_o = m$, then the gain for a short-contract firm that deviates from the candidate strategy is proportional to $D(m)$, while the gain for a long-contract firm that deviates from the candidate strategy is proportional to $-D(m)$. Since $D(m) = 0$, these gains are zero, and the candidate strategy is therefore optimal.

Regime 3: If $c_2 \leq c < c_3$, then $G(\frac{1}{2}, \frac{1}{2}) \leq 0 < G(1, 0)$ and the candidate strategy is to conclude a long contract whenever the old contract expires, which makes each firm either an even firm or an odd firm. We need to show that a long-contract firm cannot gain from a one-time deviation from the candidate strategy by concluding a short contract and afterwards always concluding long contracts. According to Lemma 2, if $n_1 = 0$, $n_e \in [1 - \hat{m}, \hat{m}]$, and $n_o = 1 - n_e$, then the gain for an even firm that deviates from the candidate strategy in period T_e is $a_{T_e} hG(n_e, 1 - n_e)$, and the gain for an odd firm that deviates from the candidate strategy in period T_o is $a_{T_o} hG(n_o, 1 - n_o)$. If $n_e \leq \hat{m}$, then $G(n_e, 1 - n_e) \leq 0$ so that an even firm does not gain from deviating, and if $n_o \leq \hat{m}$, then $G(n_o, 1 - n_o) \leq 0$ so that an odd firm does not gain from deviating. Consequently, if $n_e \in [1 - \hat{m}, \hat{m}\,]$, then $G(n_e, 1 - n_e) \leq 0$ and $G(n_o, 1 - n_o) \leq 0$ so that no firm can gain from deviating. The candidate strategy is therefore optimal.

Regime 4: If $c_3 \leq c$, then $G(1, 0) \leq 0$ and the candidate strategy is to conclude a long contract whenever the old contract expires, which makes each firm either an even firm or an odd firm. As in Regime 3, we need to show that a long-contract firm cannot gain from a one-time deviation from the candidate strategy by concluding a short contract and afterwards always concluding long contracts. According to Lemma 2, if $n_1 = 0$, $n_e \in [0, 1]$, and $n_o = 1 - n_e$, then the gain for an even firm that deviates from the candidate strategy in period T_e is $a_{T_e} hG(n_e, 1 - n_e)$, and the gain for an odd firm that deviates from the candidate strategy in period T_o is $a_{T_o} hG(n_o, 1 - n_o)$. Since $G(n_e, 1 - n_e) \leq 0$ for any n_e and $G(n_o, 1 - n_o) \leq 0$ for any n_o, no firm can gain from deviating, and the candidate strategy is therefore optimal.

This completes the proof of Theorem 1.

Appendix C

Proof of Theorem 2

To determine the effect of c on m, differentiate $D(m) = 0$ with respect to c, which yields

$$\frac{dm}{dc} = -\frac{1}{\partial D(n)/\partial n},$$

where $\partial D(n)/\partial n$ is evaluated at $n = m$. Since $\partial D(n)/\partial n < 0$, it follows that $dm/dc > 0$.

To determine the effect of c on $\hat{m}$, differentiate $G(\hat{m}, 1 - \hat{m}) = 0$ with respect to c, which yields

$$\frac{d\hat{m}}{dc} = \frac{\rho}{(1 + \rho)\partial G(n, 1 - n)/\partial n},$$

where $\partial G(n, 1 - n)/\partial n$ is evaluated at $n = \hat{m}$. Since $\partial G(n, 1 - n)/\partial n > 0$, it follows that $d\hat{m}/dc > 0$.

To determine the effect of c on s, we use the definition of s to obtain that

$$\frac{ds}{dc} = -\frac{d\hat{m}/dc}{\hat{m}^2} < 0.$$

Proof of Theorem 3

Clearly, $dm/d\rho = 0$. To determine the effect of ρ on $\hat{m}$, differentiate $G(\hat{m}, 1 - \hat{m}) = 0$ with respect to ρ, which yields

$$\frac{d\hat{m}}{d\rho} = \frac{D(1 - \hat{m})}{(1 + \rho)^2 \partial G(n, 1 - n)/\partial n},$$

where $\partial G(n, 1 - n)/\partial n$ is evaluated at $n = \hat{m}$. Since $D(1 - \hat{m}) > 0$ and $\partial G(n, 1 - n)/\partial n > 0$, it follows that $d\hat{m}/d\rho > 0$. Furthermore,

$$\frac{ds}{d\rho} = -\frac{d\hat{m}/d\rho}{\hat{m}^2} < 0.$$

References

Andersen, Torben M. 1998. "Persistency in Sticky Price Models." *European Economic Review,* 42(3-5): 593-603.

Ascari, Guido. 2000. "Optimising Agents, Staggered Wages and Persistence in the Real Effects of Money Shocks." *Economic Journal,* 110(465): 664-686.

Ball, Laurence. 1987. "Externalities from Contract Length." *American Economic Review,* 77(4): 615-629.

Ball, Laurence, and David Romer. 1989. "The Equilibrium and Optimal Timing of Price Changes." *Review of Economic Studies,* 56(4): 179-198.

Ball, Laurence, and Stephen G. Cecchetti. 1988. "Imperfect Information and Staggered Price Setting." *American Economic Review,* 78(5): 999-1018.

Blanchard, Olivier J. 1986. "The Wage Price Spiral." *Quarterly Journal of Economics,* 101(3): 543-565.

Calvo, Guillermo A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics,* 12(3): 383-398.

Cho, Jang-Ok, Thomas F. Cooley, and Louis Phaneuf. 1997. "The Welfare Cost of Nominal Wage Contracting." *Review of Economic Studies,* 64(3): 465-484.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy.* 113(1): 1-45.

Christofides, Louis N. 1990. "The Interaction between Indexation, Contract Duration and Non-contingent Wage Adjustment." *Economica,* 57(227): 395-409.

Christofides, Louis N., and Chen Peng. 2006. "Contract Duration and Indexation in a Period of Real and Nominal Uncertainty." *Labour Economics,* 13(1): 61-86.

Clarida, Richard, Jordi Gali, and Mark Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature.* 37(4): 1661-1707.

Danziger, Leif. 1988. "Real Shocks, Efficient Risk Sharing, and the Duration of Labor Contracts." *Quarterly Journal of Economics,* 103(2): 435-440.

Dye, Ronald A. 1985. "Optimal Length of Labor Contracts." *International Economic Review,* 26(1): 251-270.

Edge, Rochelle M. 2002. "The Equivalence of Wage and Price Staggering in Monetary Business Cycle Models." *Review of Economic Dynamics,* 5(3): 559-585.

Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin. 2000. "Optimal Monetary Policy with Staggered Wage and Price Contracts." *Journal of Monetary Economics,* 46(2): 281-313.

Fethke, Gary, and Andrew Policano. 1984. "Wage Contingencies, the Pattern of Negotiation and Aggregate Implications of Alternative Contract Structures." *Journal of Monetary Economics,* 14(3): 151-170.

Fethke, Gary, and Andrew Policano. 1986. "Will Wage Setters Ever Stagger Decisions?" *Quartely Journal of Economics,* 101(3): 867-877.

Fethke, Gary, and Andrew Policano. 1987. "Monetary Policy and the Timing of Wage Negotiations." *Journal of Monetary Economics,* 19(1): 89-105.

Fethke, Gary, and Andrew Policano. 1990. "Information Incentives and Contract Timing Patterns." *International Economic Review,* 31(3): 651-665.

Fuhrer, Jeff, and George Moore. 1995. "Inflation Persistence." *Quarterly Journal of Economics.* 110(1): 127-159.

Gray, Jo Anna. 1978. "On Indexation and Contract Length." *Journal of Political Economy,* 86(1): 1-18.

Harris, Milton, and Bengt Holmstrom. 1987. "On the Duration of Agreements." *International Economic Review,* 28(2): 389-406.

Huang, Kevin X. D., and Zheng Liu. 2002. "Staggered Price-Setting, Staggered Wage-Setting, and Business Cycle Persistence. *Journal of Monetary Economics,* 49(2): 405-33.

Matsukawa, Shigeru. 1986. "The Equilibrium Distribution of Wage Settlements and Economic Stability." *International Economic Review,* 27(2): 415-37.

Murphy, Kevin J. 1992. "Determinants of Contract Duration in Collective Bargaining Agreements." *Industrial and Labor Relations Review,* 45(2): 352-365.

Olivei, Giovanni, and Silvana Tenreyro. 2007. "The Timing of Monetary Policy Shocks." *American Economic Review,* 97(3): 636-663.

Olivei, Giovanni, and Silvana Tenreyro. 2008. "Wage Setting Patterns and Monetary Policy: International Evidence." CEP Discussion Paper No 872.

Phaneuf, Louis. 1990. "Wage Contracts and the Unit Root Hypothesis." *Canadian Journal of Economics.* 23(3): 580-592.

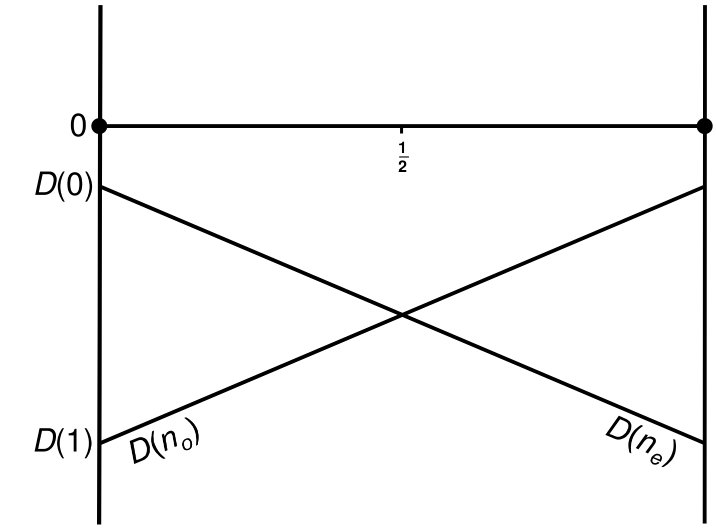
Rich, Robert, and Joseph Tracy. 2004. "Uncertainty and Labor Contract Durations." *Review of Economics and Statistics,* 86(1): 270-287.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review,* 97(3): 586-606.

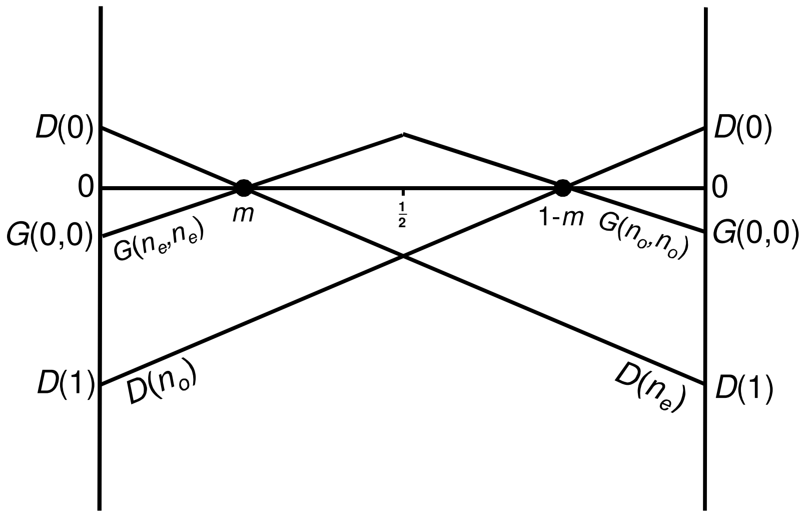
Taylor, John B. 1980. "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy,* 88(1): 1-23.

Wallace, Frederick H. 2001. "The Effects of Shock Size and Type on Labor-Contract Duration." *Journal of Labor Economics,* 19(3): 658-681.

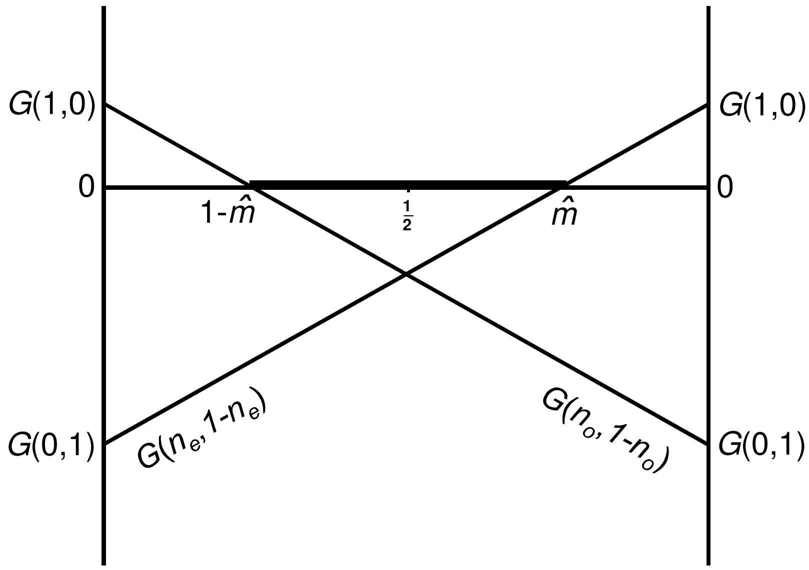
Woodford, Michael. 2003. *Interest and Prices.* Princeton: Princeton University Press.



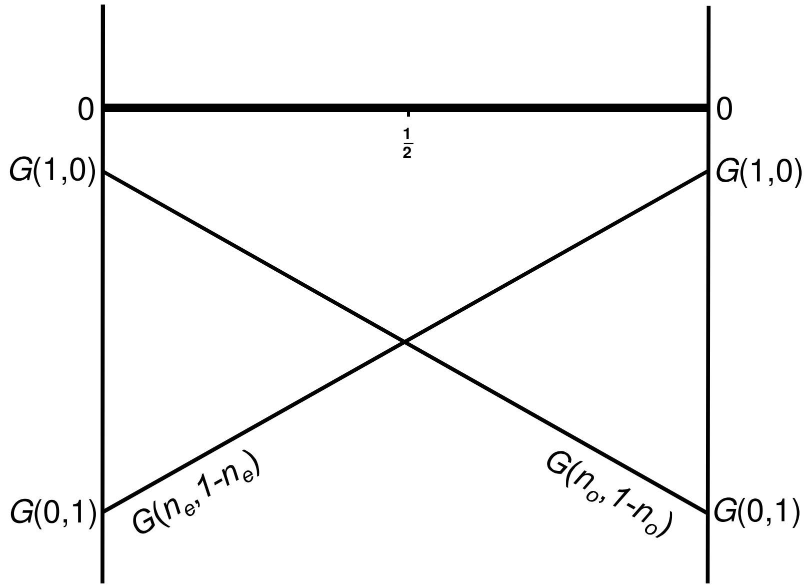
(a): Regime 1



(b): Regime 2



(c): Regime 3



(d): Regime 4

FIGURE 1: THE EQUILIBRIA

Note: The horizontal axis measures n_e from left to right and n_o from right to left