

Labor supply within the firm

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Abstract

Estimates of labor supply elasticities can be sensitive to the source of identifying variation. This paper's model of production complementarities helps to interpret conflicting evidence. Complementarities attenuate working time adjustments to idiosyncratic, or individual-specific, variation in work incentives. Complementarities do not restrict, however, responses to firm-wide shocks; the latter is mediated by preference parameters. Estimating the model using matched firm-worker data, the paper disentangles production from preference parameters. The Frisch elasticity along the intensive margin is found to be around 0.5. A quasi-experimental approach, using idiosyncratic variation in work incentives, would find an elasticity less than half this.

JEL: J22, J23, J31.

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Variation in labor input occurs along two margins. The extensive margin refers to the formation and termination of employment relationships, whereas the intensive margin describes the choice of working time conditional on being employed. Recent labor market analysis, such as in the search and matching literature, has focused on the extensive margin. But variation along the intensive margin is significant. At the aggregate level, fluctuations in working time per employee are as large as movements in employment in several European economies (Llosa et al, 2012). At the plant level, U.S. data show that the variance of changes in working time per person is equal to that of employment growth (Cooper et al, 2015).¹

This evidence on intensive-margin fluctuations appears at odds with implications of the earlier labor supply literature. The data in Cooper et al (2015) show that a one standard-deviation movement in hours amounts to about 55 hours per quarter.² Yet Hall (1999) notes that estimates of the labor supply elasticity (for men) are as low as 0.05 and typically no higher than 0.4. If these estimates were right, Hall argues, the deadweight burden of these plant-level hours fluctuations would seem to be implausibly high.

In this paper, we consider a framework that can reconcile this seemingly contradictory evidence on the intensive margin. In this setting, workers are complements in production but have heterogeneous preferences over leisure. Complementarities have important implications for the identification of the intertemporal (Frisch) elasticity of substitution in (intensive-margin) labor supply. For instance, variation in a worker's own, *idiosyncratic* labor supply incentives yields relatively small changes in working time, since the efficient response is attenuated when one's effort is not complemented by higher effort of co-workers. On the other hand, *firm-wide* variation in the return to working coordinates the responses of heterogeneous workers, revealing the true willingness to substitute effort intertemporally. The model can thus predict more significant changes in firm-wide working time without implying counterfactually large responses to idiosyncratic events. We estimate the model using employer-employee matched data from northern Italy and show how to recover the structural parameters governing the degree of complementarities and the Frisch labor supply elasticity.

Our approach has been foreshadowed (informally) in several earlier assessments of the labor supply literature. For instance, Pencavel (1986) notes that a worker's labor input is often coordinated by his employer. Relatedly, Hall (1999) contends that, "if an event occurs

¹Intensive-margin adjustments account for one-half of the aggregate variation in U.S. total labor input at a quarterly frequency (Heckman, 1984). At an annual frequency, the contribution of the intensive margin is about 20 percent.

²This can be derived from Cooper et al's estimates of the variance and persistence of log hours changes. If the latter are Normally distributed, one can use these moments to infer the variance of the *level* of hours, given an estimate of its mean. Though the authors do not report mean hours, we can use an estimate from the March Current Population Survey over the authors' sample (1972-80).

that is personal to the worker ... it is unlikely that the employer will agree to a reduction in weeks *ad hoc*" (p. 1148). These comments place the *employer* at the center of the theory of intensive-margin labor supply.³

In this paper's model, the firm does have a starring role. The firm and its workers join in long-term employment relationships, bound together by the fact that extensive-margin adjustments are costly. Working time is bargained jointly to maximize the surplus from the match. The resulting distribution of working time across employees represents a balancing of two interests—productive complementarities and heterogeneity in the disamenity from work. If the former is forceful enough, then employees agree, jointly with their employer, to vary their working time in a similar manner despite having disperse preferences.

Differences in preferences over leisure are accommodated, instead, by the earnings bargain, which is derived from a Nash-like surplus-sharing protocol. If a worker's labor input remains high despite an increase in her marginal value of time, she is compensated accordingly. Hence, under complementarities, the distribution of working-time adjustments across employees *within the firm* is compressed *relative* to the dispersion in earnings growth.

To assess our interpretation of working-time fluctuations and earnings, we introduce in Sections 2 and 3 a unique source of panel data. We use a matched worker-firm dataset that tracks the universe of workers and firms in the northern Italian region of Veneto from 1982 to 2001.⁴ The dataset includes each employee's annual days worked for each of her employers. Working days is an active margin: in a given year, over 50 percent of workers adjust their days, and among these, the typical change is between 10 and 19 days. Still, it is arguably concerning that we observe only days, not total hours. Supplementing our analysis with household data from the Italian Labor Force Survey, we find, however, that variation in working days accounts for the majority of variation in total hours. Much of this variation in working days takes the form of Saturday overtime, according to Giaccone (2009).

In Section 4, we estimate the model using the method of simulated moments. Our identification strategy relies on our ability to observe earnings and working time *inside* firms. Complementarities "squeeze out" the influence of idiosyncratic factors on working time. As a result, these factors are reflected primarily through the (within-firm) dispersion of earnings growth. We can thus infer the strength of complementarities by comparing the variance of working time adjustments across workers within firms to the variance of earnings

³Indeed, Pencavel and Hall emphasize theories in which the firm unilaterally chooses working time. Our approach still leaves room for "events personal to the worker" to influence labor supply among the employed.

⁴In Italy, these data have to be recorded because taxes and social insurance contributions are tied to days worked. Data are reported to the public social security organisation INPS. The dataset has been organized and maintained by researchers at the University of Venice.

growth (again, inside firms). In particular, if the ratio of the former to the latter is small, idiosyncratic variation is suppressed in working time. Accordingly, our model infers a high degree of complementarities, or more exactly, a low elasticity of substitution across workers in production.

Whereas we identify complementarities off within-firm variation, the intertemporal elasticity of substitution in labor supply is better informed by *firm-wide* fluctuations. In particular, this elasticity is informed by the size of fluctuations in the firm's average working time. Our approach uncovers an estimate of the Frisch elasticity of 0.536. This is near the top end of the range of estimates found in the earlier, seminal life-cycle literature (see MacCurdy, 1981; Browning et al., 1985; Altonji, 1986). It is, however, more in line with recent results summarized in Chetty, Guren, Manoli, and Weber (2011). In Section 4, we discuss the source of variation used in more recent studies and why we suspect our results align with theirs.

To highlight the implications of our results for empirical analysis, we simulate a simple policy intervention in Section 5. A fraction of a firm's workforce receives the "treatment"—a shift in their own labor supply incentives—but the remainder of the firm's workers do not. We contrast the outcome with the case in which all workers participate in the intervention. Reflecting the role of complementarities, working time declines by 50 to 115 percent more when all employees receive the treatment (depending on how the extensive margin adjusts). Furthermore, if we use the treatment effect in the case where only a fraction of the workforce participates to infer the Frisch elasticity, the implied elasticity is less than half the estimate (0.536) we uncover.

This experiment illustrates that the response of working time to an idiosyncratic event may bear little resemblance to the underlying preference parameter. This is a simple, but important, point, because many influential studies utilized this latter variation. Hall (1999) notes, for instance, that the tepid response of working time in the randomized control trials known as the Negative Income Tax (NIT) experiments greatly informed the consensus on labor supply. Yet this kind of variation—a sample of workers is selected to receive a cash grant—is clearly idiosyncratic to the worker.⁵ The same point applies to the seminal life-cycle analyses of MacCurdy (1981) and Altonji (1986), which identify the Frisch elasticity off the response of time worked to an individual's own (predictable) wage changes.⁶

⁵The NIT experiments were run in a handful of U.S. cities in the late 1960s and early 1970s. The participating households received a cash grant on a sliding scale, with the grant declining in the worker's market earnings. We discuss the NITs again in Section 5, since the size of the simulated policy intervention is based on the typical NIT.

⁶These life-cycle studies do not restrict their focus to intensive-margin movements. However, their sample consists of workers—white males, age 25–46, who are in stable marriages—who are most likely to be in relatively long-lived employer-employee relationships.

A few recent contributions touch on a number of themes presented here. Chetty, Friedman, Olsen, and Pistaferri (2011) identify evidence of coordination in working time using the “bunching” of taxable income at kinks in the tax-rate schedule. We use different data and a distinct identification strategy, but like these authors, we conclude that idiosyncratic variation in the return to working will typically fail to recover the true willingness of workers to vary their labor input. Chetty (2012) offers another approach to inference, which uses estimated elasticities to bound preference parameters even when the source of the wedge between elasticities and parameters is not explicit. Our approach is complementary: we formalize a specific reason why reduced-form estimates may not identify preference parameters and use this model to recover the parameters.⁷ Finally, Rogerson (2011) also argues that coordination may break the link between estimated elasticities and structural parameters, but studies an aggregative model in which workers coordinate their leisure.

The paper proceeds as follows. Section 1 introduces a dynamic labor demand model in which a firm and its worker bargain over working time and wages. In Sections 2 and 3, we describe our data and present the empirical moments used in estimation. Section 4 estimates the model, and Section 5 assesses the implications of our results for empirical work on the intensive margin. Section 6 examines the robustness of our results. Section 7 concludes.

1 Theory

The contribution of this section is a tractable model of a production process in which employees have to “work together” to some degree despite differences in preferences. Our formalism is parsimonious, and, as such, involves a number of assumptions that help retain tractability. These assumptions are discussed later in this section. Also note that we take as given a firm-worker match until Section 1.2.4, where we analyze the extensive margin.

1.1 The setting

Preferences. Workers’ utility is separable over consumption and leisure, and linear in consumption. A member’s disutility from time worked h is given by $\xi g(h) \equiv \xi \frac{h^{1+\varphi}}{1+\varphi}$, where $1/\varphi$ is the Frisch intertemporal elasticity of labor supply and ξ indexes the “distaste” for work. The term ξ should be interpreted broadly to encompass any shift in the worker’s marginal value of time (we return to this shortly). To simplify the exposition below, we

⁷Chetty (2012) also focuses on the steady-state Hicksian (wealth-constant) elasticity, whereas we estimate the Frisch (marginal utility of consumption constant) elasticity.

will treat ξ as the only source of heterogeneity across workers within a given firm. The quantitative model in Section 4 will incorporate a worker-specific component of productivity, and we show how to separately identify these two forces.

For tractability, we make several simplifying assumptions concerning ξ . To begin, ξ is i.i.d. Specifically, there is a (finite) number, M , of types, with the set of types denoted by $\mathcal{X} \subseteq \mathbb{R}^M$. Assuming a law of large numbers, a share $\lambda_\xi \in (0, 1)$ of a firm's workforce draws anew some $\xi \in \mathcal{X}$ in each period, where $\sum_{\xi \in \mathcal{X}} \lambda_\xi = 1$ and $\frac{1}{M} \sum_{\xi \in \mathcal{X}} \xi$ is normalized to 1.⁸ We argue later that the absence of persistence in ξ is unlikely to weaken our results regarding the degree of complementarities. Second, we assume ξ is unknown to the firm at the time of hire but perfectly observed thereafter. Accordingly, firm and worker can contract (earnings and working time) on ξ . This is plausible if the two anticipate a long-term arrangement that supports the (credible) communication of private information.⁹

Our assumption of risk neutrality over consumption is a pragmatic choice. As we shall discuss, decisions regarding time worked and wages hinge, more generally, on the value of ξ relative to the marginal value of wealth, or equivalently, the marginal utility of consumption. As we do not observe consumption, our data do not permit us to separately identify these two components. Thus, *given* the structure of our model, the assumption of risk neutrality sacrifices little. Nonetheless, we will occasionally rely on the isomorphism between shifts in ξ and marginal utility as a convenient way to interpret heterogeneity in the data, even if the latter are not explicitly modeled.¹⁰

Production structure. A firm's output is an aggregate over a continuum of jobs, which are (potentially) complements in the production of a final good. Formally, $y(i)$ is output of job i , and

$$Y = Z \left(\int_0^1 y(i)^\rho di \right)^{\alpha/\rho}, \quad (1)$$

is final output, where Z is a firm-wide profitability shock; $\alpha \in (0, 1)$ is the returns to scale at the firm level; and $\rho \in (-\infty, \alpha)$ determines the elasticity of substitution across jobs, given by $1/(1 - \rho)$. Note that under decreasing returns ($\alpha < 1$), the limiting case of perfect

⁸This removes any uncertainty regarding the share of a firm's workforce that will draw type ξ . Arguably, this means the model applies best to large firms. Though we use all firms in our baseline analysis, we look at the sub-sample of large firms in Section 3.

⁹In practice, a firm likely requires some time to "get to know" its workers, but we suspect our main message obtains as long as ξ is partially observable.

¹⁰Of course, the introduction of risk aversion would alter some aspects of the model. This is especially clear with respect to wage setting: risk neutral firms could gain by offering long-term contracts to risk averse workers, whereas we will model flexible wage bargains. Interestingly, we argue later (Section 6) that our model is likely to infer too little complementarities by abstracting from long-term contracts.

substitutes corresponds to $\rho \rightarrow \alpha$, which makes production additively separable over jobs.

Under certain simplifying assumptions, (1) takes a more tractable form. Assuming output $y(i)$ of a job i is proportional to total man-hours on that job and supposing that no worker has a comparative advantage in any one job, one can motivate a simple allocation in which an equal measure $m \equiv 1/M$ of (non-overlapping) jobs is assigned to each type.¹¹ In that case, (1) becomes

$$Y = F(\mathbf{h}, \mathbf{n}, Z) = Z \left(\sum_{\xi \in \mathcal{X}} (n_\xi h_\xi)^\rho \right)^{\alpha/\rho}, \quad (2)$$

where n_ξ is the mass of workers of type ξ ; $n_\xi h_\xi$ is total time (“man-hours”) applied to production by type ξ ; and $\mathbf{n} \equiv \{n_\xi\}$ and $\mathbf{h} \equiv \{h_\xi\}$ are vectors of, respectively, employment and time worked per worker across types. (The constant $m^{(1-\rho)\alpha/\rho}$ is subsumed into Z .) Note that for $\rho < \alpha$, time inputs of different types are q-complements, i.e., the marginal product of one type is increasing in the input of any other type. This underlies the supermodularity of the firm’s problem, a critical feature of the model (as we shall see).

Labor market environment. Last, we turn to the structure of the labor market. We consider a setting in which labor market frictions mediate the formation of employment relationships and yield rents to worker-firm matches, which will be divided by a wage bargain. Following Roys (2015), there is a matching friction that operates at an aggregate level, that is, the pace of job finding (and, job filling) is mediated by aggregate conditions. Since we analyze a firm’s problem in the aggregate steady state, we do not elaborate further on matching. There are also employment adjustment costs, which take the form of a per-capita cost of hiring, \bar{c} , and firing, c . The hiring cost includes the expense of recruiting and training, whereas the firing cost can be interpreted as mandated severance. These adjustment costs along the extensive margin imply that it will be optimal to respond to shocks, to a certain extent, via changes in time worked of incumbent employees.

1.2 Characterization

1.2.1 Working time

The choice of h_ξ is a static decision problem, and so may be solved before we introduce the dynamic program of the firm (Section 1.2.4). The privately efficient choice equates the marginal disutility of work to the marginal product of an additional unit of time, given a

¹¹Since we do not observe intra-firm job allocations, we default to this simple “rule of thumb” allocation. In the online appendix, we solve the assignment problem and contrast the optimal allocation to this rule of thumb.

vector of workers \mathbf{n} . The FOC for time worked by any worker of type ξ is given by

$$\xi h_\xi^\varphi = \frac{\partial F(\mathbf{h}, \mathbf{n}, Z)}{\partial h_\xi} \equiv \alpha Z \left(\sum_{x \in \mathcal{X}} (n_x h_x)^\rho \right)^{\frac{\alpha-\rho}{\rho}} n_\xi^\rho h_\xi^{\rho-1}.$$

Note that, in general, the left hand side would appear as $(\xi/\ell) h_\xi^\varphi$, where ℓ is the marginal value of wealth that translates ξh_ξ^φ into units of numeraire. In our estimation, we will identify variation in the *ratio* ξ/ℓ using fluctuations in working time, but, as noted above, we cannot distinguish ξ from ℓ in our data. Thus, we proceed as if $\ell \equiv 1$, but as noted, we will occasionally appeal to this isomorphism between shifts in ξ and ℓ as a way to interpret heterogeneity in our data.¹²

Now combining FOCs for types ξ and $\xi' \neq \xi$, we have

$$\frac{\xi}{\xi'} \left(\frac{h_\xi}{h_{\xi'}} \right)^{\varphi+1-\rho} = \left(\frac{n_\xi}{n_{\xi'}} \right)^\rho.$$

Using this to substitute for any $h_{\xi'}$ with $\xi' \neq \xi$ in type ξ 's FOC, we can solve for type- ξ working time,

$$h_\xi = (\alpha Z \Omega(\mathbf{n}))^{\frac{1}{\varphi+1-\alpha}} \cdot [n_\xi^{\rho-1}/\xi]^{\frac{1}{\varphi+1-\rho}}, \quad (3)$$

where $\Omega(\mathbf{n}) \equiv \left(\sum_{x \in \mathcal{X}} (n_x^\varphi/x)^{\frac{\rho}{\varphi+1-\rho}} \right)^{\frac{\alpha-\rho}{\rho}}$.¹³ For any $\rho < \alpha$, type ξ 's working time is increasing in the mass (n_x , $x \neq \xi$) of other types and decreasing in own employment (n_ξ). Substituting (3) into (2), we obtain

$$Y = \hat{F}(\mathbf{n}, Z) \equiv \alpha^{\frac{\alpha}{\varphi+1-\alpha}} Z^{\frac{\varphi+1}{\varphi+1-\alpha}} \left(\sum_{\xi \in \mathcal{X}} (n_\xi^\varphi/\xi)^{\frac{\rho}{\varphi+1-\rho}} \right)^{\frac{\alpha}{\rho} \frac{\varphi+1-\rho}{\varphi+1-\alpha}}, \quad (4)$$

which expresses revenue *after* optimization of time worked.

To put (3) in context, we note that Deardorff and Stafford (1976) and Chetty et al (2011), who also study time allocation with heterogeneous workers, assume a common work schedule across employees. Equation (3) still enables a role for idiosyncratic factors, so we can accommodate the distribution of working time changes within the firm that we observe.

¹²Card (1990) flagged changes in the marginal value of wealth as a source of variation in working time.

¹³Since we assume a law of large numbers, such that a worker is “small” relative to his cohort of type- ξ workers, the marginal product of each worker in the cohort is identical. It follows that each worker of type ξ works the same time: $h_\xi(i) = h_\xi$. This enables us to simplify and arrive at (3).

1.2.2 Earnings

Earnings are determined according to the Stole and Zwiebel (1996) bargain, which was generalized by Cahuc, Marque, and Wasmer (2008) to the case of heterogeneous workers. Cahuc et al (2008) abstracted from the intensive margin and assumed a constant rate of separations (layoffs). Our solution relaxes these restrictions.

Taking as given the participation of the remaining \mathbf{n} workers, the Stole and Zwiebel protocol is for the firm and each employee split the surplus of their match according to an exogenously given bargaining weight. In other words, the wage is set by splitting the *marginal* surplus, awarding a share, $\eta \in (0, 1)$, to the worker.¹⁴

Proposition 1 formally presents the earnings bargain. Of course, the derivation of (5) requires a treatment of the firm's dynamic employment demand problem underlying the firm's surplus. This problem is introduced shortly. We present Proposition 1 here in order to proceed more quickly to the implications of the model for earnings and working time.¹⁵

Proposition 1 *The Stole and Zwiebel bargain is given by*

$$W_\xi(\mathbf{n}, Z) = \eta \left[A \frac{\partial \hat{F}(\mathbf{n}, Z)}{\partial n_\xi} + r\underline{c} \right] + (1 - \eta) (A\xi g_\xi(\mathbf{n}) + \mu), \quad \xi \in \mathcal{X} \quad (5)$$

where $A \equiv \frac{\varphi+1-\alpha}{(\varphi+1)(1-\eta(1-\alpha))-\alpha} \geq 1$ and $g_\xi(\mathbf{n}) \equiv \frac{h_\xi(\mathbf{n})^{1+\varphi}}{1+\varphi}$.

The structure of (5) is very intuitive. The bargain is a weighted average of the worker's contribution to the firm and his outside option. The former consists of the worker's productivity plus the annuitized firing cost, $r\underline{c}$, which the worker "saves" the firm by continuing the match.¹⁶ (Here, r is the real interest rate.) The outside option also includes two pieces. One is the utility, $\xi g(h_\xi)$, that could be recovered by quitting to non-employment. The other component, μ , is the annuity, or flow, value of non-employment. The latter consists of flow payoffs from, e.g., unemployment insurance and home production as well as the expected gains from job search. Note that μ is independent of type, consistent with the assumption

¹⁴Brüggemann, Gautier and Menzio (2015) show that splitting the marginal surplus is the outcome of a game in which a firm bargains with each of its workers in sequence, and where the strategic position of each of the workers is symmetric.

¹⁵An analysis of the earnings bargain also requires us to delineate the worker's surplus from employment. This aspect of the problem is highly standard, insofar as it mirrors the treatment of risk-neutral workers in canonical models with matching frictions (Pissarides, 2000). See the appendix for more.

¹⁶The worker can use \underline{c} to negotiate a higher wage because the firm is subject to the severance cost as soon as the worker is hired. This is consistent with the labor contract that was most prevalent in Italy in our sample. See Mortensen and Pissarides (1999) for a discussion of bargaining under severance costs.

that ξ is i.i.d., and thus this period's ξ does not influence the expected future gains from search.

Interestingly, (5) also shares features with the solutions of collective bargaining games. For instance, Taschereau-Dumouchel (2015) shows that the Nash bargaining solution between a firm and its unionized workforce sets a wage for each worker that has, like (5), components that reflect labor productivity and that worker's outside option. One notable difference with respect to (5) is that the union-negotiated wage depends only on average, not marginal, product, so the variance of outside options is the sole source of earnings heterogeneity within the firm. But the quantitative importance of this distinction is limited: in our estimated model (see Section 4.2), the variance of outside options (i.e., $A\xi g_\xi(\mathbf{n}) + \mu$) accounts for 70 percent of within-firm variance in earnings growth. Thus, key aspects of (5) are robust to alternative bargaining protocols.¹⁷

To gain further insight into (5), substitute (3) and (4) into the bargain to write it as

$$W_\xi(\mathbf{n}, Z) = \text{constant} \times (\alpha Z \Omega(\mathbf{n}))^{\frac{\varphi+1}{\varphi+1-\alpha}} n_\xi^{-\frac{(\varphi+1)(1-\rho)}{\varphi+1-\rho}} \xi^{-\frac{\rho}{\varphi+1-\rho}} + \omega, \quad (6)$$

where $\text{constant} \equiv \frac{\eta\varphi+(1-\eta)\frac{\varphi+1-\alpha}{\varphi+1}}{(\varphi+1)(1-\eta(1-\alpha))-\alpha}$ and $\omega \equiv \eta r \underline{c} + (1-\eta) \mu$. For any $\rho < \alpha$, earnings are increasing in the employment of other types (via $\Omega(\mathbf{n})$) and decreasing in own employment. If workers are gross complements ($\rho < 0$), earnings are also increasing in ξ : though a higher ξ depresses working time, the rise in the wage rate, which is needed to compensate for the added disutility, is sufficient to increase earnings.

As we discuss below, this sensitivity of earnings to idiosyncratic events (ξ) is an important implication of complementarities. Is it a plausible feature of wage setting? Perhaps firms and workers in fact set wages to smooth out a portion of these fluctuations. Our empirical strategy could still be applied in this case (a point to which we return in Section 6), but *some* (positive) pass through of ξ to earnings remains critical to our reading of the data. Note that the role of ξ in wage setting does not preclude that substantial variation in earnings reflects shifts in firm and worker productivity. It does, though, point to there being a component of earnings growth that reflects the premium for supplying effort when doing so is costly.

¹⁷Unlike in (5), the heterogeneity of outside options in Taschereau-Dumouchel reflects differences in μ across workers, which derive from persistent differences in workers' productivities. In our context, differences in worker productivities that persist across employers would render the problem much less tractable.

1.2.3 Implications

A model with complementarities has implications for working time and earnings fluctuations (i) across workers inside firms and (ii) across time for the firm as a whole. To see these, there is a special case of (3) and (5) that is particularly instructive. Suppose all types are equally likely, so that $\lambda_\xi \equiv \lambda = 1/M$ for all $\xi \in \mathcal{X}$. Also, assume that the firm, in light of the costs to hire and fire, chooses to leave firm-wide employment at its initial level, N_{-1} . In that case, $n_\xi = \lambda N_{-1} \equiv n$ for any ξ . Then (3) simplifies to

$$h_\xi = (\Xi Z \alpha n^{\alpha-1})^{\frac{1}{\varphi+1-\alpha}} \times \xi^{-\frac{1}{\varphi+1-\rho}}, \quad (7)$$

where $\Xi \equiv \left(\sum_{x \in \mathcal{X}} x^{\frac{-\rho}{\varphi+1-\rho}} \right)^{\frac{\alpha-\rho}{\rho}}$. Equation (7) consists of two components. The first, $\Xi Z \alpha n^{\alpha-1}$, is a firm-wide component that shapes the marginal product of labor. This depends on firm productivity Z as well as “average” preferences (Ξ), since the latter influence co-workers’ time allocations and, via complementarities, the productivity of an individual cohort (of type ξ). The second component depends on ξ , the idiosyncratic preference.

Using (7), we can show that working time is increasingly detached from one’s own preference, ξ , as complementarity increases. To see this most clearly, we consider several special cases.

$\rho = \alpha \rightarrow 1$. This considers a particularly simple limiting case of the model, in which production tends toward linear and, thus, one worker’s output is independent of her colleagues’ effort. In this case (7) tends to

$$h_\xi = \left(\frac{Z}{\xi} \right)^{1/\varphi}. \quad (8)$$

This reveals a key restriction of this limiting case: (8) says that working time reacts symmetrically to firm-wide and worker-specific driving forces. Accordingly, one can infer the Frisch elasticity using exclusively *idiosyncratic* variation in ξ . Turning next to earnings, setting $\alpha = \rho \rightarrow 1$ in (5) and evaluating $\partial \hat{F}(\mathbf{n}, Z) / \partial n_\xi$ yields

$$W_\xi = \eta Z h_\xi + (1 - \eta) \xi g(h_\xi) + \omega. \quad (9)$$

The limiting case (8)-(9) would leave a clear imprint on certain moments of the data that we can measure. To illustrate this, consider the effect of a shift in ξ on an individual’s working time and earnings. By focusing on ξ and abstracting from shifts in Z that are common to all workers within a firm, we can measure the response of working time and

earnings to exclusively idiosyncratic influences. Differentiating (8) and (9) with respect to ξ implies

$$\frac{\partial \ln W_\xi / \partial \ln \xi}{\partial \ln h_\xi / \partial \ln \xi} = 1 - (\omega/W_\xi) < 1. \quad (10)$$

Reflecting the elasticity of working time in this special case, equation (10) indicates that the size of the (log) change in earnings due to idiosyncratic forces is smaller than the size of the corresponding (log) change in working time. Since this is true for all types ξ , it suggests that changes in ξ across workers yields a distribution of earnings growth *within the firm* that is compressed relative to the distribution of changes in working time.¹⁸ In Section 3, we show that this prediction can be tested, and rejected, using matched employer-employee data, which allows us to observe the variance of earnings and working time changes inside firms.

It is worth noting that one way of expanding the variance of earnings growth within firms is by introducing a “demand shock”, such as variation in a worker-specific component of productivity. In contrast to “supply shocks” such as ξ , this moves both elements of earnings—working time and the wage rate—in the same direction, amplifying dispersion in earnings growth.¹⁹ However, as we shall see, the data suggest a more limited role for idiosyncratic productivity shocks. In particular, we find that the covariance of individuals’ wage growth ($\Delta \ln w$) and working time changes ($\Delta \ln h$) is *negative*, echoing earlier results by Abowd and Card (1989).²⁰

$\rho = \alpha < 1$. This special case retains the assumption that workers are perfect substitutes within the firm, but introduces decreasing returns at the firm level. Equation (7) simplifies to

$$h_\xi = (\alpha Z n^{\alpha-1} / \xi)^{\frac{1}{\varphi+1-\alpha}}. \quad (11)$$

Under decreasing returns in (11), variation in ξ is still highly informative about φ , though the returns to scale α also now plays a role. Further, note that Z and ξ continue to have symmetric effects on working time.

¹⁸The inequality in (10) is weak in the limiting case where $\eta \rightarrow 1$ and $c \rightarrow 0$, which implies $W_\xi = Z h_\xi$. However, we estimate η to be significantly less than 1.

¹⁹Suppose output is now $Z\theta$, where θ is the worker-specific portion of productivity. The analogue to (10) for a given change in θ is $\frac{d \ln W_\xi}{d \ln h_\xi} = \left(1 - \frac{\omega}{W_\xi}\right)(1 + \varphi)$. If φ is sufficiently large, then $d \ln W_\xi > d \ln h_\xi$.

²⁰Though we have demonstrated (10) assuming efficient bargaining over working time, the result obtains under alternative protocols. The online appendix considers two cases: (i) the worker unilaterally chooses her working time; and (ii) the firm unilaterally makes the decision (the so-called “right-to-manage” protocol).

Turning to earnings, we first use (7) to substitute for h_ξ , simplifying the bargain to

$$W_\xi(\mathbf{n}, Z) = \text{constant} \times (\Xi Z \alpha n^{\alpha-1})^{\frac{\varphi+1}{\varphi+1-\alpha}} \times \xi^{\frac{-\rho}{\varphi+1-\rho}} + \omega. \quad (12)$$

Now setting $\alpha = \rho$, noting that Ξ collapses to 1, and using (11), we have the following result:

$$\frac{\partial W_\xi / \partial \ln \xi}{\partial h_\xi / \partial \ln \xi} = \alpha \left(1 - \frac{\omega}{W_\xi} \right) < 1.$$

Echoing the finding above, the absence of complementarities implies that changes in earnings due exclusively to idiosyncratic forces are compressed relative to corresponding responses of working time.

$\rho \rightarrow -\infty$. Last, we take the limit in the other direction, sending $\rho \rightarrow -\infty$. This implies that workers are perfect complements. Equation (7) then yields

$$h_\xi = (\alpha Z n^{\alpha-1} / M)^{\frac{1}{\varphi+1-\alpha}}, \quad (13)$$

This says that idiosyncratic variation, in the form of ξ , has *no* direct effect on a type's time worked. Under perfect complementarity, the marginal product of an individual's additional time is zero holding fixed the effort of co-workers. Accordingly, the efficient bargain implies that time worked is invariant to changes in one's marginal value of time.²¹

Working time does not respond to ξ , but earnings do. Applying L'hopital's rule to (12), a log point change in ξ increases earnings by $1 - (\omega/W_\xi)$ log points, reflecting the premium for working when one's marginal value of time is high. Thus, under perfect complementarity, shifts in ξ are accommodated *only* through changes in earnings.

Summarizing thus far, these findings suggest that the distribution of working time changes *within* the firm, which arises due to idiosyncratic driving forces, is compressed by complementarities (low ρ) relative to the corresponding distribution of earnings changes. In Section 3, these implications guide our choice of moments that are used to estimate ρ .

Complementarities also have implications for our understanding of fluctuations in *firm-wide* working time. If complementarities are strong, then (13) shows that φ is not identified off idiosyncratic variation. Rather, it is revealed by firm-wide variation. Taking logs and

²¹Equation (13) makes the even stronger claim that the level of an individual's working time does not deviate from the working time of any other employee. But, this is true only under symmetry, $n_\xi = \lambda N_{-1} \equiv n$, which is assumed here to convey the main result—the invariance of h_ξ to ξ —most simply.

expectations of each side of (13) and differentiating yields

$$\frac{\partial \mathbb{E}_\xi [\ln h_\xi]}{\partial \ln Z} = \frac{1}{\varphi + 1 - \alpha}, \quad (14)$$

where \mathbb{E}_ξ evaluates the mean over types. Hence, the elasticity of firm-wide working time is independent of ρ , and anchored exclusively by φ and the returns to scale, α . Put another way, the symmetry between Z and ξ under perfect substitutes in (11) is broken by complementarities, under which only firm-wide variation helps identify the Frisch elasticity.

1.2.4 Employment demand

Thus far, we have taken total firm employment as given. The combination of complementarities and employment adjustment frictions imply, in fact, a nontrivial dynamic labor demand problem. In this section, we shift gears to study the extensive margin of our model.

At the start of a period, the firm has a workforce of measure N_{-1} .²² Firm productivity Z is realized. At this point, the firm may hire at cost \bar{c} per position, which represents the cost to recruit and train a worker. We assume hires are anonymous, in that the new workers' types have not been drawn at the point of hire. After hires (if any) are made, the firm's workforce is denoted by \mathcal{N} . Then, all \mathcal{N} workers draw a type, ξ , and the firm and (some of) its workers may jointly decide to separate at cost \underline{c} per separation. Let s_ξ denote the number of separations of type- ξ workers. These flows out of and into the firm satisfy,

$$s_\xi = \max \{0, \lambda_\xi \mathcal{N} - n_\xi\}, \quad \text{and} \quad N = \sum_{\xi \in \mathcal{X}} n_\xi, \quad (15)$$

where n_ξ is the mass of type- ξ workers retained and N is total employment used in production. Wages and time inputs are bargained after separations (if any) are made.

As a preliminary step to analyze the firm's problem, we define the present value of a firm for a given choice, $\mathbf{n} \equiv \{n_\xi\}$. To this end, let $\pi(\mathbf{n}, Z)$ stand for profit gross of \underline{c} and \bar{c} but conditional on optimal time worked,

$$\pi(\mathbf{n}, Z) \equiv \hat{F}(\mathbf{n}, Z) - \sum_\xi W_\xi(\mathbf{n}, Z) n_\xi.$$

Then the present value given $\mathbf{n} \equiv \{n_\xi\}$, gross of adjustment costs, is

$$\tilde{\Pi}(\mathbf{n}, Z) \equiv \pi(\mathbf{n}, Z) + \beta \int \Pi(N, Z') dG(Z'|Z),$$

²²The subscript -1 denotes a one-period lag, and a prime ' denotes next-period values.

where $\beta \in (0, 1)$ is the discount factor, G is the distribution function of productivity and Π is the continuation value.

The dynamic programming problem may now be written as follows. It is instructive to work backwards, given a \mathcal{N} . The firm's problem at this stage is to decide separations, and is characterized by the Bellman equation,

$$\begin{aligned}\Pi^-(\mathcal{N}, Z) &= \max_{\mathbf{n}} \left\{ \tilde{\Pi}(\mathbf{n}, Z) - \underline{c} \sum_{\xi \in \mathcal{X}} s_\xi \right\} \\ &= \max_{\mathbf{n}} \left\{ \tilde{\Pi}(\mathbf{n}, Z) - \underline{c} \sum_{\xi \in \mathcal{X}} \max \{0, \lambda_\xi \mathcal{N} - n_\xi\} \right\},\end{aligned}\tag{16}$$

where we have used (15). Then, step back and consider the choice of hires, which brings the workforce up to a level \mathcal{N} . Since hires are anonymous, the value of the firm at this stage is

$$\Pi(N_{-1}, Z) = \max_{\mathcal{N}} \left\{ -\bar{c} \cdot \max \{0, \mathcal{N} - N_{-1}\} + \Pi^-(\mathcal{N}, Z) \right\}.\tag{17}$$

Note that (16)-(17) allow that a firm may hire and separate in the same period. For realistic \underline{c} and \bar{c} , however, this will not happen: if \underline{c} is sizable, productivity must be low to warrant any separations, and as a result, no hires will be made.²³ In that case, $\mathcal{N} = N_{-1}$ in (16), since the firm will not have hired, and the choice of \mathcal{N} in (17) directly implies the allocation of workers across types, $n_\xi = \lambda_\xi \mathcal{N}$, since the firm will not subsequently separate.

Here, we should also note that the simplicity of (16)-(17) is purchased by the assumption of i.i.d. types ξ . This implies we do not have to track individual cohorts over time. As a result, the only state variables of the problem are firm-wide objects, N_{-1} and Z . This feature is an important source of tractability.

To shed light on the optimal labor demand policy, consider the problem of a firm that has workforce $\mathcal{N} = N_{-1}$ (it does not hire). We ask if this firm should separate from workers of (arbitrary) type ξ , taking as given the participation of the remaining types. A separation is made if the marginal value of labor, evaluated at N_{-1} , is less than the separation cost,

$$\frac{\partial \pi(\boldsymbol{\lambda} N_{-1}, Z)}{\partial n_\xi} + \beta \int \Pi_1(N_{-1}, Z') dG(Z'|Z) < -\underline{c},\tag{18}$$

where $\boldsymbol{\lambda}$ is a $M \times 1$ vector of the shares λ_ξ , and the derivative of π is evaluated at the initial workforce, $\mathbf{n} \equiv \boldsymbol{\lambda} N_{-1}$. The appendix verifies that Π is supermodular in its arguments, which implies that the marginal value of labor, the left-hand side of (18), is increasing in Z . It follows that there exists a threshold, $\zeta_\xi(N_{-1})$, such that a type- ξ worker is separated if (and

²³See the appendix and the online appendix for more.

only if) Z falls below $\zeta_\xi(N_{-1})$. The type of worker separated first is the type ξ for which $\zeta_\xi(N_{-1})$ is highest.

If Z falls further, the firm separates from another type, $x \neq \xi$. As the firm does this, separations from the first type ξ continue. This reflects that workers are (q -) complements in production: as the firm reduces labor input of type x , that further reduces the marginal value of type ξ .²⁴ Thus, the optimal policy prescribes that *both* types are separated in tandem. This intuition underlies the result given below and proven in the appendix. To state the proposition, we use the notation $\xi_1, \dots, \xi_j, \dots, \xi_M$ to convey that a type ξ_j is the j th type to be separated.

Proposition 2 *There exists a ranking ξ_1, \dots, ξ_M and a corresponding sequence of functions $(\zeta_1(N_{-1}), \zeta_2(N_{-1}), \dots)$, with the latter listed in decreasing order, such that workers of all types (ξ_1, \dots, ξ_i) are separated if and only if $Z < \zeta_i(N_{-1})$.*

This proposition establishes the *existence* of the thresholds, ζ_ξ . In certain cases, we can say more about the exact map from ξ to ζ_ξ . For instance, if $\lambda_\xi = 1/M$ for all types, the low- ξ workers are the first to be separated. Intuitively, high- ξ workers supply less effort *conditional on* participation, and, as a result, their participation is valued all the more if jobs are complements. If the λ_ξ s differ across types, complementarities imply that workers from relatively abundant cohorts (all else equal) will be separated first.

Figure 1 illustrates this labor demand policy for a case with three types ($M = 3$). There is a range of Z s over which employment of each type is unchanged from its start-of-period value. This optimal range of inaction arises because of the form of the adjustment costs, \bar{c} and c (Bentolila and Bertola, 1990). To the right of this range, the firm hires, and each type's employment is increased in line with its share in the population. As Z declines to the left of this range, one type's employment is reduced, while other types' participation remains fixed. In this example, the shares λ_ξ are the same, so the first type to separate is the lowest ξ , denoted by ξ_1 . As Z falls further, a second type is separated jointly with type ξ_1 , consistent with Proposition 2.

1.3 Discussion of assumptions

A few aspects of our modeling deserve further comment.

Preferences. We have assumed ξ is i.i.d.. Consider now the implications of a persistent process. This has no direct effect on the working time decision, since the latter is a static

²⁴Different labor types are q -complements—the marginal value of one type is increasing in the quantity of the others—for any $\rho < \alpha$. Thus, this does not require gross complements, or $\rho < 0$.

choice. As for earnings, the online appendix argues that the bargain has the same form as (5), but μ is now indexed by ξ to reflect that a worker's outside option depends on her type. Specifically, the value of looking for a new job is decreasing in ξ , reflecting the lower expected gains from future employment when the distaste for working is high.²⁵ Since earnings are otherwise increasing in ξ (if $\rho < 0$), the response of μ attenuates the reaction of earnings to changes in ξ , and thereby *reduces* the dispersion of earnings growth within the firm. This is key: as we have seen, the dispersion of earnings growth relative to the dispersion of working time adjustments is revealing of the degree of complementarities. Therefore, if persistence in ξ dampens earnings changes, our model would require more complementarities to match the relative dispersion of earnings growth in the data. In other words, in a model with i.i.d. ξ , we obtain a *lower bound* on the degree of complementarities.²⁶

Production. The revenue function (4) suppresses two dimensions of heterogeneity. First, it omits individual worker productivity. The online appendix generalizes the model to include i.i.d. productivity innovations, and the quantitative model of Section 4 incorporates this source of fluctuations. Second, our focus on the notion that workers "work together" abstracts from the fact that skilled workers have tended to replace less skilled workers in the labor force. We see the latter as a longer-run trend that is not the subject of this paper. We focus on annual fluctuations, and pursue the idea that, at this frequency, workers of all skill levels likely have to coordinate their effort.

Working time. Our model misses some institutional realities of working time determination in Italy. For instance, unions negotiate limits on overtime. However, union work rules still permit at least 200 hours of overtime per year (per worker), which amounts to 25 or more eight-hour days. Also, whereas management often consults with the union regarding reductions in plant-wide time worked (Giaccone, 2009; Treu, 2007), we are not aware of attempts to compress changes in time worked *within* the firm. Moreover, a quarter of Italian workers report that they negotiate their time worked bilaterally with their firm (Giaccone, 2009).

Earnings. One may suspect that unionization in Italy forecloses any scope for decentralized bargains like (5). However, at the industry level, unions negotiate minimum wages, and in the (relatively rich) Northern region of Veneto, these typically do not bind

²⁵To illustrate this claim, suppose ξ is permanent. Then the annuity value of, or flow return to, non-employment is $\mu = r\mathcal{U}$, where \mathcal{U} represents the present value of non-employment. In a conventional search and matching setting, \mathcal{U} will depend on the expected surplus from working, which is in turn proportional to the marginal value of labor under surplus sharing. The latter is declining in ξ : a higher ξ reduces output (by lowering h_ξ) and raises the wage bill.

²⁶The persistence of ξ will also have indirect effects on working time and earnings via its implications for employment demand.

(Card et al, 2014).²⁷ Moreover, at the firm level, union representatives have negotiated pay-for-performance agreements (Damiani and Ricci, 2014). Consistent with these observations, Card et al find that, in a panel of Veneto firms, wages are responsive to fluctuations in firm value-added. More generally, wage premia in Italy are highly heterogeneous across firms (Erickson and Ichino, 1993; Cingano, 2003). In summary, it seems that, *at the margin*, bargaining is reasonably decentralized.²⁸

2 Veneto Work History Files

Our data includes a direct measure of working time for all employees of each firm in the sample, which is rare in panel datasets. Though it is imperfect, our data likely captures a substantial amount of annual variation in working time.

2.1 Data description

Our empirical analysis utilizes the Veneto Worker History (VWH) dataset. The VWH is a matched worker-firm database that covers the northern Italian region of Veneto for years 1982- 2001.²⁹ For virtually every private-sector employee in Veneto, it records each employer for which he worked at least one day. Public-sector employees and the self-employed are excluded. The full sample contains 22.245 million worker-year observations.

The VWH data has a number of features that recommend it for this analysis. Most importantly, the VWH reports for each worker the number of annual days paid and the calendar months worked with each of the individual's employers. It also gives a worker's annual earnings, from which we can compute the average daily wage.

Table 1 provides a set of summary statistics for the full sample. On average, workers work between 23 and 24 days per month (conditional on positive days worked that month). This reflects the prevalence of six-day weeks in Veneto in this period. As noted above, the sixth day, in many cases, represents overtime. The average daily wage is around 120 Euros, and on average the number of paid months per worker (per year) is 10.

²⁷Wage setting in Northern Italy also appears to be more decentralized than in the South, where written wage bargains between an individual firm and its workers' union are rare (Cella and Treu, 2009). Also, real wages in the North are notably more flexible in response to aggregate fluctuations (Peng and Siebert, 2008).

²⁸National laws are typically silent on compensation. The exception was the *scala mobile*, an indexation scheme that escalated wages with inflation and was dismantled in 1992. But since this applied uniformly to workers, it should shift the mean of the distribution of earnings changes without affecting its variance.

²⁹The region of Veneto, in the North-East of Italy, is one of the richest in Italy: its 2001 GDP ranks third among twenty Italian regions. It has a population of about 5 million, or 8 percent of Italy's total.

Table 2 zeroes in on moments of the distribution of annual changes in working days.³⁰ Many workers do not adjust days from one year to the next.³¹ At the same time, though, 33 percent change working days by more than 10 days. Moreover, conditional on changing days, the typical size of the change is between 10 and 19, depending on whether some of the largest adjustments are included.

Since our data measure *paid* work days, it is important to be precise about paid leaves of absence. In Italy, workers are typically guaranteed at least four weeks of paid vacation. If this is taken each year, we difference it out in computing annual changes in working time. Other forms of leave, such as maternity leave, will show up to some degree in our estimates of working time changes.³² We repeat our analysis below for a sub-sample with men only in order to gauge the importance of female entry and exit from paid work.

The data also identify a worker as full- or part-time, and report the type of contract under which a worker was hired. A permanent contract includes restrictions on individual dismissals. Beginning in the late 1980s, employers were allowed to hire workers under fixed-term contracts that could be terminated after two years without penalty. However, part-time and fixed-term contracts were not used widely over our sample period. On average, 7 percent of workers were part-time, and 11 percent were employed on a fixed-term contract. In our baseline analysis, then, we do not break down the workforce along these lines.³³

2.2 Measuring working time

Even though the Veneto data stand out for providing *any* information on working time, the absence of total working hours is still worrying. The reason is that our measure of earnings is, implicitly, based on total working hours. This discord between the measurement of working time and earnings can affect our analysis in two ways.

First, suppose that, in response to *firm-wide* events, workers increase both working days as well as hours per day. In that case, our data underestimate the variation in the firm-wide component of working time. We refer to this as the *coordinated* response of working time.

³⁰Table 2 pertains to the sample of workers used in our baseline analysis. See Section 3.1 for details.

³¹Our model will not replicate this degree of inaction ($\Delta h = 0$). We could introduce adjustment costs on the intensive margin, which would convert small changes into zeros. But, this comes at considerable expense in terms of tractability and is unlikely to affect inference of other structural parameters. Alternatively, one may interpret this inaction as indicative of overhead labor. We discuss the latter issue in Section 6.

³²A two-parent household has 16 months of paternity leave. This includes 5 months of leave for the mother specifically, during which Social Security pays 80 percent of her salary. The other 11 months are shared by the parents (a day of leave by either counts against this allotment). Social Security pays 30 percent of the parents' salaries for the first 6 of these 11 months. The last 5 months are unpaid (Ray, 2008).

³³Restrictions on fixed-term contracts were relaxed more substantially by Parliament in 2001. Restrictions on part-time work were relaxed in 2000. See Tealdi (2011) for more.

Understating the coordinated variation would be problematic, since we rely on these firm-wide fluctuations to identify the intertemporal (Frisch) elasticity of substitution.

The second concern has to do with how working time reacts to *idiosyncratic* events, ξ . This variation underlies the dispersion we see inside a firm. Our identification strategy will rely on comparing this within-firm dispersion in working time adjustments to the dispersion in earnings growth. This comparison is compromised if workers react to idiosyncratic events by varying daily hours rather than working days. Since changes in daily hours are not captured by our measure of working time but are reflected in earnings growth, our estimates will exaggerate the compression in working time changes.

There is no direct evidence on how coordinated and idiosyncratic working time variation are apportioned between days and daily hours in Italy. Still, we can try to gauge how much total working time variation we are likely missing in our Veneto panel.

To this end, we turn to the Italian Labor Force Survey (LFS). The LFS is administered quarterly, and is a rotating panel: each household is surveyed for two (consecutive) quarters; exits the sample for the next two quarters; and re-enters for two more quarters. The LFS asks about weekly hours and (weekly) days worked. Thus, we can calculate changes in both hours and days for the half of the LFS sample that is in their third or fourth quarter of participation. In total, we have 632,786 observations on year-over-year changes in hours and days worked over the period 1993-2003.³⁴

Our analysis of the LFS suggests that days can account for much of the variation in total hours. By definition, total weekly hours = daily hours \times weekly days. The standard deviation of changes in weekly days is 0.65. Since the typical workday is 8 hours, variation in days *alone* implies a standard deviation of changes in weekly hours of $8 \times 0.65 = 5.2$. This is about 80 percent of the actual standard deviation of hours changes, which is 6.3. We obtain similar answers if we use only full-time workers or workers who have been with their employer for at least a year.

The LFS evidence aligns with the narrative in Giaccone (2009), who reviews working time arrangements in Italy. He stresses the use of Saturday overtime as a means of varying working time. Giaccone also reports that 20 percent of the Italian workforce engages in shiftwork, which can favor the days margin over the daily hours margin as a means of adjustment. To see why, suppose a firm's daily schedule is divided into two 8-hour shifts. If this year is a "good time" to work, how might a person increase her total hours? If she is on the second shift, the only way to acquire overtime for herself is to replace an absent worker on the first

³⁴We restrict this sample to include only workers who stay with the same employer across the year. This conforms to our treatment of the Veneto data in the next section. See the discussion there for more.

shift. Unless she works 16 hours per day, this means an increase in days worked.

Lastly, the online appendix reports on an attempt to gauge (somewhat more directly) how the *coordinated* component of time worked, specifically, is varied. For this, though, we must rely on U.S. data from one industry (autos).³⁵ We find that employer-wide variation in annual working days is a very good proxy for variation in annual total working hours.

3 Estimation Strategy

We will estimate the model of Section 1 by the method of simulated moments (MSM): we stipulate moments, and select values for the parameters such that the model reproduces the observed moments. One advantage of MSM in our application is its relatively minimal data requirements. To illustrate, recall (from (14)) that one could recover the Frisch elasticity, $1/\varphi$, from a projection of (log) firm-wide time worked on $\ln Z$ (modulo the returns to scale, α). Unfortunately, our dataset does not report firm TFP or revenue per worker, the most obvious proxies for Z . But our data does include other variables, such as employment, whose volatility is informative about the variance of Z . If we can infer the latter, (14) suggests that the variance of firm-wide working time then provides substantial identifying information about φ . MSM enables us to harness this information.

3.1 Empirical moments

There are two broad themes that guide the choice of moments used in estimation. First, following on the reasoning set forth in Section 1, we want to distinguish firm-wide from the idiosyncratic (worker-specific) components of working time and earnings. Using our matched employer-employee data, we can do this using simple least squares regressions.

Second, our moments relate to *changes* in working time and earnings, rather than to their levels. To see why, suppose there are fixed differences in productivity across workers. This would support a non-degenerate distribution in time worked even under perfect complements, since time worked would be set to equate efficiency units across employees. Yet workers' time inputs would adjust by the same amount, so the distribution of changes in time worked conveys more clearly the extent of complementarities.

³⁵This uses plant-level data on hours and days worked from Bresnahan and Ramey (1994) and Ramey and Vine (2006).

3.1.1 Earnings & working time

We begin by developing the moments summarizing earnings and working time changes.

Regression framework. Our empirical analysis centers around a simple regression model designed to distinguish variation across workers within a firm from firm-wide movements in working time. Letting $\Delta \ln h_{ijt}$ denote the log change in days worked for employee i in firm j in year t , we estimate

$$\Delta \ln h_{ijt} = \boldsymbol{\chi}'_{ijt} \boldsymbol{\Gamma}_h + \phi_{jt}^h + \epsilon_{ijt}^h, \quad (19)$$

where $\boldsymbol{\chi}_{ijt}$ collects the (time-varying) worker characteristics in our data, $\boldsymbol{\Gamma}_h$ is a conformable vector of coefficients, and ϕ_{jt}^h is a firm-year effect. Equation (19) is applied to a sub-sample of workers who stay at a firm for consecutive years $t - 1$ and t (see below for more on sample selection). The elements of $\boldsymbol{\chi}_{ijt}$ consist of a cubic in the worker's tenure (measured as of $t - 1$) and the change in broad occupation (between $t - 1$ and t).³⁶ These controls help purge the data of observable persistent heterogeneity in work schedules. The variation then captured in ϕ_{jt}^h and ϵ_{ijt}^h is what is used to estimate the structural model.

The firm-year effect, ϕ_{jt}^h , in (19) measures the log change in firm j 's working time relative to the average log change among firms in year t . We interpret ϕ_{jt}^h as reflective of shocks to labor demand at the *firm level*. Accordingly, the variance of ϕ_{jt}^h is our measure of fluctuations in firm-wide working time. Recalling (14), which links firm-wide changes in working time to φ , the moment, $\text{var}(\phi_{jt}^h)$, will be highly informative as to the value of the Frisch elasticity.

It follows that the residual in (19) isolates variation across workers *within a firm*. We pool the estimated ϵ^h 's and calculate $\text{var}(\epsilon_{ijt}^h)$, which we interpret as the variance of idiosyncratic (worker-specific) working time changes. Furthermore, we can repeat this exercise by replacing $\Delta \ln h$ in (19) with the log change in earnings,

$$\Delta \ln W_{ijt} = \boldsymbol{\chi}'_{ijt} \boldsymbol{\Gamma}_w + \phi_{jt}^W + \epsilon_{ijt}^W. \quad (20)$$

The moment, $\text{var}(\epsilon_{ijt}^W) / \text{var}(\epsilon_{ijt}^h)$, compares the variances of earnings and working time changes within the firm. From our model's perspective, a high degree of complementarities means that idiosyncratic variation in preferences (or productivity) reflected in $\text{var}(\epsilon_{ijt}^W)$ is not passed through to $\text{var}(\epsilon_{ijt}^h)$. Accordingly, the ratio of these two communicates the extent to

³⁶Initial tenure helps control for the possibility that more tenured workers have less variable work schedules. As for occupation, we measure four broad categories. Blue-collar workers make up 65 percent of the sample; "clerks", or white-collar non-managerial workers, make up 31 percent; managers comprise about 1 percent; and apprentices, or interns, make up 3 percent.

which working time adjustments are compressed by complementarities, and, thus, provides critical identifying information for ρ .

Sample selection. To estimate (19)-(20), we use a sample of workers attached to a firm for consecutive years. By confining the sample to stayers, we isolate intensive-margin adjustments, i.e., changes in working time *conditional on* sustaining the employment relationship across years. We will then map the variance of these adjustments to their counterparts in our structural model (see Section 3.2 below).

More exactly, our baseline sample includes workers in the year- t cross section only if they are paid for at least one day in all months of the first (calendar) quarter of year $t - 1$ and in all months of the last quarter of year t . This restriction yields 11.8 million annual observations. We then remove workers employed at firms with only one employee; it would be awkward to discuss complementarities with these firms in the sample. Though such firms make up a substantial share of the population of firms, the number of workers involved is small; we still have well over 11 million observations.

We refer to the workers in our baseline sample as *2-year stayers*. They appear to have relatively strong attachments to their firms insofar their annual absences from their employers are not re-current. For instance, among workers who are not paid for a full month or more in year $t - 1$, most are paid for at least one day in every month of the next year.

A few more remarks on our sample selection are warranted. First, the restriction to 2-year stayers reduces the sample by about half. This seems consistent with data on worker flows in Italy. Contini et al (2009) estimates that in relatively large Italian firms (with at least 20 workers), 36 percent of a firm's workforce exits over two years.³⁷ Since turnover is lower at larger firms (Idson, 1993), we are not surprised we drop about half of the sample.

Second, whereas we require a stayer to begin year $t - 1$ and end year t with the same firm, one could instead set a criterion based on the number of months of employment in adjacent years (regardless of where in a year those months lie). To this end, we have recomputed the moments under a definition of a stayer as one who draws pay for (any) nine months in each year $t - 1$ and t . The results—and in particular, the ratio of the variances—are quite similar to those in our baseline (results available on request).

Third, one could alternatively consider a tighter definition of stayers, which requires more consistent participation at the firm. To this end, we also present results below for an alternative sample, which we refer to as the 12/12 stayers. These workers are paid for at

³⁷Contini et al estimate that over a typical 12 month period in the 1990s, the gross separation rate was about 20 percent. Therefore, among a cohort of workers at the start of year $t - 1$, $1 - (1 - 0.2)^2 = 36$ percent exit by the end of year t .

least one day in every month over years $t - 1$ and t .

Last, our use of stayers may raise concerns about selection bias. We suspect such bias is negligible vis à vis our parameters of interest, but return to this below (Section 6).

Estimates. Table 3 summarizes several key moments of the data. The first three rows pertain to within-firm (idiosyncratic) variation. Specifically, the first row reports $\text{var}(\epsilon_{ijt}^W)$; the second shows $\text{var}(\epsilon_{ijt}^h)$; and the third gives the ratio of the two. In our sample of 2-year stayers, this ratio is 2.247—idiosyncratic earnings growth is more than twice as volatile as idiosyncratic working time changes. The next three rows report the counterparts to these moments at the firm level, namely $\text{var}(\phi_{jt}^W)$, $\text{var}(\phi_{jt}^h)$, and the ratio of the two. We call attention to the value of $\text{var}(\phi_{jt}^h)$ in particular (for 2-year stayers). This variation represents 1.5-2 days per month for the typical worker.³⁸

Comparing estimates in Table 3 across 2-year and 12/12 stayers reveals clear, but intuitive, differences. For instance, idiosyncratic working-time fluctuations, as captured by $\text{var}(\epsilon_{ijt}^h)$, are larger among the 2-year stayers, which is not surprising: they include employees who can experience longer non-working spells in years $t - 1$ or t . Some of this variation in working time fluctuations is also likely reflected in the greater variance of earnings changes. The firm-wide moments are more similar. Finally, we stress that, using either sample, $\text{var}(\epsilon_{ijt}^W)$ substantially exceeds $\text{var}(\epsilon_{ijt}^h)$.

Table 4 reports on sensitivity analysis with respect to the moment, $\text{var}(\epsilon_{ijt}^W) / \text{var}(\epsilon_{ijt}^h)$, which is especially critical to our strategy. We highlight several results. First, this ratio is typically 2 or higher. Second, the ratio is higher at larger firms. This may reflect that union-bargained minimum wages are less likely to bind there, giving them greater leeway to conduct firm-level negotiations (Guiso, Pistaferri, and Schivardi 2005). Third, since women are more likely to take longer absences for family reasons, our baseline result may mask different patterns by gender. But if we restrict attention to men, the ratio for 2-year stayers is not much different.³⁹ Fourth, private firms in sectors, such as health and education, that are dominated by public enterprises may face unique environments, but their behavior is not too different from the full sample.

Last, the ratio (for 2-year stayers in particular) does not differ much across some of the largest industries in our sample. This suggests that, though the aggregation of efforts by workers in a services firm may be harder to *observe* than complementarities on, say, an assembly line, the output of the firm may rely just as critically on the combination of

³⁸The table indicates that a one standard deviation increase in log days is $\text{var}(\phi_{jt}^h)^{1/2} = 0.078$. Since the typical worker puts in about 23 days per month (Table 1), a 7.8 log point increase represents 1.8 days.

³⁹To estimate $\text{var}(\epsilon_{ijt}^W) / \text{var}(\epsilon_{ijt}^h)$ here, we use all firms but pool ϵ_{ijt}^W and ϵ_{ijt}^h across only male workers.

individuals' efforts.

3.1.2 Additional moments

The list of all seven moments that we use in estimation is given in Table 5. The first four refer to results just described. We now summarize the final three, and discuss their information content for the structural parameters.

First, we project $\Delta \ln h_{ijt}$ on the log change in the daily wage, which is calculated by dividing annual earnings (W_{ijt}) by annual working days (h_{ijt}). The estimated coefficient is -0.169 . Note here that we use observed working time and wages, rather than isolating the idiosyncratic or firm-wide component. We have verified that our estimate reflects largely variation *within* the firm: we uncover virtually the same estimate when using the idiosyncratic portion of working time and the daily wage. However, using the “raw” data aids in comparing our result to earlier findings, which documented a negative comovement of working time (hours in their case) and the (hourly) wage using household survey data. Estimates in Abowd and Card (1989) for instance imply a coefficient of -0.3 . Though these earlier results have sometimes been attributed to division bias (Borjas, 1980; Hercowitz, 2009), we are less concerned about measurement error in our administrative data.⁴⁰

The final two moments refer to employment. The first is the standard deviation of employment growth across firms. This is calculated from the employment-weighted distribution of employment growth, so that it is representative of the employment volatility faced by a typical worker. The final moment is mean firm size, exclusive of single employer firms.

3.2 Identification

Seven parameters are estimated. They include: ρ , which governs the elasticity of substitution across jobs, $1/(1-\rho)$; the Frisch elasticity, $1/\varphi$; worker bargaining power, η ; the worker's outside option, μ ; and the variance of preferences, σ_ξ^2 . Also, as previewed above, our quantitative model includes a worker-specific component of productivity, denoted by θ with variance σ_θ^2 . (See the online appendix for a statement of the firm's problem that incorporates θ .) Lastly, we recover the variance, σ_Z^2 , of innovations to firm-wide productivity, Z . The moments we aim to reproduce are derived from the sample of 2-year stayers (Table 5).

We now offer some intuition for how our moments identify the parameters. The extent of complementarities influences the dispersion of working time changes within the firm rel-

⁴⁰Of course, one distinction between these earlier studies and ours is that we do not observe the hourly wage, but rather daily earnings. We discuss this concern at length in Section 6.

ative to the dispersion in earnings changes (inside the firm). Hence, ρ maps most clearly to $\text{var}(\epsilon_{ijt}^W) / \text{var}(\epsilon_{ijt}^h)$. Second, worker bargaining power, η , helps mediate the reaction of earnings to changes in working time. It thus influences the relative variances of these objects. Since ρ bears most directly on $\text{var}(\epsilon_{ijt}^W) / \text{var}(\epsilon_{ijt}^h)$, bargaining power η is “needed” especially to target $\text{var}(\phi_{jt}^W) / \text{var}(\phi_{jt}^h)$.

Next, the variances of idiosyncratic preference (ξ) and productivity (θ) are informed in particular by two moments. The size of preference (supply) shocks *relative* to productivity (demand) shocks influences the comovement of working time and the wage, as reflected through the regression of the former on the latter. Negative comovement suggests, for instance, the prominence of “supply-side” idiosyncratic variation (i.e., ξ), which drives working time and wages in opposite directions. In addition, the size of idiosyncratic (worker-specific) movements in working time, as reflected in $\text{var}(\epsilon_{ijt}^h)$, offers further information about the variances of these idiosyncratic shocks.

The final three parameters are φ , σ_Z , and μ . As foreshadowed by (14), the Frisch elasticity, $1/\varphi$, influences the amplitude of working time fluctuations at the firm level, conditional on the size of firm-wide shocks, Z . This helps target $\text{var}(\phi_{jt}^h)$. The variance of these latter shocks are, in turn, greatly informed by the dispersion in employment growth across firms, $\Delta \ln N$. Lastly, the outside option, μ , is a critical determinant of the incentive to form new matches: if μ is large, the rents from the match are small, and so fewer hires are made. This indicates that the average size of firms, $\mathbb{E}[N]$, will help pin down μ .

4 Model Estimation

4.1 Preliminaries

To begin, we pre-set values of several parameters. We begin with the firm productivity process. Since we lack revenue data, we are inclined to parameterize this based on results in the firm dynamics literature. However, these choices have important implications for variables in our dataset whose moments we wish to replicate. Our strategy, then, is to “split the difference” and preset some parameters and estimate others. Specifically, we assume firm productivity, Z , follows a geometric AR(1),

$$\ln Z = \gamma \ln Z_{-1} + \varepsilon^Z, \quad \text{with } \varepsilon^Z \sim N(0, \sigma_Z^2),$$

and fix $\gamma = 0.8$ based on plant-level estimates of total factor productivity (TFP).⁴¹ But, we treat the standard deviation, σ_Z , as a parameter to be estimated, as discussed below.

Next, we set values for four other parameters for which there is credible external information. Our choice of the severance cost, c , amounts to 7 months of earnings. This represents an attempt to synthesize multiple separation costs in Italy (see the online appendix for calculations). We set the hiring cost, \bar{c} , at 5 percent of annual earnings, based on the range of measurements in the literature.⁴² Third, we fix $\alpha = 0.667$, which, as we discuss below, is consistent with labor's share in Italy as well as structural estimates off plant-level data (Cooper, Haltiwanger, and Willis, 2015). Lastly, we set the discount factor $\beta = 0.941$, which is consistent with the average annual real rate of interest in Italy over our sample.⁴³

One final set of parameters pertains to the number of preference types, ξ , and idiosyncratic productivities, θ , as well as the distribution of each. It seems heroic to try to identify the shape of the distribution of ξ or θ given our data. Thus, we have simply assumed that each is uniformly distributed. Next, we use $M = 3$ preferences (ξ) and 3 productivities (θ). This yields 9 pairs of (ξ, θ) within the firm, where each cohort is equally represented (i.e., $\lambda_{\xi, \theta} = 1/9$ for each (ξ, θ)). This choice is influenced by computational constraints.⁴⁴ However, our parameter estimates should not be too sensitive to the precise number of pairs *given* the variances of ξ and θ . The variance of a type (i.e., ξ) can be replicated by any $M > 1$ and (partly) anchors the variances of earnings and working time changes within the firm.

4.2 Main results

Table 5 summarizes our results. The top panel lists the empirical and model-generated moments. The model replicates the moments nearly exactly. This goodness of fit should arguably be demanded from a just-identified model, but it is, still, the first test to be passed, and the model does so. The bottom panel lists MSM estimates of the structural parameters.

Frisch elasticity. Our estimate of a Frisch elasticity ($1/\varphi$) of 0.536 is higher than in the seminal life-cycle analyses of MaCurdy (1981) and Altonji (1986). However, it is

⁴¹We draw from Foster et al (2008). We do not know of plant-level estimates for Italy.

⁴²Our choice is the average of estimates derived from (i) two U.S. surveys of employers reported, respectively, in Barron et al (1997) and Hall and Milgrom (2008); and (ii) a survey of French firms described in Abowd and Kramarz (2003). The value of \bar{c} is the sum of recruiting and training costs, though only Abowd and Kramarz (2003) offer evidence on the latter.

⁴³The real interest rate is interpreted as $r = (1 - \beta)/\beta \cong 1 - \beta$.

⁴⁴Computational time can rise considerably with the number of pairs. We evaluate every pair's labor demand FOC to find the first one to be separated. Conditional on the solution for this pair, we re-evaluate the FOCs for all other pairs to find the next one to be separated; and so on.

somewhat lower than in Pistaferri (2003), who finds an elasticity of 0.7. Pistaferri also estimates a life-cycle model, but implements a novel identification strategy: he recovers the Frisch elasticity as the response of total hours (in year t) to the survey respondent's *expected* wage growth (between years $t - 1$ and t).

From the perspective of our model, the interpretation of Pistaferri's results hinges on whether firm-wide or worker-specific (idiosyncratic) variation is more persistent. To see this, we can write the expected future earnings bargain by combining the solution for working time (3) with (6) to yield

$$\mathbb{E}[W'] = \text{constant} \cdot \mathbb{E}[\xi' h'^{1+\varphi}] + \omega,$$

where a prime ' denotes the next period value. Note that an individual's future working time h' depends on her draw of ξ' ; the firm's productivity, Z' ; and the size of the firm as summarized by $\mathbf{n}' \equiv \{n'_\xi\}$. Therefore, if the idiosyncratic element ξ' is sufficiently transitory (in our model, it is i.i.d.) and if firm productivity is sufficiently persistent, then $\mathbb{E}[W']$ will largely reflect the *firm-wide* component. In that case, Pistaferri's identifying variation would be purified of idiosyncratic influences; his estimate would reflect, like ours, the response of working time to firm-wide variation.

Elasticity of substitution. Our estimate of $\rho = -1.907$ implies an elasticity of substitution across jobs within the firm of $(1 - \rho)^{-1} \cong 0.344$. To convey the meaning of this result in more concrete terms, we can compute the reaction of working time to a (one log point) change in an individual's preference ξ , holding fixed employment of each type. If workers were perfect substitutes (see Section 1.2.3), this would coincide with the Frisch elasticity, $1/\varphi = 0.536$. Our estimate of ρ instead implies a response, using (3), equal to $(\varphi + 1 - \rho)^{-1} \cong 0.209$. Thus, a worker's reaction to idiosyncratic events is attenuated by about 60 percent relative to the perfect-substitutes case.

Worker bargaining power. We estimate that $\eta = 0.452$. This is not too different than Roys' (2014) estimate of 0.52, though we bring very different identifying information to bear on η . Roys estimates a model of dynamic labor demand that includes a Stole and Zwiebel bargain. His French firm-level panel lacks data on working time but includes sales, which enables him to use the comovement of wages and output to infer η . On the other hand, our estimate of η implies that earnings are more responsive to average product than found in Guiso, Pistaferri, and Schivardi (2005). Interestingly, as we discuss below, our estimate of η declines if we re-parameterize the process of Z to induce a persistence in revenue that is comparable to that measured by Guiso et al.

Flow outside option. To interpret our estimate of $\mu = 0.196$, suppose the annuity, or flow, value of non-employment satisfies $\mu = b + f \cdot \beta \mathbb{E} [\text{worker's surplus}]$.⁴⁵ Here, b is non-employment income; f is the transition probability into employment; and “worker’s surplus” refers to the present value surplus from a future job.⁴⁶ Since workers receive a share η of the total match surplus, it follows that

$$\mu = b + f \frac{\eta}{1 - \eta} \cdot \beta \mathbb{E} [\text{firm's surplus}] .$$

The firm’s surplus from a type- ξ worker can be computed from the estimated model. Using this, an annual transition probability, f , of 40 percent (Elsby et al, 2013), and our estimates of μ and η , we can solve for the implied value of b . Dividing this by average earnings (in the model) yields a replacement rate of 49 percent. This compares to a replacement rate in the first year of an unemployment spell in Italy of 58 percent. Since this is not a moment of the data we target, we are encouraged by the model’s performance along this dimension.⁴⁷

Shocks. Our estimate of σ_Z implies a standard deviation of the log change in firm productivity (i.e., $\text{var}(\Delta \ln Z)$) of 0.198. This is remarkably similar to estimates implied by plant-level TFP in European economies (see “France” and “Spain” in Table 2 of Asker, Collard-Wexler, and De Loecker, 2014). As for idiosyncratic heterogeneity, we find that it is slightly more substantial than firm-level dispersion: the unconditional standard deviation of firm productivity (i.e., $\sqrt{\text{var}(\ln Z)}$) is 0.315, and the standard deviation of the sum of idiosyncratic disturbances is $\sqrt{0.292^2 + 0.219^2} \cong 0.365$.

5 Implications for empirical research

We have found evidence of production complementarities within a firm. This implies that the labor supply response to idiosyncratic variation can yield a downwardly biased estimate of a worker’s willingness to substitute effort intertemporally. We illustrate this point quantitatively in this section.

We carry out a randomized control trial within the estimated model. A fraction of a firm’s workforce is “treated” with a higher distaste, ξ , for work. We then compute the change in

⁴⁵Again, this is a standard formulation in matching models. Note, though, that we do *not* need to impose this in order to estimate our structural model.

⁴⁶To be more exact, let \mathcal{U} denote the asset value of non-employment *after* receipt of the severance, \underline{c} . Then the annuity is $r(\underline{c} + \mathcal{U}) \equiv r\underline{c} + \mu$. However, since $r\underline{c}$ is small, the latter is dominated by $\mu \equiv r\mathcal{U}$.

⁴⁷Benefits in Italy are offered beyond the first year to older workers, but at a reduced rate. Thus, 58 percent overstates the replacement rate among completed spells. See the online appendix for our calculations.

working time of the treated group, and compare this to the outcome if the full workforce were treated.

As noted in Section 1, a shift in ξ is, in general, isomorphic to reducing the marginal value of wealth, ℓ . Even though our use of risk neutrality “shuts down” such fluctuations in the marginal value of wealth in our model, we can still use this general isomorphism as a tool for calibration. Specifically, we can use canonical consumer theory, and (non-unitary) estimates of the intertemporal elasticity of substitution, to map from a change in ℓ to a change in ξ in order to derive an empirically relevant “treatment”.

To proceed, suppose a lump-sum transfer is made. The magnitude of the transfer is set equal to the size of a typical grant in the U.S. Negative Income Tax (NIT) experiments, referenced in the Introduction. This implies a transfer of 37 percent of a participant’s initial (pre-NIT) earnings.⁴⁸ We assume a marginal propensity to consume out of transitory income of $1/3$ (Johnson et al, 2006).⁴⁹ We can then map from the change in consumption to the change in ℓ , assuming utility is separable in consumption, C , and leisure. In this case, optimality implies $\Delta \ln C = - (1/\phi) \Delta \ln \ell$, where the own-price elasticity, $1/\phi$, is set to $1/2$ (Hall, 2009). This yields $\Delta \ln \ell \cong -0.25$, which, in our setting, is equivalent to increasing the distaste, ξ , for working by 25 percent.

Within our model, we now “treat” one of the 9 (ξ, θ) cohorts in the firm. The treatment merely scales up the workers’ disamenity, ξ . The model implies that these employees reduce their time worked by 5.3 percent. If we viewed this reaction through the lens of a model with no complementarities (see Section 1.2.3), we would infer a Frisch elasticity of $0.053/0.25 = 0.212$. This is *less than half* the size of the Frisch elasticity that we estimate.

This effect can be contrasted to the change in working time when *all* workers receive the treatment. To illustrate, first suppose that the designer of the randomized trial can hold employment fixed. In that case, using (3), we can compute the treatment effect as $\frac{1}{\varphi+1-\alpha} \times \Delta \ln \xi = 11.4$ percent. Thus, the reduction of working time is larger by more than a factor of 2. More realistically, though, if firms can adjust on both margins of labor demand, this will take some of the burden off adjusting working time. Allowing for employment adjustments, mean working time declines by 7.8 percent. Though smaller, this is 50 percent

⁴⁸In a typical NIT trial, enrollment was restricted to families with income below a threshold, \hat{y} . A “treated” household received a lump-sum benefit, or guarantee, but a share, ϱ , of this was reduced for each \$1 in earnings. To compute our transfer, we take the average NIT guarantee and apply a ϱ of 50 percent (used in most trials) based on an income of $\hat{y}/2$, the midpoint of the eligible range. Unlike in the NIT, though, this transfer is *not* adjusted according to ϱ based on subsequent changes in earnings. Our calculations of the guarantee and \hat{y} are participation-weighted averages across NIT trials (see Burtless (1987)).

⁴⁹Carroll (2001) shows that a marginal propensity to consume of $1/3$ can be understood without resort to liquidity constraints. Therefore, we assume the worker is “on” his first order condition.

higher than what we find if only one cohort were treated.⁵⁰

To put these results in context, we can consider the average effect of the NIT on (men’s) annual hours, though the comparison is complicated. Burtless (1987) reports an average effect of 7 percent, but these changes appear to have reflected longer job search spells more so than reduced working time conditional on working (Moffit, 1981; Robins and West, 1983). Thus, the intensive-margin response was likely appreciably lower than 7 percent, arguably more in line with our model’s predictions.⁵¹

6 Robustness

This section probes the robustness of our results in three respects. First, we take up a few specific concerns about our identification of complementarities. Second, we investigate the sensitivity of our results to alternative pre-set parameters and sub-samples. Third, we confront the model with certain moments not used in estimation.

6.1 On the inference of complementarities

We noted that our use of stayers raises a question of selection bias. In response, we stress that we run the same regressions with stayers on model-generated data. If the model is correctly specified, our estimates of the structural parameters are consistent (Smith, 1993). Thus, any concern about selection bias has to do with model mis-specification.

To illustrate, suppose there is heterogeneity in complementarities across jobs within the firm—a feature we do not model. In particular, imagine a worker’s separation from a firm is indicative of an *absence* of complementarity between his job and others. Then, our sample of stayers will consist of the most complementary jobs; this will confound the inference of ρ . Perhaps an argument in favor of this hypothesis is that a firm competes more aggressively to retain workers in complementary jobs. But, by this logic, a similar firm that seeks to “poach” such a worker to fill a vacancy should also compete aggressively.⁵² This latter consideration

⁵⁰In the absence of employment adjustment costs, one might imagine that firms would learn how to identify job applicants who did not receive the treatment and hire the latter (who want to work relatively more) to replace their workers who were treated (and who want to work relatively less). But in the presence of adjustment frictions, the marginal surplus vis a vis treated workers can still be positive.

⁵¹Our model also has implications for the response of earnings to a NIT-like treatment. However, this comparison of model and data is especially hard, because job searchers may have accepted lower-wage jobs due to the benefit reduction rate (see footnote 48). This will mask earnings dynamics of incumbent workers in the data (Robins and West, 1983).

⁵²This assumes the worker will perform a similar job in the new firm, and that the new firm’s production structure is broadly comparable to the worker’s present employer.

suggests that separations (where a firm poaches a worker) may correspond to jobs with a *high* degree of complementary. A priori, then, we do not see a strong argument for why a separation would systematically reveal the complementarity of the job.

Another concern stems from our lack of data on total hours, which can compromise our use of certain moments. Consider the regression of days worked on daily earnings. This moment is critical to our strategy: the negative comovement limits the scope for worker-specific productivity shocks alone to reproduce the moment, $\text{var}(\epsilon_{ijt}^W) / \text{var}(\epsilon_{ijt}^h)$, and so points to a role for complementarities. But, daily earnings conflates movements in daily hours and hourly wages. Therefore, our estimate could reflect the negative comovement of days and daily hours, rather than the comovement of time worked and the wage per unit time. It would then be inappropriate to map the latter to its counterpart in the model.

We address this concern as follows. The least-squares coefficient from a regression of the log change in days worked on the log change in daily earnings can be decomposed as

$$-0.169 = \frac{\text{Covar}(\Delta \ln \text{ daily hours}, \Delta \ln \text{ days})}{\text{Var}(\Delta \ln \text{ daily earnings})} + \frac{\text{Covar}(\Delta \ln \text{ hourly wage}, \Delta \ln \text{ days})}{\text{Var}(\Delta \ln \text{ daily earnings})}. \quad (21)$$

The Italian Labor Force Survey (LFS) includes data on daily hours and days. We can use this to fill in an estimate for the numerator in the first term in (21).⁵³ Using the variance of the log change in daily earnings in our Veneto data, we can then calculate the first term, which summarizes the role of daily hours in driving the comovement of daily earnings and days. We estimate this term to lie between -0.045 and -0.10 .⁵⁴ Taking the midpoint of these and comparing to the estimate of -0.169 , it seems that shifts in hourly wages do drive the majority of the comovement we are capturing in the Veneto data.

One final factor that might confound our inference of complementarities is the presence of overhead labor. Since the latter does not vary its days (by much), it serves to compress the distribution of days worked movements, from which our model infers that there are complementarities. The concern is that this inference masks a flexible production structure among non-overhead labor. To assess this concern, we drop workers who report 52 weeks of paid work in adjacent years and re-estimate (19) and (20) to recover the idiosyncratic components, ϵ_{ijt}^W and ϵ_{ijt}^h . This is very generous to the notion of overhead labor, as it drops any worker who participates full time in consecutive years. As anticipated, the amount of compression in the distribution of days worked movements is diminished. And yet, $\text{var}(\epsilon_{ijt}^W) / \text{var}(\epsilon_{ijt}^h)$ is

⁵³We use observations in the LFS for Veneto residents, but results hardly change if we use the full sample.

⁵⁴The result depends on whether we use, respectively, usual daily hours or average daily hours in the reference week. One can argue for usual hours if “usual” is interpreted as average hours that year. This is in fact the concept that maps to the annual Veneto data.

1.59—well above 1, which poses a challenge to models that neglect complementarities.

6.2 Sensitivity analysis

We have explored the robustness of our results along a variety of dimensions. We condition our estimation on a higher severance, \underline{c} ; a lower persistence of productivity, γ ; and higher returns to scale, α . In a final exercise, we re-estimate the model over a certain sub-sample. The results are reported in Table 6. Taken together, they point to a Frisch elasticity between 0.283 and 0.641, and an elasticity of substitution between 0.232 and 0.460.⁵⁵

We now discuss these sensitivity analyses in detail. First, a higher severance cost and less persistent productivity push many parameters in the same direction. A severance of one year of earnings compresses changes in employment, and larger firm-wide shocks are required to regenerate the variance of $\Delta \ln N$. Less persistent productivity also induces smaller adjustments in labor demand: if employment changes are costly to reverse, firms attenuate responses to transitory shocks. As a result, when we lower γ to 0.32, which enables the model to replicate the persistence of value-added in Guiso et al's (2005) Italian firm-level data, σ_Z rises to recreate the variance of $\Delta \ln N$.⁵⁶ Larger firm-wide shocks, in turn, require a smaller Frisch elasticity and a lower bargaining power η to restrain movements in working time and earnings. For $\gamma = 0.32$, η falls to 0.231, which reduces the elasticity of earnings to average product to 0.37 (from 0.596 in our baseline). Even here, though, we find more responsive earnings than in Guiso et al (2005), who recover an elasticity closer to 0.1.⁵⁷

Many parameters react in the opposite manner when α is raised. To arrive at our choice of $\alpha = 0.824$, we reinterpret (1) as the reduced form of a monopolistically competitive firm's revenue function where α reflects both returns to scale and the product demand elasticity, ς (Cooper et al, 2015). The increase from our baseline of $\alpha = 0.667$ to $\alpha = 0.824$ can then be shown to correspond to a doubling of ς , from a benchmark of $\varsigma = 4$ (Nakamura and Steinsson, 2008) to $\varsigma = 8$.⁵⁸ This makes labor demand more elastic, which translates into a wider distribution of employment growth. Therefore, σ_Z is lowered to match the variance of

⁵⁵We do not report the model-implied moments; these match the data almost exactly.

⁵⁶Guiso et al's projection of log revenue on its lag yields a coefficient of 0.477. This is what we target.

⁵⁷This is not quite comparable to our result, since Guiso et al estimate the response of earnings to a *permanent* increase in value-added.

⁵⁸Suppose a fixed measure, s , of jobs in a firm is done by labor, and the remainder by capital. Let $N \equiv (\int_0^s y(i)^\rho di)^{1/\rho}$ represent the aggregation of jobs done by labor, and assume N and $K \equiv (\int_s^1 y(i)^\rho di)^{1/\rho}$ are joined to make output, $Y = ZN^{\bar{\alpha}}K^{1-\bar{\alpha}}$. If product demand is $Y = P^{-\varsigma}$ and if K is chosen optimally, Cooper et al show that the elasticity of revenue with respect to N is $\alpha \equiv \frac{\varsigma-1}{\varsigma+(1-\bar{\alpha})/\bar{\alpha}}$, which is the parameter that appears in (1). Here, $\bar{\alpha}$ is comparable to labor's share (though somewhat lower, because of bargaining). We recover our baseline choice of $\alpha = 2/3$ if we set $\bar{\alpha} = 2/3$, in line with OECD data, and $\varsigma = 4$.

$\Delta \ln N$. Working time changes appear larger in light of smaller shocks, so the Frisch elasticity must be raised: $1/\varphi$ is now 0.641.

Lastly, we re-estimate the model over the sub-sample, 1994-2001. This covers a period since the Italian government signed the Tripartite Agreement with employer and worker organizations, which encouraged firm-level bargaining. Consistent with a push toward decentralizing wage setting, Table 6 shows that the variance of earnings growth both within the firm and at the firm level is more volatile than in the full sample. Other changes, relative to the full sample, include somewhat smaller fluctuations in working time, and “less negative” comovement of working time and daily earnings.

These changes in the moments map intuitively to changes in structural parameters. First, the larger variance of earnings growth at the firm level drives η up to 0.569, and the smaller variance of firm-wide working time changes drives down the Frisch elasticity, $1/\varphi$, to 0.352. The increase in η also expands the variance of earnings growth within the firm, because it implies a higher pass through of the idiosyncratic component of marginal product. To offset this effect on $\text{var}(\epsilon_{ijt}^W) / \text{var}(\epsilon_{ijt}^h)$, $1/(1-\rho)$ must rise to 0.46.⁵⁹ Thus, our model infers that production processes became, in this sense, more flexible in this period (though this was not an explicit aim of the Agreement). Last, the increase in the comovement of working time and wages requires larger idiosyncratic productivity shocks.

6.3 Further tests

We close this section by discussing a few “overidentifying tests,” which confront the model with moments not used in estimation.

First, recall that the model’s wage is flexible in the sense that it does not smooth variation owing to idiosyncratic events (θ, ξ) . This means that, in a statistical (regression) sense, working time in the model can be accounted for by firm productivity (i.e., firm-year effects) and wage rates, which summarize the remaining variation coming from idiosyncratic forces. This prediction is challenged by evidence in Card (1990, 1994), which shows that wage rates add little explanatory power in working time regressions. This suggests that firms can indeed smooth out some of the idiosyncratic variation.⁶⁰ But note that this source of compression in within-firm earnings growth (relative to working time fluctuations) in the

⁵⁹Interestingly, we estimate a lower degree of complementarity even though $\text{var}(\epsilon_{ijt}^W) / \text{var}(\epsilon_{ijt}^h)$ is higher. Thus, despite the intuitive mapping between the latter moment and ρ , one must still take into account the implications of changes in other parameters, such as η , for $\text{var}(\epsilon_{ijt}^W) / \text{var}(\epsilon_{ijt}^h)$.

⁶⁰Firms can forego increasing earnings when ξ is high if they can commit to not reducing earnings when ξ is low. In the interest of tractability, our bargaining protocol rules out commitment.

data will be mistakenly attributed by our model to (a lack of) complementarities. Thus, we will *over-estimate* the elasticity of substitution.

Second, our model treats the firm as a unitary actor. This restriction is made because our data does not offer information on sub-firm organizations, such as the departments within a firm. But, this parsimony does imply strong restrictions on the data.

To illustrate, one can rearrange the FOC (3) into a simple (approximate) regression model for the change in an individual's time worked,

$$\Delta \ln h \cong \mathbb{E}[\Delta \ln h] - \frac{1}{\varphi + 1 - \rho} \Delta \ln \xi, \quad (22)$$

where $\Delta \ln \xi$ is the i.i.d. change in this worker's preference and is orthogonal to $\mathbb{E}[\Delta \ln h]$, the average log change in time worked at the firm.⁶¹ The prediction is that, *regardless* of the degree of complementarities, the coefficient on $\mathbb{E}[\Delta \ln h]$ should be 1. The reason for this reflects the restriction we have made in the production structure: there is a single common component, the shift in firm productivity $\Delta \ln Z$, and $\mathbb{E}[\Delta \ln h]$ is a proxy for this. When we run the regression (22) on Veneto data, however, we find a coefficient on $\mathbb{E}[\Delta \ln h]$ of 0.419. We conjecture that this result may point to a production structure in which there is no single, common component; rather, divisions within a firm react to their own “common” components, and these are imperfectly correlated.

Even if the production structure is more intricate, however, our estimation strategy can still identify economically important quantities. In particular, our use of the moment, $\text{var}(\epsilon_{ijt}^W) / \text{var}(\epsilon_{ijt}^h)$, to infer ρ should uncover the degree to which changes in time worked are compressed throughout the firm as a whole. In this sense, our approach should uncover a notion of the *average* degree of complementarities that operate firm-wide.

7 Conclusion

This paper has pursued the idea that an individual's labor supply is bound up with the working time choices of her colleagues within the firm. We have developed a tractable theory of earnings, working time, and employment demand that formalizes this idea. In particular,

⁶¹To derive this, calculate $\mathbb{E}[\Delta \ln h]$ implied by (3) and then use it to substitute for the terms in (3) common to all workers. The precise result will have an additional term, relative to (22). This term is $\Delta \ln n - \mathbb{E}[\Delta \ln n]$, where $\Delta \ln n$ measures the change in the size of a worker's ξ -cohort when the worker switches type from $\xi = x$ to $\xi = x'$. But this term, which isolates variation specific to the worker, is unlikely to be strongly correlated with *firm-wide* shifts in time worked, so (22) can be consistently estimated if the term is omitted.

the model expresses the intuition that, if there are sufficiently strong complementarities, working time adjustments across employees inside a firm are compressed, regardless of the true Frisch elasticity of labor supply. The Frisch elasticity is better informed in this setting by variation at the firm level; intuitively, firm-wide productivity movements serve to coordinate employees' working time and elicit the true elasticity.

We then showed how to estimate the model's structural parameters using moments from a matched employer-employee dataset from Veneto, Italy. Using the model's estimates, we carried out a simple counterfactual to explore the consequences of failing to control for complementarities in conducting inference about labor supply elasticities. We find that if one estimates the Frisch elasticity using only variation in labor supply incentives idiosyncratic to a worker, the estimate will be biased down by more than 50 percent.

We see a number of ways to further advance this line of research. First, data on total hours would be valuable. If this is not included in available firm-worker matched panels, we hope researchers and statistical agencies can field smaller-scale matched datasets that focus attention on measurement of the intensive margin. It would be particularly helpful if such datasets also included richer information on the firm (than is available in our Veneto panel), such as revenue and investment. With respect to theory, we see several profitable extensions of our framework. First, we have omitted a treatment of union work rules and legislative restrictions on work schedules. But the model could be used more widely to examine policy interventions if these features were included. Second, we have assumed flexible wage setting, which means that high-frequency variation in productivity and preferences passes through to earnings. We hope that future work can consider a richer theory of wage setting, which allows for long-term contracting.

8 References

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9 Appendix: Proofs

9.1 The firm's problem

In what follows, we need a weak restriction on the revenue function, \hat{F} .

Assumption 1 *The parameter, ρ , satisfies $\rho < \alpha$.*

This has two implications. First, it guarantees that \hat{F} is concave, that is, the Hessian, $\nabla^2 \hat{F}(\mathbf{n}, Z)$, is negative definite. Second, it implies that \hat{F} is supermodular, in that $\frac{\partial}{\partial Z} \frac{\partial \hat{F}}{\partial n_\xi} > 0$ for any type ξ and $\frac{\partial^2}{\partial n_\xi \partial n_x} \hat{F}(\mathbf{n}, Z) > 0$ for any $\xi \neq x$. We assume that these properties of \hat{F} pass to period profit, π . They will be verified once a solution for the wage bargain is obtained.

Conjecture 1 *The period profit function, $\pi(\mathbf{n}, Z)$, is concave and supermodular.*

The next lemma provides a key intermediate result in the characterization of the optimal policy. Since its proof relies on standard techniques, it is omitted here.

Lemma 1 *The value function, Π , is concave and supermodular, under Conjecture 1.*

Proof. See online appendix. ■

We are now prepared to prove Proposition 2.⁶²

⁶²Since the result of Proposition 2 is used to analyze the wage bargain, we present it first.

Proof of Proposition 2. The optimal employment level of the first-to-be separated type ξ is dictated by the first-order condition,

$$\tilde{\Pi}_{n_\xi}^-(\mathbf{n}, N_{-1}, Z) \equiv \frac{\partial \pi(n_\xi, \boldsymbol{\lambda}_{/\xi} N_{-1}, Z)}{\partial n_\xi} + \beta \mathbb{E}[\Pi_N(N, Z') | Z] + \underline{c} = 0, \quad (23)$$

where $\boldsymbol{\lambda}_{/\xi}$ is a $(M - 1) \times 1$ vector of employment shares *exclusive* of the type- ξ share and $N = n_\xi + \sum_{x \neq \xi} \lambda_x N_{-1}$. By supermodularity, the left side of (23) is increasing in Z for any n_ξ . Setting $n_\xi = \lambda_\xi N_{-1}$, it follows that there is a threshold $\zeta_\xi(N_{-1})$ such that the firm separates from type ξ when Z falls below $\zeta_\xi(N_{-1})$, driving $\tilde{\Pi}_{n_\xi}^-(\boldsymbol{\lambda} N_{-1}, N_{-1}, Z)$ below zero. At this point, the firm adjusts n_ξ according to (23). This yields a labor demand policy rule $n_\xi = \nu_\xi(N_{-1}, Z)$, where $\frac{\partial}{\partial Z} \nu_\xi > 0$.

At lower values of Z , the firm will separate from a(nother) type, denoted by $\hat{\xi} \neq \xi$, if the marginal value of that cohort falls below $-\underline{c}$,

$$\frac{\partial \pi(\nu_\xi(N_{-1}, Z), \boldsymbol{\lambda}_{/\xi} N_{-1}, Z)}{\partial n_{\hat{\xi}}} + \beta \mathbb{E}[\Pi_N(N, Z') | Z] < -\underline{c}, \quad (24)$$

where $N \equiv \nu_\xi(N_{-1}, Z) + \sum_{x \neq \xi} \lambda_x N_{-1}$. Note that since the FOC (23) remains in effect as Z falls below $\zeta_\xi(N_{-1})$, this derivative is evaluated at the optimal size of cohort ξ , $\nu_\xi(N_{-1}, Z)$. Therefore, at lower Z , the left side declines, for two reasons: the direct effect of lower productivity, and the indirect effect of a reduction in a complementary factor, n_ξ . It follows that, at some lower Z , (24) will take hold, and the firm will separate from type $\hat{\xi}$.

When separations of $\hat{\xi}$ -workers begins, the firm continues to separate from type- ξ workers. This follows immediately from the supermodularity of $\tilde{\Pi}$: if $n_{\hat{\xi}}$ is reduced, the marginal value of type- ξ labor declines, and n_ξ must be reduced to enforce the FOC (23).

Summarizing, there exists functions $\zeta_{\hat{\xi}}(N_{-1}) < \zeta_\xi(N_{-1})$ such that the firm separates from *both* type ξ and $\hat{\xi}$ workers if $Z < \zeta_{\hat{\xi}}(N_{-1})$. Since type ξ is the first type to separate, it is the rank-1 type and denoted by ξ_1 . Similarly, we refer to $\hat{\xi}$ as the rank-2 type and set $\hat{\xi} \equiv \xi_2$. It is straightforward to repeat this analysis for the other types, thereby establishing the ordering of types from rank 1 to rank M . ■

In line with our notation from Proposition 2, we will, in what follows, refer to an arbitrary type as type- ξ if its rank within the firm is unimportant in the context of the discussion. Otherwise, we will refer to a type as type- j , where j denotes its rank.

To complete the description of the optimal policy, we consider when the firm will hire. The supermodularity of the problem implies that a firm will hire only if $Z > \zeta_0(N_{-1})$, where ζ_0 is the critical point at which the marginal value of labor, assessed at N_{-1} , is equal to \bar{c} , the cost of a new hire. Formally, $\zeta_0(N_{-1})$ solves the indifference relation, $\frac{\partial \Pi^-(N, \zeta_0(N_{-1}))}{\partial N} \Big|_{N=N_{-1}} = \bar{c}$. The only matter that remains is to verify if the firm will hire and separate simultaneously.

Under certain conditions, it will not. In the interest of space, however, the proof of this next claim is omitted here.

Lemma 2 *If $\bar{c} + \underline{c}$ is sufficiently large, then the firm will never hire if it also separates.*

Proof. See online appendix. ■

9.2 Wage bargaining

Under Stole and Zwiebel, all workers and the firm can reopen a pairwise negotiating session at any time in the bargaining round. In that session, they split their match surplus. It follows that, at the conclusion of bargaining, the earnings agreement of any type ξ solves the surplus sharing rule,

$$\mathcal{W}_\xi(\mathbf{n}, Z) - \mathcal{U} = \eta (\mathcal{W}_\xi(\mathbf{n}, Z) - \mathcal{U} + \mathcal{J}_\xi(\mathbf{n}, Z) + \underline{c}), \quad (25)$$

where \mathcal{W}_ξ is the present value to a type- ξ employee of working at a firm of productivity Z taking as given the firm's remaining workers, \mathbf{n} ; \mathcal{U} is the value of non-employment (including the flow value of leisure as well as the present value of job search); and $\mathcal{J}_\xi(\mathbf{n}, Z)$ is the value to a firm of employing a worker of type ξ . Proposition 1 asserts that (25) rule yields the wage bargain (5) in the main text.

Proof of Proposition 1. The marginal contribution of any type- ξ worker to the firm, *gross* of the separation cost \underline{c} , is

$$\mathcal{J}_\xi(\mathbf{n}, Z) \equiv \pi_\xi(\mathbf{n}, Z) + \beta \int \Pi_N(N, Z') dG(Z'|Z), \quad (26)$$

where $\pi_\xi(\mathbf{n}, Z)$ is the marginal effect of type- ξ labor on period profit:

$$\pi_\xi(\mathbf{n}, Z) \equiv \frac{\partial \hat{F}(\mathbf{n}, Z)}{\partial n_\xi} - \left[W_\xi(\mathbf{n}, Z) + \frac{\partial W_\xi(\mathbf{n}, Z)}{\partial n_\xi} n_\xi + \sum_{x \neq \xi} \frac{\partial W_x(\mathbf{n}, Z)}{\partial n_\xi} n_x \right]. \quad (27)$$

The expected marginal value of labor in (26) can be decomposed using Leibniz's rule,⁶³

$$\int \Pi_N(N, Z') dG = \sum_{j=1}^M \int_{\zeta_{j+1}(N)}^{\zeta_j(N)} \Pi_N^j(N, Z') dG + \int_{\zeta_1(N)}^{\zeta_0(N)} \Pi_N^0(N, Z') dG + \int_{\zeta_0(N)}^{\infty} \Pi_N^+(N, Z') dG, \quad (28)$$

⁶³We will often abbreviate $dG(Z'|Z)$ by dG .

where the term Π^j , with $j = 1, \dots, M$, denotes the value of the firm in states of the world in which it separates from *all* types indexed by $i \leq j$.⁶⁴ The value of the firm in states of the world in which it freezes is given by Π^0 . If the firm hires, it is valued at Π^+ .

We next describe the marginal value of labor if the firm adjusts. If the firm hires, the Envelope theorem implies,

$$\Pi_N^+ (N, Z') = \bar{c}. \quad (29)$$

To treat the case of separations, return to (16) and consider the state in which the firm separates only from type-1 labor, that is, workers with taste ξ_1 . The value of the firm is

$$\Pi^1 (N, Z') = \pi (\nu_1 (N, Z'), \boldsymbol{\lambda}_{/1} N, Z') - \underline{c} [\lambda_1 N - \nu_1 (N, Z')] + \beta \int \Pi (N', Z'') dG,$$

where $\nu_1 (N, Z')$ denotes the optimal choice of type-1 labor conditional on adjusting; $\boldsymbol{\lambda}_{/1} \equiv (\lambda_2, \dots, \lambda_M)$ is the vector of labor shares exclusive of type-1 labor; and $N' = \nu_1 (N, Z') + \sum_{i=2}^M \lambda_i N$. By the Envelope theorem,

$$\Pi_N^1 (N, Z') = -\lambda_1 \underline{c} + \sum_{i=2}^M \lambda_i \widehat{\mathcal{J}}_i^1 (N, Z'), \quad (30)$$

where

$$\widehat{\mathcal{J}}_i^1 (N, Z') \equiv \frac{\partial \pi (\nu_1 (N, Z'), \boldsymbol{\lambda}_{/1} N, Z')}{\partial n_i} + \beta \int \Pi_{N'} (N', Z'') dG (Z'' | Z').$$

An additional worker exacts a cost on the firm, $-\underline{c}$, with probability λ_1 but otherwise contributes a marginal increase in firm value $\widehat{\mathcal{J}}_i^1 (N, Z')$ with probability λ_i . Generalizing from (30), we have that for any state $Z \in [\zeta_{j+1} (N), \zeta_j (N)]$ with $j \geq 1$,

$$\Pi_N^j (N, Z') = -\Lambda_j \underline{c} + \sum_{i=j+1}^M \lambda_i \widehat{\mathcal{J}}_i^j (N, Z'), \quad (31)$$

where $\Lambda_j \equiv \sum_{i=1}^j \lambda_i$ and

$$\widehat{\mathcal{J}}_i^j (N, Z') \equiv \frac{\partial \pi (\boldsymbol{\nu}_j (N, Z'), \boldsymbol{\lambda}_{/j} N, Z')}{\partial n_i} + \beta \int \Pi_{N'} (N', Z'') dG. \quad (32)$$

The marginal value of labor in the “freezing” regime, $\Pi_N^0 (N, Z')$, can be obtained as follows. Forwarding (16)-(17) one period, setting $s'_\xi = 0 \forall \xi$ and $\mathcal{N} = N_{-1}$, noting that $\mathbf{n}' = \mathbf{n} = \boldsymbol{\lambda} N$ in this case, and differentiating with respect to N yields

$$\Pi_N^0 (N, Z') = \sum_{\xi \in X} \lambda_\xi \frac{\partial \pi (\mathbf{n}, Z')}{\partial n_\xi} + \beta \int \Pi_N (N, Z'') dG (Z'' | Z'), \quad (33)$$

⁶⁴The ordering of types from 1 to M follows their ranking described in Proposition 2, and the sequence, $\{\zeta_j\}_{j=1}^M$, represents the corresponding thresholds governing separation. We define $\zeta_{M+1} (N) \equiv \min \{Z\}$, the minimum of the support of Z . The firm then separates from all types if $Z < \zeta_M (N)$.

But now recalling (26), evaluating the latter at $\mathbf{n} = \boldsymbol{\lambda}N$, and taking a weighted average of J_ξ across cohorts reveals that

$$\Pi_N^0(N, Z') = \sum_{\xi \in X} \lambda_\xi J_\xi(\boldsymbol{\lambda}N, Z') = \sum_{j=1}^M \lambda_j J_j(\boldsymbol{\lambda}N, Z'). \quad (34)$$

Substituting (29), (31), and (34) into (28) and inserting the resulting expression into (26) gives

$$\begin{aligned} J_\xi(\mathbf{n}, Z) &\equiv \pi_\xi(\mathbf{n}, Z) \\ -\beta c \sum_{j=1}^M \lambda_j G(\zeta_j(N) | Z) + \beta \sum_{j=1}^M \sum_{i=j+1}^M \lambda_i \int_{\zeta_{j+1}(N)}^{\zeta_j(N)} \widehat{\mathcal{J}}_i^j(N, Z') dG \\ + \beta \int_{\zeta_1(N)}^{\zeta_0(N)} \sum_{j=1}^M \lambda_j J_j(\boldsymbol{\lambda}N, Z') dG + \beta \bar{c}(1 - G(\zeta_0(N)) | Z), \end{aligned} \quad (35)$$

where we have used

$$\sum_{j=1}^M \Lambda_j [G(\zeta_j(N) | Z) - G(\zeta_{j+1}(N) | Z)] = \sum_{j=1}^M \lambda_j G(\zeta_j(N) | Z).$$

We next characterize the employee's surplus. The instantaneous return on working equals earnings less the cost of effort, $W_\xi(\mathbf{n}, Z) - \xi g_\xi(\mathbf{n})$, where $g_\xi(\mathbf{n}) \equiv \frac{h_\xi(\mathbf{n})^{1+\varphi}}{1+\varphi}$. In the next period, a worker will draw one of M types. Conditional on some $\xi_j \in X$, the worker is separated if $Z' < \zeta_j(N)$. Accordingly, the present value of working at a firm (\mathbf{n}, Z) is

$$\mathcal{W}_\xi(\mathbf{n}, Z) = \frac{W_\xi(\mathbf{n}, Z) - \xi g_\xi(\mathbf{n})}{+ \beta \sum_{i=1}^M \lambda_i [\mathbb{E}_{Z'}[s_i(N, Z') \cdot \mathcal{U} + (1 - s_i(N, Z')) \cdot \mathcal{W}_i(\mathbf{n}', Z')]]},$$

where s_j is the probability that an individual worker of type ξ_j is separated. Rearranging terms, this can be written in terms of the surplus from work, $\mathcal{S}_\xi^W(\mathbf{n}, Z) \equiv \mathcal{W}_\xi(\mathbf{n}, Z) - \mathcal{U}$,

$$\mathcal{S}_\xi^W(\mathbf{n}, Z) = W_\xi(\mathbf{n}, Z) - \xi g_\xi(\mathbf{n}) - r\mathcal{U} + \beta \sum_{i=1}^M \lambda_i \mathbb{E}_{Z'}[(1 - s_i(N, Z')) \mathcal{S}_i^W(\mathbf{n}', Z')], \quad (36)$$

where $r \equiv 1 - \beta$.

To assess (36), suppose the worker is type (or, rank) 1. Then by (25),

$$\mathbb{E}_{Z'}[(1 - s_1(N, Z')) \mathcal{S}_1^W(\mathbf{n}', Z')] = \frac{\eta}{1-\eta} \mathbb{E}_{Z'}[(1 - s_1(N, Z')) \cdot [\text{firm's surplus}(N, Z') + \underline{c}]].$$

The firm's surplus in this expression can be derived as follows. By Proposition 2, separations occur if $Z' < \zeta_1(N)$, and the marginal value of type-1 labor in any such state must be $-\underline{c}$. If, on the other hand, $Z' \in [\zeta_1(N), \zeta_0(N)]$, the employer "freezes" and earns $J_1(\boldsymbol{\lambda}N, Z')$, given by (26). Lastly, if $Z' > \zeta_0(N)$, the firm hires, and the marginal value of labor must

be \bar{c} . Collecting these pieces and noting that $s_1(N, Z') = 0$ if $Z' \geq \zeta_1(N)$, we have

$$\mathbb{E}_{Z'} [(1 - s_1(N, Z')) \mathcal{S}_1^W(\mathbf{n}', Z')] = \frac{\eta}{1-\eta} \left\{ \underline{c} \int_{\zeta_1(N)}^{\zeta_1(N)} dG + \int_{\zeta_1(N)}^{\zeta_0(N)} \mathcal{J}_1(\boldsymbol{\lambda}N, Z') dG + \int_{\zeta_0(N)} \bar{c} dG \right\}. \quad (37)$$

Next, consider the continuation value of a rank-2 worker. The only difference with respect to (37) is that the worker is not subject to separation in states $Z' \in (\zeta_2(N), \zeta_1(N))$. Instead, the firm “freezes” rank-2 labor, and earns a surplus of $\widehat{\mathcal{J}}_2^1(N, Z')$, as given by (32). Applying this line of reasoning to higher-ranked types shows that

$$= \frac{\eta}{1-\eta} \left\{ \underline{c} \int^{\zeta_i(N)} dG + \sum_{j=1}^{i-1} \int_{\zeta_{j+1}(N)}^{\zeta_j(N)} \widehat{\mathcal{J}}_i^j(N, Z') dG + \int_{\zeta_1(N)}^{\zeta_0(N)} \mathcal{J}_i(\boldsymbol{\lambda}N, Z') dG + \int_{\zeta_0(N)} \bar{c} dG \right\}$$

for any type i . Substituting these results into (36) and collecting terms,

$$\begin{aligned} \mathcal{S}_\xi^W(\mathbf{n}, Z) &= W_\xi(\mathbf{n}, Z) - \xi g_\xi(\mathbf{n}) - r\mathcal{U} \\ &+ \beta \frac{\eta}{1-\eta} \left\{ \underline{c} \sum_{i=1}^M \lambda_i [1 - G(\zeta_i(N) | Z)] + \sum_{i=1}^M \lambda_i \sum_{j=1}^{i-1} \int_{\zeta_{j+1}(N)}^{\zeta_j(N)} \widehat{\mathcal{J}}_i^j(N, Z') dG \right. \\ &\quad \left. + \sum_{i=1}^M \int_{\zeta_1(N)}^{\zeta_0(N)} \lambda_i \mathcal{J}_i(\boldsymbol{\lambda}N, Z') dG + \bar{c} [1 - G(\zeta_0(N) | Z)] \right\} \end{aligned} \quad (38)$$

Now substituting from (35) and (38) into (25) and using (27), we have

$$W_j(\mathbf{n}, Z) = \eta \left\{ \frac{\partial \hat{F}(\mathbf{n}, Z)}{\partial n_j} - \sum_{i=1}^M \frac{\partial W_i(\mathbf{n}, Z)}{\partial n_j} n_i + r\underline{c} \right\} + (1 - \eta) (\xi g_j(\mathbf{n}) + r\mathcal{U}), \quad j = 1, \dots, M. \quad (39)$$

The solution to this system of partial differential equations is (Cahuc et al, 2008))

$$W_j(\mathbf{n}, Z) = \eta \left[A \frac{\partial \hat{F}(\mathbf{n}, Z)}{\partial n_j} + r\underline{c} \right] + (1 - \eta) (A \xi g_j(\mathbf{n}) + r\mathcal{U}), \quad (40)$$

where $A \equiv \frac{\varphi+1-\alpha}{(\varphi+1)(1-\eta(1-\alpha))-\alpha}$. Using (4) and the solution for working time, one can calculate period profit and confirm Conjecture 1. ■

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