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Abstract

De Paoli, Scott, and Weeken [2010, Asset pricing implications of a New Keynesian model. Journal of Economic Dynamics and Control 34, 2056-73] study equity and bonds prices in a New Keynesian model with sticky nominal prices. This note argues that their model generates a behavior of the labor market variables that is contrary to empirical evidence and, as remedy for this deficiency, suggests a model with both sticky nominal wages and prices.

JEL-Code: E440, E430, G120.

Keywords: equity premium, New Keynesian Model, nominal rigidities.

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1 Introduction

De Paoli, Scott, and Weeken (2010) (PSW henceforth) examine the behavior of asset prices in a New Keynesian model with sticky prices. In order to generate a sizeable equity premium, real rigidities in the form of habit formation in consumption and labor as well as capital adjustment costs are introduced in the standard business cycle model. In addition, they study the effects of nominal rigidities on the risk premium and show that it depends on the nature of the shock. While the risk premium is reduced vis-a-vis the flexible-price model if cycles are driven by productivity shocks, it increases in the case of monetary shocks.

PSW argue that the matching of asset prices is "... *potentially important because, as New Keynesian models are used more often in policy-making institutions such as central banks, increasing demands will be placed on their ability to tell stories about the behavior of asset markets as well as goods and labor markets.*"

In the following, we concentrate on the labor market. Our results are as follows: 1) We show that the model of PSW has deficiencies in replicating key labor market statistics. In particular, output and hours are negatively correlated. 2) For some of the parameters namely those describing the habits of households and the adjustment costs of capital, little empirical evidence is available. PSW set these parameters exogenously. However, it has become standard practice in DSGE modeling to choose unobserved parameters so that the model replicates certain empirical facts. If we follow this strategy, the model that comes closest to our targets features sticky prices and has still undesirable labor market implications. 3) However, by combining sticky nominal wages with sticky prices and choosing the free parameters optimally yields predictions that are in good accordance with empirical observations.

The paper is structured as follows. In Section 2, we present the sticky-price model of PSW (2010). Section 3 introduces rigid wages in the New Keynesian model. Section 4 concludes. The Appendix covers the mathematical details of both models.

2 The Model of PSW

In this section, we consider a simplified version of the model of De Paoli, Scott, and Weeken (2010). Since we are interested in certain facts of the real business cycle we suppress the modeling of the real and nominal term structure and of money holdings. Those parts of the PSW model do not interact with the real side and can, thus, be safely neglected.[1]

Households. Households enter the current period t with a given amount of firm shares S_t, nominal bonds B_t^n, and real bonds B_t^r. The current price level is P_t. Both kinds of bonds have a maturity of one period. Real bonds can be purchased at the current price v_t^{rb} and pay one unit of consumption in period $t+1$. Nominal bonds have a price of $1/P_t$ and pay the flexible gross nominal return R_t^{nb}. The real share price is v_t^e and real dividend payments per share are d_t. Firms pay the real wage w_t per unit of working hours N_t. Thus,

$$v_t^e(S_{t+1} - S_t) + v_t^{rb} B_{t+1}^r + \frac{B_{t+1}^n}{P_t} \leq w_t N_t + d_t S_t + B_t^r + R_t^{nb} \frac{B_t^n}{P_t} - C_t \tag{2.1}$$

is the household's budget constraint.

The household maximizes

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{(C_{t+s} - \chi^C C_{t+s-1})^{1-\gamma^C} - 1}{1-\gamma^C} - \theta^N \frac{(N_{t+s} - \chi^N N_{t+s-1})^{1+\gamma^N} - 1}{1+\gamma^N} \right]$$

subject to (2.1), where previous period consumption and labor supply are treated as given (exogenous habits). From the first-order conditions we obtain the gross returns of equities, the real bond (i.e., the risk-free real rate), and the relation between the real and the nominal gross return:[2]

$$R_{t+1}^e = \frac{v_{t+1}^e + d_{t+1}}{v_t^e}, \tag{2.2a}$$

$$R_t^{rb} = \frac{1}{v_t^{rb}}, \quad v_t^{rb} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}, \tag{2.2b}$$

$$1 = \mathbb{E}_t \frac{R_{t+1}^{nb}}{R_t^{rb} \pi_{t+1}}, \quad \pi_t = \frac{P_t}{P_{t-1}}, \tag{2.2c}$$

where Λ_t denotes the Lagrange multiplier of the budget constraint.

Firms. There is a unit mass of firms, each producing a single good $Y_t(j)$ according to the function

$$Y_t(j) = Z_t N_t(j)^{1-\alpha} K_t(j)^\alpha, \quad \alpha \in (0,1). \tag{2.3}$$

[1]See the Appendix for the complete model. Gauss and Fortran programs that perform the simulations are available from the authors upon request.

[2]The time index denotes the date at which the return becomes known to the household.

The economy wide level of total factor productivity Z_t is governed by the AR(1)-process

$$\ln Z_t = \rho^Z \ln Z_{t-1} + \epsilon_t^Z, \quad \epsilon_t^Z \sim N\left(0, \sigma_{\epsilon^Z}^2\right). \tag{2.4}$$

The demand for good $j \in [0, 1]$ is

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\eta} Y_t, \tag{2.5}$$

where P_t and Y_t denote the price level and aggregate output, respectively. The producer finances investment $I_t(j)$ out of retained earnings and distributes the remaining surplus $D_t(j) = Y_t(j) - w_t N_t(j) - I_t(j)$ as dividends to the household. He maximizes[3]

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[Y_{t+s}(j) - w_{t+s} N_{t+s}(j) - I_{t+s}(j) - \frac{\chi^P}{2} \left(\frac{P_{t+s}(j)}{\pi P_{t+s-1}(j)} - 1 \right)^2 Y_{t+s} \right]$$

subject to (2.3), (2.5), and the law of capital accumulation

$$K_{t+1}(j) = (1-\delta) K_t(j) + \left[\frac{a_1}{1-\chi^K} \left(\frac{I_t}{K_t} \right)^{1-\chi^K} + a_2 \right] K_t(j), \quad \chi^K > 0. \tag{2.6}$$

Monetary Policy. The central bank sets the nominal interest rate R_{t+1}^{nb} according to the Taylor rule

$$R_{t+1}^{nb} = \left(R_t^{nb}\right)^{\theta^R} \left(\frac{\pi}{\beta}\right)^{1-\theta^R} \left(\frac{\pi_t}{\pi}\right)^{\theta^\pi} e^{\epsilon_t^R}, \quad \delta_1 \in [0, 1), \quad \epsilon_t^R \sim N(0, \sigma_{\epsilon^R}). \tag{2.7}$$

The elasticity of R_{t+1}^{nb} with respect to the deviation of the inflation factor π_t from its steady state value π will be chosen so that the equilibrium is determinate. Usually, this requires $\theta^\pi > 1$.

Calibration. If not noted otherwise, our simulations of the model use the parameter values of PSW presented in Table 2.1. Parameters which are not shown in this table, as, e.g., θ^N, are implied by the model's equilibrium conditions.

Equity Premium and Second Moments. We compute the gross rate of return on the shares of the representative firm from (2.2a). Our estimate of R_{t+1}^e is the average from a simulated time series of 1,000,000 observations.

[3] π denotes the inflation factor in the stationary equilibrium.

Table 2.1
Calibration of the PWS Model

Preferences	β=0.99	χ^C=0.82	χ^N=0.82	γ^C=5	γ^N=2.5
	N=1/3				
Production	α=0.27	δ=0.025	χ^K=1/0.30	η=6.0	χ^P=77
	ρ^Z=0.95	σ_{ϵ^Z}=0.01			
Taylor Rule	π=1.0	θ^R=0.75	θ^π=1.5	σ_{ϵ^R}=0.01	

In order to compute the gross risk-free rate of return R_t^{rb} defined in (2.2b), we integrate the second-order approximate solution of Λ_{t+1} using the six-point Gauss-Hermite quadrature formula.[4]

Table 2.2 displays second moments from simulations of the model. The moments in the first panel are from simulations that assume perfectly flexible prices, i.e., $\psi = 0$. The equity premium in the model with flexible prices amounts to 2.40 percentage points. Sticky prices reduce the volatility of output, investment, and hours, and, thus, asset returns become less risky. As a consequence, the equity premium shrinks to 0.35 percentage points.

As is evident from the inspection of Table 2.2, output is more volatile in the flexible price case than in the sticky price case. Moreover, output and hours as well as hours and the real wage are negatively correlated if prices are flexible. The negative correlations disappear if prices are sticky and the business cycle is only driven by monetary shocks. However, in this case, the relative volatility of the real wage exceeds its empirical value by a factor of more than fifty.

In the PSW model the steady state Frisch elasticity of labor supply equals $\tau = (1 - \chi^N)/\gamma^N$ so that the values from Table 2.1 imply $\tau = 0.072$, which is quite small as compared to estimates for the US.[5] If we employ $\tau = 0.3$, and thus a more elastic response of hours with respect to the real wage, the equity premium is reduced to about one third of its value implied by $\tau = 0.072$.

[4]PSW use a Monte-Carlo simulation to compute the risk-free rate. Our results only differ from theirs with respect to the third digit.

[5]Heer and Maußner (2008), p. 649 review estimates of this elasticity and conclude that 0.3 is a conservative estimate.

Table 2.2

Summary of Results

R^{rb}	R^{eq}	$R^{eq} - R^{rb}$	s_Y	s_I/s_Y	s_N/s_Y	s_w/s_Y	r_{YN}	r_{wN}	Score
Data									
0.8	6.98	6.18	1.72	2.97	0.98	0.44	0.78	0.21	
Models									
Flexible Prices, Productivity Shocks Only, $\tau = 0.072$									
3.07	5.47	2.40	0.78	2.70	1.08	2.04	−0.93	−0.98	21.33
	(3.06)	(5.49)	(2.43)						
Flexible Prices, Productivity Shocks Only, $\tau = 0.3$									
3.74	4.53	0.79	0.43	2.66	3.15	4.11	−0.95	−1.00	51.79
Sticky Prices, Productivity Shocks Only, $\tau = 0.072$									
3.88	4.23	0.35	0.41	2.08	3.59	33.95	−0.81	0.42	1167.34
(3.89)	(4.24)	(0.35)							
Sticky Prices, Monetary Shocks Only, $\tau = 0.072$									
3.97	4.19	0.22	0.29	2.80	1.55	25.05	1.0	0.75	641.69
(3.96)	(4.19)	(0.23)							
Optimal Choice of Free Parameters									
3.82	4.24	0.42	0.68	2.66	1.56	1.90	−0.81	−0.58	38.93
Sticky Price and Wage Model									
1.73	8.05	6.33	2.16	3.06	1.43	0.38	0.80	0.02	0.28

Notes: s_x:=Standard deviation of time series x, where $x \in \{Y, I, N, w\}$ and Y, I, N, and w denote output, investment, hours, and the real wage, respectively. s_x/s_y:=standard deviation of variable x relative to standard deviation of output y. r_{NY}:=Cross-correlation of variable hours with output, r_{wN}:=Cross-correlation of the real wage with hours. The column Score presents the sum of squared differences between the moments from simulations of the model and the moments from the data.
The second moments implied by the model are averages from 500 simulations. The length of each simulated, HP-filtered time series is 200. The filter weight is 1600. The asset returns are averages from a simulation with 1,000,000 observations.
Entries in parenthesis are from de Paoli, Scott, and Weeken (2010), Tables 3, 5 and 6.

Optimal Choice of Free Parameters. The PSW model has several unobserved parameters: the habit parameters χ^C and χ^N, the adjustment cost parameter χ^K and the parameter that determines the degree of price stickiness χ^P. PSW take $\chi^C = 0.82$ and $\chi^K = 1/0.3$ from Jermann (1998) and $\chi^P = 77$ from Ireland (2001)[6] and assume $\chi^N = \chi^C$. Many researchers, including Jermann (1998) and, more recently, Uhlig (2007), however determine the free parameters of their models to match certain empirical targets as close as possible. Here we will employ the equity premium, the volatility of investment, hours, and the real wage relative to the volatility of output as well as the correlations between hours and output and hours and the real wage (see Table 2.2).[7]

In addition to χ^C, χ^N, χ^K, and χ^P we will also allow the relative importance of monetary shocks $\psi = \sigma_{\epsilon^R}/\sigma_{\epsilon^Z}$ to vary. On a coarse grid over $\chi^C \in [0.01, 0.95]$, $\chi^N \in [0.01, 0.95]$, $\chi^K \in [0.1, 4.5]$, $\chi^P \in [0.01, 80]$, and $\psi \in [0.2, 5]$ we searched for those parameter values that yield the minimal value of the sum of squared deviations between the moments implied by the model and our six targets.[8]

The model that comes closest to our targets features a strong consumption habit ($\chi^C = 0.80$), a negligible habit in hours ($\chi^N = 0.01$), moderate adjustment costs of capital ($\chi^K = 1.8$), sticky prices $\chi^P = 70$, and a small role played by monetary shocks ($\psi = 0.2$). Yet, even with this choice, the model has many counterfactual implications: the equity premium is smaller than one-tenth of its empirical value and the correlations between output and hours as well as between hours and the real wage are significantly negative (see 2.2).[9]

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[6] Ireland (2001) estimates χ^P from a model that differs in several respects from the PSW model. In particular, utility is non-separable in consumption and real balances and adjustment costs are modeled differently. Thus, it is questionable whether his estimate applies to the PSW model too.

[7] The sources for the respective empirical values of these moments are Mehra and Prescott (1985) for the equity premium, Galí and van Rens (2010) for the correlation between hours and the real wage, and Cooley and Prescott (1995) for the remaining moments.

[8] In our search over the parameter space we adjust the parameter γ^N so that the steady state Frisch elasticity of labor supply equals 0.3.

[9] There is a model with the same set of parameters, except $\chi^P = 0.01$, whose score of 39.42 is only slightly worse.

3 Sticky Wages and Sticky Prices

In this section, we augment the previous model with wage staggering as introduced by Erceg, Henderson, and Levin (2000). Our motivation in doing so is Uhlig (2007), who demonstrates that a real business cycle model with real wage rigidity, adjustment costs of capital, and exogenous habits in consumption and leisure is well able to account for both the equity premium and the labor market statistics. In order to permit a recursive formulation of the optimality condition for wage setting, we must set $\chi^N = 0$.[10]

The model (which is spelled out in the Appendix) has two additional free parameters: the elasticity of the demand for labor of type $h \in [0, 1]$ and the fraction of the members of the representative household who are not allowed to adjust their nominal wage $W_t(h)$ optimally. We set ϵ_w equal to 4 as in Erceg, Henderson, and Levin (2000) and treat ϕ_w as an additional free parameter which we choose to minimize our score statistic.

Calibration and Results. The minimizer, found on a coarse grid, implies a strong consumption habit, $\chi^C = 0.80$, moderate adjustment costs, $\chi^K = 2.0$, rigid wages and prices, $\phi_w = 0.82$ and $\chi^P = 90$, and a predominance of monetary as compared to productivity shocks, $\psi = \sigma^Q/\sigma = 4$. As can be seen from Table 2.2 the model provides both, an equity premium close the one reported by Mehra and Prescott (1985) and labor market statistics close to those found for the US economy. With an overall score of 0.28 the model clearly outperforms the different calibrations of the PSW model.

4 Conclusion

Dynamic stochastic general equilibrium (DSGE) models are increasingly used for monetary policy evaluation. We thus agree with PSW, who argue that these models should be able to "tell stories about the behavior of asset markets *as well as goods and labor markets*"[11]

We show that the PSW model is at odds with the stylized facts of the US labor market, even if we do not choose the free parameters of their model ad hoc but to match the

[10] In the light of the results of the parameter search for the PSW model, this seems a harmless choice.

[11] Italics added by the authors of this note.

equity premium and several second moments that characterize this market as closely as possible.

As an alternative, we present a model with both sticky goods prices and wages that mimics the stylized facts considered very accurately. Therefore, we conclude that wage and price stickiness seem to be an integral part of any story about asset as well as goods and labor market facts.

References

Cooley, Thomas F. and Edward C. Prescott. 1995. Economic Growth and Business Cycles. in: Thomas F. Cooley (Ed.), Frontiers of Business Cycle Research. Princeton, NJ: Princeton University Press. pp. 1-38

De Paoli, Bianca, Alasdair Scott and Olaf Weeken. 2010. Asset Pricing Implications of a New Keynesian Model. *Journal of Economic Dynamics and Control*. Vol. 34. pp. 2056-2073

Erceg, Christopher J., Dale W. Henderson and Andrew D. Levin. 2000. Optimal Monetary Policy with Staggered Wage and Price Contracts. *Journal of Monetary Economics*. Vol. 46. pp. 281-313

Galí, Jordi and Thijs van Rens. 2010. The Vanishing Procyclicality of Labor Productivity, Mimeo, Universitat Pompeu Fabra and Barcelona GSE.

Heer, Burkhard and Alfred Maußner. 2008. Computation of Business Cycle Models: A Comparison of Numerical Methods. *Macroeconomic Dynamics*. Vol. 12. pp. 641-663

Ireland, Peter N. 2001. Sticky-Price Models of the Business Cycle: Specification and Stability. *Journal of Monetary Economics*. Vol. 47. pp. 3-18

Jermann, Urban J. 1998. Asset Pricing in Production Economies. *Journal of Monetary Economics*. Vol. 41. pp. 257-275

Mehra, Rajnish and Edward C. Prescott. 1985. The Equity Premium: A Puzzle. *Journal of Monetary Economics*. Vol. 15. pp. 145-161

Uhlig, Harald. 2007. Explaining Asset Prices with External Habits and Wage Rigidities in a DSGE Model. *American Economic Review, Papers & Proceedings*. Vol. 97. pp. 239-243.

Appendix

In this Appendix, we provide the formal details of our simplified version of the PSW model and the model with wage and price rigidity.

A.1 The PSW Model

Households In the PSW model the representative household maximizes

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{\left(C_{t+s} - \chi^C C_{t+s-1}\right)^{1-\gamma^C} - 1}{1-\gamma^C} - \frac{\theta^N \left(N_{t+s} - \chi^N N_{t+s-1}\right)^{1+\gamma^N} - 1}{1+\gamma^N} \right.$$
$$\left. + \frac{\theta^M \left(M_{t+s+1}/P_{t+s}\right)^{1-\gamma^M} - 1}{1-\gamma^M} \right\}$$

subject to the sequence of budget constraints $s = 0, 1, \ldots, \infty$:

$$v_s^e(S_{t+s+1} - S_{t+s}) + v_{t+s}^{rb} B_{t+s+1}^r + \frac{M_{t+s+1} + B_{t+s+1}^n}{P_{t+s}}$$
$$\leq w_{t+s} N_{t+s} + d_{t+s} S_{t+s} + B_{t+s}^r + R_{t+s}^{nb} \frac{B_{t+s}^n}{P_{t+s}} + \frac{M_{t+s}}{P_{t+s}} + Tr_{t+s} - C_{t+s}.$$

The notation follows PSW and is described in the body of the paper except for the inclusion of end-of-period money balances M_{t+1} in the utility function of the household and real government transfers Tr_t in the budget constraint.

The first-order conditions of this problem are:

$$\Lambda_t = \left(C_t - \chi^C C_{t-1}\right)^{-\gamma^C}, \tag{A.1.1a}$$

$$\Lambda_t w_t = \theta^N \left(N_t - \chi^N N_{t-1}\right)^{\gamma^N}, \tag{A.1.1b}$$

$$v_t^e \Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1} \left(v_{t+1}^e + d_{t+1}\right), \tag{A.1.1c}$$

$$\Lambda_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1} R_{t+1}^{nb}}{\pi_{t+1}}, \quad \pi_t \equiv \frac{P_t}{P_{t-1}}, \tag{A.1.1d}$$

$$\Lambda_t = \mathbb{E}_t \left(\theta^M m_{t+1}^{-\gamma^M} + \beta \frac{\Lambda_{t+1}}{\pi_{t+1}}\right), \quad m_{t+1} \equiv \frac{M_{+1}}{P_t}, \tag{A.1.1e}$$

$$v_t^{rb} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}, \tag{A.1.1f}$$

where Λ_t is the Lagrange multiplier of the budget constraint.

Firms Firm $j \in [0,1]$ maximizes

$$V_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ Y_{t+s}(j) - w_{t+s} N_{t+s}(j) - I_{t+s}(j) - \frac{\chi^P}{2} \left(\frac{P_{t+s}(j)}{\pi P_{t+s-1}(j)} - 1 \right)^2 Y_{t+s} \right\}$$

subject to

$$\begin{aligned}
Y_{t+s}(j)) &= Z_{t+s}(N_{t+s}(j))^{1-\alpha}(K_{t+s}(j))^{\alpha}, \\
Y_{t+s}(j) &= \left(\frac{P_{t+s}(j)}{P_{t+s}} \right)^{-\eta} Y_{t+s}, \\
K_{t+s+1}(j) &= (1-\delta)K_{t+s}(j) + \omega(I_t(j)/K_t(j))K_t(j),
\end{aligned}$$

where the function $\omega(\cdot)$ is parameterized as

$$\omega(I_t(j)/K_t(j)) = a_1 \left(\frac{I_{t+s}(j)}{K_{t+s}(j)} \right)^{1-\chi^K} + a_2.$$

In a symmetric equilibrium the first-order conditions of this problem reduce to

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \tag{A.1.2a}$$

$$K_{t+1} = \omega\left(I_t/K_t\right) K_t + (1-\delta)K_t, \tag{A.1.2b}$$

$$w_t = (1-\alpha)\Gamma_t \frac{Y_t}{N_t}, \tag{A.1.2c}$$

$$q_t = \frac{1}{\omega'(I_t/K_t)}, \tag{A.1.2d}$$

$$q_t = \beta \mathbb{E}_t \Lambda_{t+1} \left[(1-\alpha)\Gamma_{t+1} \frac{Y_{t+1}}{K_{t+1}} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1} \left[(1-\delta + \omega(I_{t+1}/K_{t+1}) \right] \right], \tag{A.1.2e}$$

$$\begin{aligned}
0 = {} & (1-\eta)Y_t + \eta\Gamma_t Y_t - \psi \left(\frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} Y_t \\
& + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \psi \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} Y_{t+1},
\end{aligned} \tag{A.1.2f}$$

where q_t and Γ_t denote Tobin's q and marginal costs and are equal to the Lagrange multipliers of the capital accumulation equation and the demand function, respectively.

Monetary Authority. The central bank targets the nominal interest factor R_t^{nb} and supplies whatever amount of nominal money balances is demanded at this rate. The ensuing seignorage is distributed lump-sum to the household sector. Accordingly, the following equations describe its behavior:

$$R_{t+1}^{nb} = \left(R_t^{nb}\right)^{\theta^R} \left(\frac{\pi}{\beta} \right)^{1-\theta^R} \left(\frac{\pi_t}{\pi} \right)^{\theta^\pi} e^{\epsilon_t^R}, \quad \theta^R \in [0,1), \quad \epsilon_t^R \sim N(0, \sigma_{\epsilon^R}), \tag{A.1.3a}$$

$$T_t = \frac{M_{t+1} - M_t}{P_t}, \tag{A.1.3b}$$

$$m_{t+1} = \frac{\mu_t}{\pi_t} m_t, \tag{A.1.3c}$$

$$\mu_t = \frac{M_{t+1}}{M_t}. \tag{A.1.3d}$$

Equation (A.1.3c) is just another way to write the definition of end-of-period real money balances $m_t = M_{t+1}/P_t$ given the definition of the money growth factor μ_t in equation (A.1.3d) and the inflation factor π_t.

Temporary Equilibrium. In equilibrium the supply of bonds is zero, $B_t^r = B_t^n = 0$ and the supply of shares is constant. Therefore, aggregate real dividends as well as dividends per share equal

$$d_t = Y_t - w_t N_t - I_t, \tag{A.1.4a}$$

and the household's budget constraint simplifies to

$$Y_t = C_t + I_t. \tag{A.1.4b}$$

The dynamics of the model are described by the set of equations (A.1.1), (A.1.2), (A.1.3a), (A.1.3c), and (A.1.4).

Since the nominal interest rate R_{t+1}^{nb} is determined in period t, it is non-stochastic with respect to the conditional expectations operator $\mathbb{E}_t$. Thus, condition (A.1.1d) can be written as

$$\frac{\Lambda_t}{R_{t+1}^{nb}} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\pi_{t+1}},$$

which allows one to reduce the first-order condition (A.1.1d) to a static equation by using the definition (A.1.3c) and the Taylor rule (A.1.3a).

Stationary Equilibrium. As usual, the stationary equilibrium is defined by setting the shocks equal to their unconditional means and by assuming $x_{t+1} = x_t = x$ for all variables x of the model. In this case, equation (A.1.2f) simplifies to

$$\Gamma = \frac{\eta - 1}{\eta}. \tag{A.1.5a}$$

As usual, we assume that capital adjustment costs play no role in the stationary equilibrium, which amounts to specifying $\omega'(I/K) = 1$ and $\omega(I/K) = \delta$. This is accomplished by choosing

$$a_1 = \delta^{\chi^K}, \tag{A.1.5b}$$

$$a_2 = \frac{\chi^K}{\chi^K - 1}\delta. \tag{A.1.5c}$$

With these assumptions, equation (A.1.2e) reduces to

$$\frac{Y}{K} = \frac{1 - \beta(1 - \delta)}{\alpha\beta\Gamma}, \tag{A.1.5d}$$

so that for given N the stationary stock of capital equals

$$K = N(Y/K)^{\frac{1}{\alpha-1}}. \tag{A.1.5e}$$

and output Y is determined by (A.1.2a). Given the properties of the adjustment cost function, (A.1.2b) implies

$$I = \delta K, \tag{A.1.5f}$$

and we get the stationary value of consumption from the resource constraint (A.1.4b). Given the solution for C we can compute the solution for Λ from (A.1.1a). The stationary real wage follows from equation (A.1.2c). This allows us to determine the parameter θ^N:

$$\theta^N = \Lambda w \left(N - \chi^N N\right)^{-\gamma^N}. \tag{A.1.5g}$$

Dividends d follow from equation (A.1.4a). The stationary share price derives from (A.1.1c):

$$v = \frac{\beta}{1 - \beta}d. \tag{A.1.5h}$$

In the stationary equilibrium, the Taylor rule (A.1.3a) fixes the nominal interest rate factor R^{nb} for a given inflation target π:

$$Q = \frac{\pi}{\beta}, \tag{A.1.5i}$$

and (A.1.3c) implies $\mu = \pi$. Finally, given θ^N, equation (A.1.1e) can be used to determine the stationary end-of-period level of real money balances m:

$$m = \left(\frac{\Lambda(1 - (\beta/\pi))}{\theta^M}\right)^{-1/\gamma^M}. \tag{A.1.5j}$$

A.2 Sticky Wages and Sticky Prices

Labor Demand. Labor input N_t in production $Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}$ is an index of the different types of labor $N_t(h)$ supplied by the members $h \in [0,1]$ of the representative household:

$$N_t = \left[\int_0^1 N_t(h)^{\frac{\epsilon_w - 1}{\epsilon_w}} dh \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \quad \epsilon_w > 1. \tag{A.2.1}$$

Let W_t denote the nominal wage rate at date t and $W_t(h)$ the wage paid to labor of type h. Minimizing the wage bill

$$W_t N_t = \int_0^1 W_t(h) N_t(h) dh$$

subject to (A.2.1) yields the demand function for labor and the wage index:

$$N_t(h) = \left(\frac{W_t(h)}{W_t} \right)^{-\epsilon_w} N_t, \tag{A.2.2}$$

$$W_t = \left[\int_0^1 W_t(h)^{1-\epsilon_w} \right]^{\frac{1}{1-\epsilon_w}}. \tag{A.2.3}$$

Wage Setting. Each member $h \in [0,1]$ of the household sector has preferences[12]

$$u(C_t(h), N_t(h)) = \frac{(C_t(h) - \chi^C C_{t-1})^{1-\gamma^C} - 1}{1 - \gamma^C} - \theta^N \frac{N_t(h)^{1+\gamma^N} - 1}{1 + \gamma^N}. \tag{A.2.4}$$

With probability $1 - \phi_w$ individual h gets the opportunity to choose his nominal wage $W_t(h)$ optimally otherwise he can raise his wage only according to the steady state inflation rate π. In the former case he chooses $W_t(h)$ to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta \varphi_w)^s u(C_{t+s}(h), N_{t+s}(h)) \tag{A.2.5}$$

subject to his individual budget constraint and the demand function (A.2.2).

As usual in this literature, we assume that the members of the household pool their income so that consumption and portfolio allocation follow the same conditions as in the representative household framework considered in the previous section.

[12]We neglect money in the utility function. As the previous model shows, they do not contribute to the equilibrium dynamics of the equity premium and the labor market variables. Further, we set $\chi^N = 0$ to allow for a recursive representation of the first-order condition of the optimal nominal wage.

The Optimal Relative Wage. Substituting from (A.2.2) in (A.2.5) and (2.1) yields the Lagrangian for choosing the optimal wage:

$$
\begin{aligned}
\mathscr{L} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_w)^s \Bigg\{ \Bigg[& \frac{(C_{t+s}(h) - bC_{t+s-1})^{1-\gamma^C} - 1}{1-\gamma^C} \\
& - \frac{\theta^N}{1+\gamma^N} \left(\frac{\pi^s W_t(h)}{W_{t+s}} \right)^{-\epsilon_w(1+\gamma^N)} N_{t+s}^{1+\gamma^N} + \theta^M \frac{\left(\frac{M_{t+s+1}(h)}{P_{t+s}}\right)^{1-\gamma^M} - 1}{1-\gamma^M} \Bigg] \\
& + \Lambda_{t+s}(h) \Bigg[\frac{\pi^s W_t(h)}{P_{t+s}} \left(\frac{\pi^s W_t(h)}{W_{t+s}} \right)^{-\epsilon_w} N_{t+s} + S_{t+s}(h) d_{t+s} + B_{t+s}^{rb}(h) \\
& + R_{t+s}^{nb} \frac{B_{t+s}^{nb}(h)}{P_{t+s}} + \frac{M_{t+s}(h)}{P_{t+s}} + Tr_{t+s}(h) - C_{t+s}(h) \\
& - \frac{M_{t+s+1}(h) + B_{t+s+1}^{n}(h)}{P_{t+s}} - v_{t+s}^{rb} B_{t+s+1}^{rb}(h) \\
& - v_{t+s}^{e} (S_{t+s+1}(h) - S_{t+s}(h)) \Bigg] \Bigg\}.
\end{aligned}
$$

Differentiating with respect to $W_t(h)$ and setting the ensuing expression equal to zero delivers

$$
\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_w)^s \left\{ N_{t+s}(h) \left[\theta^N N_{t+s}(h)^{\gamma^N} - \frac{\epsilon_w - 1}{\epsilon_w} \Lambda_{t+s}(h) \frac{\pi^s W_t(h)}{P_{t+s}} \right] \right\}.
$$

We assume that there is a sufficiently rich set of contingent security markets so that a representative agent exists. Thus, $\Lambda_{t+s}(h) = \Lambda_{t+s}$ and all wage setters will opt for the same relative wage $w_{At} \equiv \frac{W_t(h)}{W_t}$. Therefore, the preceding condition can be stated as:

$$
w_{At} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{\Gamma_{1t}}{\Gamma_{2t}}, \tag{A.2.6a}
$$

$$
\Gamma_{1t} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_w)^s \theta^N \left(\frac{\pi^s W_t(h)}{W_{t+s}} \right)^{-\epsilon_w(1+\gamma^N)} N_{t+s}^{1+\gamma^N}, \tag{A.2.6b}
$$

$$
\Gamma_{2t} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_w)^s \Lambda_{t+s} \frac{\pi^s W_t}{P_{t+s}} \left(\frac{\pi^s W_t(h)}{W_{t+s}}^{-\epsilon_w} \right) N_{t+s}. \tag{A.2.6c}
$$

The auxiliary variables Γ_{1t} and Γ_{2t} have a recursive definition. Consider (A.2.6b):

$$
\begin{aligned}
\Gamma_{1t} = \mathbb{E}_t \Bigg\{ & \theta^N \left(\frac{W_t(h)}{W_t} \right)^{-\epsilon_w(1+\gamma^N)} N_t^{1+\gamma^N} + (\beta\phi_w)\theta^N \left(\frac{\pi W_t(h)}{W_{t+1}} \right)^{-\epsilon_w(1+\gamma^N)} N_{t+1}^{1+\gamma^N} \\
& + (\beta\phi_w)^2 \theta^N \left(\frac{\pi^2 W_t(h)}{W_{t+2}} \right)^{-\epsilon_w(1+\gamma^N)} N_{t+2}^{1+\gamma^N} + \dots \Bigg\}
\end{aligned}
$$

(A.2.7)

Therefore,

$$\Gamma_{1t+1} = \mathbb{E}_{t+1}\Bigg\{\theta^N \left(\frac{W_{t+1}(h)}{W_{t+1}}\right)^{-\epsilon_w(1+\gamma^N)} N_{t+1}^{1+\gamma^N} + (\beta\phi_w)\theta^N \left(\frac{\pi W_{t+1}(h)}{W_{t+2}}\right)^{-\epsilon_w(1+\gamma^N)} N_{t+2}^{1+\gamma^N} + (\beta\phi_w)^2\theta^N \left(\frac{\pi^2 W_{t+1}(h)}{W_{t+3}}\right)^{-\epsilon_w(1+\gamma^N)} N_{t+3}^{1+\gamma^N} + \dots\Bigg\}$$

From the perspective of period $t+1$ the variables $W_t(h)$, $W_{t+1}(h)$, and W_{t+1} are non-random. Thus, multiplying the previous equation on both sides by

$$(\beta\phi_w)\left(\pi\frac{(W_t(h)/W_t)}{W_{t+1}(h)/W_{t+1}}\frac{W_t}{W_{t+1}}\right)^{-\epsilon_w(1+\gamma^N)} \equiv (\beta\phi_w)\left(\frac{\pi w_{At}}{w_{At+1}}\frac{1}{\omega_{t+1}}\right)^{-\epsilon_w(1+\gamma^N)}$$

and taking expectations as of period t yields (since $\mathbb{E}_t\mathbb{E}_{t+1}\{\cdot\} = \mathbb{E}_t\{\cdot\}$ by the law of iterated expecations)

$$(\beta\phi_w)\mathbb{E}_t\left(\frac{\pi w_{At}}{\omega_{t+1}w_{At+1}}\right)^{-\epsilon_w(1+\gamma^N)}\Gamma_{1t+1} = \mathbb{E}_t\Bigg\{(\beta\phi_w)\theta^N\left(\frac{\pi W_t(h)}{W_{t+1}}\right)^{-\epsilon_w(1+\gamma^N)} N_{t+1}^{1+\gamma^N} + (\beta\phi_w)^2\theta^N\left(\frac{\pi^2 W_t(h)}{W_{t+2}}\right)^{-\epsilon_w(1+\gamma^N)} N_{t+2}^{1+\gamma^N} + \dots\Bigg\}.$$

Together with (A.2.7) this establishes:

$$\Gamma_{1t} = \theta^N w_{At}^{-\epsilon_w(1+\gamma^N)} N_t^{1+\gamma^N} + \beta\varphi_w\mathbb{E}_t\left(\frac{\pi w_{At}}{\omega_{t+1}w_{At+1}}\right)^{-\epsilon_w}\Gamma_{1t+1}, \tag{A.2.8a}$$

Analogously, the recursive definition of the auxiliary variable Γ_{2t},

$$\Gamma_{2t} = \Lambda_t w_t w_{At}^{-\epsilon_w} N_t + \beta\varphi_w\left(\frac{\pi}{\omega_{t+1}}\right)^{1-\epsilon_w}\left(\frac{w_{At}}{w_{At+1}}\right)^{-\epsilon_w}\Gamma_{2t+1}, \tag{A.2.8b}$$

can be derived, where

$$w_t = \frac{W_t}{P_t}, \tag{A.2.8c}$$

$$\omega_t = \frac{W_t}{W_{t-1}}. \tag{A.2.8d}$$

Finally, note that $W_{t-1}(h) = W_{t-1}$ for those that cannot adjust their wage optimally. Thus, equation (A.2.3) implies:

$$W_t^{1-\epsilon_w} = (1-\varphi_w)W_{At}^{1-\epsilon_w} + \varphi_w(\pi W_{t-1})^{1-\epsilon_w}$$

or

$$1 = (1-\varphi_w)w_{At}^{1-\epsilon_w} + \varphi_w(\pi/\omega_t)^{1-\epsilon_w}. \tag{A.2.9}$$

Firms. The production sector is the same as depicted in PSW model so that equations (A.1.2) describe the optimal decisions of producer j in a symmetric equilibrium.

Equilibrium Dynamics. The full model, thus, is described by the following set of equations:

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\gamma^C}, \tag{A.2.10a}$$

$$w_t = (1-\alpha)\Gamma_t Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{A.2.10b}$$

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \tag{A.2.10c}$$

$$Y_t = C_t + I_t, \tag{A.2.10d}$$

$$q_t = \frac{1}{\omega'(I_t/K_t)}, \tag{A.2.10e}$$

$$d_t = Y_t - w_t N_t - I_t, \tag{A.2.10f}$$

$$w_{At} = \frac{\epsilon_w}{\epsilon_w - 1}\frac{\Gamma_{1t}}{\Gamma_2 t}, \tag{A.2.10g}$$

$$1 = (1-\varphi_w) w_{At}^{1-\epsilon_w} + \varphi_w(\pi/\omega_t)^{1-\epsilon_w}, \tag{A.2.10h}$$

$$w_t = \frac{\omega_t}{\pi_t} w_{t-1}, \tag{A.2.10i}$$

$$K_{t+1} = (1-\delta)K_t + \omega(I_t/K_t)K_t, \tag{A.2.10j}$$

$$v_t^e = \beta \mathbb{E}_t \Lambda_{t+1}(v_{t+1}^e + d_{t+1}), \tag{A.2.10k}$$

$$\Lambda_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1} R_{t+1}^{nb}}{\pi_{t+1}}, \quad \pi_{t+1} = \frac{P_{t+1}}{P_t}, \tag{A.2.10l}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\Big\{\alpha \Gamma_{t+1} Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1}\big[\omega(I_{t+1}/K_{t+1}) + 1 - \delta\big]\Big\} \tag{A.2.10m}$$

$$\Gamma_{1t} = \theta^N w_{At}^{-\epsilon_w(1+\gamma^N)} N_t^{1+\gamma^N} + \beta\varphi_w \mathbb{E}_t \left(\frac{\pi w_{At}}{\omega_{t+1} w_{At+1}}\right)^{-\epsilon_w} \Gamma_{1t+1}, \tag{A.2.10n}$$

$$\Gamma_{2t} = \Lambda_t w_t w_{At}^{-\epsilon_w} N_t + \beta\varphi_w \left(\frac{\pi}{\omega_{t+1}}\right)^{1-\epsilon_w} \left(\frac{w_{At}}{w_{At+1}}\right)^{-\epsilon_w} \Gamma_{2t+1}, \tag{A.2.10o}$$

$$\begin{aligned} 0 &= (1-\eta)Y_t + \eta\Gamma_t Y_t - \psi\left(\frac{\pi_t}{\pi} - 1\right)\frac{\pi_t}{\pi} Y_t \\ &\quad + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\psi\left(\frac{\pi_{t+1}}{\pi} - 1\right)\frac{\pi_{t+1}}{\pi} Y_{t+1}. \end{aligned} \tag{A.2.10p}$$

$$R_{t+1}^{nb} = \left(R_t^{nb}\right)^{\theta^R} \left(\frac{\pi}{\beta}\right)^{1-\theta^R} \left(\frac{\pi_t}{\pi}\right)^{\theta^\pi} e^{\epsilon_t^R}. \tag{A.2.10q}$$

Stationary Equilibrium. The stationary equilibrium of the model differs only with respect to the real wage from the one of the PSW model. Equations (A.1.5) still apply. For $w_t = w_{t-1}$, equation (A.2.10i) implies that the wage inflation ω_t equals the price inflation π_t so that (from (A.2.10h)) the stationary optimal relative wage equals one. Equations (A.2.10n), (A.2.10o), and (A.2.10g), then, imply

$$\Lambda w = \frac{\epsilon_w}{\epsilon_w - 1} \theta^N N^{\gamma^N}. \tag{A.2.11}$$

We use this equation to determine θ^N for given N and γ^N.