ifo WORKING PAPERS

274
2018

November 2018

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Impressum:

ifo Working Papers
Publisher and distributor: ifo Institute – Leibniz Institute for Economic Research at the University of Munich
Poschingerstr. 5, 81679 Munich, Germany
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Optimal Carbon Pricing and Income Taxation Without Commitment*

Abstract

At what rate should a government price carbon emissions? This paper analyzes optimal carbon pricing while taking into account interactions with the taxation of labor and capital income. In an otherwise standard climate-economy model, the policy maker has to resort to a distortionary tax on labor and capital income, and is unable to commit to future policies. I show that the optimal time-consistent carbon price is in general not at its Pigouvian level, that is, at the level of marginal damages induced by climate change. This is due to the presence of costs and benefits of emitting carbon that only materialize in the presence of income taxes. Quantitatively, I find that in a standard calibration of the model, this tax-interaction effect accounts for deviation of the optimal tax from the level of marginal climate damages in the ballpark of 10%, due to the second-best effects partially offsetting each other. Compared to a setting with lump-sum income taxes, I observe a smaller optimal carbon price without commitment, with the average differences over time amounting to 14%.

JEL Code: E61, E62, H21, H23, Q54
Keywords: Climate-economy modeling, carbon tax, optimal income taxation

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November 12, 2018

* I would like to thank John Hassler, Per Krusell, Philippe Aghion, Lint Barrage, Niko Jaakkola, Torsten Persson, Karen Pittel and Rick van der Ploeg for useful comments. An earlier version of this paper appeared as “Optimal Carbon and Income Taxation” in Schmitt (2014).

1 Introduction

What price should a policy maker put on emissions of greenhouse gases such as CO_2 in order to internalize climate change? In this paper, I address this question while taking into account that climate change policy is set by governments that resort to income taxation to finance public goods. Specifically, I show how the optimal carbon price is affected when adding two real-world features, distortionary income taxation and the inability of the government to commit to future policies, to an otherwise standard climate-economy model. Qualitatively, I find that the interaction between taxing income and pricing polluting production inputs that was previously established in a static framework (Bovenberg and Goulder, 1996) generalizes in an intuitive way to a dynamic setting with persistent environmental damages such as climate change. In more detail, compared to a setting with lump-sum taxes, emitting carbon is linked to additional effects – positive or negative – on welfare under distortionary income taxation. These "second-best" benefits or costs are caused by the emission level affecting the households' labor supply and savings decisions, which are distorted by the income tax. As a consequence, the optimal carbon price – which must equal the marginal social cost of carbon (SCC) in order to fully internalize climate change – is in general not at the level of marginal climate change impacts on welfare caused by the emission of an additional unit of carbon (MCD). This deviation of the optimal carbon fee from its Pigouvian level is referred to as the "tax-interaction effect".

However, a quantitative assessment shows that the aforementioned second-best costs and benefits tend to offset each other. Depending on how impacts from climate change are modeled – specifically, whether climate change affects the economy through the level of productivity or has direct welfare impacts – I find that in reasonable calibrations of the model, the optimal second-best carbon price is about $2-11\%$ below the MCD level in 2010; this deviation increases over time to around $9-15\%$ by 2100. Hence, the interaction between fiscal policy and climate change mitigation does not seem an important factor in the design of optimal climate policy. This also implies that studies on optimal carbon pricing that have abstracted from distortionary income taxation can be seen as a valid approximation for guiding policy makers.

This paper builds on "integrated assessment models" (IAM), such as, for example, the DICE model (Nordhaus, 2008) or the model of Golosov et al. (2014), that are widely used to inform policy makers about the size of the social cost of carbon and hence the optimal carbon fee.[1,2] These models typically prescribe a Pigouvian tax that equals the marginal global damage caused by climate change, which is defined as the present value of the damage caused globally by emitting an additional unit of carbon.

As in these studies, I analyze optimal climate change mitigation in a standard deterministic neoclassical growth model. However, I take into account that a government's role is not limited to implementing climate policy, but it must also raise revenue in order to finance expenditures on public goods, by

[1]In this paper, a carbon fee could be either a direct carbon tax or the price of tradable emission permits. I will use the terms "price", "tax" and "fee" as synonyms. Note that in order to optimally correct a pollution externality, the emission fee must be set equal to the (marginal) social cost of pollution evaluated at the efficient emission level (Kolstad, 2000). The question of what carbon tax is optimal is therefore equivalent to asking what is (a good estimate for) the marginal social cost of carbon.

[2]For example, an Interagency Working Group of the US government published a report determining the social cost of carbon to be used in cost-benefit analysis (IWG, 2010, 2013). They have used DICE as well as the PAGE model (Hope, 2006, 2008) and the FUND model (Tol, 2002a,b; Anthoff et al., 2009).

taxing labor and capital income.[3] This introduces distortions into the economy that make the first-best allocation unfeasible, even if the government is assumed to be benevolent. When modeling distortionary taxation, I assume that all tax rates are optimally chosen.[4] In addition, it is well-known that in models with distortionary income taxes, one has to take a stand on whether the government is able to credibly commit to future policies (Klein et al., 2008). I assume that there is no such commitment device.[5] This assumption is arguably more realistic and plausible than allowing a government to commit to all future tax rates.

I provide an analytical characterization of the optimal second-best carbon price which shows that it is in general not equal to the marginal climate damage and hence is not at the Pigouvian rate. This tax-interaction effect is caused by additional effects of taxing carbon that only materialize in a setting with distortionary income taxes, and that are caused by the interaction of carbon emissions with current and future "wedges", that is, distortions of a first-best margin. For example, under certain conditions, an emission tax leads to a decrease in labor supply, which results in a welfare loss if the intratemporal labor-leisure margin is distorted by a labor income tax. This second-best cost of regulating emissions causes the optimal emission fee to deviate from the Pigouvian rate. This was a prominent finding in earlier studies, which consid-

[3]Throughout this paper, I assume that the government is restricted to a total income tax, and has no access to lump-sum taxation. If a lump-sum tax were feasible, a Pigouvian carbon price would be optimal.

[4]There is a large literature in environmental economics that instead analyzes partial tax reforms, where non-environmental tax rates, and possibly the pollution tax, are exogenously given. An example is Glomm et al. (2008) for a dynamic growth model. Compare also the quantitative results in Barrage (2016).

[5]Note that throughout this paper, I focus on Markov-perfect equilibria when solving the model with distortionary taxation.

ered second-best environmental taxation in a static model with a labor income tax (Bovenberg and de Mooij, 1994; Parry, 1995; Bovenberg and Goulder, 1996) and hence focused on the labor-leisure wedge.[6]

In a dynamic model, however, taking into account only the second-best effect of carbon emissions on current labor supply gives an incomplete characterization of the optimal carbon price. For example, taxing carbon has a negative effect on household savings, thereby exacerbating the intertemporal distortion caused by the tax on capital income. This effect represents an additional second-best cost of emission reductions and thus further decreases the carbon price. In addition, I show that current carbon emissions also impact labor and savings margins in subsequent periods, as their effect is propagated into the future by capital accumulation and temperature change dynamics. This propagation induces further cost and benefits of current carbon emissions.

To investigate the quantitative importance of these second-best effects, I solve a calibrated climate-economy model using recursive methods. This exercise shows that the deviation of the optimal carbon price from the level of marginal damage from climate change appears to be small, although it depends on how climate impacts are modeled.

The remainder of this paper is structured as follows. Section 2 discusses some related literature. Section 3 presents the framework. In sections 4 and 5, I analyze a global climate-economy model with distortionary taxation. Section 6 concludes the paper.

[6]Those studies usually found that the tax-interaction effect dominates "revenue recycling", that is, using the revenue raised with the emission tax to lower distortionary income taxes, which results in an efficiency gain.

2 Related Literature

This paper is related to different strands of the literature, both in climate-economy modeling and public finance. In the context of climate change economics, numerous studies have used IAMs to compute the social cost of carbon and the optimal carbon tax, often in a global planner model without distortionary taxation.[7] One of the earliest and most influential IAMs is the DICE model (Nordhaus, 2008, 2011), which features a neoclassical growth model similar to the one used below.[8,9] The estimates for the SCC in DICE have increased over the past versions. The most recent version, DICE-2013R, finds an optimal carbon price of 66\$/tC in 2015 (Nordhaus, 2013). A multi-region version of this framework is the RICE model (Nordhaus and Yang, 1996; Nordhaus, 2010). Here, in a cooperative regime, a planner sets carbon prices optimally in all regions to maximize global welfare for a given set of welfare weights.[10] Nordhaus (2010) finds an optimal carbon tax of 29\$/tC in 2010.

Golosov et al. (2014) consider a climate-economy model similar to DICE, although with a different formulation of the carbon cycle and an explicit modeling of fossil fuel use. Notably, they derive a closed-form expression for the expected social cost of carbon under certain conditions - in particular, loga-

[7]One example of a "second-best" model is Gerlagh and Liski (2018). They compute Markov-perfect optimal carbon fees in a setting without distortionary taxes, but where the government is unable to commit to future policies and has time-inconsistent preferences.

[8]A notable difference is that in DICE, fuel use is not explicitly modeled. Instead, carbon emissions are linked proportionally to output. The planner can invest in abatement, which reduces the amount of pollution for a given output level (Nordhaus, 2008).

[9]Other examples of frequently used IAMs are the PAGE model (Hope, 2006, 2008) and the FUND model (Tol, 2002a,b; Anthoff et al., 2009). Note that these are not optimal growth models, but instead take output scenarios as given.

[10]Hence, in the cooperative regime, the RICE model does not explicitly model climate agreements. See Barrett (2005) for an overview of the literature on environmental agreements and Harstad (2012, 2013) for recent work on agreements in a dynamic climate model.

rithmic utility and full depreciation - and show that the optimal tax as a share of contemporaneous output is constant. Quantitatively, they find an optimal carbon tax 57$/tC in 2010.

A complementary analysis to this paper is done by Barrage (2016). While the motivating research question is the same – what is the optimal carbon price under distortionary income taxation? – her analysis differs from this paper with respect to assumptions and, as a consequence, methods and results. Importantly, she studies a setting where the government is assumed to be able to commit to future income tax rates. Under these conditions, as shown by Judd (1985) and Chamley (1986), it is optimal to set the tax on capital income to zero. In her main result, she then shows that there is a qualitative difference between internalizing impacts from climate change that have a direct effect on utility and impacts that affect productivity, with only the latter being fully internalized.

In this paper, the lack of commitment and the assumption of a total income tax prevent equilibria that feature zero capital tax rates. Hence, the distinction between productivity and utility damages is less important, even though there are some qualitative differences, as discussed below. Instead, I focus on the different channels, most of them dynamic, that introduce wedges between the optimal carbon price and the level of marginal climate change damage. Quantitatively, the findings by Barrage (2016) are of a similar order as in this paper.[11]

As indicated above, the present model is a generalization of the literature

[11] Specifically, she finds that in a setting where both income and carbon taxes are set optimally – as they are in this paper – the optimal carbon price is between 4% and 18% below the marginal climate damage level. It should be noted that her COMET model differs from the model in this paper in a number of dimensions.

on the interaction between income and pollution taxation in static models initiated by Bovenberg and de Mooij (1994). While Bovenberg and de Mooij (1994) analyze a model with a pollution-intensive consumption commodity, Parry (1995) and Bovenberg and Goulder (1996) examine the case of a “dirty” input in the production process, similar to the setting in this paper.[12]

Methodologically, this paper is related to Klein et al. (2008), Azzimonti et al. (2009), and Martin (2010) who analyze time-consistent Markov-perfect equilibria in a standard neoclassical growth model without environmental quality.[13] Analogous to Klein et al. (2008), I derive the current government's generalized Euler equations, which are weighted sums of intertemporal and intratemporal wedges that the government trades off against each other. Similar to Azzimonti et al. (2009), this paper analyzes a *stock* of a public good, rather than a pure flow.[14] Note that while the application here is with respect to an environmental public good, the analysis would be similar to the case of the stock of a non-environmental public good. For example, one could think of infrastructure such as public roads and buildings as a persistent public good, i.e., expenditures today are of importance for the stock tomorrow.

3 The Climate-Economy Model

In this section, I introduce a simple dynamic framework in which I analyze optimal carbon and income taxation. Consider the standard neoclassical growth

[12]Compare also Bovenberg and van der Ploeg (1994), Goulder (1995), Goulder et al. (1997) and Bovenberg and Goulder (2002).

[13]See Fischer (1980) and Lucas and Stokey (1983) for earlier work on the time inconsistency of optimal policy in the presence of distortionary incomes taxes. Kehoe (1989) extended the model in Fischer (1980) to a two-country setting.

[14]Battaglini and Coate (2007, 2008) also consider an environment with distortionary income taxation and public good provision, but their focus is on the political economy of fiscal policy, in particular on legislative decision making.

model, extended by "fossil fuel", which is used as a factor of production, in addition to capital and labor. Burning fuel causes emissions of a pollutant, here carbon dioxide (CO_2). The amount of emissions generated in the production process is a determinant of climate change, which affects both the utility function of the representative household – as in the static second-best literature following Bovenberg and de Mooij (1994) – and the productivity of the representative firm, as, for example, in Golosov et al. (2014). Producers do not take into account how their decisions affect the climate; hence, carbon emissions represent an externality.

Note that throughout this paper, to keep the analysis tractable, I employ a deterministic model and abstract from all uncertainty related to the climate or economic development. Moreover, technological growth is exogenous.[15]

3.1 Utility and Household Problem

In each period t, there are L_t identical households. The population size grows with an exogenous rate g_L: $L_t = L_0 \exp(g_L t)$. An individual household's per-period utility is given by $u(C/L, h, G/L, T)$, where C/L denotes private per-capita consumption of a final good; $h \leq 1$ the share of hours worked in the total time endowment, which is normalized to unity; and G public consumption. T is an indicator of climate change. Specifically, it represents the change in mean global surface temperature relative to the preindustrial period. The two latter variables are not chosen by the household; hence, they represent public goods. The utility function is increasing in its first three arguments and decreasing in T. In other words, a higher T corresponds to a "worse" state of the climate. There are several channels through which permanently warmer temperatures

[15]These assumptions are discussed in more detail in the conclusion.

cause disutility (Barrage, 2016), for example, by affecting health and general well-being, as well as through the possible loss of biodiversity. Note that in contrast to many papers in public finance, I have assumed that the public consumption good is valued by the household; therefore, the amount provided is a choice variable of the government.

Denote $\mathcal{C}_t = C_t/L_t$. A household maximizes its dynasty's lifetime utility, subject to its budget constraint, taking price and tax sequences as given:

$$\max_{\{\mathcal{C}_t, h_t, \mathcal{I}_t, \mathcal{K}_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t L_t u(\mathcal{C}_t, h_t, G_t/L_t, T_t), \tag{1}$$

subject to

$$\mathcal{C}_t + \mathcal{I}_t \leq (1 - \tau_t)(r_t \mathcal{K}_t + W_t h_t) + \tau_t \delta \mathcal{K}_t. \tag{2}$$

and

$$\mathcal{K}_{t+1} \exp(g_L) = (1 - \delta)\mathcal{K}_t + \mathcal{I}_t. \tag{3}$$

$\mathcal{K}_t$ denotes the household's asset holdings in period t, while $\mathcal{I}_t$ are its net savings. δ denotes the rate of capital depreciation. r_t and W_t are the factor prices per unit of capital and per unit of time ("hours") spent working, respectively, while τ_t is a linear tax rate on capital and labor income. Note that in the former case, the tax base is the capital income net of depreciation. Solving this problem yields two standard optimality conditions, one intertemporal (consumption-savings) and one intratemporal (consumption-leisure).

As is standard in the environmental economics literature, I let preferences between private consumption (of goods and leisure) and public goods be additively separable (for example Cremer and Gahvari, 2001):

$$u(\mathcal{C}, h, G, T) = u(v(\mathcal{C}, h), G/L, T). \tag{4}$$

The assumption of additivity is convenient, since it facilitates the analytical and numerical analysis. Moreover, with respect to environmental quality, relaxing this assumption is not straightforward. While there is some evidence that higher temperatures have an effect on the marginal utility of leisure (for example Graff Zivin and Neidell, 2014), it is unclear whether, on aggregate, leisure and climate are substitutes or complements. In addition, how to specify a non-separable utility function with temperature change in a macroeconomic model is an open research question.

3.2 Production, Fuel Use and Firm's Problem

The consumption good is produced with a technology represented by a function F, which uses capital, labor and fossil fuel as inputs. Burning fuel causes CO_2 emissions. I assume that there is a proportional relationship between the amount of fuel used in production and the level of CO_2 emissions. Hence, for simplicity, I model emissions E_t as a direct input into the production process. Let gross output Y_t be given by:

$$Y_t = F(K_t, A_{H,t}H_t, A_{E,t}E_t), \tag{5}$$

where F is assumed to exhibit constant returns to scale. H_t is the total amount of labor supplied in period t. With a total population size of L_t and each household having a total time endowment normalized to one, $H_t = L_t h_t$. Similarly, K_t denotes the economy's capital stock, with $K_t = L_t \mathcal{K}_t$. $A_{H,t}$ denotes labor-augmenting productivity, while $A_{E,t}$ captures energy-augmenting productivity, which could also be interpreted as "energy efficiency". Both grow exogenously with rate $g_{A,j}$: $A_{j,t} = A_{j,0} \exp(g_{A,j}t)$.

Temperature change T does not only affect utility, but also has an impact on the production process. "Net" output - taking damages from climate change into account - is given by $\tilde{Y}_t = \tilde{F}(K_t, A_{H,t}H_t, A_{E,t}E_t, T_t)$. Following Nordhaus (2008) and Golosov et al. (2014), I assume that T enters the production function multiplicatively:

$$\tilde{Y}_t = \tilde{F}(K_t, A_{H,t}H_t, A_{E,t}E_t, T_t) = [1 - d(T_t)]F(K_t, A_{H,t}H_t, A_{E,t}E_t) \quad (6)$$

The "damage function" d captures damages to productivity, with $0 \leq d(T) \leq 1$. It is assumed to be convex and increase in temperature ($d_T > 0$, $d_{TT} > 0$). A common functional form for the damage function is:

$$d(T) = \frac{b_1 T^2}{1 + b_1 T^2}. \quad (7)$$

This specification is used, for example, in the DICE model (Nordhaus, 2008).

Moreover, I assume that the *private* marginal cost of emissions is paid in terms of the final good.[16] It is given by κ_t and grows with rate g_κ: $\kappa_t = \kappa_0 \exp(g_\kappa t)$. This captures, for example, the cost for extracting fossil fuel, an activity which is commonly expected to become more expensive over time.

The economy's resource constraint then reads:

$$C_t + G_t + \kappa_t E_t + K_{t+1} = [1 - d(T_t)]F(K_t, A_{H,t}L_t h_t, A_{E,t}E_t) + (1 - \delta)K_t. \quad (8)$$

A representative firm solves the following problem:

$$\max_{K_t, H_t, E_t} [1 - d(T_t)]F(K_t, A_{H,t}H_t, A_{E,t}E_t) - \kappa_t E_t - r_t K_t - W_t L_t h_t - \tilde{\theta}_t E_t,$$

where $\tilde{\theta}_t$ denotes a tax on carbon emissions. From the corresponding first-order condition, it follows that the carbon tax satisfies:

$$\tilde{\theta}_t = [1 - d(T_t)]F_E(t) - \kappa_t. \quad (9)$$

[16] Alternatively, one could let the cost be a function of the resources left in the ground, or model the production of fossil fuel as a production sector that uses labor and possibly capital as in Golosov et al. (2014) or Barrage (2016).

3.3 Climate Change

In addition to the private extraction cost, using fuel has a social cost: it causes carbon emissions, which negatively affect the state of the climate by increasing the mean global surface temperature. Many IAMs model this mechanism in two steps (Nordhaus, 2008; Golosov et al., 2014): first, past (and possibly current) carbon emissions, plus a vector of initial carbon concentrations in the atmosphere and other reservoirs like the upper and lower oceans, translate into current carbon concentrations. Second, the current vector of carbon stocks, $\mathbf{s}_t$, maps into the mean global temperature change in period t: $T_t = \mathcal{F}(\mathbf{s}_t)$.[17] One consequence of this modeling strategy is that many IAMs typically feature multiple variables summarizing the state of the climate (Nordhaus, 2008; Cai et al., 2015).

To keep the number of state variables low, I use a more reduced-form approach in this paper. Specifically, I assume a direct mapping $T_t = \tilde{q}(\mathbf{E}^t)$, where $\mathbf{E}^t = \{E_t, E_{t-1}, ..., E_{t_0}\}$ denotes the history of past global CO_2 emissions back to period t_0, and $\partial \tilde{q}/\partial E_j > 0 \, \forall j$. In words, the current *flow* of carbon has an impact on the state of the climate in future periods. This is a reasonable assumption, given that carbon stays in the atmosphere for a very long time.

The functional form for $\tilde{q}$ that is used below is based on Matthews et al. (2009). They define the "climate-carbon response" (CCR) as the ratio of global mean temperature change and total cumulative carbon emissions over some period of time. Using both historical emission data and an ensemble of climate models, they show that this variable is almost constant over time and, in par-

[17] Carbon is sometimes referred to as a "stock pollutant", since the stock in the atmosphere, rather than the flow, matters for climate change.

ticular, it is independent of the atmospheric carbon concentration. From this observation, it follows that the increase in the global mean surface temperature can be written recursively as:

$$T_{t+1} = T_t + \text{CCR} \cdot E_t. \tag{10}$$

For the quantitative analysis, this specification is convenient, since it summarizes the climate side of the model in one state variable, T_t, while abstracting from the carbon concentrations in the different reservoirs. Note that in contrast to other climate-economy models (Nordhaus, 2008; Golosov et al., 2014; Gerlagh and Liski, 2018), this specification implies that an increase in the global mean temperature due to carbon emissions is irreversible. This has quantitative implications for the size of the optimal carbon tax, as discussed below.

3.4 Government

The government must finance the public good by taxing labor and capital income. By assumption, lump-sum taxes are not available. Its budget constraint reads:

$$G_t \leq \tau_t[(r_t - \delta)K_t + W_t H_t] + \tilde{\theta}_t E_t. \tag{11}$$

Note that throughout this paper, I assume that the government has to balance its budget in every period. In other words, it can neither borrow from nor lend to households. The latter assumption is crucial, as pointed out by Azzimonti et al. (2006). They show that if the government were allowed to accumulate assets, it would be able to dispense with distortionary taxation after a finite number of periods. Hence, even in the absence of commitment, a government would set a zero tax rate on capital income in the long run. The intuition for this result is that the government could confiscate all income in the first period, and then lend to households every period and accumulate assets over time. After a sufficiently large number of periods, the government's wealth would be large enough to finance the public good without resorting to distortionary taxation. Therefore, since I want income tax rates to be non-zero in every period, I abstract from government assets.

3.5 Quasi-Stationary Transformation

Next, I transform the model variables into units such that *in the absence of climate change* (i.e. with $d(T_t) = 0$), this transformed "quasi-stationary" economy converges towards a steady state. In other words, the original model is assumed to move along a balanced growth path.

Inspecting the resource constraint (8), we can see that a sufficient condition for quasi-stationarity is that output, private and public consumption, capital and emission expenditures $\kappa_t E_t$ grow with the same rate:

$$g_Y = g_C = g_G = g_K = g_\kappa + g_E. \tag{12}$$

In addition, given that F is assumed to display constant returns to scale, a sufficient (but not necessary) set of conditions is that energy efficiency $A_{E,t}$ grows with the same rate as κ_t ($g_\kappa = g_{A,E}$) and that $g_Y = g_L + g_{A,H}$.

In the baseline specification of the model below, I will assume a Cobb-Douglas production function:

$$F(K_t, A_{H,t}H_t, A_{E,t}E_t) = K_t^\alpha (A_{E,t}E_t)^\gamma (A_{H,t}L_t h_t)^{1-\alpha-\gamma}. \tag{13}$$

In this case, it can be shown that using (12), balanced growth is characterized by:

$$g_A = \frac{\gamma}{1-\alpha-\gamma}(g_{A,E} - g_\kappa) + g_{A,H}, \tag{14}$$

where g_A is the growth rate of output per capita Y_t/L_t: $g_Y = g_L + g_A$.[18]

From this, it follows that a transformation of the model variables can be achieved in the following way: for variable $X \in \{Y, K, I, C, G\}$, define x_t as

$$x_t \equiv \frac{X_t}{L_t A_{H,t} A_{E,t}^{\frac{\gamma}{1-\alpha-\gamma}} \exp(-g_\kappa t)^{\frac{\gamma}{1-\alpha-\gamma}}} = \frac{X_t}{L_t A_t}. \tag{15}$$

Moreover, define E_t as

$$e_t \equiv \frac{E_t}{L_t A_{H,t} A_{E,t}^{\frac{\gamma}{1-\alpha-\gamma}} \exp(-g_\kappa t)^{\frac{1-\alpha}{1-\alpha-\gamma}}} = \frac{E_t}{L_t A_t \exp(-g_\kappa t)}. \tag{16}$$

A_t can be interpreted as factor-augmenting productivity, which grows with rate g_A as defined above. It can be shown that dividing both sides of the resource constraint (8) by $L_t A_t$ - the level of "effective population" - gives:

$$c_t + g_t + \kappa_0 e_t + k_{t+1} \exp(g_A) \exp(g_L) = [1 - d(T_t)] F(k_t, h_t, e_t) + (1-\delta) k_t. \tag{17}$$

Moreover, I can rewrite the household's problem in units per effective capita:

$$\max_{\{c_t, h_t, i_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t L_t u(A_t c_t, h_t, A_t g_t, T_t), \tag{18}$$

subject to

$$c_t + i_t \leq (1-\tau_t)(r_t k_t + w_t h_t) + \tau_t \delta k_t. \tag{19}$$

and

$$k_{t+1} \exp(g_A) \exp(g_L) = (1-\delta) k_t + i_t. \tag{20}$$

[18] Compare the appendix for a proof. Also note that if $g_\kappa = g_{A,E}$, this condition implies $g_Y = g_L + g_{A,H}$, which is the sufficient condition in the general case.

Hence, the first-order conditions are given by:

$$u_c(t) - \beta u_c(t+1)[1 + (1 - \tau_{t+1})(r_{t+1} - \delta)] = 0 \tag{21}$$

$$u_h(t) + u_c(t)(1 - \tau_t)w_t = 0 \tag{22}$$

Note that $w_t \equiv W_t/A_t$ is the price per unit of effective labor. Since w_t is assumed to be constant in the transformed economy's steady state, W_t must grow with g_A along the (hypothetical) balanced growth path. Therefore, the product $W_t H_t$ – the factor payments to labor – grow with $g_A + g_L = g_Y$, the same as output.

Finally, note that the relationship between emissions per effective capita e_t and temperature change can be written as:

$$T_{t+1} = q(T_t, e_t, t). \tag{23}$$

since by (16), emissions are a function of e_t and period t:

$$E_t = e_t L_0 A_0 \exp((g_L + g_A - g_\kappa)t).$$

3.6 Social Planner's Problem and First-Best Allocation

The social planner maximizes the present value of the population's welfare. Expressed in units per effective capita, her problem can be written as:

$$\max_{\{c_t, h_t, k_{t+1}, e_t, g_t, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t L_t u(A_t c_t, h_t, A_t g_t, T_t), \tag{24}$$

subject to the resource constraint (17) and the law of motion for temperature change (23).

The first-best equilibrium is characterized by the following set of equations:

$$\omega_{CS}(t) \equiv u_c(t) - \beta u_c(t+1)[\tilde{F}_k(t+1) + 1 - \delta] = 0 \tag{25}$$

$$\omega_{LL}(t) \equiv u_h(t) + u_c(t)\tilde{F}_h(t) = 0 \tag{26}$$

$$\omega_{PG}(t) \equiv u_g(t) - u_c(t) = 0 \tag{27}$$

and

$$\begin{aligned}\omega_{CC}(t) \equiv\ & u_c(t)(\tilde{F}_e(t) - \kappa) \\ & + q_e(t) \sum_{j=t+1}^{\infty} \hat{\beta}^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) [u_T(j) + u_c(j)\tilde{F}_T(j)] = 0\end{aligned} \tag{28}$$

where $u_c(t) = \frac{\partial u(A_t c_t, h_t, A_t L_t g_t, T_t)}{\partial c_t}$ etc.

In the expressions above, I define "wedges" for the consumption-savings margin (ω_{CS}), the labor-leisure margin (ω_{LL}), and the public-private good margin (ω_{PG}) and the "climate change" margin (ω_{CC}). The first-best equilibrium is characterized by all these wedges being simultaneously zero, and hence all of the margins being undistorted.

The first term on the right-hand side of (28) captures the marginal benefit of current carbon emissions in utils, net of marginal private cost. Without a climate externality, $\tilde{F}_e(t) = \kappa$ in the optimum. With climate change, the difference between the marginal benefit and the marginal private cost must be the marginal social cost of emitting carbon, which is commonly referred to as the "social cost of carbon" (SCC). I divide by by the current marginal utility $u_c(t)$ to express the SCC in monetary terms:

$$SCC(t) \equiv \tilde{F}_e(t) - \kappa. \tag{29}$$

The second term on the left-hand side of (28) represents the discounted sum of future disutility and productivity damages caused by emitting an additional unit of carbon in period t, measured in utils. Dividing this term by the current marginal utility $u_c(t)$ gives the marginal global climate change damage caused by emitting carbon in monetary units:

$$MCD(t) \equiv -\frac{q_e(t)}{u_c(t)} \sum_{j=t+1}^{\infty} \hat{\beta}^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) [u_T(j) + u_c(j)\tilde{F}_T(j)]. \tag{30}$$

Hence, I can write (28) more concisely as $SCC(t) = \text{MCD}(t)$. In other words, in first best, the social cost of carbon is equal the marginal climate damage in every period, which is equivalent to saying that the climate change margin is undistorted and climate change mitigation, a public good, is provided at the first-best margin.

Finally, note that in a decentralized economy, the price on carbon emissions, θ_t, must be equal to the SCC in order to internalize the climate externality. This can be seen from the firms' optimality condition which respect to emissions, which reads:

$$\tilde{F}_e(t) = \kappa + \theta_t \tag{31}$$

Hence, the first-best carbon fee equals the MCD and hence represents a *Pigouvian* price:

$$\theta_t^p = -\frac{q_e(t)}{u_c(t)} \sum_{j=t+1}^{\infty} \hat{\beta}^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) (u_T(j) + u_c(j)\tilde{F}_T(j)). \tag{32}$$

In the next section, I show that in dynamic models without commitment, climate policy interacts with distortionary taxation and the optimal carbon price is in general not at the Pigouvian level.

4 Decomposing the Tax-Interaction Effect

4.1 Preliminaries

Consider a decentralized equilibrium of the economy outlined in the last section. If the government has access to lump-sum taxes in order to finance public consumption expenditures, the equilibrium is characterized by equations (25) - (28), and the first-best allocation is attained. In this section, I relax the assumption that lump-sum taxation is feasible. Instead, the government must resort to a distortionary tax on labor and capital income for financing public consumption. In addition, I assume that it does not have access to a technology that allows it to commit to future tax rates. When solving for the outcome under lack of commitment, I look for the time-consistent differentiable Markov-perfect equilibrium in this economy.[19] The basic idea of this equilibrium concept is that only current payoff-relevant states, but not the history of states and actions, matter for a player's action choice.

Note that in this setting, the current government plays a game with its successor.[20] This implies that the current government takes into account the optimal behavior of next period's government when solving its problem. While the current government cannot directly choose policies in the following period, it can affect them by choosing the economy's future state variables.

I define the Markov-perfect equilibrium in a setting where the government has access to a total income tax. That is, it is restricted to impose the same

[19] An alternative approach would be to look for all sustainable equilibria, along the lines of Phelan and Stacchetti (2001) or Reis (2011).

[20] If governments in different periods are identical, the current government actually plays a game "against itself". That is, even though I have the same government making the decisions in both periods, it must be treated as different players, due to the lack of commitment. Equivalently, announcements in the current period about how the government will behave in the following period are not credible.

tax rate on labor and capital income. This assumption ensures a setting where both the intertemporal consumption-savings margin and the intratemporal labor-leisure margin are distorted simultaneously, i.e. where I have positive tax rates on both labor and capital income. As shown by Martin (2010), in a setting where the government has ex-ante access to two separate tax rates, the equilibrium features a zero tax rate on labor income, assuming that labor taxes are bounded to be non-negative and that the capital income tax is not bounded from above. This is intuitive: recall that taxes on capital are ex post non-distortionary and thus, can be considered as de facto lump-sum taxes. Hence, in the presence of such a tax, assuming that it is unbounded, it cannot be optimal to have a positive distortionary tax on labor income.[21] Empirically, a setting with a total income tax appears more realistic than an equilibrium in which the government finances its expenditures only using a tax on capital or on labor income.[22]

The analysis is a straightforward extension of Klein et al. (2008) and Azzimonti et al. (2009), adding a second public good, environmental quality, which in contrast to the other good does not only affect utility, but also the production process. Hence, there is a second state variable in addition to capital, here the current state of the climate T. Moreover, by comparing the Markov-perfect carbon tax to the outcome under commitment, I will show that in the presence of distortionary taxes, the optimal pollution price is in general not

[21]In other words, the government is not allowed to subsidize labor. Martin (2010) shows that subsidizing labor is optimal in a setting with unrestricted separate tax rates on labor and capital. However, this appears to be empirically less relevant than zero labor taxes.

[22]There are other ways of modifying the model such that one would get positive tax rates on both production factors in equilibrium. Martin (2010) considers an exogenous upper bound on the capital tax, as well as making the utilization rate of capital endogenous. The former is somewhat unsatisfying since it leaves the origin of the bound unmodeled. The latter slightly changes the logic of the mechanism in this paper.

time-consistent.

Also note that the tax schedule set by a government which had access to a commitment device is different than the one chosen under lack of commitment. Note that this time inconsistency is due to the interaction between environmental and non-environmental taxes: as seen above, the optimal pollution price depends on the optimal tax structure. If other taxes are time-inconsistent - for example a positive tax on labor income in a scenario where separate tax rates on labor and capital are feasible - so is the carbon tax.

4.2 Definitions

To save on notation, define let $\mathbf{x}$ denote the vector of state variables and $\mathbf{z}$ the vector of all inputs in the production function:

$$\mathbf{x}_t = (k_t, T_t, t)$$
$$\mathbf{z}_t = (k_t, h_t, e_t, T_t).$$

Moreover, from the resource constraint (17), I define a function C that gives private consumption as a function of the remaining variables:

$$\mathsf{C}(\mathbf{z}_t, k_{t+1}, g_t) = \tilde{F}(\mathbf{z}_t) + (1-\delta)k_t - g_t - \kappa_0 e_t - k_{t+1}\exp(g_A)\exp(g_L) \quad (33)$$

Similarly, from a quasi-stationary version of the government's budget constraint,

$$g_t \leq \tau_t[(r_t - \delta)k_t + w_t h_t] + \theta_t e_t,$$

let function T give the income tax rate that balances the government's budget:

$$\mathsf{T}(\mathbf{z}_t, g_t) = \frac{g_t - (\tilde{F}_e(\mathbf{z}_t) - \kappa_0)e_t}{(\tilde{F}_k(\mathbf{z}_t) - \delta)k_t + \tilde{F}_h(\mathbf{z}_t)h_t}. \quad (34)$$

With these auxiliary equations, we can write the household's first-order conditions, (21) and (22), from the government's perspective. The government takes into account that equilibrium factor prices equal marginal products, and that the income tax rate τ_t is given by (34). Thus, the Euler equation can be written as:

$$u_c(t) - \beta u_c(t+1)\left[1 + \big(1 - \mathsf{T}(\mathbf{z}_t, g_t)\big)\big(\tilde{F}_k(\mathbf{z}_t) - \delta\big)\right] = 0, \tag{35}$$

where

$$u_c(t) = u_c\big(A_t\mathsf{C}(\mathbf{z}_t, k_{t+1}, g_t), h_t, A_t g_t, T_t\big). \tag{36}$$

Similarly, the intratemporal optimality condition can be written as:

$$\frac{u_h(t)}{u_c(t)} + \big(1 - \mathsf{T}(\mathbf{z}_t, g_t)\big)\tilde{F}_h(\mathbf{z}_t) = 0, \tag{37}$$

with

$$u_h(t) = u_h\big(A_t\mathsf{C}(\mathbf{z}_t, k_{t+1}, g_t), h_t, A_t g_t, T_t\big). \tag{38}$$

4.3 Equilibrium Definition

Before giving a formal definition of the stationary Markov-perfect equilibrium concept used in the remainder of the paper, I provide a verbal exposition. As it is standard with models of optimal income taxation, both under commitment and in the absence of commitment, the current government maximizes the representative household's welfare, taking the household's first-order conditions – the Euler equation (35) and the intratemporal optimality condition (37) – as constraints.

A lack of commitment implies that the current government cannot control the behavior of the subsequent government, even if it remains in power. In other words, as outlined above, the current government plays a game against its successor, possibly itself. Hence, it takes the strategies of the future government as given. Assume these strategies are given by the policy functions ψ (for public expenditures), ϕ (for energy use), n^k (for savings) and n^h (for hours worked). Then, following the one-stage deviation principle, these policy functions constitute an equilibrium if they prescribe optimal actions for the current government, i.e. if

$$g_t = \psi(\mathbf{x}_t),\ e_t = \phi(\mathbf{x}_t),\ k_{t+1} = n^k(\mathbf{x}_t),\ h_t = n^h(\mathbf{x}_t)$$

maximize its objective function for all realizations of the state vector $\mathbf{x}_t$, subject to all relevant constraints and taking the strategies of the future government as given. In other words, assuming that the future government chooses policies according to the equilibrium decision rules, it must be optimal for the current government to follow the same policy functions.

Formally, a stationary Markov-perfect equilibrium is defined as a value function v and differentiable policy functions ψ, ϕ, n^k and n^h such that for all $\mathbf{x}_t = (k_t, T_t, t)$, $\psi(\mathbf{x}_t)$, $\phi(\mathbf{x}_t)$, $n^k(\mathbf{x}_t)$ and $n^h(\mathbf{x}_t)$ solve

$$\max_{g_t, e_t, k_{t+1}, h_t, T_{t+1}} u(A_t \mathsf{C}(\mathbf{z}_t, k_{t+1}, g_t), h_t, A_t g_t, T_t) + \beta v(k_{t+1}, T_{t+1}, t+1), \quad (39)$$

subject to

$$\begin{aligned} &\beta u_c\left(A_{t+1}\mathsf{C}\left(\begin{array}{c} k', n^k(\mathbf{x}_{t+1}), n^h(\mathbf{x}_{t+1}), \\ \psi(\mathbf{x}_{t+1}), \phi(\mathbf{x}_{t+1}), T_{t+1} \end{array}\right), n^h(\mathbf{x}_{t+1}), A_{t+1}\psi(\mathbf{x}_{t+1}), T_{t+1}\right) \cdot \\ &\cdot \left\{1 + \left[1 - \mathsf{T}\left(\begin{array}{c} k_{t+1}, n^h(\mathbf{x}_{t+1}), \psi(\mathbf{x}_{t+1}), \\ \phi(\mathbf{x}_{t+1}), T_{t+1} \end{array}\right)\right]\left[\tilde{F}_k\left(\begin{array}{c} k_{t+1}, n^h(\mathbf{x}_{t+1}), \\ \phi(\mathbf{x}_{t+1}), T_{t+1} \end{array}\right) - \delta\right]\right\} \\ &- u_c\big(A_t \mathsf{C}(\mathbf{z}_t, k_{t+1}, g_t), h_t, A_t g_t, T_t\big) = 0, \end{aligned} \quad (40)$$

$$\frac{u_h\big(A_t\mathsf{C}(\mathbf{z}_t,k_{t+1},g_t),h_t,A_tg_t,T_t\big)}{u_c\big(A_t\mathsf{C}(\mathbf{z}_t,k_{t+1},g_t),h_t,A_tg_t,T_t\big)}+[1-\mathsf{T}(\mathbf{z}_t,g_t)]\tilde{F}_h(\mathbf{z}_t)=0, \tag{41}$$

and $T_{t+1}=q(T_t,e_t,t)$. Moreover, for all $\mathbf{x}_t=(k_t,T_t,t)$,

$$v(\mathbf{x}_t)=u\left[A_t\mathsf{C}\begin{pmatrix} k_t,n^k(\mathbf{x}_t),n^h(\mathbf{x}_t),\\ \psi(\mathbf{x}_t),\phi(\mathbf{x}_t),T_t\end{pmatrix},n^h(\mathbf{x}_t),A_t\psi(\mathbf{x}_t),T_t\right] \\ +\beta v\big(n^k(\mathbf{x}_t),q(T_t,\phi(\mathbf{x}_t),t+1),t+1\big).$$

Note that an alternative definition of the equilibrium has k_{t+1} and h_t be explicitly chosen by the households. Denote the left-hand side of (40) as $\eta(k_t,T_t,g_t,e_t,k_{t+1},h_t,t)$ and the left-hand side of (41) as $\epsilon(k_t,T_t,g_t,e_t,k_{t+1},h_t)$, respectively. Define the functions $\mathsf{K}(\mathbf{x}_t,g_t,e_t)$ and $\mathsf{H}(\mathbf{x}_t,g_t,e_t)$ implicitly as

$$\eta(k_t,T_t,g_t,e_t,\mathsf{K}(\mathbf{x}_t,g_t,e_t),\mathsf{H}(\mathbf{x}_t,g_t,e_t),t)=0 \tag{42}$$

$$\epsilon(k_t,T_t,g_t,e_t,\mathsf{K}(\mathbf{x}_t,g_t,e_t),\mathsf{H}(\mathbf{x}_t,g_t,e_t))=0. \tag{43}$$

K and H can be interpreted as the households' response function for savings (hours worked) to the current government's policy choice, assuming that future governments follow the equilibrium policies: it gives the household's optimal savings level if the current governments set expenditures g and a carbon tax that results in emission level e. In equilibrium,

$$\mathsf{K}(\mathbf{x}_t,\psi(\mathbf{x}_t),\phi(\mathbf{x}_t))=n^k(\mathbf{x}_t)$$
$$\mathsf{H}(\mathbf{x}_t,\psi(\mathbf{x}_t),\phi(\mathbf{x}_t))=n^h(\mathbf{x}_t).$$

4.4 The Generalized Euler Equation

Using the households' response function K and H defined above, I can write the goverment's problem more compactly as:

$$\max_{g_t,e_t} u\big[A_t\mathsf{C}\big(k_t,\mathsf{H}(\mathbf{x}_t,g_t,e_t),e_t,T_t,\mathsf{K}(\mathbf{x}_t,g_t,e_t),g_t\big),\mathsf{H}(\mathbf{x}_t,g_t,e_t),A_tg_t,T_t] \\ +\beta v[\mathsf{K}(\mathbf{x}_t,g_t,e_t),q(T_t,e_t,t),t+1\big], \tag{44}$$

where, as before,

$$v(\mathbf{x}_t) = u\left[A_t \mathsf{C}\begin{pmatrix} k_t, n^k(\mathbf{x}_t), n^h(\mathbf{x}_t), \\ \psi(\mathbf{x}_t), \phi(\mathbf{x}_t), T_t \end{pmatrix}, n^h(\mathbf{x}_t), A_t\psi(\mathbf{x}_t), T_t\right] \\ + \beta v\big(n^k(\mathbf{x}_t), q(T_t, \phi(\mathbf{x}_t), t+1)\big).$$

Taking first-order conditions with respect to e_t and g_t, rearranging and using the definitions for $\omega_{LL}(t)$ and $\omega_{PG}(t)$ yields the following equations characterizing a stationary Markov-perfect equilibrium:

$$\omega_{PG}(t) + \omega_{LL}(t)\mathsf{H}_g(t) + [\hat{\beta} v_k(t+1) - u_c(t)\exp(g_A)\exp(g_L)]\mathsf{K}_g(t) = 0 \quad (45)$$

$$\tilde{F}_e(t) - \kappa + \hat{\beta}\frac{q_e(t)}{u_c(t)} v_T(t+1) + \omega_{LL}(t)\frac{\mathsf{H}_e(t)}{u_c(t)} \\ + [\hat{\beta} v_k(t+1) - u_c(t)\exp(g_A)\exp(g_L)]\frac{\mathsf{K}_e(t)}{u_c(t)} = 0. \quad (46)$$

Note that the optimality conditions (45) and (46) contain the derivatives of the policy functions for savings and labor and hence, following Klein et al. (2008), I refer to them as *generalized* Euler equations (GEE). In the following, I show that these equations can be written as a linear combination of present and future wedges ω_j.[23]

To facilitate the interpretation of the first-order condition (46), in appendix A.2 I show how to substitute for the derivatives v_k and v_T of the value function. This results in a characterization of the difference between the optimal carbon price and the level of marginal climate change damage, which is summarized in proposition 1:

Proposition 1. *In a Markov-perfect equilibrium, the government's first-order*

[23] In the interest of brevity, I focus on (46) which relates to the climate change wedge. An analogous argument can be made for (45) and the public consumption wedge (cp. Klein et al., 2008).

with respect to emissions can be written in the following way:

$$\begin{aligned}
MCD(t) - \theta(t) &= \omega_{LL}(t)\frac{\mathsf{H}_e(t)}{u_c(t)} + \omega_{CS}(t)\frac{\mathsf{K}_e(t)}{u_c(t)} + \\
&+ \frac{\mathsf{K}_e(t)}{u_c(t)} \sum_{j=t+1}^{\infty} \hat{\beta}^{j-t} \left[\left(\prod_{i=t+1}^{j} \mathsf{K}_k(i) \right) \omega_{CS}(j) + \left(\prod_{i=t+1}^{j-1} \mathsf{K}_k(i) \right) \mathsf{H}_k(j)\omega_{LL}(j) \right] \\
&+ \frac{q_e(t)}{u_c(t)} \sum_{j=t+1}^{\infty} (\hat{\beta})^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \mathsf{H}_T(j)\omega_{LL}(j) \\
&+ \frac{q_e(t)}{u_c(t)} \sum_{j=t+1}^{\infty} (\hat{\beta})^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \mathsf{K}_T(j) \left[\hat{\beta} v_k(j+1) - u_c(j)\exp(g_A)\exp(g_L) \right]
\end{aligned} \tag{47}$$

with

$$\begin{aligned}
&\hat{\beta} v_k(t+1) - u_c(t)\exp(g_A)\exp(g_L) \\
&= \omega_{CS}(t) + \sum_{j=t+1}^{\infty} \hat{\beta}^{j-t} \left[\left(\prod_{i=t+1}^{j} \mathsf{K}_k(i) \right) \omega_{CS}(j) + \left(\prod_{i=t+1}^{j-1} \mathsf{K}_k(i) \right) \mathsf{H}_k(j)\omega_{LL}(j) \right]
\end{aligned}$$

Proof: cp. Appendix, section A.2.

In the following, I denote the difference between the MCD and the carbon price as $\mathcal{W}$, which equals the (negative) climate change wedge defined earlier, in terms of the final good rather than in utils:

$$\mathcal{W}(t) \equiv \text{MCD}(t) - \theta(t) = -\frac{\omega_{CC}(t)}{u_c(t)}$$

The interpretation of (47) is straightforward: in equilibrium, the government trades off wedges. In first best, as $\omega_{CS} = \omega_{LL} = 0$, (47) reduces to $\mathcal{W}(t) = \text{MCD}(t) - \theta(t) = 0$. In contrast, if the government has to resort to distortionary taxation, the household's optimality conditions (40) and (41) imply that the consumption-savings and labor-leisure wedges are positive. Assuming that the derivatives of the best-response functions are non-zero, it follows from (47) that $\mathcal{W}(t)$ cannot be zero in general: in optimum, climate change mitigation is not provided at the first-best margin, which is equivalent to the social cost of carbon not being equal to the marginal climate damage. As the difference between the MCD and the optimal carbon price is caused by the interaction of climate change mitigation and income taxation, I will refer to $\mathcal{W}$ as a measure of the "tax-interaction" effect.

What is the intuition behind this result? Proposition 1 illustrates that $\mathcal{W}$ is composed of a number of terms, each of which captures a (sum of) second-best benefit and cost from carbon emissions. The following paragraphs will describe these terms in more detail.

The Second-Best Labor Effect I refer to the second-best effect of emitting carbon on welfare through the channel of current labor supply as the "second-best labor effect" (SBL). It is formally captured by:

$$\text{SBL}(t) = \omega_{LL}(t)\frac{\mathsf{H}_e(t)}{u_c(t)} \tag{48}$$

The SBL can be positive or negative and hence represent a second-best benefit or cost of emitting carbon.

For illustration, assume that the other terms on the right hand side of (47) are zero, and hence I have:[24]

$$\mathcal{W}(t) = \omega_{LL}(t)\frac{\mathsf{H}_e(t)}{u_c(t)} \tag{49}$$

[24]Note that this assumption and the resulting GEE would hold in a static one-period version of the model. Specifically, using the definition of the wedges above, the GEE reads:

$$u_c(\tilde{F}_e - \kappa) + q_e(u_T + u_c F_T) + \mathsf{H}_e(u_c\tilde{F}_h + u_h) = 0.$$

This expression illustrates that, in equilibrium, the government trades off wedges. In first best, since both $\mathcal{W}$ and ω_{LL} are zero, (49) holds. In second best where $\tau > 0$, I have that $\omega_{LL} > 0$ and hence the environmental wedge cannot be zero unless $\mathsf{H}_e = 0$. More precisely, if $\mathsf{H}_e > 0$ ($\mathsf{H}_e < 0$), $\mathcal{W}$ must be positive (negative) for (49) to be satisfied. In other words, there is an interaction between climate policy and income taxation, in the sense that in the presence of a distortionary tax on labor income, climate change mitigation is not provided at the first-best margin. Whether or not the climate externality is less than fully internalized, i.e. whether or not $\mathcal{W} > 0$, depends on the sign of H_e.

For the sake of the argument, assume that $\mathsf{H}_e(t) > 0$. That is, a marginal increase in carbon emissions raises the labor supply. In first best, where $\omega_{LL} = 0$, a marginal change in hours worked does not affect welfare. In contrast, in second best, it leads to a first-order welfare gain since, in equilibrium, the benefit from a marginal increase in hours worked, $u_c f_h$, is greater than the marginal disutility of working more ($-u_h$). This is only the case if $\omega_{LL} > 0$, i.e. as long as the income tax is positive. In this sense, emitting carbon attenuates the intratemporal distortion caused by the income tax. This positive effect on current labor supply is an additional benefit of carbon emissions, besides the usual benefit of increasing output and consumption. Thus, it reduces the social cost of carbon below the value of the marginal damage - or, in terms of tax rates, the optimal second-best tax is lower than the corresponding Pigouvian fee.[25] In other words, the margin between private consumption and climate

[25] The same logic applies if $\mathsf{H}_e < 0$. In that case, there is an additional second-best cost of using fuel, since it decreases labor supply and hence exacerbates the intratemporal distortion.

change mitigation is distorted compared to the first best.

Finally, note that in the static case where (49) holds, the sign of H_e and hence the direction of the tax-interaction effect can be determined analytically.

The Second-Best Savings Effect The "second-best savings effect" (SBS) is represented by the term

$$\text{SBS}(t) = \omega_{CS}(t)\frac{\mathsf{K}_e(t)}{u_c(t)}. \tag{50}$$

As before, assume that the remaining terms on the right hand side of (47) are zero. Then,

$$\mathcal{W}(t) = \omega_{CS}(t)\frac{\mathsf{K}_e(t)}{u_c(t)}. \tag{51}$$

This equation shows that again, climate policy interacts with fiscal policy. This implies that as long as current savings are affected by current fuel use ($\mathcal{K}_e \neq 0$), climate change mitigation is not at its first best level. As before, the sign of the climate wedge depends on the sign of $\mathcal{K}_e$: if current savings increase (decrease) in current fuel use, $\mathcal{W}$ is positive (negative), i.e., the SCC is below (above) its Pigouvian level.

The intuition is similar to the static case above. First, note that since households understand that capital will be taxed in the following period and thus their return to savings will be lower, they consume more and save less than in first best. Then, if $\mathcal{K}_e > 0$, by increasing emissions and thus "under-providing" climate change mitigation, i.e. by not fully internalizing climate damages, the government can increase current savings. This has a first-order welfare gain in second best since $\omega_{CS} > 0$ and hence the discounted marginal increase in utility due to more consumption in the subsequent period is higher than the marginal utility loss due to less consumption today. It follows that emitting carbon has an additional benefit which is not present in first best and hence the social cost of carbon does not fully internalize the utility damage caused by the climate externality.

In the case of the SBL effect, emitting carbon affects labor supply through a change in the return to labor. Here, the mechanism is slightly different. The return to current savings depends on the amount of carbon emitted in the next period, which is not directly affected by the current government. Instead, more energy use and hence emissions today affects the amount of resources available to the household by increasing today's capital income, i.e. the return to past savings. For $\mathsf{K}_e > 0$, this allows the household to move more resources to the next period, thereby mitigating the intertemporal distortion.

Note that this result is analogous to Klein et al. (2008) who only consider not-environmental public consumption. In general, underproviding a public good today dampens underinvestment and thus mitigates the intertemporal distortion caused by the positive tax on capital income.

The Propagation-through-Capital Effect The "propagation-through-capital effect" (PPC) is formally captured by the following term:

$$\mathrm{PPC}(t) = \frac{\mathsf{K}_e(t)}{u_c(t)} \sum_{j=t+1}^{\infty} \hat{\beta}^{j-t} \left[\left(\prod_{i=t+1}^{j} \mathsf{K}_k(i) \right) \omega_{CS}(j) + \left(\prod_{i=t+1}^{j-1} \mathsf{K}_k(i) \right) \mathsf{H}_k(j) \omega_{LL}(j) \right] \tag{52}$$

The PPC works through the following channels: a marginal increase in current emissions has an effect on today's savings ($\mathsf{K}_e(t)$) and hence on tomorrow's capital stock. This, in turn, affects tomorrow's economic activity and hence causes changes in both labor supply and the level of savings, which triggers a second-best labor and savings effect in period $t+1$ ($\mathsf{H}_k(t+1)\omega_{LL}(t+1)$ and $\mathsf{K}_k(t+1)\omega_{CS}(t+1)$, respectively). Moreover, a change in tomorrow's savings ($\mathsf{K}_k(t+1)$) impacts the capital stock in period $t+2$, which again affects economic activity, labor supply and savings. This causal chain continues over time. In other words, the impact from a marginally higher emission level in the current period is propagated into the future by affecting the capital stocks and savings choice in all subsequent periods. Assuming that proportional income taxation continues in the future, there are future second-best costs and benefits which are analogous to the SBL and SBS effects described above.

The Propagation-through-Temperature Effect Finally, the "propagation-through-temperature effect" (PPT)

$$\begin{aligned}\text{PPT}(t) =&\frac{q_e(t)}{u_c(t)}\sum_{j=t+1}^{\infty}(\hat{\beta})^{j-t}\left(\prod_{i=t+1}^{j-1}q_T(i)\right)\mathsf{H}_T(j)\omega_{LL}(j)\\ &-\frac{q_e(t)}{u_c(t)}\sum_{j=t+1}^{\infty}(\hat{\beta})^{j-t}\left(\prod_{i=t+1}^{j-1}q_T(i)\right)\mathsf{K}_T(j)\left[\frac{u_c(j)}{K_e(j)}[\text{SBS}(j)+\text{PPC}(j)]\right],\end{aligned} \tag{53}$$

where I have made use of (72) implying that

$$\hat{\beta}v_k(t+1)-u_c(t)\exp(g_A)\exp(g_L)=\frac{u_c(t)}{K_e(t)}[\text{SBS}(t)+\text{PPC}(t)]$$

The PPT works in a similar fashion to the PPC, but the effect of a marginally higher emission level in the current period is propagated into the future via higher temperatures rather than the capital stock. Emitting more carbon today impacts tomorrow's level of temperature change, as well as the extent to which temperature increases in all future periods, through the persistency of T captured by its law of motion in (10). Through affecting productivity, temperature change in turn has an effect on economic activity and thus on the households' labor supply and savings choice. Again assuming that income tax rates are positive in the future, this causes an SBL $(\mathsf{H}_T(j)\omega_{LL}(j))$, as well as an SBS and PPC in every future period.

With the definitions provided above, I can characterize the *overall tax-interaction effect*, measured by the difference between the MCD and the optimal carbon price, as the *sum of the individual effects of a marginal increase in emissions*:

$$\mathcal{W}(t) = \text{MCD}(t) - \theta(t) = \text{SBL}(t) + \text{SBS}(t) + \text{PPC}(t) + \text{PPT}(t) \qquad (54)$$

For any of the terms on the right hand side, they represent a benefit (cost) of emitting carbon when positive (negative), which decreases (increases) the optimal carbon price.

Figure 1 illustrates the tax-interaction effect graphically. It displays a scenario where these different effect work in opposite directions and hence partly offset each other. One way to read this graph is the following: start with the dashed curve that represents the level of marginal climate damage over time. In a setting without distortionary taxation, the optimal carbon price would always lie on this curve. Here, under income taxation, the propagation-through-temperature by itself would imply that the carbon price exceeds its Pigouvian level and lies on the dotted curve; in other words this channel would constitute a second-best cost of emitting carbon. At the same time, the other three channels are assumed to capture second-best benefits of emitting carbon, and hence push the optimal carbon price downwards to the solid curve, which in the example here is located below the Pigouvian curve.

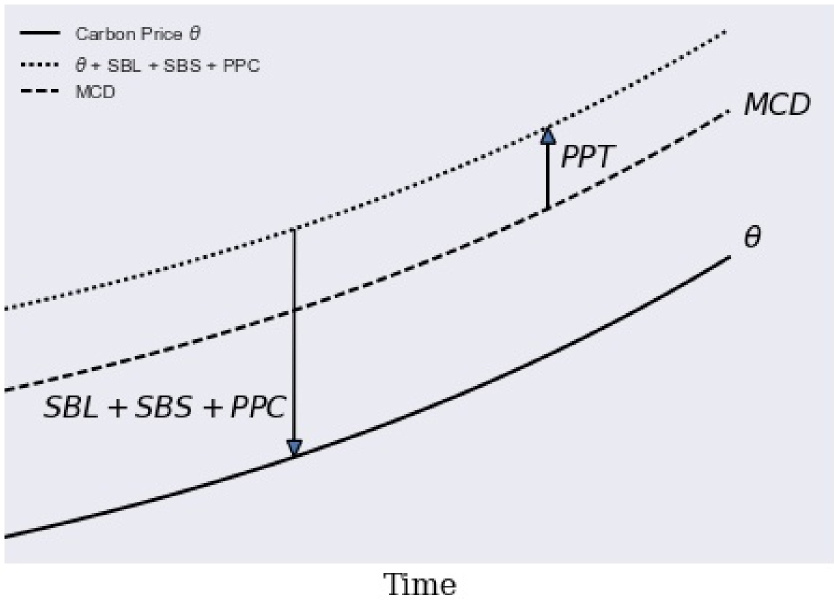

Figure 1: Tax-Interaction Effect - Illustration

Of course, other scenarios where the effects reinforce each other are also possible. As we will see in the next section, however, figure 1 reflects what seems to be the most plausible constellation when evaluating the tax-interaction effect quantitatively.

Equation (54) encapsulates the main takeaway from this section: the interaction between income taxation and climate change mitigation causes a deviation of the optimal second-best carbon price from marginal climate change damage and hence from its Pigouvian level. This deviation can be attributed to four distinct effects, each of which captures a particular channel through which current carbon emissions affect present and future welfare in second best.

The questions of whether there is a tax-interaction effect and in what direction it goes can therefore be asked in terms of (54): is the combined effect of the components of $\mathcal{W}(t)$ different from zero, and if so, what is the sign? In other words, to what extent do the terms on the right hand side of (54) move in the same direction or cancel each other out?

Note that the wedges and derivatives that show up in (54) are endogenous and solved for simultaneously when computing the equilibrium. Moreover, in general, it is not possible to determine their sign analytically. Hence, the questions just asked are ultimately quantitative questions and therefore require a numerical analysis. This is done in the next section.

5 Quantitative Analysis

In the previous section, I have characterized the tax-interaction effect in a dynamic infinite-horizon model without commitment. I have identified the second-best benefits and costs of emissions that affect the optimal carbon price. However, this section has given no indication of how important the tax-interaction effect is and in what direction it goes. As argued above, this is a quantitative question. Therefore, this section computes the OCP in the Markov-perfect equilibrium of the calibrated climate-economy model and compares it to the Pigouvian rate, as well as the OCP in first best and under commitment.

5.1 Calibration

5.1.1 Economic Module

For the baseline calibration, I choose a utility function which is additively separable in all arguments:

$$u(C_t/L_t, h_t, G_t/L_t, T_t) = \log(C_t/L_t) - \eta \frac{h_t^{1+\xi}}{1+\xi} + \zeta_g \log(G_t/L_t) - \frac{\zeta_T}{2} T_t^2 \quad (55)$$

Moreover, as outlined in (13), I use a Cobb-Douglas specification for the production function F. This results in four parameters in both the utility function (η, ξ, ζ_g, ζ_T) and the production function (α, γ, $g_{A,H}$, $g_{A,E}$). In addition, I need to choose values for the discount rate β and the depreciation rate δ.

To calibrate the economic parameters of the model, I focus on the period between 1970 and 2010. There are several simplifying assumptions underlying the calibration strategy:

1. Climate change does not occur until 2010, and hence there was no need for climate policy in this period.

2. The stationary version of the global economy outlined above is in a steady state for most of this period.

The first assumption is motivated by the observation that while climate change has had an effect on human and natural system before 2010 (IPCC, 2014), the bulk of climate change impacts is expected to occur towards the middle of the 21st century and beyond. Moreover, while meaningful climate policies have yet to be implemented on a global level, the few quantitatively important regional initiatives, such as the European Union Emissions Trading System, did not start until well into the first decade of this century.

I use the World Bank's "DataBank"[26] to obtain time series for global output, consumption, investment, government expenditures, population and CO_2 emissions between 1970 and 2010. The first step of the calibration is to obtain a time series for the stock of physical capital, which is not included in the database. Following the literature (for example DeJong and Dave, 2011), I use the "perpetual inventory method", which is based on the law of motion for the capital stock

$$K_{t+1} = (1 - \delta)K_t + I_t, \tag{56}$$

and employs beginning-of-sample averages to find K_0. To apply (56), I set the annual capital depreciation rate to $\delta = 0.08$ throughout all scenarios.

In addition to δ, a second parameter value that I need to choose ex ante is the discount factor β, which is determined by the time discount rate (also referred to as the pure rate of time preference). Ever since the publication of the Stern review (Stern, 2006), there has been much debate in the literature about the "right" value for the social discount rate, and hence for the pure rate of time preference (e.g. Weitzman, 2007; Dasgupta, 2008), noting that the level and growth of the optimal carbon price depends to a large degree on discounting. Since the focus of this paper is less on the absolute level of the optimal carbon price, but instead on its deviation from both the Pigouvian and the first-best level, the choice of a value for β is not of first-order importance. In the baseline calibration of my second-best model, I follow the more conservative approach advocated for example by Nordhaus (2011). I set $\beta = 0.9835$, which implies a social discount rate of 4%, as outlined below.

Next, I employ least squares regression to determine the parameters of the

[26]http://databank.worldbank.org/data/home.aspx

production function. In a model with a Cobb-Douglas production function, I can get a linear regression equation by taking logs:

$$\begin{aligned}\log\left(\frac{Y_t}{H_t}\right) &= \gamma\log(A_{E,0}) + (1-\alpha-\gamma)\log(A_{H,0}) + [g_E\gamma + g_A(1-\alpha-\gamma)]t \\ &\quad + \alpha\log\left(\frac{K_t}{H_t}\right) + \gamma\log\left(\frac{E_t}{H_t}\right) \\ &= TFP_0 + dt + \alpha\log\left(\frac{K_t}{H_t}\right) + \gamma\log\left(\frac{E_t}{H_t}\right),\end{aligned} \tag{57}$$

with total factor productivity

$$TFP_0 \equiv \gamma\log(A_{E,0}) + (1-\alpha-\gamma)\log(A_{H,0}). \tag{58}$$

and $d \equiv \gamma g_E + g_A(1-\alpha-\gamma)$. To ensure consistency with the household's first-order conditions in steady state, as outlined below, I fix a value for the income share of capital α, and estimate the remaining parameters γ, TFP and d from (57), using global data. From the latter two, I can compute the initial level and the growth rate for factor-augmenting productivity A, as defined above. I set $\alpha = 0.338$, which is consistent with a social discount rate of 4% in a second-best setting.

In the next steps, I use the first-order conditions from the model in steady state to pin down the remaining parameters. With a Cobb-Douglas production function, the private cost of emissions in the initial period, κ_0, is found as:

$$\kappa_0 = \gamma\frac{Y_0}{E_0}.$$

For the remaining parameters, it matters whether I use a model with or without income taxes for calibration. In the former case, I use the government's budget constraint in (11):

$$\tau_t = \frac{G_t}{(r_t-\delta)K_t + W_tH_t}, \tag{59}$$

where $\tilde{\theta}_t = 0$ due to the absence of climate policy. Assuming a steady state, I use sample averages between 1970 and 2010 for government expenditures and the capital stock. Due to the lack of data, I assume a steady share for hours worked of 0.2. Finally, I compute the marginal product of capital and labor with the calibrated Cobb-Douglas production function. This gives a steady-state income tax rate of $\bar{\tau} = 0.245$.

For the baseline utility function (55), the Euler equation in steady state can be written as:

$$\frac{\bar{Y}}{\bar{K}} = \frac{\beta^{-1}\exp(g_A) - 1 + (1-\bar{\tau})\delta}{(1-\bar{\tau})\alpha}. \tag{60}$$

This equation demonstrates the connection between the social discount rate, which is given by $\beta^{-1}\exp(g_A) - 1$, and the income share of capital α: assuming a social discount rate of 4%, the data is consistent with $\alpha = 0.338$, which was used in the least squares regression above.

Similarly, the first-order condition with respect to labor is given by

$$\frac{L_t}{C_t}(1-\tau_t)(1-\alpha-\gamma)K_t^{\alpha}(A_{E,t}E_t)^{\gamma}(A_t L_t h_t)^{-\alpha-\gamma}A_t L_t = L_t \eta h_t^{\xi}. \tag{61}$$

Rearranging and again using sample averages, I can compute η as:

$$\eta = (1-\bar{\tau})\frac{1-\alpha-\gamma}{\bar{h}^{1+\xi}}\frac{\bar{Y}}{\bar{C}}. \tag{62}$$

Finally, the parameter ζ can be computed from (55) by:

$$\zeta = \gamma\frac{\bar{G}}{\bar{C}}.$$

The third-column in the following table summarizes the economic parameter values for the baseline second-best calibration.

Parameter	Description	Second-best	First-best	Source
Determined Ex Ante				
δ	depreciation rate	.08	.08	-
β	time discount rate	.9835	.9835	Target SDR 4%
Calibrated				
α	income share capital	.338	.3	First-order condition
γ	income share fuel	.0411	.068	Least Squares
g_A	growth rate output p.c.	.0181	.0177	Least Squares
η	utility weight hours	20.34	27.43	First-order condition
ζ_g	utility weight public good	.318	.318	First-order condition
κ	fuel cost	.0525	.0868	First-order condition

Table 1: Parameter Values in the Baseline Calibration

As outlined in previous sections, in addition to comparing the optimal carbon price (after 2010) in second best to its Pigouvian level, I am also interested in relating it to the optimal price coming out of a first-best model without distortionary income taxation. The straightforward approach here is to solve a planner's problem in the economy calibrated above, under the assumption that lump-sum taxes are available. This would reflect a "tax reform", i.e. a setting in which distortionary income taxes are used until the base year (here 2010) and then abolished. As a consequence, there is a considerable rise in economic activity, that is, in labor and savings (due to the abolition of taxes), and hence in output, consumption and emissions. This, in turn, has impacts on the optimal carbon price, which distorts the comparison with the second-best carbon price in the absence of a tax reform.

In order to account for such effects, below I also compute the first-best carbon price in an economy calibrated to a (hypothetical) setting without distortionary taxes between 1970 and 2010. In other words, I repeat the same calibration steps as above, assuming that the income tax rate is zero. The resulting parameter values are summarized in the fourth column in table 1.

Most of the differences are driven by a slightly lower capital income share α: since I continue to target a social discount rate of 4%, this implies a lower gross return to capital without an income tax. As illustrated by (60), this would translate into a smaller α. This has an impact on the regression results, leading a higher emission income share γ and, in turn, to a higher fuel cost parameter κ. In addition, in order to get a steady state level of 0.2 for hours worked, abolishing income taxation requires a higher disutility weight on working (η).

5.1.2 Climate Module

With respect to modeling climate change, I use the law of motion for temperature change from Matthews et al. (2009) and the damage function from the DICE model (Nordhaus, 2008), as outlined in (7) and (10), respectively. Hence, I need to choose values for the parameters b_1 and b_2 in the damage function, capturing climate change impacts to total factor productivity, and for ζ_T, the weight on temperature change in the utility function. Following DICE, I assume that the damage function is quadratic in temperature change ($b_2 = 2$). With respect to b_1 and ζ_T, I choose a number of different combinations and examine the effects on the model outcome when changing one or both of these parameters as part of the sensitivity analysis. For the baseline calibration, I set $b_1 = 0.0028$ as in the 2008 version of the DICE model. I then find ζ_T such that mean global temperature does not exceed 2°C by 2100.

5.1.3 Solution Method

Numerically, the equilibrium with lump-sum income taxation is straightforward to solve for, by maximizing over all variables for a large number of periods, possibly given some terminal conditions. With distortionary income taxation and under lack of commitment, this method is not feasible. Instead, I resort to dynamic programming, following, among others, Jensen and Traeger (2014) and Cai et al. (2015).

More specifically, I use value-function iteration (VFI) to solve the incumbent's problem given by (39)-(41). In each iteration, the optimization in (39) is executed for a given value function v and given policy functions (ψ, ϕ, n^k, n^h), reflecting that, when choosing its optimal policy, the current government takes as given its successor's time-consistent decision rules. When solving the maximization problem, I approximate the value and the policy functions using Chebyshev polynomials. Computations are performed in AMPL, using the KNITRO optimization solver (Ziena, 2013).

Note that following Jensen and Traeger (2014), the incumbent government's problem is set up as an infinite-horizon problem. Since in the presence of temperature change and damages, the model cannot be transformed into a stationary framework, a time indicator is included as a state variable.[27]

5.2 Results

In the following, I present results from different model runs for the optimal second-best carbon price in the absence of commitment, and relate it to (i) the Pigouvian level in second best, and (ii) the optimal carbon price in a first-best setting with lump-sum taxes. The size and direction of the first relationship is determined by the tax-interaction effect and was the focus of section 4. The main results will be illustrated in graphs in the spirit of figure 1, displaying

[27]An alternative approach, used for example by Cai et al. (2015), would be resorting to finite-horizon dynamic programming, with a continuation value after some period P. This would result in a smaller number of state variables, while at the same time require assumptions about the economy after period P, in order to compute the continuation value.

how these variables of interest evolve over time, for different scenarios with respect to how damages from climate change are modeled.

5.2.1 Baseline

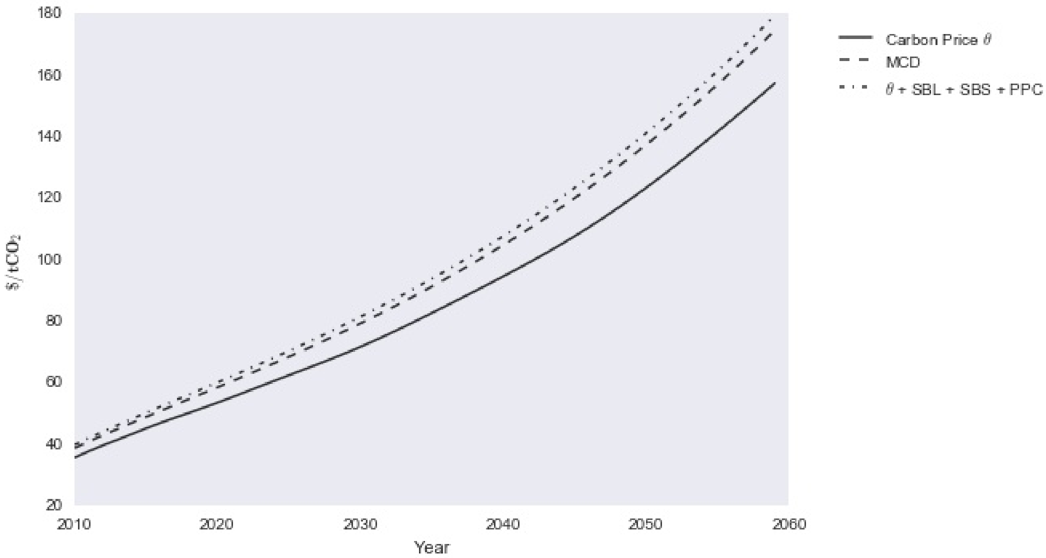

Figure 2: Optimal Carbon Price - Baseline Scenario

Figure 2 shows the time paths of the optimal carbon price in the second-best Markov-perfect equilibrium between 2010 and 2060, as well as the corresponding Pigouvian fee, i.e. the level of marginal climate damage (MCD). In addition, following the stylized illustration of the tax-interaction effect in figure 1, it also displays the time path for the sum of the optimal carbon price and three of the second-best effects discussed in the previous section, the second-best labor, second-best savings and propagation-through-capital effects. Recall that from (54), it follows that

$$\text{MCD}(t) - \text{PPT}(t) = \theta(t) + \text{SBL}(t) + \text{SBS}(t) + \text{PPC}(t). \tag{63}$$

Starting with a (hypothetical) carbon price given by the dashed MCD curve, the propagation-through-temperature effect by itself would push it upwards to the dotted curve; from the definition above, it follows that $PPT(t) < 0$, indicating a second-best cost of emitting carbon. However, the sum of the remaining three effects is positive, thus moving the carbon price from the dotted curve all the way to the solid curve. In other words, the SBL, SBS and PPC effects represent second-best benefits, at least in the aggregate, that outweighs the size of the PPT effect, and hence pushes the optimal carbon price below the MCD, and hence below its Pigouvian level.

In terms of the size of the deviation, the gap between the optimal second-best carbon price and the Pigouvian fee increases slightly over time, from about 8% in 2010 to about 11% in 2110 in the baseline calibration. In other words, the main takeaway from figure 2 is that the optimal global carbon price under distortionary taxation over the next 100 years is in the ballpark of 10% lower than the corresponding Pigouvian fee. Note that in absolute terms, the optimal second-best carbon price amounts to $35.5\$/tCO_2$ in 2010.

Figure 3 shows the optimal carbon price under distortionary taxation compared to the carbon fee in a first-best setting. Recall from above that the first-best allocation is computed using a different model calibration, to exclude effects stemming from an increase in economic activity when income taxation is abolished. The graph shows that the second-best carbon price is constantly below the first-best fee. The relative gap between the optimal carbon price with and without distortionary income taxation decreases slightly over the next 100 years, from about 14% to about 4% in 2110.

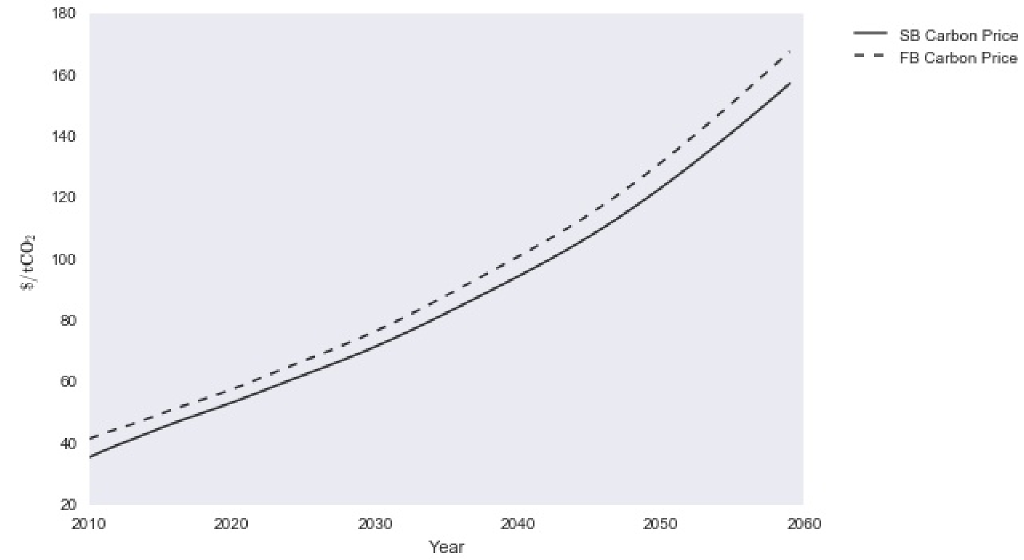

Figure 3: Optimal Carbon Price - Second Best vs. First Best

5.2.2 Sensitivity Analysis: Climate Change Damage

Figure 5 compares the optimal second-best carbon price with the Pigouvian fee for the DICE damage function (7), but without a direct disutility caused by temperature change. In other words, compared to the baseline calibration, I set $\zeta_T = 0$. Qualitatively, the resulting graph is similar to figure 2. Given that climate change is assumed to be less damaging, the absolute level of the optimal carbon price is lower than in the baseline. Moreover, the gap to the MCD is smaller, going from 2% in 2010 to about 9% in 2110. At the same time, figure 5 indicates that the PTT effect is relatively more important for productivity-only damages.

Figure 6 shows the other extreme, a setting with climate change damages only to utility, and hence $b_1 = 0$. Here, I have set $\zeta_T = 0.0185$, which, as the baseline model, corresponds to a temperature change of 2°C in 2100.

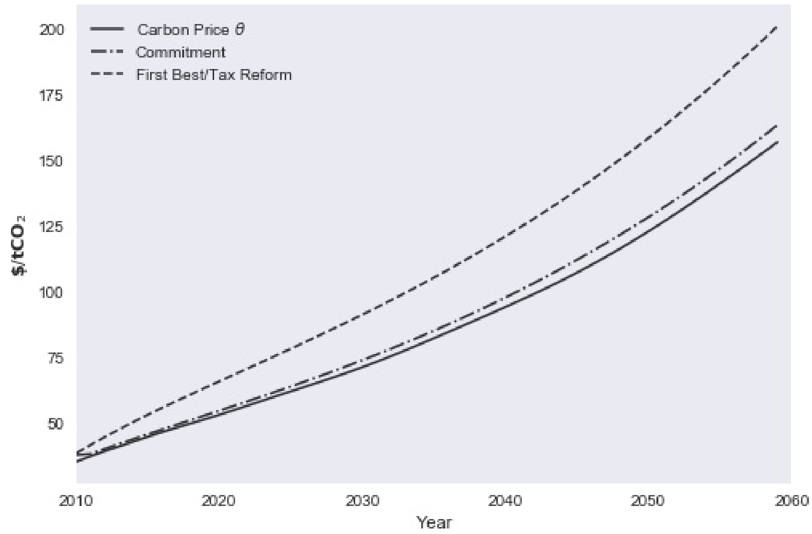

Figure 4: Optimal Carbon Price - Comparison

Hence, the absolute level of the optimal carbon price is comparable to figure 2. Note that in the absence of productivity impacts from climate change, the PTT effect is zero, which is reflected by the MCD curve lying on top of the dotted curve in figure 6.[28] Quantitatively, the gap between the OCP and the Pigouvian fee is between 11% and 15% for the time between 2010 and 2110.

As a final robustness check with respect to the parameterization of climate change impacts, I calibrate b_1 and ζ_T such that temperature change in first best reaches 1.5°C rather than 2°C by 2100.[29] Quantitatively, as illustrated in figure 7, this implies a considerably higher carbon price in absolute. However, with respect to the deviation of the OCP to the Pigouvian fee, the result is qualitatively the same as in the other settings: the gap between the OCP and the Pigouvian fee is around 10% in the period up to 2110.

[28]The small deviation between the two is caused by approximation errors.

[29]For simplicity, I start with the baseline calibration and scale up both b_1 and ζ_T by a factor of 4.

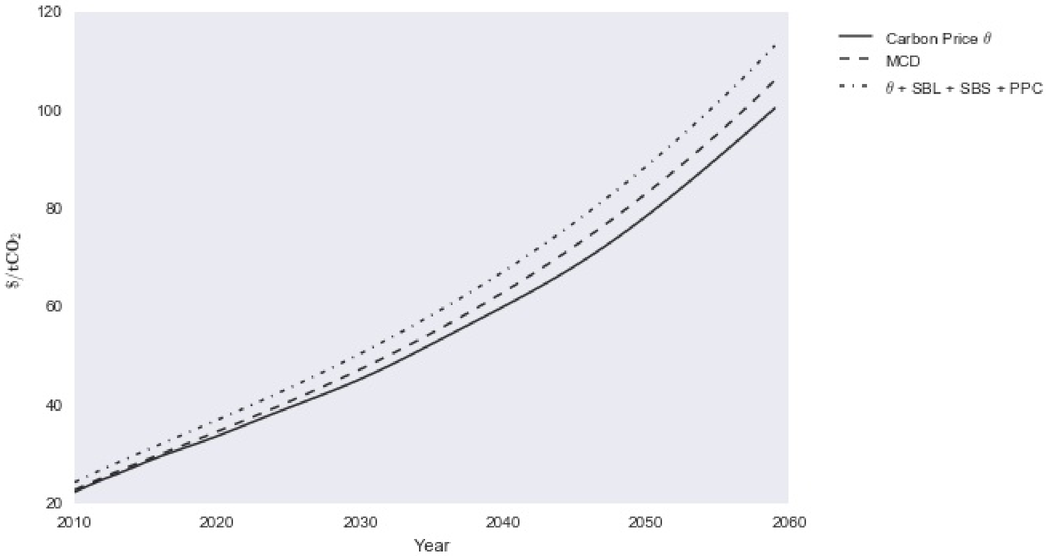

Figure 5: Optimal Carbon Price - DICE Damage Function

6 Conclusion

This paper has analyzed optimal carbon pricing in a world where governments have to resort to distortionary taxation to finance public expenditures and cannot commit to future policies. I have added these features to an otherwise standard dynamic climate-economy model and computed optimal carbon price schedules in Markov-perfect equilibria.

The main findings of this paper are the following. First, I have characterized optimal policy analytically in a global planner model. I have illustrated that the optimal second-best carbon price is in general not at the Pigouvian level, due to the presence of additional costs and benefits of carbon emissions that only materialize under distortionary income taxation. In contrast to previous studies, I have shown that it is not only the current labor-leisure margin that is affected by climate policy, but there are other current and future wedges that

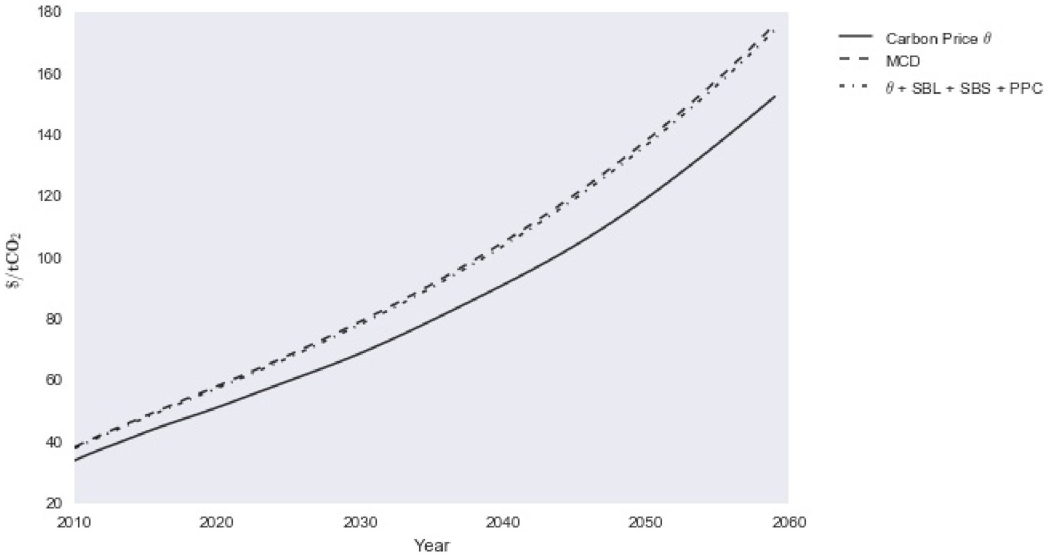

Figure 6: Optimal Carbon Price - Utility Damages Only

interact with carbon taxes. Second, a quantitative analysis has shown that the overall effect decreases the optimal carbon price by magnitudes around 10%, both compared to the Pigouvian level and the corresponding first-best outcome, with variations depending on how impacts from climate change are modeled.

In order to facilitate the analysis, I have made a number of simplifying assumptions. Relaxing those assumptions could result in potentially interesting extensions of the above framework. Most notably, in this paper I have considered a global economy. Since fiscal policy is typically set at the country level, extending the model to multiple regions would deliver more reliable quantitative results. Moreover, the framework in this paper is deterministic and has abstracted from uncertainty, both with respect to climate change and long- and short-run economic growth. Regarding the latter, it is straightforward to

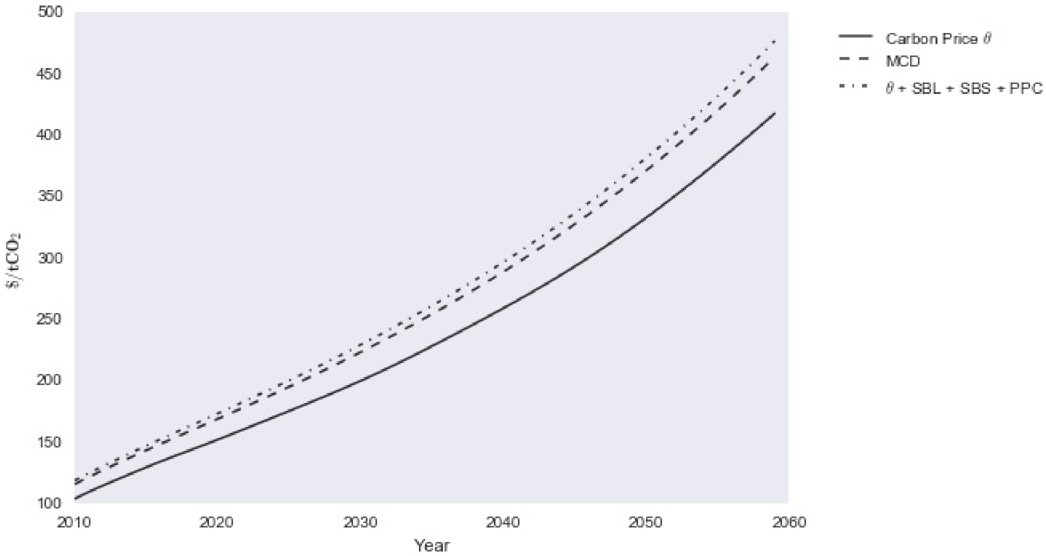

Figure 7: Optimal Carbon Price - Utility Damages Only

add exogenous productivity or taste shocks to the model, in the tradition of the real business cycle literature. Previous work by Heutel (2012) has shown that in a first-best setting, climate policy is procyclical in the sense that a carbon price optimally increases during expansion, while it must be reduced in a recession. The framework used in this paper would allow me to analyze how robust this finding is to the introduction of distortionary income taxes in the economy.

Given its long-term nature and its complexity, climate change gives rise to many types of uncertainty, both related to the science and to the economic damages.[30] Integrated assessment models are usually either deterministic, or consider some form of parametric uncertainty (Nordhaus, 2008; Golosov et al., 2014). Some recent papers instead focus on uncertainty caused by the random occurrence of exogenous events. In particular, studies by Lemoine and Traeger

[30]Compare Stern (2013) for a recent summary.

(2013) and Cai et al. (2015) incorporate so-called "tipping points", defined as irreversible and abrupt shifts in the climate system, in stochastic versions of the DICE model. Jensen and Traeger (2014) and Cai et al. (2015) analyze optimal carbon mitigation under long-term growth uncertainty. While such questions are beyond the scope of this paper, it is important to keep in mind that these channels have a potentially large quantitative impact on optimal carbon taxes.

Integrated assessment models such as the one used in this paper have other limitations. Two important areas of ongoing research are the modeling of economic growth and the representation of climate damages. The above framework has built upon the neoclassical growth model, assuming an exogenously given progress of both labor productivity and energy efficiency. A recent strand of the literature has instead considered optimal environmental policy in endogenous-growth models. Using a model of directed technical change, Acemoglu et al. (2012) and Acemoglu et al. (2016) endogenize productivity growth for both a clean and a dirty production input. They show that optimal climate policy in such a setting consists of both a carbon tax and a research subsidy to the clean type of energy.[31] These results suggest that incorporating endogenous growth along similar lines would affect the results in this paper qualitatively and quantitatively.

Moreover, Stern (2013) notes that integrated assessment models usually assume that economic growth is not affected by climate damages. Instead, the multiplicative damage function that is used in this paper, as well as in many

[31] Hemous (2013) embeds this framework in a setting with two regions and analyzes unilateral policy. He also finds that research subsidies are an important component of optimal policy.

others studies, relates contemporaneous damages to the current flow of output, but not to, for example, the stock of capital or other factors determining the growth potential of the economy.[32] This shortcoming is exacerbated by the fact that the damage function as used in the DICE model yields quantitatively small damages.[33] These issues are summarized by Stern (2013), arguing that the "exogeneity of a key driver of growth, combined with weak damages" is one of the key weaknesses of current integrated assessment modeling. While this paper had a different focus and did not attempt to contribute to advances in climate-economy modeling in these areas, one should keep these limitations in mind when interpreting its results.

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[32]Moyer et al. (2013) consider a model in which climate change has direct effects on productivity. They find that this leads to a considerable increase in the social cost of carbon.

[33]As shown by Ackerman et al. (2010), the damage function in Nordhaus (2008) would imply a decrease in output by only 50% when the temperature increases by 19°C relative to the preindustrial level. However, it should be noted that William Nordhaus warns that there is not sufficient evidence to reliably use this damage function for temperature changes above 3°C (Stern, 2013).

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A Appendix

A.1 Quasi-Stationary Transformation

Consider the Cobb-Douglas function

$$Y_t = F(K_t, A_{H,t}H_t, A_{E,t}E_t) = K_t^{\alpha}(A_{E,t}E_t)^{\gamma}(A_{H,t}L_th_t)^{1-\alpha-\gamma}. \tag{64}$$

Let $Y_{t+1} = \exp(g_Y)Y_t$, $A_{H,t+1} = \exp(g_{A,H})A_{H,t}$ and so on. Moreover, following the condition for a balanced-growth path in (12), $g_Y = g_K$. Then,

$$
\begin{aligned}
Y_{t+1} &= \exp(\rho_Y)Y_t \\
&= (\exp(\rho_Y)K_t)^{\alpha}(\exp(\rho_{A,E})A_{E,t}\exp(\rho_E)E_t)^{\gamma}(\exp(\rho_{A,H})A_{H,t}\exp(\rho_L)L_t h_t)^{1-\alpha-\gamma} \\
&= \exp(\rho_Y)^{\alpha}(\exp(\rho_{A,E})\exp(\rho_E))^{\gamma}(\exp(\rho_{A,H})\exp(\rho_L))^{1-\alpha-\gamma}Y_t
\end{aligned}
$$

It follows that

$$(1-\alpha)\rho_Y = \gamma\rho_{A,E} + \gamma\rho_E + (1-\alpha-\gamma)\rho_{A,H} + (1-\alpha-\gamma)\rho_L$$

Next, from (12), $g_E = g_Y - g_\kappa$. Inserting this above and rearranging gives:

$$\rho_Y - \rho_L = \frac{\gamma(\rho_{A,E} - \rho_\kappa)}{1-\alpha-\gamma} + \rho_{A,H}$$

With ρ_A denoting the growth rate of output per capita, we have

$$\rho_A = \frac{\gamma(\rho_{A,E} - \rho_\kappa)}{1-\alpha-\gamma} + \rho_{A,H} \tag{65}$$

As done in the main text, define A_t as

$$A_t = A_{H,t}A_{E,t}^{\frac{\gamma}{1-\alpha-\gamma}}\exp(-g_\kappa t)^{\frac{\gamma}{1-\alpha-\gamma}} \tag{66}$$

We then have:

$$A_{t+1} = \exp(\rho_{A,H})\exp(\rho_{A,E})^{\frac{\gamma}{1-\alpha-\gamma}}\exp(-g_\kappa)^{\frac{\gamma}{1-\alpha-\gamma}}A_t \tag{67}$$

Hence, A_t grows with the rate ρ_A found in (65).

A.2 Deriving the GEE

A.2.1 First-order Conditions

The government's problem is given by (44). Taking f.o.c. yields:

$$-u_c(\mathsf{K}_g\exp(g_A)\exp(g_L) - \tilde{F}_h\mathsf{H}_g + 1) + u_h\mathsf{H}_g + u_g + \beta v'_k\mathsf{K}_g = 0 \tag{68}$$

$$-u_c[\mathsf{K}_e\exp(g_A)\exp(g_L) - \tilde{F}_h\mathsf{H}_e - (\tilde{F}_e - \kappa)] + u_h\mathsf{H}_e + \beta[v'_k\mathsf{K}_e + v'_T q_e] = 0. \tag{69}$$

Rearranging and using the definitions for ω_{LL} and ω_{PG} in (26) and (27), respectively, gives expressions (45) and (46) in the main text.

A.2.2 Derivative v_k

The derivative of the value function $v(k, T, t)$ with respect to k reads:

$$\begin{aligned} v_k(t) = & u_c(t)[\tilde{F}_k(t) + 1 - \delta - \mathsf{K}_k(t)\exp(g_A)\exp(g_L) + \tilde{F}_h(t)\mathsf{H}_k(t)] \\ & + u_h(t)\mathsf{H}_k(t) + \beta\mathsf{K}_k(t)v_k(t+1). \end{aligned} \tag{70}$$

Using this derivative at $t+1$ in $[\beta v_k(t+1) - u_c(t)\exp(g_A)\exp(g_L)]$ gives:

$$\begin{aligned} & \beta v_k(t+1) - u_c(t)\exp(g_A)\exp(g_L) \\ & = -u_c(t)\exp(g_A)\exp(g_L) + \beta u_c(t+1)\left[\tilde{F}_k(t+1) + 1 - \delta\right] \\ & \quad + \beta\mathsf{H}_k(t+1)\left[u_c(t+1)\tilde{F}_h(t+1) + u_h(t+1)\right] \\ & \quad + \beta\mathsf{K}_k(t+1)\left[\beta v_k(t+2) - u_c(t+1)\exp(g_A)\exp(g_L)\right] \\ & = \omega_{CS}(t) + \beta\mathsf{H}_k(t+1)\omega_{LL}(t+1) \\ & \quad + \beta\mathsf{K}_k(t+1)\left[\beta v_k(t+2) - u_c(t+1)\exp(g_A)\exp(g_L)\right] \end{aligned} \tag{71}$$

Define

$$\begin{aligned} x(t) & \equiv \beta v_k(t+1) - u_c(t)\exp(g_A)\exp(g_L), \\ a(t) & \equiv \omega_{CS}(t) + \beta\mathsf{H}_k(t+1)\omega_{LL}(t+1). \end{aligned}$$

Then, (71) can be more compactly written as:

$$\begin{aligned} x(t) & = a(t) + \beta\mathsf{K}_k(t+1)x(t+1) \\ & = a(t) + \beta\mathsf{K}_k(t+1)\left[a(t+1) + \beta\mathsf{K}_k(t+2)x(t+2)\right] \quad = ... \\ & = a(t) + \sum_{j=t+1}^{\infty}\beta^{j-t}\left(\prod_{i=t+1}^{j}\mathsf{K}_k(i)\right)a(j) \end{aligned}$$

Hence, we have

$$\begin{aligned} & \beta v_k(t+1) - u_c(t)\exp(g_A)\exp(g_L) \\ & = \omega_{CS}(t) + \beta\mathsf{H}_k(t+1)\omega_{LL}(t+1) \\ & \quad + \sum_{j=t+1}^{\infty}\beta^{j-t}\left(\prod_{i=t+1}^{j}\mathsf{K}_k(i)\right)\left[\omega_{CS}(j) + \beta\mathsf{H}_k(j+1)\omega_{LL}(j+1)\right] \\ & = \omega_{CS}(t) + \sum_{j=t+1}^{\infty}\beta^{j-t}\left[\left(\prod_{i=t+1}^{j}\mathsf{K}_k(i)\right)\omega_{CS}(j) + \left(\prod_{i=t+1}^{j-1}\mathsf{K}_k(i)\right)\mathsf{H}_k(j)\omega_{LL}(j)\right] \end{aligned}$$

$$ \tag{72} $$

A.2.3 Derivative v_T

(72) can be used to rewrite (46):

$$
\begin{aligned}
&u_c(t)[\tilde{F}_e(t) - \kappa] + \beta q_e(t) v_T(t+1) + \omega_{LL}(t)\mathsf{H}_e(t) + \omega_{CS}(t)\mathsf{K}_e(t) \\
&\quad + \mathsf{K}_e(t) \sum_{j=t+1}^{\infty} \beta^{j-t} \left[\left(\prod_{i=t+1}^{j} \mathsf{K}_k(i) \right) \omega_{CS}(j) + \left(\prod_{i=t+1}^{j-1} \mathsf{K}_k(i) \right) \mathsf{H}_k(j) \omega_{LL}(j) \right] = 0.
\end{aligned}
$$

The derivative of the value function $v(k, T, t)$ with respect to T is given by:

$$
\begin{aligned}
v_T(t) = {} & u_c(t)[\tilde{F}_T(t) - \mathsf{K}_T(t) \exp(g_A) \exp(g_L) + \tilde{F}_h(t) \mathsf{H}_T(t)] \\
& + u_h(t) \mathsf{H}_T(t) + u_T(t) + \beta[\mathsf{K}_T(t) v_k(t+1) + q_T(t) v_T(t+1)].
\end{aligned}
$$

and hence, in $t+1$,

$$
\begin{aligned}
v_T(t+1) = {} & u_c(t+1)\tilde{F}_T(t+1) + u_T(t+1) + \mathsf{H}_T(t+1)\omega_{LL}(t+1) \\
& + \mathsf{K}_T(t+1) \left[\beta v_k(t+2) - u_c(t+1) \exp(g_A) \exp(g_L) \right] \\
& + \beta q_T(t+1) v_T(t+2).
\end{aligned}
$$

With the assumption that q_T is constant, $v_T(t+1)$ can be written as an infinite sum:

$$
\begin{aligned}
v_T(t+1) = {} & \sum_{j=t+1}^{\infty} (\beta)^{j-(t+1)} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \left[u_c(j) \tilde{F}_T(j) + u_T(j) \right] \\
& + \sum_{j=t+1}^{\infty} (\beta)^{j-(t+1)} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \mathsf{H}_T(j) \omega_{LL}(j) \\
& + \sum_{j=t+1}^{\infty} (\beta)^{j-(t+1)} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \mathsf{K}_T(j) \left[\beta v_k(j+1) - u_c(j) \exp(g_A) \exp(g_L) \right]
\end{aligned}
$$

and hence

$$\beta q_e(t) v_T(t+1) = q_e(t) \sum_{j=t+1}^{\infty} (\beta)^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \left[u_c(j) \tilde{F}_T(j) + u_T(j) \right]$$
$$+ q_e(t) \sum_{j=t+1}^{\infty} (\beta)^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \mathsf{H}_T(j) \omega_{LL}(j)$$
$$+ q_e(t) \sum_{j=t+1}^{\infty} (\beta)^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \mathsf{K}_T(j) \left[\beta v_k(j+1) - u_c(j) \exp(g_A) \exp(g_L) \right]$$

Dividing both sides of this expression by $u_c(t)$, the first-term on the right hand side equals the present value of the sum of future marginal damages when increasing current emissions, and hence the negative MCD as defined in (30):

$$\beta \frac{q_e(t)}{u_c(t)} v_T(t+1) = -MCD(t) + \frac{q_e(t)}{u_c(t)} \sum_{j=t+1}^{\infty} (\beta)^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \mathsf{H}_T(j) \omega_{LL}(j)$$
$$+ \frac{q_e(t)}{u_c(t)} \sum_{j=t+1}^{\infty} (\beta)^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \mathsf{K}_T(j) \left[\beta v_k(j+1) - u_c(j) \exp(g_A) \exp(g_L) \right] \tag{73}$$

I can substitute (72) and (73) in (46) to get a characterization of the difference between the optimal carbon price and the level of marginal climate change damage:

$$\tilde{F}_e(t) - \kappa - MCD(t) + \frac{q_e(t)}{u_c(t)} \sum_{j=t+1}^{\infty} (\beta)^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \mathsf{H}_T(j) \omega_{LL}(j)$$
$$+ \frac{q_e(t)}{u_c(t)} \sum_{j=t+1}^{\infty} (\beta)^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \mathsf{K}_T(j) \left[\beta v_k(j+1) - u_c(j) \exp(g_A) \exp(g_L) \right]$$
$$+ \omega_{LL}(t) \frac{\mathsf{H}_e(t)}{u_c(t)} + \left[\beta v_k(t+1) - u_c(t) \exp(g_A) \exp(g_L) \right] \frac{\mathsf{K}_e(t)}{u_c(t)} = 0.$$

Rearranging and using the definition for θ_t, as well as (72), gives

$$
\begin{aligned}
&\mathrm{MCD}(t) - \theta(t) = \omega_{LL}(t)\frac{\mathsf{H}_e(t)}{u_c(t)} + \omega_{CS}(t)\frac{\mathsf{K}_e(t)}{u_c(t)} \\
&+ \frac{\mathsf{K}_e(t)}{u_c(t)} \sum_{j=t+1}^{\infty} \beta^{j-t} \left[\left(\prod_{i=t+1}^{j} \mathsf{K}_k(i) \right) \omega_{CS}(j) + \left(\prod_{i=t+1}^{j-1} \mathsf{K}_k(i) \right) \mathsf{H}_k(j)\omega_{LL}(j) \right] \\
&+ \frac{q_e(t)}{u_c(t)} \sum_{j=t+1}^{\infty} (\beta)^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \mathsf{H}_T(j)\omega_{LL}(j) \\
&+ \frac{q_e(t)}{u_c(t)} \sum_{j=t+1}^{\infty} (\beta)^{j-t} \left(\prod_{i=t+1}^{j-1} q_T(i) \right) \mathsf{K}_T(j) \left[\beta v_k(j+1) - u_c(j) \exp(g_A) \exp(g_L) \right]
\end{aligned}
$$

□

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