CESifo WORKING PAPERS

6872
2018

January 2018

Rationality in Economics: Theory and Evidence

Sanjit Dhami, Ali al-Nowaihi

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editors: Clemens Fuest, Oliver Falck, Jasmin Gröschl
www.cesifo-group.org/wp

An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: www.CESifo-group.org/wp

CESifo *Working Paper No. 6872*
Category 13: Behavioural Economics

Rationality in Economics: Theory and Evidence

Abstract

We examine the various senses in which economist use the term "rationality" and then outline some of the commonly drawn implications and auxiliary assumptions. Finally, we confront the implications with the empirical evidence, drawing on the insights from the exciting new field of behavioral economics.

JEL-Codes: B400.

Keywords: rationality, self-regarding preferences, efficient markets, heuristics, optimization.

Sanjit Dhami
Division of Economics
School of Business
University of Leicester
University Road
United Kingdom – Leicester LE1 7RH
Sanjit.Dhami@le.ac.uk

Ali al-Nowaihi
Division of Economics
School of Business
University of Leicester
University Road
United Kingdom – Leicester LE1 7RH
aa10@le.ac.uk

24 January 2018
This is a version of an article that is forthcoming in the MIT Handbook of Rationality.

1. Introduction

Rationality has diverse meanings in the social sciences and in common discourse. The dominant paradigm in economics, *neoclassical economics*, defines rationality in a precise way. This forms the basis of a coherent body of economic theory that allows for sharp and testable predictions. However, economists typically justify their assumption of rationality on grounds of plausibility and tractability rather than on the only admissible grounds in science: Conformity with the empirical evidence.

The plan of this essay is as follows. In Section 2, we specify the meaning of rationality in economics. Section 3 considers some auxiliary assumptions that are either regarded as part of a broader concept of rationality or are sometimes mistakenly believed to be consequences of rationality, as defined in Section 2. Finally, in Section 4, we pit the neoclassical view of rationality against the evidence that has emerged over the last several decades from the exciting and fastest growing field in economics: behavioral economics. It is impossible to cite all the papers in such a short essay, so we alert the reader to surveys that contain other relevant references.

2. Rationality in economics

In this section we explain the precise nature of rationality in economics. In each case, the primitives are the preferences that individuals have over appropriately defined sets of objects.

2.1. The assumption of rationality under certainty

Suppose that an individual has a set, $X \subset \mathbb{R}^n$, of choices, where $\mathbf{x} \in X$ may be interpreted as a bundle of n goods or services. Let $\succeq$ be a binary preference relation: for $\mathbf{x}, \mathbf{y} \in X$, $\mathbf{x} \succeq \mathbf{y}$ means that $\mathbf{x}$ is considered 'at least as good as' $\mathbf{y}$.

Definition 1 : *The individual is rational, or equivalently, the preference relation $\succeq$ is rational, if the following hold:*
1. (Completeness) For all $\mathbf{x}, \mathbf{y} \in X$ we have either $\mathbf{x} \succeq \mathbf{y}$ or $\mathbf{y} \succeq \mathbf{x}$.
2. (Transitivity) Let $\mathbf{x}, \mathbf{y}, \mathbf{z} \in X$, $\mathbf{x} \succeq \mathbf{y}$ and $\mathbf{y} \succeq \mathbf{z}$. Then $\mathbf{x} \succeq \mathbf{z}$.

Definition 2 : *If $\mathbf{x} \succeq \mathbf{y}$ then we say that $\mathbf{x}$ is at least as good as $\mathbf{y}$. If $\mathbf{x} \succeq \mathbf{y}$ and $\mathbf{y} \succeq \mathbf{x}$ then we say that $\mathbf{x}$ is indifferent to $\mathbf{y}$ and we write this as $\mathbf{x} \sim \mathbf{y}$. On the other hand, if $\mathbf{x} \succeq \mathbf{y}$ but it is not the case that $\mathbf{y} \succeq \mathbf{x}$, then we say that $\mathbf{x}$ is strictly preferred to $\mathbf{y}$ and we write $\mathbf{x} \succ \mathbf{y}$.*

The *assumption of rationality* states that a decision maker is rational in the sense of Definition 1, i.e., a decision maker has a preference relation, $\succeq$, on the set of choices, X, such that pairwise comparisons between any two choices can be made (completeness) and preferences are transitive.

Note that a violation of rationality (Definition 1) in itself does not entail a violation of any law of logic, nor does it entail that the individual is irrational in the legal or psychological senses.

There is a widespread consensus among economists that the assumption of rationality is plausible. However, the crucial question is whether it conforms to human behavior. We examine the evidence in Section 4.

Also, note that the definition of rationality, Definition 1, can be stated for *any* non-empty set X, not just $X \subset \mathbb{R}^n$. For example, X could be a set of lotteries (see subsection 2.2, below) or consumption streams (see subsection 2.3, below). However, restricting X to be a subset of $\mathbb{R}^n$ allows us to introduce a useful continuity property (without explicitly defining a topology on X).

Definition 3 *(Continuity): The preference relation $\succeq$ is continuous if, for each $\mathbf{y} \in X$, both the sets $\{\mathbf{x} \in X : \mathbf{y} \succeq \mathbf{x}\}$ and $\{\mathbf{z} \in X : \mathbf{z} \succeq \mathbf{y}\}$ are closed.*

A *utility function* is simply a useful alternative, and equivalent, way of expressing the preference relation $\succeq$.

Definition 4 *(Utility function) A utility function $u : X \to \mathbb{R}$ represents the preference relation $\succeq$, if for $\mathbf{x}, \mathbf{y} \in X$, $\mathbf{x} \succeq \mathbf{y}$ if, and only if, $u(\mathbf{x}) \geq u(\mathbf{y})$.*

But does such a utility function exist?

Proposition 1 *: Suppose that the preference relation $\succeq$ on X is complete, transitive and continuous. Then, there exists a continuous utility function u that represents $\succeq$.*

The converse of Proposition 1 also holds.

Proposition 2 *: If a continuous function u represents the preference relation $\succeq$ on X, then $\succeq$ must be complete, transitive and continuous.*

It is often more convenient to work with a utility function than with preferences.

2.2. Rationality under risk

Risk is pervasive in most economic problems. Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed finite set of real-valued outcomes such that $x_1 \leq x_2 \leq ... \leq x_n$.[1] A *simple lottery*, or *simple gamble*, L, is a shorthand way of capturing risk.

$$L = (x_1, p_1; x_2, p_2; ...; x_n, p_n), \tag{2.1}$$

where $p_1, ..., p_n$ are the respective probabilities corresponding to the outcomes $x_1, ..., x_n$ such that $p_i \in [0, 1]$ and $\sum_{i=1}^{n} p_i = 1$. The lottery $(x_i, 1)$ denotes an outcome x_i received with certainty; this is the situation considered in subsection 2.1.

We shall also consider compound lotteries, where the outcomes $L_1, L_2, ..., L_n$ (or prizes) are themselves lotteries:

$$L = (L_1, p_1; L_2, p_2; ...; L_n, p_n), \tag{2.2}$$

thus, starting with simple lotteries, (2.1), we can compound lotteries to any level.

Let $\mathcal{L}$ be the set of all lotteries over X (simple and compound). Which lottery will the decision maker choose?

The most widely used theory under risk in economics is *expected utility theory* (EU). Suppose that the decision maker is endowed with a binary preference relation over $\mathcal{L}$ denoted by $\succeq$. So, for $L_1, L_2 \in \mathcal{L}$, $L_2 \succeq L_1$ means that the lottery L_2 is 'at least as good as' the lottery L_1.

Consider the following axioms on behavior under risk:

Axiom 1 *(Order) Order requires the following two conditions.*
1. Completeness: For all lotteries L_1, L_2, either $L_2 \succeq L_1$ or $L_1 \succeq L_2$.
2. Transitivity: For all lotteries $L_1, L_2, L_3 : L_3 \succeq L_2$ and $L_2 \succeq L_1 \Rightarrow L_3 \succeq L_1$.

Two further binary relations between lotteries can be defined in terms of $\succeq$:

$$\begin{cases} \text{Indifference:} & L_1 \sim L_2 \Leftrightarrow L_2 \succeq L_1 \text{ and } L_1 \succeq L_2. \\ \text{Strict preference:} & L_2 \succ L_1 \Leftrightarrow \text{it is not the case that } L_1 \succeq L_2. \end{cases}$$

Axiom 2 *(Best and worst): $x_n \succ x_1$ (i.e., $(x_n, 1) \succ (x_1, 1)$).*

Axiom 3 *(Continuity): For each lottery, L, there is a $p \in [0, 1]$ such that $L \sim (x_1, 1 - p; x_n, p)$.*

Axiom 4 *(Independence): For all lotteries L_1, L_2, L, and all $p \in [0, 1]$, $L_2 \succeq L_1 \Leftrightarrow (L_2, p; L, 1 - p) \succeq (L_1, p; L, 1 - p)$.*

[1] For ease of exposition, we restrict ourselves to a fixed finite set, X, of real valued outcomes. Extensions to vector valued outcomes are possible and to sets X that are not, necessarily, finite. See, for example, Kreps (1990); who also makes a strong case for exploring alternatives to EU.

Axiom 5 *(Reduction, or law of compounding lotteries): Let* p_1, p_2, $p \in [0,1]$. *Let* $L_1 \sim (x_i, 1-p_1; x_j, p_1)$ *and* $L_2 \sim (x_i, 1-p_2; x_j, p_2)$. *Then*

$$\begin{aligned} (L_1, p; L_2, 1-p) &\sim ((x_i, 1-p_1; x_j, p_1), p; (x_i, 1-p_2; x_j, p_2), 1-p) \\ &\sim (x_i, (1-p_1)p + (1-p_2)(1-p); x_j, pp_1 + (1-p)p_2). \end{aligned}$$

The axioms of completeness, transitivity and continuity introduced above are analogous to those introduced in subsection 2.1. From Axiom 2, under certainty, a decision maker strictly prefers receiving the highest to the lowest outcome. The continuity property in Axiom 3 asserts that every lottery (no matter how complex) is equivalent to a simple lottery obtained by probabilistically mixing the highest outcome, x_n, and the lowest outcome, x_1. The independence axiom, Axiom 4, requires that if a lottery, L_2, is preferred to a lottery, L_1, then mixing each with a third lottery, L, (with the same mixing probability, p) does not alter the preference.

The next definition gives 'Rationality' a precise meaning under risk.

Definition 5 *(Axioms of rationality under risk): The following axioms: Order, best and worst, continuity, independence and reduction, collectively, are termed as the axioms of rationality under risk.*

We need to point out that an implicit assumption is that a decision maker follows standard probability theory (Kolmogorov, 1950, original German, 1933). More on this in Section 4, below.

Definition 6 *(Representation): Let* $\succeq$ *be a binary relation on* $\mathcal{L}$ *and let* $EU : \mathcal{L} \to \mathbb{R}$ *be such that for all* $L_1, L_2 \in \mathcal{L}$, $L_2 \succeq L_1$ *if, and only if,* $EU(L_2) \geq EU(L_1)$. *Then the function* EU *is said to represent* $\succeq$ *and we say that* $\succeq$ *is induced by* EU.

Definition 7 *(Expected utility function): A utility function* $EU : \mathcal{L} \to \mathbb{R}$ *has the expected utility form (or the von Neumann-Morgenstern form) if we can assign a real number,* $u(x_i)$, *to each possible outcome,* x_i, *such that for the lottery* L *in (2.1), we can write*

$$EU(L) = \textstyle\sum_{i=1}^{n} p_i u(x_i). \tag{2.3}$$

We now state the main result, the existence of a von Neumann-Morgenstern expected utility representation.

Proposition 3 *: Suppose that the binary relation,* $\succeq$, *on the set of lotteries* $\mathcal{L}$, *satisfies the axioms of rationality under risk (Definition 5). Then there is a von Neumann-Morgenstern utility function,* $EU : \mathcal{L} \to \mathbb{R}$, *that represents* $\succeq$. *If* $EU(L) = \sum_{i=1}^{n} p_i u(x_i)$ *and* $EV(L) = \sum_{i=1}^{n} p_i v(x_i)$ *both represent* $\succeq$, *then there are real numbers* a *and* $b > 0$, *such that* $v(x_i) = a + bu(x_i)$.

The converse of Proposition 3 also holds.

Proposition 4 : *The binary relation, $\succeq$, on the set of lotteries $\mathcal{L}$, induced by a von Neumann-Morgenstern utility function satisfies the axioms of rationality (Definition 5).*

We note that this framework can be extended to uncertainty (where probabilities are subjective, rather than objective, but space limitations prevent an exposition of this subject matter (Savage, 1954; Dhami, 2016, Section 1.3).

2.3. Rationality in choices made over time

Let $Z \subset \mathbb{R}$ be the set of possible outcomes and let $\Gamma \subset \mathbb{R}_+$ be the set of corresponding times at which these outcomes are realized. Elements of the set Z are denoted by z_j, $j = 1, 2, ...$, and elements of the set Γ by t_j, $j = 1, 2, ...$ Denote by (z_j, t_j) an *outcome-time pair*, which *guarantees* the outcome z_j at time t_j. Let $\succeq$ denote the binary preference ordering, 'at least as good as', on the Cartesian product $X = Z \times \Gamma$. Notice that we have now redefined the set X in subsection 2.1 as outcome-time pairs. Also, let $\succ$, $\sim$ denote, respectively, the binary relations 'strictly preferred to' and 'indifferent to' on the set $X = Z \times \Gamma$. The following five axioms, B1-B5, in Fishburn and Rubinstein (1982) are typically called the *axioms of rationality for time discounting.*

Definition 8 *(Axioms of rationality for time discounting): Let $z_m, z_n \in Z$, and $t_p, t_q \in \Gamma$.*
B1. Ordering: $\succeq$ is complete and transitive.
B2. Monotonicity: If $z_m > z_n$, then $(z_m, t) \succ (z_n, t)$ for any $t \in \Gamma$ (larger rewards are preferred, keeping time fixed).
B3. Impatience: Let $t_q < t_p$, and $z > 0$, $z \in Z$. Then, $(z, t_q) \succ (z, t_p)$ (the same outcome is preferred at an earlier date). Also $(0, t_q) \sim (0, t_p)$ (no time preference for zero rewards).
B4. Continuity: Fix some outcome-time pair $(z_j, t_j) \in Z \times \Gamma$. Then $\{(z, t) : (z, t) \succeq (z_j, t_j)\}$ and $\{(z, t) : (z_j, t_j) \succeq (z, t)\}$ are closed.
B5. Stationarity: If $(z_m, t_q) \sim (z_n, t_p)$, then $(z_m, t_q + t) \sim (z_n, t_p + t)$, for any $t \in \Gamma$ (indifference between two outcomes depends only on the length of the time difference between the two).

We have already encountered the analogues of Axioms B1 and B4 in subsection 2.1. Axioms B2 and B3 are eminently reasonable. Axiom B5 is crucial to producing the additive separability under exponential discounting. Fishburn and Rubinstein (1982, p. 681) write: "However, we know of no persuasive argument for stationarity as a psychologically viable assumption."

The utility representation of the *axioms of rationality for time discounting* is given next.

Proposition 5 *(Fishburn and Rubinstein, 1982, p. 682): If B1-B5 hold, then for any* $\delta \in (0,1)$, *there exists a continuous increasing real valued function,* u, *defined on the domain* Z *such that:*

(i) For all (z_m, t_q), (z_n, t_p) *in* $Z \times \Gamma$, $(z_m, t_q) \succeq (z_n, t_p)$ *if, and only if,*

$$\delta^{t_q} u(z_m) \geq \delta^{t_p} u(z_n).$$

(ii) The outcome-time pair $(0, t)$ *is treated as* $\delta^t u(0) = 0$.

(iii) Given δ, u *is unique up to multiplication by a positive constant.*

The *axioms of rationality for time discounting* are testable; the evidence is considered in Section 4.

2.4. Rationality in game theory

Game theory deals with strategic situations in which individuals mutually influence the payoffs of others through their actions and beliefs. Rationality plays a key role in the development of the central concepts in game theory. These are already developed in Chapters 9.1, 9.2 and 9.3 in this Handbook, so we omit this discussion. We assume a knowledge of the basic equilibrium concepts (see, e.g., Fudenberg and Tirole, 1991).

In the simplest setting of a *static game of complete information*, a Nash equilibrium places no restrictions on the beliefs of players. Such belief restrictions are studied in the *epistemic foundations of a Nash equilibrium* and they turn out to be extremely demanding and unlikely to be met in practice. Suppose that the following conditions hold: Mutual knowledge of the payoffs, mutual knowledge of rationality of players, common knowledge of the beliefs (or conjectures) each player has about the others and common priors. Then, Aumann and Brandenburger (1995) showed that the common conjectures about what other players will do are in agreement and constitute a Nash equilibrium of the game. Polak (1999) showed that if mutual knowledge of the payoffs is strengthened to common knowledge of payoffs, then the Aumann-Brandenburger conditions imply common knowledge of rationality. Recent research offers an unsettled view of the epistemic foundations (Gintis, 2009).

If the predictions of a Nash equilibrium and its refinements are empirically violated by the empirical evidence, then so are the epistemic conditions. We briefly summarize the empirical evidence in Section 4.

A Nash equilibrium is often justified as the outcome of a learning process. However, such processes do not guarantee convergence of play to a Nash equilibrium and, when convergence does takes place, there is no guarantee that the underlying learning process is plausible (Fudenberg and Tirole, 1991, p. 25-29). Even when convergence does take place under a plausible learning process, there is no guarantee that it does so in a reasonable amount of time that is representative of actual real world interactions.

3. Some auxiliary assumptions

In this section, we briefly outline some auxiliary assumption that are commonly used in economics in addition to the assumption of rationality, given in Section 2. These assumptions are sometimes, mistakenly, believed to be consequences of the rationality assumptions of Section 2. At other times, they are regarded as part of a broader concept of rationality.

3.1. Individuals have self-regarding preferences

Individual i is said to have *other-regarding preferences* (or *social preferences*) if, the individual cares about own-consumption, x_i, and the consumption of others, x_{-i}, i.e., individual $i's$ utility function is $u_i(x_i, x_{-i})$. In Definitions 1, 4 and Propositions 1, 2, preferences are purely *self-regarding* since the utility of any individual i only depends on own-consumption, i.e., we have $u_i(x_i)$. This has led to a common narrative that rationality implies self-regarding preferences.

This is not correct. One may define the initial set X in any way one wishes to. For instance, for individual i, elements $x \in X$ may be appropriately defined as pairs of the form (x_i, x_{-i}). One can then proceed, as in Section 2, to derive a utility function with other-regarding preferences. However, in actual practice, neoclassical economics has not taken this direction.[2]

All this has changed with the advent of behavioral economics in which other-regarding preferences play a central role. We now know a great deal about the nature of other-regarding preferences through lab, field and neuroeconomic evidence. Behavioral models of other-regarding preferences have made several precise and confirmed predictions. Experiments have played a key role in the precise identification of other-regarding preferences. Experimental games such as the dictator game, the ultimatum game, public goods game, trust game and the gift exchange game, among many others, have allowed stringent tests of competing theories. For surveys, the interested reader may consult Fehr and Schmidt (2006) and Dhami (2016, Part 2).

3.2. There is no role for emotions in decision making.

Rationality in economics has also been taken to imply cold, emotionless, deliberation. This does appear to be an indirect implication of Definitions 1, 5, and 8. None of the axioms explicitly takes account of emotional states. In contrast, there has been growing interest in modelling emotions in behavioral economics.

[2] To be sure there are some models of keeping up with the Joneses and snob consumption but these have played only a fringe role in modern economics.

In principle, one could modify the classical framework by introducing a set of emotional states M and, for any $m \in M$, index the preference relation $\succsim$ in Section 2 by m to get $\succsim_m$; this applies to certainty, uncertainty and to the time dimension. This allows one's preferences to be dependent on emotions. In an emotional state, one may make erratic or inconsistent decisions. In anger, one could signal credibly one's own future intended actions. *Anxiety* is an important emotion that has been used to explain the equity-premium puzzle in finance (Caplin and Leahy, 2001). *Projection-bias*, the tendency to project our current emotional states to future states of the world, explains why those awaiting kidney transplants have relatively lower reported well-being as compared to a healthy control group, yet after the transplant, both groups report similar well-being (Loewenstein et al., 2003). For a large number of other applications and for the relevant evidence, the interested reader can consult Dhami (2016, Part 6). Behavioral game theory has used emotions to successfully explain diverse economic phenomena under the rubric of *psychological game theory* (Dhami, 2016, Section 13.5).

3.3. Perfect attention, unlimited computing power and no misperceptions

The choice set, X, whether it be the space of outcomes, lotteries, or time-outcome pairs, can be infinite (Section 2). Thus, the rationality assumption requires individuals to make a large number of pairwise comparisons in order to choose the most preferred or maximal element in the set X. In addition, transitivity must not be violated. This may require enormous computation power, memory, and attention. In contrast, the evidence shows that people are subject to inattention, misperception, and limited computation powers (see Section 4 below).

Consider the following typical example of what underlies almost all areas of economics.

Example 1 : *A self-regarding individual lives for T time periods and cares only about the dated stream of his consumption, i.e., $c_1, c_2, ..., c_T$. At time t, $t = 1, 2, ..., T$, he inherits a stock of savings, S_{t-1}, from the previous period. This gives him the interest payment rS_{t-1}, where $r \geq 0$ is the return on one-period bonds. In addition, he has income y_t from other sources (e.g., labour income). Thus, his total period t income is $y_t + rS_{t-1}$. This is spent on consumption c_t and the balance, $s_t = y_t + rS_{t-1} - c_t$, is saved. Thus $S_t = S_{t-1} + s_t$. Let his utility from period t consumption be $u(c_t)$ and his subjective discount factor be $\delta \in (0, 1)$. His objective at time t is to choose $c_t, c_{t+1}, ..., c_T$ so as to maximize the present value of his stream of utilities $\sum_{i=t}^{i=T} \delta^{t-i} u(c_i)$. At the time of his death, in period T, he cannot be in debt, so we impose the condition $S_T \geq 0$.*

The problem formulated in Example 1 can be given the following mathematical form that we refer to as Problem-T:

For each time t, $t = 1, 2, ..., T$, choose $c_t, c_{t+1}, ..., c_T$, so as to maximize

$$\sum_{i=t}^{i=T} \delta^{t-i} u(c_i) \text{ (objective function)}$$

subject to the following conditions:

$$\begin{cases} \text{Initial condition:} & S_{t-1} \text{ given} \\ \text{Dynamic equation:} & S_i = y_i + (1+r)\, S_{i-1} - c_i,\ i = t, t+1, ..., T \\ \text{Terminal condition:} & S_T \geq 0 \end{cases}$$

Problem-T is a simple example of a *dynamic optimization problem*. It is an *optimization problem* because we have an objective function we want to maximize. It is a *dynamic problem* because excessive consumption (say) in the early periods will not leave enough money for adequate consumption in the final periods. The initial state of the consumer is his initial stock of savings, S_0. His final state is zero savings (because it is suboptimal to leave unused savings). The control variables are the consumption levels.

The solution method to Problem-T is by *backward induction*. The consumer should start with the final period T. Then for *every* possible value of the stock of savings, S_{T-1}, inherited from the penultimate period, $T-1$, find the optimal level of consumption by choosing c_T so as to maximize $u(c_T)$ subject to $y_T + (1+r)\, S_{T-1} - c_T = 0$. Then move back in time and repeat for period $T-1$. The consumer should continue until reaching the first period, $t = 1$, and then substitute for the initial value for savings, S_0. This will then determine the optimal consumption levels $c_1^*, c_2^*, ..., c_T^*$. There are, however, three problems which we now turn to.

1. Although backward induction is conceptually simple, it requires a huge number of calculations. For example, consider a 40 year horizon and a period length of one year (so $T = 40$). Suppose that income y_t at each possible time t can take two possible values, high and low, and corresponding to this, we have just two possible optimal levels of consumption, low and high. Depending on the values of consumption, income and initial wealth, this could require the consumer to carry out up to $2^{40} > 10^{12}$ (one trillion) calculations.

2. Even when the problem is simplified, say, for a 3 year horizon, so that the total number of calculations is, at most, $2^3 = 8$, experimental subjects do not use backward induction (see Dhami, 2016, Part 4).

3. Any real life situation is far more complex than Example 1. For example, individual might be uncertain of the time of death, future incomes, and future rates of return. Bequests might have to be calculated optimally. Decisions might be taken in hot, and unanticipated, emotional states, and capital markets might be imperfect.

The maintained assumption in neoclassical economics is that the man on the street can solve these hard problems literally, in his head, in an instant (analytically or numerically), or behave in a manner *as if* he can solve the problem. However, when tested, the assumptions and predictions of many leading neoclassical models fail (see Section 4).

Furthermore, in many real life situations one might be distracted by other pressing issues (limited time, information overload, deadlines, marital discord, emotional lows) that make the supposed optimization exercise even more challenging. Yet, in real life, individuals routinely make complex decisions in the presence of uncertainty. So how do they do it? Could it be that they do not use the sort of optimization that economists believe that they engage in, and use instead simple rules of thumbs and various bounded rational methods? See subsection 4.5, below, and Dhami (2016, Part 7)

3.4. We can ignore irrationality in financial and other markets.

In economics, financial markets are typically held out to be the epitome of efficiency. The dominant tradition in finance argues that we can ignore any irrationality (in the sense considered in Section 2) in financial markets. This argument is based on two assumptions, but with little empirical support (Dhami, 2016; chapter 21). First, although irrational individuals, if any, may make errors, these errors will cancel out in the aggregate. Second, irrational market participants (or *noise traders*), if any, will be driven out of the market by the actions of rational market participants (or *arbitrageurs*).

The assumption of Bayesian rationality in finance is central to the *efficient markets hypothesis* (EMH), which is the cornerstone of modern finance.[3] The EMH asserts that the price of an asset at any point in time equals the present discounted value of the stream of future incomes from that asset, i.e., its fundamental value. The evidence, however, is that humans, even professionals, are poor at Bayesian updating, and many assets can deviate for long stretches of time from their fundamental values (Dhami, 2016, Section 19.7, 19.18 and Chapter 21).

4. The evidence on rationality in economics

In this section, we briefly consider the evidence on the rationality assumptions of Section 2 and the auxiliary assumptions of Section 3.

[3]Bayesian rationality augments the rationality assumptions in Section 2 with the assumption that individuals update their beliefs when new information arrives, by using Bayes' rule.

4.1. Evidence on the basic assumptions of rationality

The most basic definition of rationality (Definition 1) requires that preferences be complete and transitive. Both are violated in experiments. Iyengar and Lepper (2000) exposed subjects to either a limited choice condition (6 varieties of jams) or an intensive choice condition (24 varieties of jams). There was no difference in the percentage of jams sampled in both conditions. However, nearly 30% of the consumers in the limited choice condition purchased a jam jar. The corresponding number in the intensive condition was only 3%. This suggests that individuals may find it more difficult to make choices when there are too many options.

The most famous of the violations of transitivity occurs in the experiments on *preference reversals*; this is a problem in the domain of risk. Consider two probabilities $p < P$, two outcomes $z < Z$, and the following pair of lotteries.

$$P\text{-bet} = (0, 1 - P; z, P),\ \$\text{-bet} = (0, 1 - p; Z, p). \tag{4.1}$$

The P-bet has a higher probability, P, of winning and the \$-bet has a higher prize, Z. The decision maker is given the following two tasks.

1. Direct choice between the P-bet and the \$-bet.

2. Assign *certainty equivalents* to the P-bet and the \$-bet; these are denoted, respectively, by C_P and $C_\$$.[4]

The typical empirical finding, much replicated, is that the P-bet is chosen over the \$-bet in the first task and $C_P < C_\$$ in the second task (Lichtenstein and Slovic, 2006). Transitivity fails because \$-bet $\sim C_\$ \succ C_P \sim P$-bet $\succ$ \$-bet (where $\succ$ denotes strict preference).

Unlimited attention is implicit in the assumption of rationality (Definition 1). However, there is growing interest in the implications of *limited attention* that reflects limited cognitive abilities of humans. Here are some findings from this literature (Dhami, 2016, Section 19.17). Consumers give less attention to taxes that are not listed on price stickers but added at cash registers; displaying information about taxes on price stickers reduces sales. Individuals pay relatively less attention to electronic toll payments, allowing operators to levy higher tolls as compared to, say, cash paid at manual toll collections, which make the payment salient. Car sales and car prices are sensitive to integer multiples of the 10,000 mile threshold, so there is too large a gap in prices of cars with mileage readings of 9,999

[4] The certainty equivalent is a guaranteed sum of money which makes the decision maker indifferent between accepting the money or the lottery. Thus, \$-bet $\sim C_\$$ and $C_P \sim P$-bet, where $\sim$ denotes indifference.

and 10,000 miles because consumers pay too much attention to the leftmost digit. Limited attention is also increasingly used to explain suboptimal economic decisions of the poor whose attention is diverted by the basic problems of food, shelter and clean water; a form of *cognitive taxes* that reduce *mental bandwidth* (Datta and Mullainathan, 2014; World Development Report, 2015). There are now promising theoretical attempts to formalize limited attention in which consumers pay differential attention to various attributes of a good (Gabaix, 2014).

Despite this evidence, the common consensus in economics, and in behavioral economics, is to maintain a minimum degree of rationality as dictated by Definition 1. A compromise could be to retain rationality in the sense of Definition 1, but only within a suitably narrow frame or context (Gintis, 2009, 2017). However, it is not clear if all evidence that contradicts complete and transitive preferences could be explained by invoking different contexts and frames.

4.2. Evidence on rationality in risk and uncertainty

Consider Problems 3 and 4 in Kahneman and Tversky (1979), which illustrate the work of Allais (1953). These examples refute expected utility theory (EU). Let the utility function be u and assume $u(0) = 0$. Denote by $\succ$ the strict preference relation under EU.

Problem 3: (95 subjects) Choose between the two lotteries[5]

$$b_1 = (3000, 1) \text{ and } b_2 = (4000, 0.8),$$

80% chose b_1, while 20% chose b_2.

Problem 4: (95 subjects) Choose between the two lotteries

$$b_3 = (3000, 0.25) \text{ and } b_4 = (4000, 0.2),$$

35% chose b_3, while 65% chose b_4.

Thus, for the vast majority of subjects we have $b_1 \succ b_2$ and $b_4 \succ b_3$. The EU of lotteries b_1, b_2, b_3, b_4 is:

$$EU(b_1) = u(3000), \ EU(b_2) = 0.8u(4000), \ EU(b_3) = 0.25u(3000), \ EU(b_4) = 0.2u(4000). \tag{4.2}$$

It follows that

$$b_1 \succ b_2 \Leftrightarrow 0.8u(4000) < u(3000).$$

Dividing both sides by 4, we get

$$0.2u(4000) < 0.25u(3000) \Leftrightarrow b_3 \succ b_4.$$

[5]Recall that the lottery (x, p) means an outcome x received with probability $p \in [0, 1]$. Equivalently, and more fully, we can write (x, p) as $(0, 1-p; x, p)$.

Hence under EU, the prediction is $b_1 \succ b_2 \Rightarrow b_3 \succ b_4$, yet for most subjects $b_1 \succ b_2$ and $b_4 \succ b_3$ (also $b_2 \succ b_1 \Rightarrow b_4 \succ b_3$). So at least some decision makers must have violated EU. This phenomenon, *the Allais paradox*, is a violation of the independence axiom of EU (Axiom 4) and these findings have been widely replicated.

The Allais paradox can be explained as follows. In problem 4, the decision maker codes the probabilities 0.2 and 0.25 to be very close. Hence, he prefers b_4 to b_3 because $4000 > 3000$. In problem 3, the probability 0.8 is not coded as close to 1; the certain outcome is particularly salient (*certainty effect*), i.e., the decision maker *underweights* the probability 0.8 heavily relative to the probability of 1. Hence, *non-linear weighting of probabilities* (NLPW) explains the Allais paradox. On the other hand, EU requires that probabilities enter linearly into the utility function.

The evidence for NLPW has grown enormously over the years (Fehr-Duda and Epper, 2012). NLPW plays a central role in the modern alternatives to EU, such as *rank dependent utility* (RDU) due to Quiggin (1982) and *prospect theory* (PT) due to Kahneman and Tversky (1979) and Tversky and Kahneman (1992). PT has been enormously successful in explaining the evidence, particularly where both EU and RDU fail. PT has been successfully incorporated into formal economic models in many areas of economics; for surveys, see Kahneman and Tversky (2000) and Dhami (2016, Part 1).

These are by no means the only refutations of EU.[6] Other refutations include the following (Dhami, 2016, Chapter 3): unreasonable attitudes towards risk; the endowment effect; anomalies of tax evasion; the efficacy of goal-setting behavior; backward bending labour supply curve of taxi drivers; the equity-premium puzzle; contract choice and renegotiation; and the pricing of assets and the skewness of asset returns. Furthermore, close genetic relatives (e.g., primates such as capuchin monkeys) exhibit similar violations of EU and their behavior supports PT. The implication is that preferences supporting PT were hardwired into the common ancestors of humans and capuchin monkeys.

4.3. Evidence on rationality in time discounting

The utility representation of rational time preferences takes the form of the EDU model (Proposition 5). Under EDU, the discount factor, δ, is constant. Under EDU the same discount factor, δ, applies independent of (1) the magnitude of an outcome (absence of *magnitude effect*), and (2) the sign, positive or negative, of an outcome (absence of *gain-loss asymmetry*). Furthermore, preferences are independent of the shapes of consumption profiles.

Strong empirical evidence refutes each of these features of the EDU model. Behavioral

[6] The interested reader can pursue Kahneman and Tversky (2000), Starmer (2000) and Dhami (2016) for the details.

models of time discounting have been proposed that are able to account for all of these violations. Consider the following violations of the EDU model reported in Thaler (1981) that have been much replicated; see Dhami (2016, Chapter 9) for the updated evidence.

1. *Magnitude effect*: Subjects reported being indifferent to \$15 received now and \$60 in a year's time, \$250 received now and \$350 received in a year's time, \$3000 received now and \$4000 in a year's time. The implied respective annual discount factors are 0.25, 0.71 and 0.75; these are not identical as would be required by EDU. Thus, larger amounts are discounted less.

2. *Gain-Loss asymmetry*: The discount factors for losses turned to be larger than those for gains, so losses are discounted less than gains. This could be because losses are more salient than gains.

3. *Common difference effect*: When asked how much money in one month, one year, ten years, respectively, would make them indifferent to receiving \$15 now, the median response of subjects was \$20, \$50, and \$100, respectively. The respective implied annual discount factors are 3%, 30%, 83%. Hence, shorter the horizon, the lower the implied annual discount factor (greater impatience); in contrast, under EDU, the annual discount rate factor stays constant.

The common difference effect suggests a taste for immediate gratification. A major thrust of the behavioral economics literature is to explain this phenomenon. A tractable and popular alternative to EDU is *quasi-hyperbolic discounting* (Phelps and Pollak, 1968; Laibson, 1997) in which time preference are represented by:[7]

$$U\left(c_0, ..., c_T\right) = u\left(c_0\right) + \beta \sum_{t=1}^{T} \delta^t u\left(c_t\right), \ 0 < \beta < 1. \tag{4.3}$$

In (4.3), other than the presence of $\beta \in (0,1)$ the utility function is identical to the EDU model. β shrinks the value of future lifetime utility relative to current utility, thereby creating an additional *present bias for current consumption*, or a taste for immediate gratification. It also leads to a pattern of increasing discount rates as we approach the present that potentially explain the common difference effect.

A taste for immediate gratification creates self-control problems. Further, people have imperfect awareness about their future self-control problems, hence, they often take inadequate preventive measures. This also leads to the *time inconsistency problem* (i.e., choices made now about a future date t need not be optimal when the individual re-optimizes at time t). Classically time inconsistent choices were considered to be irrational in economics (and are impossible under EDU) but evidence suggests that they often occur.

This framework resolves several problems that are unexplained by the axioms of rationality for time discounting (Definition 8). Some of these are as follows (for details see

[7] See Dhami (2016, Ch. 10) for other alternative models that explain the common difference effect.

Dhami, 2016, Ch. 11) Why does consumption track income so closely? Why do individuals undersave for retirement? Why is there a sharp drop in consumption at retirement? Why do individuals hold illiquid assets and credit card debt simultaneously? What causes addictions? Why do some smokers pay more to buy smaller packs of cigarettes, or support sales taxes on cigarettes? Why do some people buy annual gym memberships when they could save money by paying on a pay-as-you-go basis

4.4. Evidence on rationality in strategic interaction

Classical game theory (CGT) makes precise and testable predictions. However, behavior in the early rounds of games and in games unfamiliar to the players is often inconsistent with the predictions. Learning and experience may or may not produce data consistent with the predictions (Camerer, 2003; Dhami, 2016, Parts 4, 5). Humans make many important decisions only a few times in their lives and experience limited learning (the choice of a University degree/marriage partner/house/consumer durables/pension plan). Firms also make many important decisions rarely (capital restructuring, mergers, sunk costs in major machinery and equipment and choice of a new product).

Even in decisions that are taken frequently, the environment is ever-changing and uncertain, hence, novel. Thus, arguably the choices made in the early rounds of an experiment are vital in understanding strategic human behavior, yet these choices do not typically conform to the predictions of CGT. Such data is, in many cases, much better explained by *behavioral models of game theory* such as *level-k models*, *quantal response equilibrium*, *analogy based equilibrium*, *cursed equilibrium*, *team reasoning*, and *evidential reasoning* (for details see, Dhami, 2016; Ch. 13).

We summarize some of the evidence here. It is impractical to cite all the references in such a short article; see the book length treatments in Camerer (2003) and Dhami (2016, Part 4) for the details.

1. In games requiring more than 2 steps of iterated deletion of dominated strategies, a majority of the players violate the predictions of CGT.

2. Extensive form games and their equivalent normal form representations often elicit different behavioral responses.

3. The backward induction prediction of CGT in dynamic games of full information is refuted by the evidence, even when subjects highly trained in backward induction (e.g., chess grandmasters) play centipede games.

4. Even when the simplest mechanisms are tested in experiments, e.g., the Abreu-Matsushima and the Glazer-Perry mechanisms that are dominance solvable, the

results are unsupportive of CGT. This casts doubt on the implementability of more complex mechanisms.

5. Subjects play mixed strategies in experiments, but not in the proportions predicted by a *mixed strategy Nash equilibrium*. There is some positive evidence from some sports contexts, but there are many confounding factors in such experiments and it is not clear if such motor skills have external validity to other economic contexts (e.g., choice of pension plans).

6. In games with multiple equilibria, none of the selection principles such as *payoff dominance* or *risk dominance* in CGT account fully for the data. The equilibria are strongly *history dependent*. *Preplay communication* enhances coordination.

7. The CGT predictions of alternating-offers bargaining games are not supported by the evidence. When players reject an offer, they often make counteroffers that give them an even lower share (disadvantageous counteroffers). The predictions of bargaining under one-sided asymmetric information in CGT are not supported, but there is surprising and unexplained support for the predictions of two-sided asymmetric information under CGT. Under unstructured bargaining, which is more realistic, factors unimportant in CGT, such as *self-serving bias*, explain *bargaining impasse* under full information.

8. The pattern of searches and lookups in Rubinstein's alternative offer bargaining game using the MOUSELAB software do not support the predictions of CGT.

9. In signaling games there is no support for the predictions of CGT beyond the Cho-Kreps *intuitive criterion*.

10. There is evidence that several emotions such as guilt-aversion, surprise-seeking, anger, and inferring intentions behind the actions of others, directly enter into the utility functions of players. This runs counter to CGT and forms the subject matter of the promising approach in *psychological game theory*.

4.5. Heuristics and biases: An alternative to optimization

As noted in subsections 2.2 and 3.3 the dominant view in economics is that people are *fully rational Bayesian decision makers*. By this it is meant that they use unbounded rationality, have unlimited attention and computing abilities, optimize using all appropriate mathematical and statistical tools and obey the laws of standard (Kolmogorov) probability theory (not just Bayes' rule). Tversky and Kahneman (1971, 1974) put this view to the test and this has spawned an enormous literature, called the *heuristics and biases* research program.

The evidence showed that individuals are far from fully rational Bayesian decision makers, but their behavior is not random either. Individuals do not take account of all available information, do not correctly use Bayes' law and other laws of statistics, and eschew mathematical optimization in favour of simple *rules of thumb* or *heuristics*. Most heuristics are *fast*, in terms of the computation time required, and *frugal* in the use of information. Since these heuristics do not optimize , their performance is usually suboptimal. Even statistically sophisticated researchers and experts typically rely on these judgment heuristics.[8] Gerd Gigerenzer and collaborators have shown that when there is uncertainty about the optimal solution, the performance of heuristics compares favorably with more complex mathematical or statistical methods; see Dhami (2016; Section 19.15).

When individuals use the *representativeness heuristic*, they compare the likeness of even small samples with features of the population distribution. Hence, they assign too high a probability that small samples share the properties of large samples and behave 'as if' they obeyed a *law of small numbers*, rather than the statistically correct, *law of large numbers*. Thus, when asked to produce random sequences, the constructed sequences show up too much *negative autocorrelation*. This creates the *gambler's fallacy* (reluctance to bet on previously winning numbers). The converse, a belief in *positive autocorrelation* in outcomes, when there is none, is known as the *hot hands fallacy* (basketball and football players who score heavily in a game are often perceived to be on a hot streak even when there is none). The implications for financial markets (and generally for economics) are profound and this provides a much better explanation of behavior in financial markets (Dhami, 2016, Part 7).

Other heuristics that are increasingly used in economics and finance to explain human behavior include the *availability heuristic*, the *anchoring heuristic*, *base rate neglect* in using Bayes' Law, *conservatism* or *underweighting of the likelihood of a sample*, *hindsight-bias*, *confirmation-bias*, and *regression to the mean*. [9] The failure of Bayes' law is problematic for all areas of economic theory because Bayesian rationality lies at the heart of modern economic theory.

It is often claimed that presenting information in a *frequency format* rather than in a *probability format* eliminates biases arising from using heuristics (Cosmides and Tooby, 1996; Gigerenzer and Hoffrage, 1995). While for some heuristics, this is true, the case for the efficacy of the frequency format is vastly overstated (Dhami, 2016; Ch. 19). Further,

[8] For the claims in this para, see Gilovich et al. (2002), Kahneman (2003), Kahneman (2011), Tetlock (2006), Dhami (2016, Part 7).

[9] For a discussion of these heuristics, evidence, incorporation into new theoretical models and economic applications, see Dhami (2016, Ch. 19).

most real world economic information is arguably presented in a probability format.

Cognitive limitations force individuals to group different events into separate categories and to treat all events in the same category in an identical manner. This insight plays a central role in new behavioral equilibrium concepts, such as an *analogy based equilibrium* and *cursed equilibrium*, as well as for understanding *persuasion* and *advertising* (Dhami, 2016, Section 13.7, 19.12).

People form *mental models* of the world that can take the form of beliefs about causal relations, social identities, categorizing disparate information into coarse categories, and social worldviews. Mental models are pervasive, help economize on cognitive costs, are transmitted from generation to generation, once formed they are typically inertial, and can both assist and deter optimal decisions, depending on their accuracy (Datta and Mullainathan, 2014, WDR, 2015). Humans are hardwired with some mental models, while in other cases, history plays an important role in their formation. This provides another alterative to optimization in making decisions (Dhami, 2016, Section 19.3).

Another alternative to Bayesian rationality is Herbert Simon's seminal approach to bounded rationality (Simon, 1978). In a direct reference to optimization, Simon (1978) famously notes: "But there are no direct observations that individuals or firms do actually equate marginal costs and revenues." The Nobel Prize winner, Reinhard Selten, draws attention to the fact that economic problems are *NP-Complete*, i.e., the number of steps in the required solution grows exponentially with the size of the problem, e.g., the travelling salesman problem.[10] Even for apparently simple problems of this sort, the computing time required to solve the problem is very high. It then becomes a leap of faith that the man on the street can solve this problem or act 'as if' he could. Yet, neoclassical economics does make that leap of faith.

Herbert Simon proposes *satisficing behavior* whereby individuals have an *aspiration level* and adjust gradually in its direction through a sequence of steps (Simon, 1978, 2000; Selten, 2001). Empirical evidence is supportive of the theory (Caplin et al., 2011); subjects appear to search sequentially and stop searching when their aspiration level is achieved. This can demonstrate how the cooperative outcome could be achieved as a solution to the prisoner's dilemma game (Karandikar et al., 1998).

Finally, the approach of Gerd Gigerenzer and colleagues on fast and frugal heuristics is another worthwhile alternative to Bayesian rationality (see Gigerenzer and Hoffrage, 1995; Dhami, 2016, Sections 19.14, 19.15). They show that heuristics can often outperform selected optimization methods, but in many cases the optimization benchmark in these problems is far from clear. In some cases, these heuristics are derived from evolutionary

[10] Suppose that a salesman is provided with a list of cities and the distances between each pair of cities. The objective, starting with a home city, is to find the shortest possible route such that the salesman visits each city exactly once and returns to the home city.

adaptation but mostly these are experimenter provided.

4.6. Evidence on the efficiency of financial markets

The efficient markets hypothesis, EMH (see Section 3.4), is rejected by a very large body of empirical evidence. Each of the following empirical findings rejects EMH.[11] Prices of substitute assets differ consistently; there is gradual, rather than instantaneous, flow of information in the stock market; asset prices exhibit momentum at the short horizon and mean reversion at the long horizon; asset bubbles particularly in the housing market are well documented; stock markets crash even when there is "no news"; and asset prices exhibit underreaction and overreaction to new news. Indeed a mistaken belief in EMH can be disastrous for financial markets, as demonstrated by the decision of the Chairman of the Federal Reserve, Alan Greenspan, to not intervene in financial markets before the crash of 2007, believing that markets are efficient.

The refutation of both assumptions in the opening paragraph of subsection 3.4 and the discrediting of EMH on empirical grounds has led to the fast growing discipline of *behavioral finance*. Behavioral finance recognizes that individuals are boundedly rational and follow a range of judgement heuristics that can give biased solutions (subsection 4.5). Contrary to the assertion in classical finance, the errors of noise traders often aggregate and pile up in the same direction.

In the event of a departure of prices from fundamental values, and a strong belief on the part of noise traders that this mispricing will persist, rational investors will bet on the mispricing getting worse. Hence, the normal forces bringing prices back in line with fundamental values might not work and we might observe bubbles on asset prices, or extended periods during which prices depart from fundamental values. The presence of institutional investors does not solve the problem either because the deposits they receive from small investors depend on their past returns (*performance-based arbitrage*). Hence, they may liquidate their loss making assets prematurely before the end of the year to demonstrate high returns, even when they ought to have held these assets longer.

There is also a growing literature on corporate finance that applies the behavioral evidence on humans to CEOs of companies (Dhami, 2016, Section 21.7). For instance, it has been demonstrated that overconfident CEOs may undertake mergers even when there is no economic case for doing so. This might well explain why so many mergers fail.

[11] For the claims made in this section and the relevant references, the interested reader can consult the surveys in Shleifer (2000) and Dhami (2016, Ch. 21).

5. Conclusions

Neoclassical economics, which is still the mainstream, does well to propose precise and testable assumptions and rigorously derive their logical conclusions. However, the evidence (Section 4) is not consistent with the basic assumptions of rationality (Section 2) and the auxiliary assumptions (Section 3). Building on evidence and insights from cognitive psychology, behavioral economics has made great strides in building a new economics that is as rigorous as neoclassical economics, but is in better conformity with the evidence.

References

[1] Allais, M. (1953). Le comportement de l'homme rationnel devant le risque, critique des postulats et axiomes de l'ecole Américaine. *Econometrica*, 21, 503-546.

[2] Aumann, R. J. and Brandenburger, A. (1995). Epistemic Conditions for Nash Equilibrium, *Econometrica*, 63:1161–1180.

[3] Camerer, C. F. (2003). *Behavioral Game Theory: Experiments in Strategic Interaction*. Princeton: Princeton University Press.

[4] Caplin, A., Dean, M. and Martin, D. (2011). Search and Satisficing. *American Economic Review*, 101(7): 2899-2922.

[5] Caplin, A. and Leahy, J. (2001). Psychological Expected Utility and Anticipatory Feelings. *Quarterly Journal of Economics* 116, 55-79.

[6] Cosmides, L. and Tooby, J. (1996) Are humans good intuitive statisticians after all? Rethinking some conclusions from the literature on judgment under uncertainty. *Cognition*, 58:1-73.

[7] Datta, S. and Mullainathan, S. (2014). Behavioral Design: A New Approach to Development Policy. *Review of Income and Wealth* 60 (1): 7-35.

[8] Dhami, S. (2016) *Foundations of Behavioral Economic Analysis*, Oxford University Press: Oxford.

[9] Fehr, E., and Schmidt, K. (2006) The economics of fairness, reciprocity and altruism: Experimental evidence and new theories. in Kolm S. C. and Ythier, J. M. (Eds.) *Handbook of the Economics of Giving, Altruism and Reciprocity*, Volume 1., Elsevier.

[10] Fehr-Duda, H., and Epper, T. (2012). Probability and risk: foundations and economic implications of probability-dependent risk preferences. *Annual Review of Economics*. 4: 567-593.

[11] Fishburn, P.C., and Rubinstein, A. (1982). Time preference. *International Economic Review*. 23(3): 677–694.

[12] Fudenberg D., Tirole J. (1991) *Game theory*, MIT Press: Massachusetts.

[13] Gabaix, X. (2014). A Sparsity-Based Model of Bounded Rationality, *Quarterly Journal of Economics*, 129(4):1661-1710.

[14] Gigerenzer, G. and Hoffrage, U. (1995) How to improve Bayesian reasoning without instruction: Frequency formats. *Psychological Review* 102:684–704.

[15] Gilovich, T., Griffin, D. W., and Kahneman, D. (Eds). (2002). *Heuristics and Biases: The Psychology of Intuitive Judgment*. New York: Cambridge University Press.

[16] Gintis, H. (2009). *The Bounds of Reason: Game Theory and the Unification of the Behavioral Sciences*. Princeton: Princeton University Press.

[17] Gintis, H. (2017) *Individuality and Entanglement: The Moral and Material Bases of Social Life*. Princeton University Press: Princeton.

[18] Iyengar, S. S. and Lepper, M. R. (2000) When Choice is Demotivating: Can One Desire Too Much of a Good Thing? *Journal of Personality and Social Psychology*, 79(6): 995-1006.

[19] Kahneman, D. (2003). Maps of bounded rationality: Psychology of behavioral economics. *American Economic Review* 93, 1449-1475.

[20] Kahneman, D. (2011). *Thinking fast and slow*. New York, NY: Farrar, Strauss, Giroux.

[21] Kahneman, D., and Tversky, A. (1979). Prospect theory: an analysis of decision under risk. *Econometrica*. 47(2): 263–291.

[22] Kahneman, D., and Tversky, A. (2000). *Choices, Values and Frames*. Cambridge: Cambridge University Press.

[23] Karandikar, R., Mookherjee, D., Debraj, R., Vega-Redondo, F., (1998). Evolving Aspirations and Cooperation, *Journal of Economic Theory*, vol. 80(2) pages 292-331.

[24] Kolmogorov A. N. (1950). *Foundations of the Theory of Probability*, New York: Chelsea Publishing Co. Original German, 1933.

[25] Kreps, D. M. (1990) *A course in microeconomic theory*. Harvester Wheatsheaf: New York, London, Toronto, Sydney, Tokyo.

[26] Laibson, D. (1997). Golden eggs and hyperbolic discounting. Quarterly Journal of Economics. 112(2): 443-478.

[27] Lichtenstein, S., and Slovic, P. (Eds.) (2006). The Construction of Preference. New York, NY: Cambridge University Press.

[28] Loewenstein, G., O'Donoghue, T., Rabin, M. (2003). Projection bias in predicting future utility. *Quarterly Journal of Economics*, 118, pp. 1209-48.

[29] Phelps, E., and Pollack, R.A. (1968). On second best national savings and game equilibrium growth. Review of Economic Studies. 35(2): 185–199.

[30] Polak, B. (1999). Epistemic conditions for Nash equilibrium, and common knowledge of rationality. *Econometrica* 67, 673–676.

[31] Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behavior and Organization*, 3, 323-343.

[32] Savage, L. J. (1954). *The Foundations of Statistics*. New York: John Wiley and Sons. Revised edition 1972, New York: Dover Publications.

[33] Selten, R. (2001) What is bounded rationality? In G. Gigerenzer, & R. Selten (Eds.) *Bounded rationality : the adaptive toolbox*. Dahlem Workshop Report, (pp. 1-12). Cambridge, Mass.: MIT Press.

[34] Shleifer, A. (2000). *Inefficient Markets: An Introduction to Behavioral Finance.* Oxford University Press.

[35] Simon, H. A. (1978). *Rational decision-making in business organizations.* Nobel Memorial Lecture, 8 December, 1978.

[36] Simon, H. A. (2000). Bounded rationality in social science: Today and tomorrow. *Mind & Society*, 1(1) 25-39.

[37] Starmer, C. (2000). Developments in non-expected utility theory: The hunt for a descriptive theory of choice under risk. *Journal of Economic Literature.* 38: 332-382.

[38] Tetlock, P. E. (2006). *Expert political judgment: How good is it? How can we know?* Princeton, NJ: Princeton University Press.

[39] Thaler, R. H. 1981. Some empirical evidence on dynamic consistency. *Economics Letters* 8, 201-207.

[40] Tversky, A., and Kahneman, D. (1971). Belief in the law of small numbers. *Psychological Bulletin*. 76: 105-110.

[41] Tversky, A., and Kahneman, D. (1974). Judgement under uncertainty: Heuristics and biases. *Science*. 185 (4157), 1124-1131.

[42] Tversky, A., and Kahneman, D. (1992). Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk and Uncertainty*. 5(4): 297-323.

[43] World Development Report (2015) *Mind, society, and behavior*. The World Bank: Washington DC.